# Trinity Grammar School

Mathematics Department

# 2015

## YEARLY EXAMINATION

PRELIMINARY ASSESSMENT TASK 4

# Year 11

# Mathematics Extension 1

**General Instructions**

- Reading time – 5 minutes
- Working time – 2 hours
- Write using black or blue pen
- Only approved calculators for this course are allowed in this task
- Show all necessary working
- Write your Board of Studies Student Number (Year 12 HSC) or Name (Year 11) and your Class teacher on the question paper and on any answer sheets or writing booklets used to write your responses to the questions submitted
- If you do not attempt a question you must submit an answer sheet or writing booklet for that question clearly indicating **N/A** and your Student Number or Name.

- **Preliminary Assessment Weighting:** 45%

**Board of Studies Student Number**
**(Year 12 only)**

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| | | | | | | | |

**Class Teacher:**

……

**Name:**
**(Year 11 only)**

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***Do NOT write solutions on this question paper. Any working on this question paper will NOT be marked.***

**Date of Examination:**

**Wednesday 2nd September 2015**

**Total marks – 70**

**Section I**

**10 marks**

- Attempt all Questions
- Allow about **15** minutes for this section

**Section II**

**60 marks**

- Attempt all Questions
- Allow about **1** hour **45** minutes for this section

**Section I** **10 marks**

- Shade the correct response on the multiple-choice answer sheet for Questions 1 – **10**
- Each question is worth 1 mark

**1.** Which expression is the derivative of $\frac{1}{x\sqrt{x}} + x$?

(A) $-\frac{3}{2x^2\sqrt{x}} + 1$

(B) $-\frac{3}{2\sqrt{x}} + 1$

(C) $\frac{3}{2x^3\sqrt{x}} + 1$

(D) $-\frac{3}{2x\sqrt{x}} + 1$

**2.** What is the solution to the equation $|x-2| = 2x+1$?

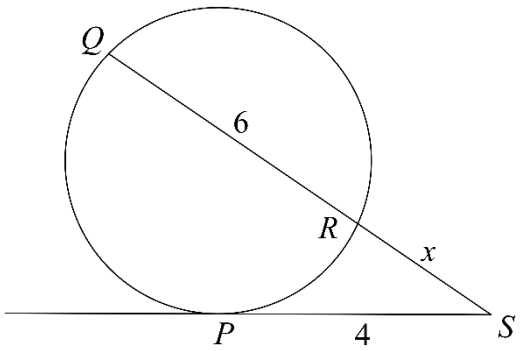
(A) $x = -3$

(B) $x = -\frac{1}{3}$

(C) $x = \frac{1}{3}$

(D) $x = 3$

**3.** Line $SP$ is a tangent to the circle at $P$ and $SQ$ is a secant meeting the circle at $Q$ and $R$.




Given that $SP = 4$, $QR = 6$ and $SR = x$, what is the value of $x$?

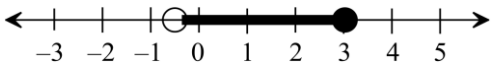
(A) -8

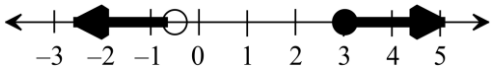
(B) 2

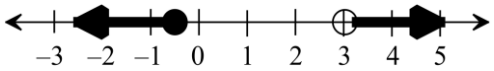
(C) 3

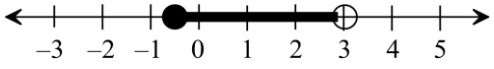
(D) 4

4. Which diagram displays the solution to $\frac{7}{3-x} \geq 2$ ?

(A) 


(B) 


(C) 


(D) 


**5.** Which solution contains all asymptotes of the function $y = \frac{-7}{x^2 - 2x - 3}$?

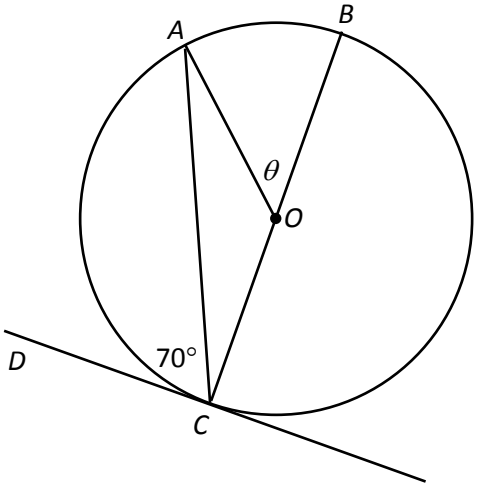
(A) $x = 3, x = -1$

(B) $x = 3, x = -1, y = 0$

(C) $x = 3, x = -1, y = -7$

(D) $y = x^2 - 2x - 3$

**6.** The diagram shows a circle with centre $O$ and diameter $BC$.
The line $DC$ is a tangent to the circle at $C$ and $\angle ACD = 70°$.




Find the size of the angle $\theta$.

(A) 70°

(B) 40°

(C) 35°

(D) 10°

**7.** Which of the following contains all solutions of the equation $\sin 2x = 2\sin x$ in the domain $0 \le x \le 2\pi$.

(A) $x = 0, \frac{\pi}{2}, 2\pi$

(B) $x = 0, \frac{3\pi}{2}, 2\pi$

(C) $x = 0, 2\pi$

(D) $x = 0, \pi, 2\pi$

**8.** Given that $x^2 = 12y$ , the coordinates of the focus and equation of the directrix respectively are:

(A) (0,3) and $y = -3$

(B) (0,-3) and $y = 3$

(C) (3,0) and $x = -3$

(D) (-3,0) and $x = 3$

**9.** What is the exact value of $\cos 195°$ ?

(A) $-\frac{\sqrt{3}+1}{2\sqrt{2}}$

(B) $\frac{1-\sqrt{3}}{2\sqrt{2}}$

(C) $-\frac{1}{\sqrt{2}}$

(D) $\frac{1}{\sqrt{2}}$

***10.*** $\alpha$ and $\beta$ are third quadrant angles where $\sin\alpha = -\frac{4}{5}$ and $\tan\beta = \frac{1}{2}$.

Evaluate $\tan(\alpha - \beta)$.

(A) $\frac{10}{11}$

(B) $2$

(C) $\frac{2}{5}$

(D) $\frac{1}{2}$

**End of Section I**

**Section II commences on the next page.**

**Section II** **60 marks**

- Begin each question in a SEPARATE writing booklet or answer sheet
- Show all relevant mathematical reasoning and/or calculations

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**Question 11 (15 marks)** Use a SEPARATE writing booklet **Marks**

(a) The point $P(x, y)$ divides the interval joining the points $A(1, 3)$ and $B(9, -5)$ externally in the ratio $-1:5$. **2**

Find the coordinates of $P$.

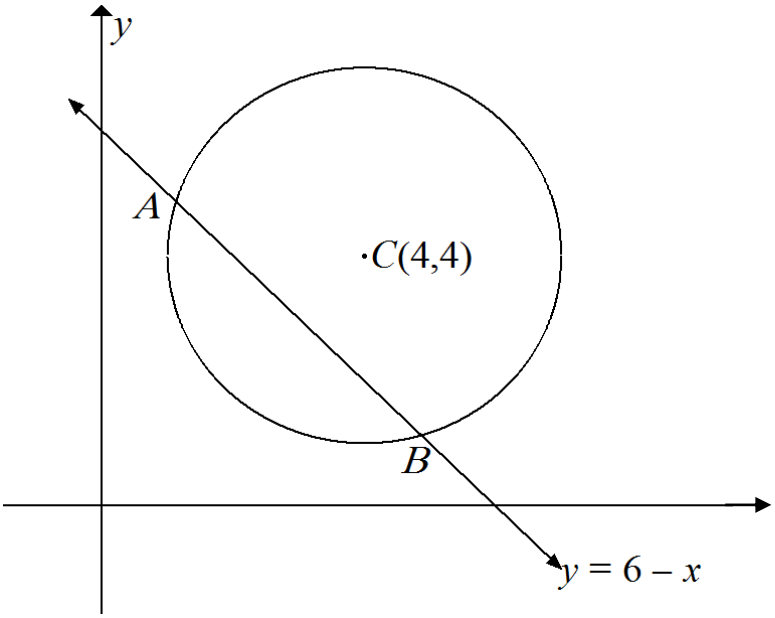
(b) Find the size of the acute angle between the lines $y = 3x + 4$ and $y = -2x - 1$. **2**

(c) (i) Neatly and clearly sketch the graphs of $y = |2x - 1|$ and $y = x^2 - 1$. **2**

(ii) Solve $x^2 - 1 < |2x - 1|$. **3**

(d) Solve the inequality $\frac{2x-5}{x+2} > 4$. **3**

(e) The diagram shows a circle with centre $C(4,4)$. $AB$ is a chord on the circle with length $2\sqrt{7}$ units. $AB$ lies on the line $y = 6 - x$.

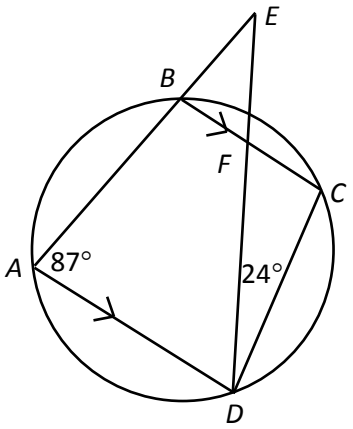



Find the radius of the circle. **3**

**End of Question 11**

**Question 12 (15 marks)** Use a SEPARATE writing booklet **Marks**

(a) *A*, *B*, *C* and *D* are four points on a circle. *AD* is parallel to *BC*.
*AB* is extended to the point *E*. *DE* cuts *BC* at the point *F*.




(i) Find the size of $\angle ADC$. **2**

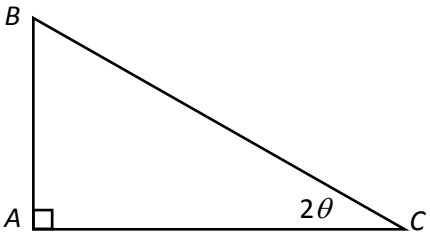
(ii) Find the size of $\angle BEF$. **2**

(iii) Explain why *ABFD* is not a cyclic quadrilateral. **1**

(b) Prove: $\cos 3A = 4\cos^3 A - 3\cos A$. **3**

(c) Solve: $\sin^2 x = \cos x$ for $0^\circ \le x \le 360^\circ$. **3**

(d) The diagram shows the right angled triangle *ABC*, where
$\angle A = 90°$ and $\angle C = 2\theta$.



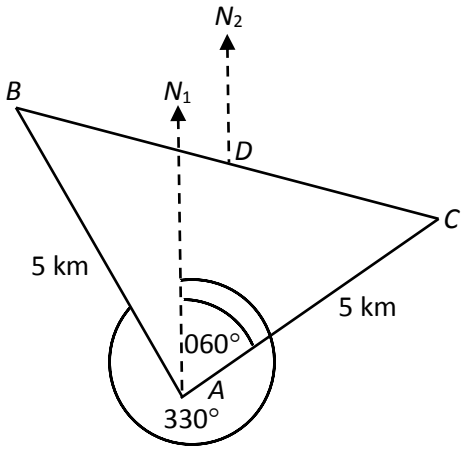

(i) Use the formula $\tan 2\theta = \dfrac{2\tan\theta}{1-\tan^2\theta}$ to express the sides *AB* and *AC* **2**
in terms of $\tan\theta$.

(ii) Hence show that $BC = \sec^2\theta$. **2**

**End of Question 12**

**Question 13 (15 marks)** Use a SEPARATE writing booklet **Marks**

(a) In the diagram, two yachts start at *A*. One yacht sails 5 kilometres on a bearing of 330° to *B*. The other sails 5 kilometres on a bearing of 060° to *C*. The yachts then sail in a straight line toward each other so that they meet at *D*, the midpoint of *BC*.




(i) Show that $\angle ABC = 45°$. **2**

(ii) Find the bearing of *C* from *D*. **2**

(b) Given that $\tan\frac{\theta}{2} = t$, express $\frac{\tan\theta - \cot\theta}{\tan\theta + \operatorname{cosec}\theta}$ in terms of *t*. **2**

(c) (i) Show that $\frac{1+\cos 2x}{\sin 2x} = \cot x$ **2**

(ii) Hence, show the exact value of $\cot 15^\circ = 2+\sqrt{3}$. **1**

(d) Show that the equation $x^2 + (2+k)x + k = 0$ has real roots for all values of *k*. **2**

**Question 13 continues on the next page.**

**Question 13 (continued)** **Marks**

(e) If the equation $x^2 + 2x + 4 = 0$ has solutions $\alpha$ and $\beta$, find the value of

$$(\alpha + \beta)\left(\frac{1}{\alpha} + \frac{1}{\beta}\right).$$ **2**

(f) The interval from (1, -4) to (3, 2) is divided internally in the ratio *a* : *b*.at the point

$\left(1\frac{3}{4}, -1\frac{3}{4}\right)$ .

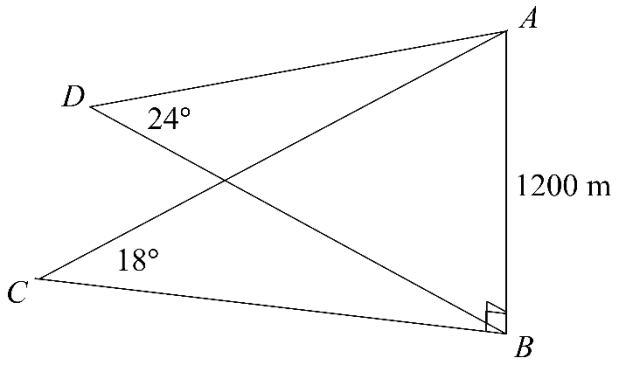
Find the values of *a* and *b*. **2**

**End of Question 13**

**Please turn over**

**Question 14 (15 marks)** Use a SEPARATE writing booklet Marks

(a) A mountain *AB* is 1200 metres above sea level. Daniel is on a boat with a bearing of 140˚ to the mountain and angle of elevation to the top of the mountain of 24˚. Chloe is on a boat with a bearing of 110˚ to the mountain and angle of elevation to the top of the mountain of 18˚.




What is the distance between Daniel and Chloe?
Answer to the nearest metre.

**3**

(b) Find the value of *A*, *B* and *C* if $x^2+5x-3\equiv Ax(x+1)+B(x+1)^2+Cx$ **3**

(c) Consider the parabola $y^2-4y-12x-8=0$.

(i) Find the coordinates of the vertex, showing all calculations. **2**

(ii) Find the coordinates of the focus. **1**

(iii) Find the equation of the directrix. **1**

(iv) Sketch the parabola, showing all features. **2**

(d) The parabola with equation $y=x^2-6x+8$ has a tangent at the point where $x=a$.

This tangent passes through the origin, find the value(s) of *a*. **3**

**End of Question 14**

**End of Section II**
**End of Examination**

# 2015 Yearly Examination (Task 4)

## Year 11 Ext 1

Total marks - 70

### Multiple Choice

1.) $\frac{d}{dx}\left(\frac{1}{x\sqrt{x}} + x\right)$

$= \frac{d}{dx}\left(x^{-3/2} + x\right)$

$= -\frac{3}{2}x^{-5/2} + 1$

$= \frac{-3}{2x^2\sqrt{x}} + 1$ 1 (A)

2.) $|x-2| = 2x+1$

$x - 2 = 2x + 1$

$-3 = x$

Test Solutions:

$x = -3$

$|-5| = -5$

incorrect

$-x + 2 = 2x + 1$

$1 = 3x$

$\frac{1}{3} = x$

Test: $x = \frac{1}{3}$

$|\frac{1}{3} - 2| = \frac{2}{3} + 1$

$|\frac{2}{3}| = 1\frac{2}{3}$

correct

2 (C)

3.) $SP^2 = SQ \times SR$

$4^2 = (x+6)x$

$0 = x^2 + 6x - 16$

$0 = (x+8)(x-2)$

$x = -8$ OR $x = 2$

Not a solution

$\therefore x = 2$ 3 (B)

4 $\frac{7}{3-x} \geq 2$

$x \neq 3$ A and B cannot be correct

$7 = 2(3-x)$

$7 = 6 - 2x$

$2x = -1$

$x = -\frac{1}{2}$

$-\frac{1}{2}$ 3

test $x = 0$

$\therefore -\frac{1}{2} \leq x < 3$

4 (D)

5.) $y = \frac{-7}{x^2 - 2x - 3}$

$= \frac{-7}{(x-3)(x+1)}$

$x \neq 3$ $x \neq -1$

$0 = \frac{-7}{x^2 - 2x - 3}$ $\left(\lim_{x\to\infty} \frac{-\frac{7}{x^2}}{1 - \frac{2}{x} - \frac{3}{x^2}}\right)$

has no solution

$\therefore y = 0$ is the horizontal asymptote

5 (B)

6.) $\angle DCB = 90° - 70 = 20°$

$\therefore \theta = 40°$

6 (B)

7) $\sin 2x = 2\sin x$

$2\sin x \cos x = 2\sin x$

$2\sin x(\cos x - 1) = 0$

$\sin x = 0$ OR $\cos x = 1$

$x = 0, \pi, 2\pi$ $x = 0, 2\pi$

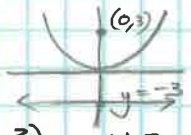
$\therefore x = 0, \pi, 2\pi$

7 (D)

8) $x^2 = 12y$

$12 = 4a$

$a = 3$




(0,3) $y = -3$ (A)

9) $\cos 195°$

$= -\cos 15°$

$= -(\cos(45° - 30°)$

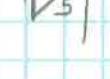
$= -(\cos 45° \cdot \cos 30° + \sin 45° \sin 30°)$

$= -\left(\left(\frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \times \frac{1}{2}\right)\right)$

$= -\left(\frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}}\right)$

$= \frac{-\sqrt{3} + 1}{2\sqrt{2}}$ (A)

10) $\sin\alpha = -\frac{3}{5}$ $\tan(\alpha - \beta) = \frac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}$




$\tan\beta = \frac{1}{2}$

$= -\frac{3}{4} - \frac{1}{2}$ / $1 + (-\frac{3}{4})(\frac{1}{2})$

$= \frac{1}{2}$ (D)

TOTAL : 10

## Section II

### Question 11:

a) A(1,3) B(9,−5)

1 : 5

$P(x,y) = \left(\frac{1(-5)+9(1)}{-4}, \frac{3(-5)+(-5)(1)}{-4}\right)$ ✓

$= \left(\frac{4}{-4}, \frac{-20}{-4}\right)$

$= P(-1, +5)$ ✓ (2)

b) $y = 3x + 4$ $\quad m_1 = 3$

$y = -2x - 1$ $\quad m_2 = -2$

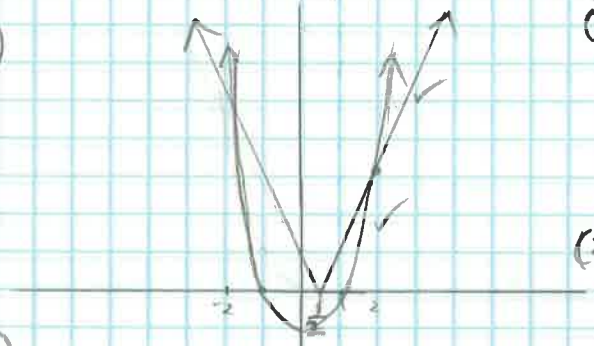
$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$

$= \left|\frac{(3) - (-2)}{1 + 3(-2)}\right|$ ✓

$= 1$ ✓

$\therefore \theta = 45°$ (2)

c) i)




✓ ✓ (2)

ii) $x^2 - 1 = |2x - 1|$

Intersections: $x^2 - 1 = 2x - 1$

$x^2 - 2x = 0$

$x(x-2)$

$x \neq 0 \quad x = 2$ ✓

OR

$x^2 - 1 = -2x + 1$

$x^2 + 2x - 2 = 0$

$x = \frac{-2 \pm \sqrt{4 - 4(-2)}}{2}$

$= \frac{-2 \pm \sqrt{12}}{2}$

$= \frac{-2 \pm 2\sqrt{3}}{2}$ ✓

$x = -1 \pm \sqrt{3}$

but $x = -1 - \sqrt{2}$

$\therefore$ Solution:

$-1 - \sqrt{3} < x < 2$ . ✓ (3)

d) $\frac{2x - 5}{x + 2} > 4$

undefined when $x = -2$

let $\frac{2x-5}{x+2} = 4$

$2x - 5 = 4x + 8$

$2x = -13$ ✓

$x = -6.5$

Test: False | True | False ✓

$-6.5 \quad -2$

$\therefore -6.5 < x < -2$ . ✓ (3)

e) C(4,4) $\quad x + y - 6 = 0$

Perp distance from C to the line:

$d = \left|\frac{(4 \times 1) + (4 \times 1) - 6}{\sqrt{1^2 + 1^2}}\right|$

$= \frac{2}{\sqrt{2}}$

$= \sqrt{2}$ ✓

Given $AB = 2\sqrt{7}$

$\therefore \frac{1}{2}(AB) = \sqrt{7}$

$r^2 = (\sqrt{2})^2 + (\sqrt{7})^2$ ✓ (Pythagoras)

$= 2 + 7$

$= 9$

$r = 3$ ✓ (3)

## Question 12:

a i) $\angle ABC + 87° = 180°$ (co-int ∠s in ll lines)
$\angle ABC = 93°$ ✓

$\angle BCD + 87° = 180°$ (opp ∠s of cyclic quad are suppl)
$\angle BCD = 93°$

$\angle ADC + 87° + 93° + 93° = 360°$ (∠ sum of a quadrilateral)
$\angle ADC = 87°$ ✓ (2)

ii) $\angle ADF + 24° = 87°$ (adj ∠s)
$\angle ADF = 63°$

$\angle BEF + 87° + 63° = 180°$ (∠ sum of triangle) } ✓
$\angle BEF = 30°$

$\angle ADF + 24° = 87°$ (adjacent ∠s)
$\angle ADF = 63°$

$\angle BEF + 87° + 63° = 180°$ (∠ sum of a triangle) } ✓ (2)
$\angle BEF = 30°$

iii) $\angle BFD + 63° = 180°$ (co-int ∠s in ll lines)
$\angle BFD = 117°$

$\angle BFD + \angle BAD = 117° + 87° \neq 180°$ ✓ (1)

∴ ABFD is NOT cylic (opp ∠s are NOT supplementary)

b) Proof: $\cos 3A$
$= \cos(2A + A)$
$= \cos 2A \cos A - \sin 2A \cdot \sin A$ ✓
$= (2\cos^2 A - 1)\cos A - 2\sin A \cdot \cos A \cdot \sin A$
$= 2\cos^3 A - \cos A - 2\cos A(\sin^2 A)$ ✓
$= 2\cos^3 A - \cos A - 2\cos A(1 - \cos^2 A)$
$= 2\cos^3 A - \cos A - 2\cos A + 2\cos^3 A$ ✓ (3)
$= 4\cos^3 A - 3\cos A$
$= RHS$

c) $\sin^2 x = \cos x$
$1 - \cos^2 x = \cos x$
$0 = \cos^2 x + \cos x - 1$
let $k = \cos x$
$0 = k^2 + k - 1$
$k = \frac{-1 \pm \sqrt{1 - 4(1)(-1)}}{2}$
$k = \frac{-1 \pm \sqrt{5}}{2}$

∴ $\cos x = \frac{-1 + \sqrt{5}}{2}$
$\cos x = 0.618$
$x = 51.8$ OR $x = 308.17$
OR
$\cos x = \frac{-1 - \sqrt{5}}{2}$
$\cos x = -1.6$
NO SOLUTION

d) i) $\tan 2\theta = \frac{AB}{AC} = \frac{2\tan\theta}{1-\tan^2\theta}$ let $\tan\theta = t$

$\therefore AB = 2\tan\theta$ ✓

$\therefore AC = 1 - \tan^2\theta$ ✓ (2)

ii) $BC^2 = (2\tan\theta)^2 + (1-\tan^2\theta)^2$

$= 4\tan^2\theta + 1 - 2\tan^2\theta + \tan^4\theta$ ✓

$= \tan^4\theta + 2\tan^2\theta + 1$

$= (\tan^2\theta + 1)^2$ ✓

$BC^2 = (\sec^2\theta)^2$ $(\tan^2\theta + 1 = \sec^2\theta)$

$BC = \sec^2\theta$ (2)

## Question 13:

13a) i) $\angle BAN_1 = 30°$ (angles at a pt)
$\angle BAC = 90°$ (adjacent angles) } ✓
$\angle ABC = 45°$ (base angle of isosceles $\Delta$) ✓ (2)

ii) $\alpha = 105°$ (angle sum of $\Delta$)
$\beta = 105°$ (vertically opposite angles) } ✓
$\angle N_2DC = 105°$ (corresp angles, $\parallel$ lines) ✓ (2)

$\therefore$ bearing of C from D
$= 105°$.

b) $\tan\frac{\theta}{2} = t$

$$\frac{\tan\theta - \cot\theta}{\tan\theta + \operatorname{cosec}\theta} = \frac{\frac{2t}{1-t^2} - \frac{1-t^2}{2t}}{\frac{2t}{1-t^2} + \frac{1+t^2}{2t}} \quad ✓$$

$$= \frac{\frac{4t^2 - 1 + 2t^2 - t^4}{2t - 2t^3}}{\frac{4t^2 + 1 - t^4}{2t - 2t^3}}$$

$$= \frac{6t^2 - 1 - t^4}{4t^2 + 1 - t^4} \quad ✓ \quad (2)$$

c) i) $LHS = \frac{1 + \cos 2x}{\sin 2x}$

$= \frac{2\cos^2 x}{2\sin x \cos x}$ ✓

$= \frac{\cos x}{\sin x}$ ✓

$= \cot x$

$= RHS$. (2)

ii) $\cot 15° = \frac{1 + \cos 30°}{\sin 30°}$

$= \frac{1 + \frac{\sqrt{3}}{2}}{\frac{1}{2}}$ ✓

$= 2 + \sqrt{3}$. (1)

13d) $x^2+(2+k)x+k=0$

$\Delta = (2+k)^2-4k$ ✓

$= 4+4k+k^2-4k$

$= k^2+4$

$> 0$

$\therefore$ Roots are real ✓ (2)

e) $x^2+2x+4=0$

$\alpha+\beta=-2$

$\alpha\beta=4$

$\left(\alpha+\beta\right)\left(\frac{1}{\alpha}+\frac{1}{\beta}\right) = \left(\alpha+\beta\right)\left(\frac{\alpha+\beta}{\alpha\beta}\right)$

$= \frac{(-2)^2}{4}$ ✓

$= 1$ ✓ (2)

f) $1\frac{3}{4} = \frac{a\times 3+b\times 1}{a+b}$

$\frac{7}{4} = \frac{3a+b}{a+b}$

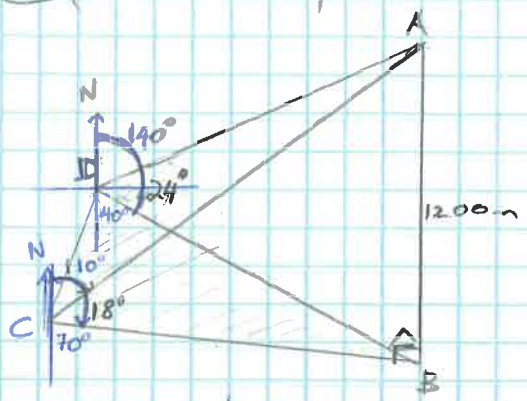
$7a+7b = 12a+4b$ ✓

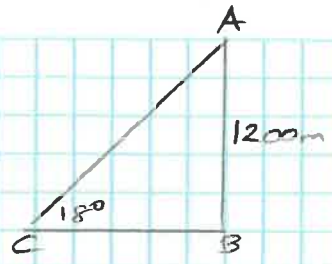
$3b = 5a$

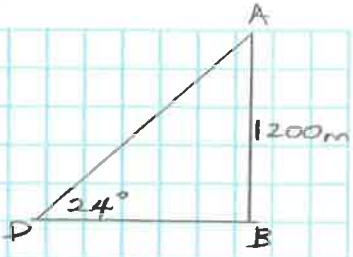
$\frac{a}{b} = \frac{3}{5}$ ✓

$a:b = 3:5$ (2)

# Question 14:




$\tan 18° = \frac{1200}{CB}$

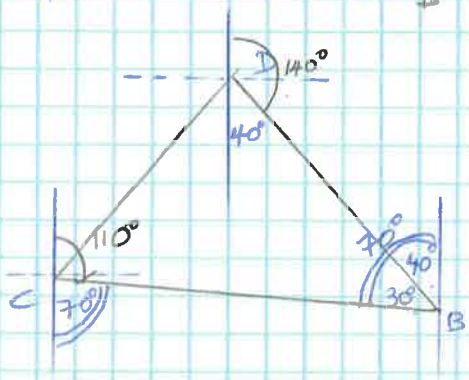
$CB = \frac{1200}{\tan 18°}$

$CB = 3693.22$

$\tan 24° = \frac{1200}{DB}$

$DB = \frac{1200}{\tan 24°}$

$DB = 2695.24$




$DC^2 = DB^2 + CB^2 - 2 \times DB \times CB \times \cos 30°$ ✓

$= 2695.24^2 + 3693.22^2 - (2 \times 2695.24 \times 3693.22 \times \cos 30°)$ ✓

$DC = 1913.9376...$

$= 1914m$ ✓ (3)

∴ Distance between Chloe and Daniel is 1914 metres.

b) $x^2 + 5x - 3 \equiv Ax(x+1) + B(x+1)^2 + Cx$

$\equiv Ax^2 + Ax + B(x^2 + 2x + 1) + Cx$

$= Ax^2 + Ax + Bx^2 + 2Bx + B + Cx$

$= (A+B)x^2 + (A + 2B + C)x + B$

∴ $A + B = 1$

$B = -3$ ✓

∴ $A = 3 + 1$

$A = 4$ ✓

$A + 2B + C = 5$

$(4) + 2(-3) + C = 5$

$4 - 6 + C = 5$

$C = 5 - 4 + 6$ ✓

$C = 7$ ✓ (3)

c) $y^2 - 4y - 12x - 8 = 0$

$y^2 - 4y + (-2)^2 = 12x + 8 + 4$

$(y-2)^2 = 12x + 12$

$(y-2)^2 = 12(x+1)$

$4a = 12$ ✓

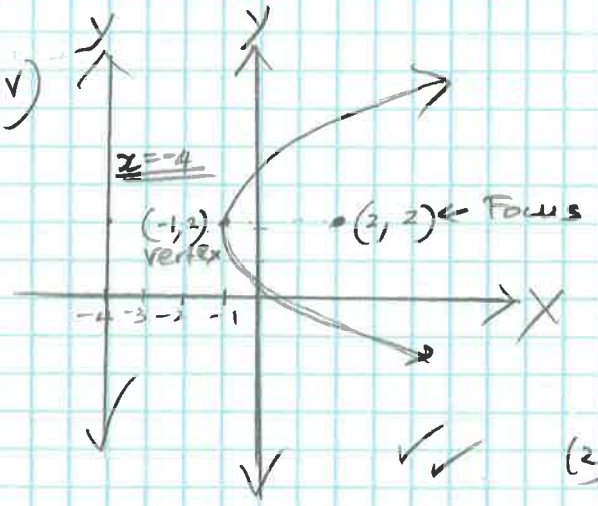
$\therefore a = 3$

i) Vertex $(-1, 2)$ ✓ (1)

ii) focus: $(2, 2)$ ✓ (1)

iii) Directrix: $x = -4$ ✓ (1)

iv)




✓✓ (2)

d) $\frac{dy}{dx} = 2x - 6$. if $y = x^2 - 6x + 8$

gradient of tangent at a:

$(x = a)$ $m = 2a - 6$ ✓

$y - (a^2 - 6a + 8) = (2a - 6)(x - a)$

substitute the point $(0, 0)$ for $x$ and $y$. ✓

$-a^2 + 6a - 8 = -2a^2 + 6a$

$a^2 - 8 = 0$

$a^2 = 8$

$a = \pm\sqrt{8}$

$a = \pm 2\sqrt{2}$ ✓ (3)