Trinity Grammar School

Mathematics Department

2016

YEARLY EXAMINATION

PRELIMINARY ASSESSMENT TASK 4

Year 11

Mathematics Extension 1

General Instructions

- Reading time – 5 minutes
- Working time – 2 hours
- Write using black or blue pen
- BOSTES approved calculators for this course may be used
- A Reference Sheet for this course is supplied
- In Questions **11 – 14** , show all relevant mathematical reasoning and/or calculations
- Write your BOSTES Student Number (Year 12 HSC) or Name (Year 11) and your Class teacher on the question paper and on any answer sheets or writing booklets used to write your responses to the questions submitted
- If you do not attempt a question you must submit an answer sheet or writing booklet for that question clearly indicating **N/A** and your Student Number or Name.

- **Preliminary Assessment Weighting:** 40%

BOSTES Student Number
(Year 12 only)

Class Teacher:

.....

Name:
(Year 10 or 11 only)

.....

Do NOT write solutions on this question paper. Any working on this question paper will NOT be marked.

Date of examination:

Monday, 5 September 2016

Total marks – 70

Section I

10 marks

- Attempt all Questions
- Allow about **15** minutes for this section

Section II

60 marks

- Attempt all Questions
- Allow about **1** hour **45** minutes for this section

Blank page

Section I **15 marks**

- Attempt Questions 1 – 10
- Allow about 15 minutes for this section
- Shade the correct response on the multiple-choice answer sheet for Questions 1 – **10**
- Each question is worth 1 mark

---

1 If $\dfrac{3\sqrt{2}+2}{2\sqrt{2}-1} = a + \sqrt{b}$, what are the values of a and b?

(A) $a = 2, b = 2$

(B) $a = 14, b = 7$

(C) $a = 14, b = 98$

(D) $a = 2, b = 1$

2 Factorise fully $2a^4 - 32b^4$.

(A) $2(a^4 - 16b^4)$

(B) $2(a^2 - 4b^2)(a^2 + 4b^2)$

(C) $2(a - 2b)(a + 2b)(a^2 + 4b^2)$

(D) $2(a - 2b)(a + 2b)^3$

3 The simplification of $3\left(\log_a\left(\dfrac{m}{n}\right) + \log_a\left(\dfrac{n}{m}\right)\right)$ is:

(A) 3

(B) $3\log_a\left(\dfrac{m+n}{n+m}\right)$

(C) $\log\left(\dfrac{m+n}{n+m}\right)^3$

(D) 0

4 Which expression is equivalent to $\sin 4x$?

(A) $4\sin 2x\cos 2x$

(B) $4\sin x\cos x\cos 2x$

(C) $4\sin x\cos x$

(D) $8\sin x\left(\cos^2 x-\sin^2 x\right)$

5. Find the $\lim_{x\to\infty}\dfrac{3x^2-2x+7}{-2x^2+6x-9}$

(A) 0

(B) $-\dfrac{3}{2}$

(C) $\dfrac{1}{3}$

(D) $\dfrac{7}{9}$

6. What are the coordinates of the focus of the parabola $4y=x^2-8$?

(A) (0, -8)

(B) (0, -7)

(C) (0, -2)

(D) (0, -1)

7 Given the function $f(x) = -\frac{4}{\sqrt{x}}$, what is the solution to the equation $f'(x) = 128$?

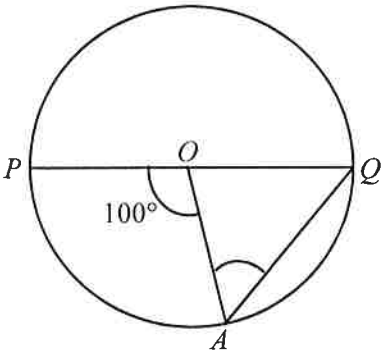
(A) $\frac{1}{2}$

(B) 2

(C) $\frac{1}{16}$

(D) 4

8 The diagram shows the point A on a circle with centre O and diameter PQ.
The size of $\angle POA$ is 100°.



NOT TO SCALE

What is the size of $\angle OAQ$?

(A) 80°

(B) 40°

(C) 50°

(D) 30°

9 For what values of k does the equation $x^2 - 6x - 3k = 0$ have real roots?

(A) $k \geq -3$

(B) $k \leq -3$

(C) $k \geq 3$

(D) $k \leq 3$

10 What is the natural domain of the function $f(x) = \frac{1}{\sqrt{x^2-9}}$?

(A) $x < -3$ and $x > 3$

(B) $x \leq -3$ and $x \geq 3$

(C) $-3 < x < 3$

(D) $-3 \leq x \leq 3$

End of Section I

Section II **60 marks**

- Attempt Questions 11 – 14
- Allow about 1 hours and 45 minutes for this section
- Answer each question in a SEPARATE writing booklet or answer sheet. Extra writing booklets are available.

In Questions 11 – 14, your responses should include all relevant mathematical reasoning and/or calculations

---

Question 11 (15 marks) Use a SEPARATE writing booklet. Marks

(a) Find the coordinates of the point $P(x, y)$ that divides AB internally in the ratio $3:4$ given that A has co-ordinates $(6,-2)$ and B has the co-ordinates $(-7,5)$. **2**

(b) If $f(x) = (2x+1)(x^4-3)$ find $f'(-1)$. **3**

(c) Find the obtuse angle between $4x+y+2=0$ and $x+y-1=0$, correct to the nearest degree. **3**

(d) Find the value of a, b and c if $2x^2 - x - 1 \equiv ax(x+1) + b(x+1) + c$. **3**

(e) If α and β are the roots of $x^2 - 6x + 3 = 0$, without solving, find the values of :

(i) $(\alpha+1)(\beta+1)$ **2**

(ii) $\alpha^2 + \beta^2$ **2**

End of Question 11

Question 12 (15 marks) Use a SEPARATE writing booklet. Marks

(a) For what values of M does the equation $x^2 + Mx + (3 - M) = 0$ have equal roots? **2**

(b) Solve $3^{2x} - 10(3^x) + 9 = 0$. **3**

(c) Find the centre and radius of the circle $x^2 - 6x + y^2 + 2y - 6 = 0$. **2**

(d) Let $f(x) = x^3 - 4x + 1$.

(i) Expand $(x + h)^3$. **1**

(ii) Use the formula $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$ to show that the derivative of $f(x)$ is $3x^2 - 4$. **3**

(iii) The tangent to the curve of $f(x)$ at the point $P(1, -2)$ is parallel to the tangent at a point Q on the same curve. Find the coordinates of Q. **3**

(e) Solve $\sin x = \cos x$ for $0 \leq x \leq 360^\circ$. **1**

End of Question 12

Question 13 (15 marks) Use a SEPARATE writing booklet. Marks

(a) Solve $\sin 2x = \frac{1}{\sqrt{2}}$, for $0 \leq x \leq 360^\circ$. **2**

(b) Solve: $\frac{a+4}{a-2} \geq 4$ **3**

(c) If $\log_a 3 = 0.477$ and $\log_a 4 = 0.602$, find:

(i) $\log_a 12$ **1**

(ii) $\log_a 3a^2$ **2**

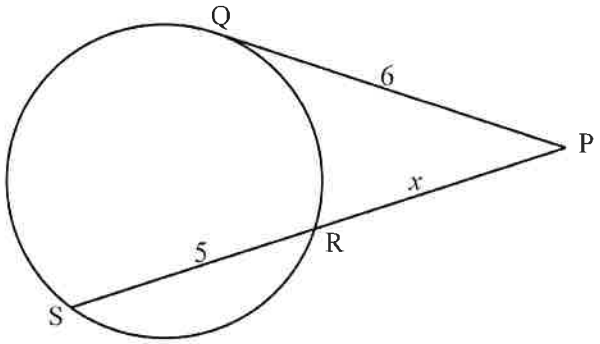
(d) Differentiate with respect to x:

(i) $\sqrt[3]{x} + 5x^{-3}$ **1**

(ii) $\sqrt[5]{5-6x^3}$, giving your answer without negative or fractional indices. **2**

(iii) $\frac{x+1}{3x^2-7}$ **2**

(e) PQ is a tangent to a circle QRS, while PRS is a secant intersecting the circle in R and S, as in the diagram.



NOT TO SCALE

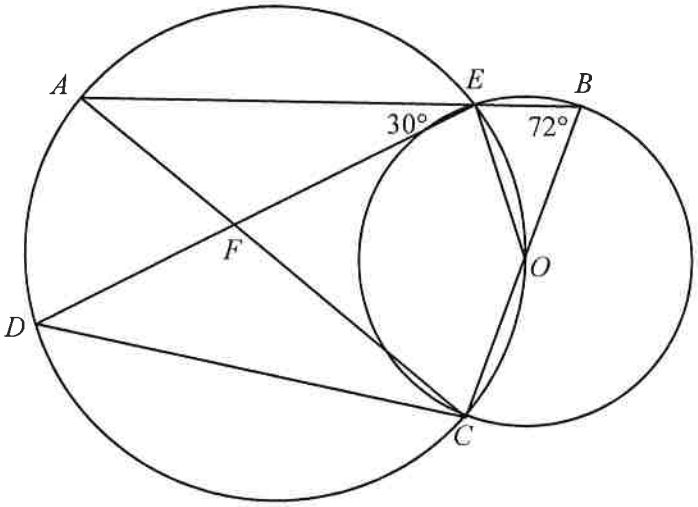
Given that $PQ = 6$, $RS = 5$, $PR = x$, find the length PR. **2**

End of Question 13

Question 14 (15 marks) Use a SEPARATE writing booklet. Marks

(a) In the diagram, the points B, C and E lie on the circle with centre O. BC is a diameter of the circle. $\angle EBO = 72°$.

The points A, E, O, C and D lie on the second circle in the diagram. AEB, AFC and DFE are straight lines. $\angle AEF = 30°$.



NOT TO SCALE

(i) Find the value of $\angle EOC$. **1**

(ii) Find the value of $\angle CDE$. **1**

(iii) Find the value of $\angle DFC$. **2**

(b) (i) Find the equation of the axis of symmetry and co-ordinates of the vertex of the parabola,

$$y = 2x^2 + 4x - 6$$ **2**

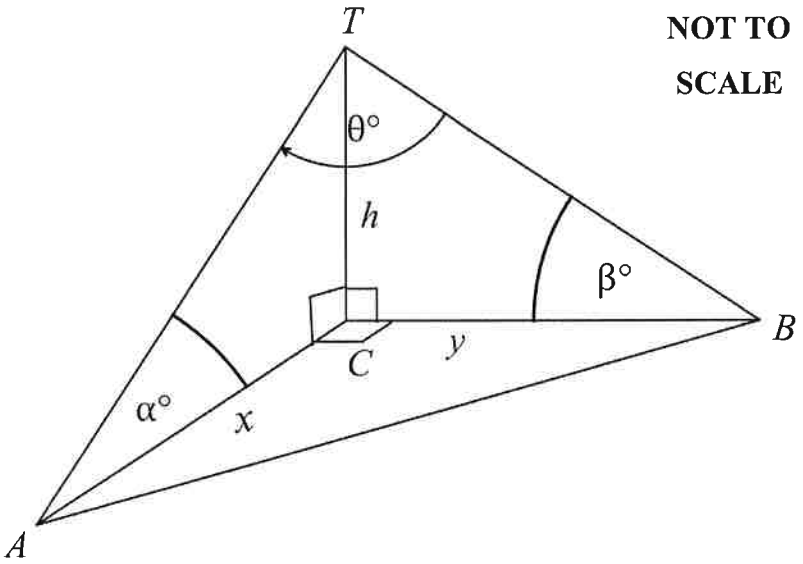
(ii) Find the y intercept and sketch the graph. **2**

Question 14 continues on the following page.

(c) In the diagram $AC = x$, $BC = y$ and $TC = h$.

$\angle ATB = \theta$, $\angle TBC = \beta$ and $\angle TAC = \alpha$.

There are three right angles at C as shown.



(i) Show that $x = h \cot \alpha$ and write a similar expression for y. **1**

(ii) Show that $AT = \sqrt{x^2 + h^2}$ and write a similar expression for BT. **1**

(iii) Use the cosine rule to show that $\cos\theta = \dfrac{h^2}{\sqrt{(x^2 + h^2)(y^2 + h^2)}}$ **3**

(iv) Hence show that $\sin \alpha \sin \beta = \cos \theta$. **2**

End of Question 14

End of paper

TRINITY GRAMMAR SCHOOL
2016, Year 11 Mathematics Extension 1
Yearly Examination
SECTION I
ANSWER SHEET

Student Number: ..... **Class Teacher:** .....

Be sure to write your answers for Section I on this answer sheet. After you have selected an answer, **CIRCLE** the correct answer. To change an answer, erase your previous mark completely, and then record your new answer. Mark only one answer for each question.

Q1.	A	B	C	D
Q2.	A	B	C	D
Q3.	A	B	C	D
Q4.	A	B	C	D
Q5.	A	B	C	D
Q6.	A	B	C	D
Q7.	A	B	C	D
Q8.	A	B	C	D
Q9.	A	B	C	D
Q10.	A	B	C	D

SOLUTIONS

TRINITY GRAMMAR SCHOOL
2016, Year 11 Mathematics Extension 1
Yearly Examination
SECTION I
ANSWER SHEET

Student Number: ..... **Class Teacher:** .....

Be sure to write your answers for Section I on this answer sheet. After you have selected an answer, **CIRCLE** the correct answer. To change an answer, erase your previous mark completely, and then record your new answer. Mark only one answer for each question.

Q1.	(A)	B	C	D
Q2.	A	B	(C)	D
Q3.	A	B	C	(D)
Q4.	A	(B)	C	D
Q5.	A	(B)	C	D
Q6.	A	B	C	(D)
Q7.	A	B	(C)	D
Q8.	A	B	(C)	D
Q9.	(A)	B	C	D
Q10.	(A)	B	C	D

Question 11

a) $A(6,-2)$ $B(-7,5)$ $3:4$

$$x_1 = \frac{3(-7)+4(6)}{3+4} \qquad y_1 = \frac{3(5)+4(-2)}{3+4}$$

$$= \frac{3}{7} \qquad = 1$$

The point P is $(\frac{3}{7}, 1)$

b) $f'(x) = (x^4-3) \times 2 + (2x+1) \times 4x^3$

$= 2x^4 - 6 + 8x^4 + 4x^3$

$= 10x^4 + 4x^3 - 6$

$f'(-1) = 10(-1)^4 + 4(-1)^3 - 6$

$= 10 - 4 - 6$

$= 0$

c) $4x+y+2=0$

$y = -4x-2$

$\therefore m_1 = -4$

$x+y-1=0$

$y = -x-1$

$m_2 = -1$

$$\tan\theta = \left| \frac{-4--1}{1+(-4)(-1)} \right|$$

$$= \frac{3}{5}$$

$\therefore$ obtuse angle $= 180° - \tan^{-1}(\frac{3}{5})$

$= 149°$

d) $ax(x+1) + b(x+1) + c = ax^2 + ax + bx + b + c$

$= ax^2 + (a+b)x + (b+c)$

$\therefore a = 2, \quad a+b=-1, \quad b+c=-1$

$2+b=-1 \qquad -3+c=-1$

$b=-3 \qquad c=2$

$\therefore$ Solution is $a=2, b=-3, c=2$

e) $a=1,\ b=-6,\ c=3$

$\therefore\ \alpha+\beta = \frac{-b}{a} = \frac{6}{1} = 6$

$\alpha\beta = \frac{c}{a} = \frac{3}{1} = 3$

(i) $(\alpha+1)(\beta+1) = \alpha\beta + \alpha + \beta + 1$

$= 6 + 3 + 1$

$= 10$

(ii) $\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$

$= 6^2 - 2(3)$

$= 30$

Question 12

a) $x^2 + mx + 3 - m = 0$

$\Delta = b^2 - 4ac = 0$ for equal roots

$m^2 - 4(1)(3-m) = 0$

$m^2 + 4m - 12 = 0$

$(m+6)(m-2) = 0$

$m = -6$ or 2

b) $9^{2x} - 10(3^x) + 9 = 0$

Let $m = 3^x$

$\therefore m^2 - 10m + 9 = 0$

$(m-9)(m-1) = 0$

$m = 9$ or $m = 1$

$3^x = 9$ $\quad$ $3^x = 1$

$x = 2$ $\quad$ $x = 0$

c) $x^2 - 6x + y^2 + 2y - 6 = 0$

$(x-3)^2 - 9 + (y+1)^2 - 1 - 6 = 0$

$(x-3)^2 + (y+1)^2 = 16$

$\therefore$ Centre is at $(3, -1)$

and radius is 4

d) $f(x) = x^3 - 4x + 1$

(i) $(x+h)^3 = x^3 + 3x^2h + 3xh^2 + h^3$

(ii) $f'(x) = \lim_{h \to 0} \dfrac{(x+h)^3 - 4(x+h) + 1 - (x^3 - 4x + 1)}{h}$

$= \lim_{h \to 0} \dfrac{x^3 + 3x^2h + 3xh^2 + h^3 - 4x - 4h + 1 - x^3 + 4x - 1}{h}$

$= \lim_{h \to 0} \dfrac{h(3x^2 + 3xh + h^2 - 4)}{h}$

$= \lim_{h \to 0} 3x^2 + 3xh + h^2 - 4$

$= 3x^2 - 4$

(iii) $f'(1) = 3(1)^2 - 4$

$= -1$ (gradient at $x = 1$)

but gradient at Q is parallel

$\therefore 3x^2 - 4 = -1$

$3x^2 = 3$

$x = \pm 1$

Now, $f(-1)' = (-1)^3 - 4(-1) + 1$

$= 4$

$\therefore$ Q is at $(-1, 4)$

e) $\sin x = \cos x$

$\dfrac{\sin x}{\cos x} = 1$

$\tan x = 1$

$\therefore x = 45°, 225°$

Question 13

a) $\sin 2x = \frac{1}{\sqrt{2}}$

$2x = 45°, 135°, 405°, 495°$

$x = 22.5°, 67.5°, 202.5°, 247.5°$

b) $\frac{a+4}{a-2} \geq 4$

Let $\frac{a+4}{a-2} = 4$

$a + 4 = 4(a-2)$

$a + 4 = 4a - 8$

$12 = 3a$

$\therefore a = 4$

Denominator, $a - 2 \neq 0$

$a \neq 2$

Now, test $a = 0$: $\frac{0+4}{0-2} = -2 \neq 0$

$\therefore$ Solution is $2 < a \leq 4$

c) (i) $\log_a 12 = \log_a (3 \times 4)$

$= \log_a 3 + \log_a 4$

$= 0.477 + 0.602$

$= 1.079$

(ii) $\log_a 3a^2 = 2 \log_a 3a$

$= 2(\log_a 3 + \log_a a)$

$= 2(0.477 + 1)$

$= 2.954$

d) (i) $\sqrt[3]{x} + 5x^{-3} = x^{\frac{1}{3}} + 5x^{-3}$

$$\frac{d}{dx}\left(x^{\frac{1}{3}} + 5x^{-3}\right) = \frac{1}{3}x^{-\frac{2}{3}} - 15x^{-4}$$

$$= \frac{1}{3\sqrt[3]{x^2}} - \frac{15}{x^4}$$

(ii) $y = (5 - 6x^3)^{\frac{1}{5}}$

$$\frac{dy}{dx} = \frac{1}{5}(5-6x^3)^{-\frac{4}{5}} \times -18x^2$$

$$= \frac{-18x^2}{5\sqrt[5]{(5-6x^3)^4}}$$

(iii) $$\frac{d}{dx}\left(\frac{x+1}{3x^2-7}\right) = \frac{(3x^2-7)\times 1 - (x+1)\times 6x}{(3x^2-7)^2}$$

$$= \frac{-3x^2 - 6x - 7}{(3x^2-7)^2}$$

e) $36 = x(x+5)$

$36 = x^2 + 5x$

$0 = x^2 + 5x - 36$

$0 = (x+9)(x-4)$

$x = -9$ or $x = 4$

$\therefore$ Solution is $x = 4$ $(x > 0)$

Question 14

a) (i) $\angle EOC = 144^\circ$ (angle at centre is twice angle at circumference)

(ii) OCDE is a cyclic quadrilateral

$\therefore \angle CDE + 144^\circ = 180^\circ$ (opposite angles of cyclic quadrilateral are supplementary)

$\therefore \underline{\angle CDE = 36^\circ}$

(iii) $\angle ACD = 30^\circ$ (angles in the same segment are equal)

$\angle DFC + 30^\circ + 36^\circ = 180^\circ$ (angle sum of triangle)

$\therefore \underline{\angle DFC = 114^\circ}$

b) (i) axis of symmetry : $x = -\frac{b}{2a}$

$= -\frac{4}{2\times 2}$

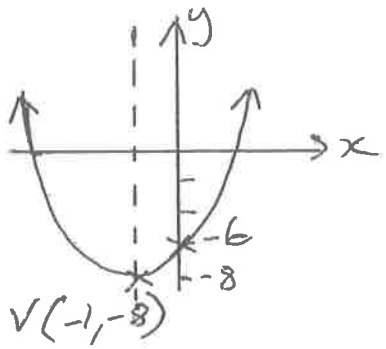
$x = -1$

When $x=-1$, $y = 2(-1)^2 + 4(-1) - 6$

$= -8$

$\therefore$ Vertex is at $(-1,-8)$

(ii) y intercept is -6.



e) (i) $\tan\alpha = \frac{h}{x}$, $\tan\beta = \frac{h}{y}$

$x = \frac{h}{\tan\alpha}$, $y = \frac{h}{\tan\beta}$

$\therefore x = h\cot\alpha$, $y = h\cot\beta$

(ii) In $\triangle ABC$,

$(AB)^2 = x^2 + y^2$

$(AT)^2 = x^2 + h^2$

$\therefore AT = \sqrt{x^2+h^2}$

$(BT)^2 = y^2 + h^2$

$BT = \sqrt{y^2+h^2}$

(iii) In $\triangle ATB$

$(AB)^2 = (AT)^2 + (BT)^2 - 2(AT)(BT)\cos\theta$

$x^2 + y^2 = x^2 + h^2 + y^2 + h^2 - 2\sqrt{(x^2+h^2)(y^2+h^2)}\cos\theta$

$0 = 2h^2 - 2\sqrt{(x^2+h^2)(y^2+h^2)}\cos\theta$

$\cos\theta = \frac{2h^2}{2\sqrt{(x^2+h^2)(y^2+h^2)}}$

$= \frac{h^2}{\sqrt{(x^2+h^2)(y^2+h^2)}}$

(iv) $\sin\alpha\sin\beta = \frac{h}{AT} \times \frac{h}{BT}$

$= \frac{h}{\sqrt{x^2+h^2}} \times \frac{h}{\sqrt{y^2+h^2}}$

$= \frac{h^2}{\sqrt{(x^2+h^2)(y^2+h^2)}}$

$= \cos\theta$ from (iii)