Student Number

Barker College

2022 TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Extension 1

Staff Involved:

- GDH • GPF • JZT • DZP*
- KJL • ESP • AXD

Friday 12th August 2022

100 copies

General Instructions:

- Reading time - 10 minutes
- Working time - 2 hours
- Write using black pen
- Calculators approved by NESA may be used
- A separate reference sheet is provided
- For questions in Section II, show relevant mathematical reasoning and / or calculations
- Marks may not be awarded for careless or poorly arranged working
- Diagrams are not to scale unless specifically stated
- Write your Student Number at the top of this page, and on all writing booklets

Total marks: 70

Section I – 10 marks (pages 2 - 5)

- Attempt Questions 1 - 10
- Allow about 20 minutes for this section.

Section II – 60 marks (pages 6 - 9)

- Attempt Questions 11 - 14
- Allow about 1 hours and 40 minutes for this section.
- Answer each question in a SEPARATE writing booklet.

Section I

10 marks
Attempt Questions 1 – 10
Allow about 20 minutes for this section

Use the multiple-choice answer sheet for Questions 1 – 10.

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1 The equation $x^3+bx^2+cx+d=0$ has roots $1+\sqrt{2}$, $1-\sqrt{2}$ and 4. What is the value of b?

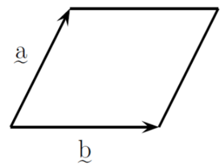
A. -6

B. -4

C. 4

D. 6

2 The diagram below shows a rhombus, spanned by the two vectors $\underset{\sim}{a}$ and $\underset{\sim}{b}$.



Which of the following statements is true?

A. $\underset{\sim}{a}\cdot\underset{\sim}{b}=0$

B. $(\underset{\sim}{a}+\underset{\sim}{b})\cdot(\underset{\sim}{a}-\underset{\sim}{b})=0$

C. $|\underset{\sim}{a}+\underset{\sim}{b}|=|\underset{\sim}{a}-\underset{\sim}{b}|$

D. $2\underset{\sim}{a}+2\underset{\sim}{b}=0$

3 For what values of a and b does $y=e^{2x}+e^{-2x}$ satisfy the equation $\frac{d^2y}{dx^2}+a\frac{dy}{dx}+by=0$?

A. $a=-1,\ b=-2$

B. $a=-2,\ b=0$

C. $a=0,\ b=-4$

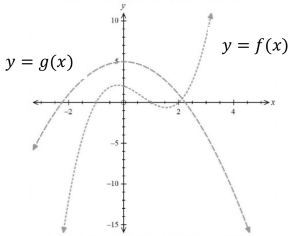
D. $a=-2,\ b=-1$

4 Which is the vector resulting from the projection of $\underset{\sim}{a} = \begin{pmatrix} -4 \\ 1 \end{pmatrix}$ onto $\underset{\sim}{b} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$?

A. $\begin{pmatrix} \frac{-2}{\sqrt{5}} \\ \frac{-4}{\sqrt{5}} \end{pmatrix}$ B. $\begin{pmatrix} \frac{-2}{5} \\ \frac{-4}{5} \end{pmatrix}$

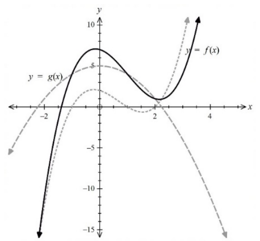
C. $\begin{pmatrix} \frac{8}{\sqrt{17}} \\ \frac{-2}{\sqrt{17}} \end{pmatrix}$ D. $\begin{pmatrix} \frac{8}{17} \\ \frac{-2}{17} \end{pmatrix}$

5 The graphs of $y = f(x)$ and $y = g(x)$ are shown.

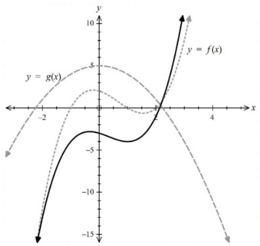


Which diagram shows the graph of $y = f(x) + g(x)$?

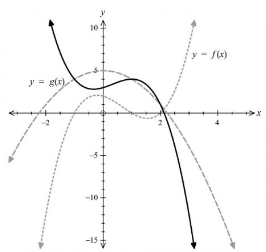
A.



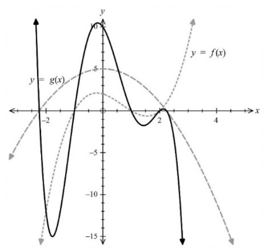
B.



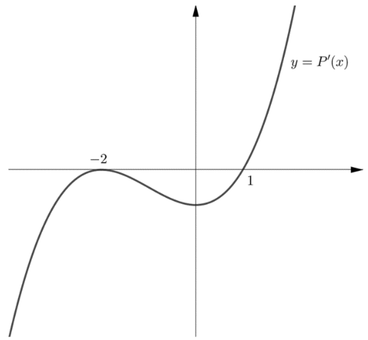
C.



D.



6 The graph of $\boldsymbol{y = P'(x)}$ is shown, where $P(x)$ is a polynomial.



If $x = -2$ is a root of $P(x)$, what is the multiplicty of this root for $P(x)$?

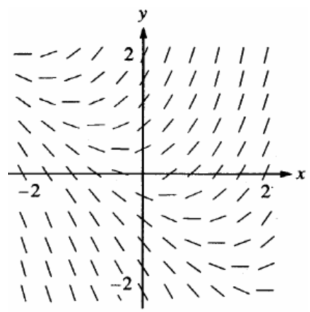
A. 0

B. 1

C. 2

D. 3

7 Which of the following differential equations best represents this direction field?



A. $\frac{dy}{dx} = 1 + x$ B. $\frac{dy}{dx} = x^2$

C. $\frac{dy}{dx} = x + y$ D. $\frac{dy}{dx} = \frac{x}{y}$

8 What is the value of $\sin^{-1}(\cos\theta)$, given $\frac{\pi}{2} < \theta < \pi$?

A. $\frac{\pi}{2} - \theta$

B. $\pi - \theta$

C. $\frac{\pi}{2} + \theta$

D. $\pi + \theta$

9 Ten cards numbered from 1 to 10 are laid face down. What is the least number of cards that must be picked to ensure that one of the numbers is twice the other?

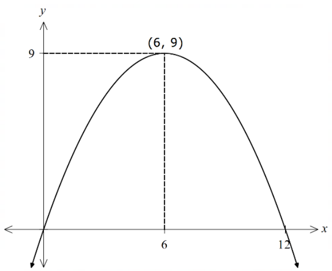
A. 6

B. 7

C. 8

D. 9

10



Which of the parametric equations below represents the parabola above?

A. $x = 12t, y = 9t^2$

B. $x = 12t, y = 9 - t^2$

C. $x = 6 - 2t, y = 9 - t^2$

D. $x = 2t - 6, y = 9t - t^2$

End of Section I

Section II

60 marks
Attempt Questions 11-14
Allow about 1 hour and 40 minutes for this section

Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.

In Questions 11 - 14, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) **[START A NEW BOOKLET]**

(a) Solve the inequality $\frac{3x}{x-2} \geq 2$. **3**

(b) Determine the coefficient of x^3 in the expansion of $(2-3x)^8$. **2**

(c) Prove the identity $\sin(3x) + \sin x = 4\sin x\cos^2 x$. **2**

(d) A state senate sub-committee is to be formed by choosing 3 senators from among 6 rural senators and 6 urban senators. How many different sub-committees can be formed if at least 1 rural and 1 urban senator must be chosen? **2**

(e) Find the exact value of $\int_0^3 \frac{1}{9+x^2}\,dx$. **1**

(f) For the function $f(x) = \frac{\pi}{2} - sin^{-1}\left(\frac{x}{3}\right)$

(i) State the domain and range of the function. **1**

(ii) Sketch the function. **2**

(g) Simplify $\frac{{}^{12}C_r}{{}^{11}C_{r-1}}$. **2**

Question 12 (15 marks) **[START A NEW BOOKLET]**

(a) Solve the initial value problem where $\frac{dy}{dx} = 2xe^{-y}$ given $y(0) = \ln 4$. **3**
Express your answer with y as the subject.

(b) The area bound by the curve $y = \sin^{-1} x$ and the y-axis between $y = 0$ and $y = \frac{\pi}{2}$ is rotated about the y-axis. Calculate, in exact form, the volume of the generated solid. **3**

(c) According to Newton's Law of Cooling, the rate at which a substance cools is proportional to the difference between the temperature of the substance and that of the surrounding air. This can be represented by the first order differential equation $\frac{dT}{dt} = k(T - A)$ where T is the temperature of the substance in degrees Celsius, t is the elapsed time in minutes, A is the air temperature and k is the cooling constant.

(i) By solving the differential equation $\frac{dT}{dt} = k(T - A)$, show that $T = A + P_0e^{kt}$ is a solution, where P_0 is a constant. **2**

(ii) If the temperature of the air is 20˚C and a substance cools from 100˚C to 50˚C in 15 minutes, find the values of P_0 and k (to 4 decimal places). **2**

(iii) Hence find the time it will take for the temperature of the substance to cool from 100˚C to 40˚C. Round your answer to the nearest minute. **1**

(d) A projectile is launched across a level plain at 30° to the horizontal and at an initial speed of 60 m/s. Taking the origin as the launching point and $g = 10$ m/s², the displacement vector $\underset{\sim}{s}(t)$ for the projectile is given by:

$$\underset{\sim}{s}(t) = (30\sqrt{3}t)\underset{\sim}{i} + (30t - 5t^2)\underset{\sim}{j} \quad \text{(Do NOT prove this)}$$

(i) Find the velocity vector $\underset{\sim}{v}(t)$ for the projectile. **1**

(ii) Find the maximum height of the projectile. **2**

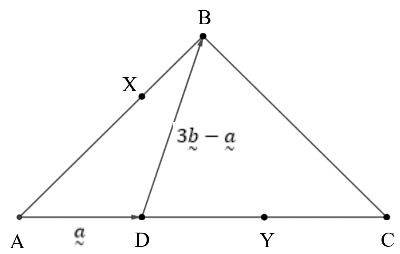
(iii) Find the exact speed of the projectile one second after launch. **1**

Question 13 (15 marks) **[START A NEW BOOKLET]**

(a) By rewriting $\sin x + \cos x$ in the form $R\sin(x + \alpha)$, solve the equation: **3**

$$\frac{\sin x + \cos x}{\sqrt{2}} = \frac{1}{2} \quad \text{for } 0 \le x \le 2\pi.$$

(b) On the diagram below, $\overrightarrow{AD} = \underset{\sim}{a}$, $\overrightarrow{DB} = 3\underset{\sim}{b} - \underset{\sim}{a}$ and AC and AB are straight lines. **3**



Given that $AD : DY : YC = 1 : 1 : 1$ and $AX : XB = 2 : 1$, show that XY is parallel to BC.

(c) Let the function $f(x) = \sec x$ be defined for the restricted domain $0 \le x < \frac{\pi}{2}$, so that its inverse $f^{-1}(x)$ is also a function.

(i) Sketch the graph of $f(x) = \sec x$ within $0 \le x < \frac{\pi}{2}$. **1**

(ii) Show that $f^{-1}(x) = \cos^{-1}\left(\frac{1}{x}\right)$ and clearly state the domain of $f^{-1}(x)$. **2**

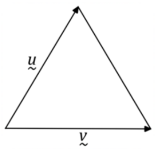
(iii) Find $\frac{d}{dx}\left(f^{-1}(x)\right)$, writing your answer in **fully** simplified form. **2**

(iv) Hence or otherwise, describe the behaviour of the graph $y = f^{-1}(x)$ as $x \to \infty$. **1**

(d) A 10 metre ladder is leaning against a wall. The bottom of the ladder is sliding away from the wall at a rate of 4 m/s. Find the rate at which the top of the ladder is moving down the wall when the bottom of the ladder is 8 metres from the wall. **3**

Question 14 (15 marks) **[START A NEW BOOKLET]**

(a) An equilateral triangle of side 4 units is shown below. Vectors $\underset{\sim}{u}$ and $\underset{\sim}{v}$ are represented in the diagram below. **1**



Prove that $2|\underset{\sim}{u} - \underset{\sim}{v}| = \underset{\sim}{u} \cdot \underset{\sim}{v}$

(b) Prove by Mathematical Induction that:

$$2(1!)+5(2!)+10(3!)+\ldots+(n^2+1)n! = n(n+1)! \text{ for positive integers } n \geq 1$$ **3**

(c) (i) Show that: $x^n(1+x)^n\left(1+\frac{1}{x}\right)^n = (1+x)^{2n}$ **1**

(ii) Hence prove that: **3**

$$1+\binom{n}{1}^2+\binom{n}{2}^2+\binom{n}{3}^2+\ldots+\binom{n}{n}^2=\binom{2n}{n}$$

(d) (i) Find $\frac{d}{dx}\left(xe^x\right)$ **1**

(ii) Using the substitution $u = e^x$ and the result from (i), find the exact value of: **3**

$$\int_0^1 e^{2x}e^{(e^x)}\,dx$$

(e) A total of five players are selected at random from four sporting teams. Each of the teams consists of ten players numbered from 1 to 10.

(i) What is the probability that of the five selected players, three are numbered '6' and two are numbered '8'? **1**

(ii) What is the probability that the five selected players contain at least four players from the same team? **2**

End of Paper

1. Σ roots $= -b$

$\therefore 1+\sqrt{2}+1-\sqrt{2}+4 = -b$

$6 = -b$

$b = -6$ $\therefore$ A

2. 

as a rhombus diagonals bisect at $90°$

$\therefore$ Dot product $= 0$

$(\underset{\sim}{a}+\underset{\sim}{b})\cdot(\underset{\sim}{a}-\underset{\sim}{b}) = 0$ B

3. $y = e^{2x}+e^{-2x}$

$y' = 2e^{2x}-2e^{-2x}$

$= 2(e^{2x}-e^{-2x})$

$y'' = 4e^{2x}+4e^{-2x}$

$= 4(e^{2x}+e^{-2x})$

Subbing into eq^n

$4e^{2x}+4e^{-2x}+2ae^{2x}-2ae^{-2x}+be^{x}+be^{-2x}=0$

$e^{2x}(4+2a+b)+e^{-2x}(4-2a+b)=0$

$\therefore a=0 \; b=-4$ C

4. $\text{proj}_{\underset{\sim}{b}}\underset{\sim}{a} = \frac{\underset{\sim}{a}\cdot\underset{\sim}{b}}{|b|}\underset{\sim}{b}$

$= -\frac{2}{5}\underset{\sim}{b}$

$= \begin{pmatrix} -\frac{2}{5} \\ -\frac{4}{5} \end{pmatrix}$ B

5. Add the y intercepts only possibility is A

6. Double root of $P'(x)$ is a triple root of $P(x)$ D

7. Horizontal lines at line $x=-y$ $\therefore \frac{dy}{dx} = x+y$ C

$\therefore$ Gradient 0

8. $\sin^{-1}(\cos\theta) = \sin^{-1}(-\cos(\pi-\theta))$ not acute

$= -\sin^{-1}(\cos(\pi-\theta))$ as odd

$= -(\pi-\frac{\pi}{2}-(\pi-\theta))$

$= -(-\frac{\pi}{2}-\theta)$

$= \theta-\frac{\pi}{2}$ A

or sub in value of θ & check

9. Sets without doubles:

1 3 4 5 7 9
2 3 5 7 8 9
2 6 7 8 9 10
1 6 7 8 9 10
1 3 5 7 8 9

all length 6

$\therefore$ need 7 to ensure B

10. Check $(0,0)$

A $t=0,0$ ✓

B $t=0,\pm3$ ✗

C $t=3,3$ ✓

D $t=3,0$ ✗

Check $(12,0)$

A $t=1,0$ ✗

C $t=-3,-3$ ✓ $\therefore$ C

11 a) $x \neq 2$

$3x(x-2) \geq 2(x-2)^2$

$3x^2 - 6x \geq 2x^2 - 8x + 8$

$x^2 + 2x - 8 \geq 0$

$(x+4)(x-2) \geq 0$

$x \leq -4,\ x > 2$

b) General term

$\binom{8}{k} 2^{8-k} (-3x)^k$

$\therefore x^3$ coeff $= \binom{8}{3} 2^5 (-3)^3$

$= -48384$

c) LHS $= \sin(x+2x) + \sin x$

$= \sin x \cos 2x + \cos x \sin 2x + \sin x$

$= \sin x(2\cos^2 x - 1) + 2\sin x \cos^2 x + \sin x$

$= 4 \sin x \cos^2 x =$ RHS

d) $2R1U = \binom{6}{2} \times \binom{6}{1}$

$= 90$

$1R2U = 90 \therefore$ total 180

e) $= \frac{1}{3} \tan^{-1} \frac{x}{3} \Big]_0^3$

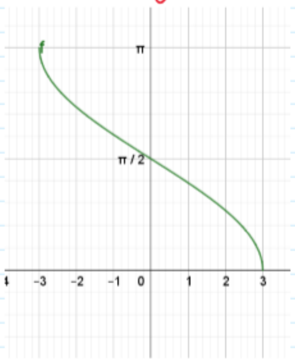
$= \frac{\pi}{12} - 0$

$= \frac{\pi}{12}$

f) (i) D: $-3 \leq x \leq 3$

R: $0 \leq y \leq \pi$

(ii)



g) $\dfrac{{}^{12}C_r}{{}^{11}C_{r-1}} = \dfrac{12!}{r!(12-r)!} \div \dfrac{11!}{(r-1)!(12-r)!}$

$= \dfrac{12!}{r!(12-r)!} \times \dfrac{(r-1)!(12-r)!}{11!}$

$= \dfrac{12}{r}$

12. a) $dy\, e^y = 2x\, dx$

$\int e^y dy = \int 2x\, dx$

$e^y = x^2 + C$

Sub in $0, \ln 4$

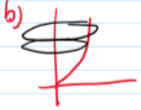
$\therefore 4 = 0 + C$

$C = 4$

$e^y = x^2 + 4$

$y = \ln|x^2 + 4|$

b)



$V = \pi \int_0^{\frac{\pi}{2}} x^2\, dy$

$y = \sin^{-1} x$

$x = \sin y$

$x^2 = \sin^2 y$

$\therefore V = \pi \int_0^{\frac{\pi}{2}} \sin^2 y\, dy$

$= \frac{\pi}{2} \int_0^{\frac{\pi}{2}} (1 - \cos 2y)\, dy$

$= \frac{\pi}{2} \left[y - \frac{\sin 2y}{2} \right]_0^{\frac{\pi}{2}}$

$= \frac{\pi}{2} \left[\left(\frac{\pi}{2} - 0 - (0 - 0) \right) \right]$

$= \frac{\pi^2}{4}\ u^2$

c) (i) $\frac{dT}{dt} = k(T - A)$

$\frac{dT}{k(T-A)} = dt$

$\int \frac{dT}{k(T-A)} = \int dt$

$\frac{1}{k} \ln|T - A| = t + C$

$\ln|T - A| = k(t + C)$

$T - A = e^{kt + kC}$

$T = A + e^{kt + kC}$

$= A + e^{kt} \times e^{kC}$

e^{kC} is a constant

$\therefore T = A + P_0 e^{kt}$

(ii) $A = 20$

$t = 0\ T = 100$

$\therefore P_0 = 80$

$t = 15\ T = 50$

$\therefore 30 = 80 e^{15k}$

$k = \frac{\ln \frac{3}{8}}{15} = -0.0654$

(iii) $T = 40 = 20 + 80 e^{kt}$

$t = 21.2$ mins

$= 21$ min

d) (i) $\underset{\sim}{v}(t) = 30\sqrt{3}\,\underset{\sim}{i} + (30 - 10t)\underset{\sim}{j}$

(ii) max height when $\dot{y} = 0$

$\therefore 30 - 10t = 0$

$t = 3$

$\therefore$ height $= 30 \times 3 - 5 \times 3^2$

$= 45$m

(iii) $t = 1\quad v = 30\sqrt{3}\,\underset{\sim}{i} + 20\,\underset{\sim}{j}$

$x = \sqrt{3100}$ m/sec

$= 10\sqrt{31}$ m/sec

13 a) $\sin x + \cos x = \sqrt{2}\sin\left(x+\frac{\pi}{4}\right)$

$$\frac{\sin x + \cos x}{\sqrt{2}} = \frac{1}{2}$$

$$\sin x + \cos x = \frac{1}{\sqrt{2}}$$

$45 \le x \le 405$

$$\therefore \sqrt{2}\sin\left(x+\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$$

$$\sin\left(x+\frac{\pi}{4}\right) = \frac{1}{2}$$

$$x+\frac{\pi}{4} = \frac{5\pi}{6}, \frac{13\pi}{6}$$

$$x = \frac{7\pi}{12}, \frac{23\pi}{12}$$

b) $\vec{AB} = \underset{\sim}{a} + 3\underset{\sim}{b} - \underset{\sim}{a}$

$= 3\underset{\sim}{b}$

$\therefore \vec{AX} = 2\underset{\sim}{b}$

$\vec{AO} = \underset{\sim}{a} = \vec{OY} = \vec{YZ}$

$\vec{XY} = -2\underset{\sim}{b} + 2\underset{\sim}{a}$

$= 2(\underset{\sim}{a} - \underset{\sim}{b})$

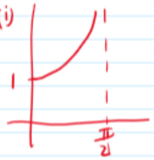
$\vec{BC} = \underset{\sim}{a} - 3\underset{\sim}{b} + 2\underset{\sim}{a}$

$= 3(\underset{\sim}{a} - \underset{\sim}{b})$

$\therefore \vec{XY}$ and $\vec{BC}$ are parallel

as they are scalar multiples of each other

c)(i)



(ii) $x = \sec y$

$$x = \frac{1}{\cos y}$$

$$\frac{1}{x} = \cos y$$

$$y = \cos^{-1}\frac{1}{x}$$

$D: x \ge 1$

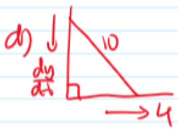
(iii) $$\frac{d}{dx}\cos^{-1}\frac{1}{x} = -\frac{-x^{-2}}{\sqrt{1-\left(\frac{1}{x}\right)^2}}$$

$$= \frac{1}{x\sqrt{x^2-1}}$$

(iv) from (iii) no turning points

y approaches $\frac{\pi}{2}$ as $x \to \infty$

d)



$$y^2 = 100 - x^2$$

$$y = (100-x^2)^{\frac{1}{2}}$$

$\frac{dx}{dt} = 4$ $\quad \frac{dy}{dx} = \frac{1}{2}(100-x^2)^{-\frac{1}{2}} \times -2x$

$$= \frac{-x}{\sqrt{100-x^2}}$$

$$\frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt}$$

$$= \frac{-x}{\sqrt{100-x^2}} \times 4$$

$$= \frac{-4x}{\sqrt{100-x^2}}$$

when $x = 8$

$$\frac{dy}{dt} = \frac{-32}{6}$$

$$= \frac{-16}{3} \text{ m/s}$$

$\therefore \frac{16}{3}$ m/s down the wall

14 a) $RHS = \underset{\sim}{u} \cdot \underset{\sim}{v}$

$\underset{\sim}{u} \cdot \underset{\sim}{v} = |u||v|\cos\theta$

$= 16\cos 60$

$= 8$ units

$LHS = 2|\underset{\sim}{u} - \underset{\sim}{v}|$

$= 2 \times 4$

$= 8$

b) Let $n=1$ $LHS = 2$

$RHS = 2$

Assume $n=k$

$2(1!) + 5(2!) + 10(3!) + \dots (k^2+1)k! = k(k+1)!$

Let $n = k+1$

RTP

$2(1!) + 5(2!) + 10(3!) + \dots (k^2+1)k! + ((k+1)^2+1)(k+1)! = (k+1)(k+2)!$

$LHS = k(k+1)! + (k^2+2k+2)(k+1)!$

$= (k+1)!(k^2+3k+2)$

$= (k+1)!(k+2)(k+1)$

$= (k+1)(k+2)! = RHS$

$\therefore$ True by PMI

c)(i) $LHS = x^n(1+x)^n\left(\frac{x+1}{x}\right)^n$

$= (1+x)^n(1+x)^n$

$= (1+x)^{2n}$

(ii) $(1+x)^n = 1 + \binom{n}{1}x + \binom{n}{2}x^2 + \dots \binom{n}{n}x^n$

$$\left(1+\frac{1}{x}\right)^n = 1 + \binom{n}{1}\frac{1}{x} + \binom{n}{2}\frac{1}{x^2} + \dots \binom{n}{n}\frac{1}{x^n}$$

$$(1+x)^{2n} = 1 + \binom{2n}{1}x + \binom{2n}{2}x^2 + \dots \binom{2n}{n}x^n + \dots \binom{2n}{2n}x^{2n}$$

Looking at coefficients of x^n

$$LHS = 1 + \binom{n}{1}^2 + \binom{n}{2}^2 + \binom{n}{3}^2 + \dots \binom{n}{n}^2$$

$$RHS = \binom{2n}{n}$$

$$\therefore 1 + \binom{n}{1}^2 + \binom{n}{2}^2 + \binom{n}{3}^2 + \dots \binom{n}{n}^2 = \binom{2n}{n}$$

Qu 14 ii) $LHS = x^n(1+x)^n(1+\frac{1}{x})^n$

$= x^n\left({}^nC_0 + {}^nC_1 x + {}^nC_2 x^2 + \dots + {}^nC_n x^n\right)\left({}^nC_0 + {}^nC_1 \frac{1}{x} + {}^nC_2 \frac{1}{x^2} + \dots {}^nC_n \frac{1}{x^n}\right)$

To get coefficients of x^n, $(1+x)^n \times (1+\frac{1}{x})^n$ must be a constant.

That happens when you multiply their 1st terms together, 2nd terms, ..., last terms.

e.g $\quad {}^nC_0 \times {}^nC_0 = \binom{n}{0}^2$

${}^nC_1 x \times {}^nC_1 \frac{1}{x} = \binom{n}{1}^2$

$\vdots$

$\binom{n}{n}x^n \times \binom{n}{n}\frac{1}{x^n} = \binom{n}{n}^2$

RHS: Coefficient of x^n in $(1+x)^{2n}$ is $\binom{2n}{n}$

$\binom{2n}{0} + \binom{2n}{1}x + \binom{2n}{2}x^2 + \dots + \binom{2n}{n}x^n + \dots + \binom{2n}{2n}x^{2n}$

Equating coefficients of x^n:

$\therefore \quad 1 + \binom{n}{1}^2 + \binom{n}{2}^2 + \binom{n}{3}^2 + \dots + \binom{n}{n}^2 = \binom{2n}{n}$

d) (i) $u = x \quad v = e^x$

$u' = 1 \quad v' = e^x$

$\therefore \frac{d}{dx} x e^x = x e^x + e^x$

$= e^x(x+1)$

(ii) $u = e^x$

$\frac{du}{dx} = e^x$

$x=1 \quad x=0$

$u=e \quad u=1$

$\int_1^e u^2 e^u \frac{du}{u}$

$= \int_1^e u e^u \, du$

now $\int u e^u \, du + \int e^u = u e^u$ from (i)

$\int_1^e u e^u \, du = \left. u e^u - e^u \right]_1^e$

$= e \times e^e - e^e - (e - e)$

$= e^{e+1} - e^e$

$= e^e(e-1)$

e)(i) $P = \frac{{}^4C_3 \times {}^4C_2}{{}^{40}C_5}$

$= \frac{1}{27417}$

(ii) $P = \frac{{}^{10}C_5 \times 4 + {}^{10}C_4 \times {}^{30}C_1 \times 4}{{}^{40}C_5}$

$= \frac{28}{703}$