
Student Number

Barker College

2024 TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Extension 1

Staff Involved:

PM WEDNESDAY 21ST AUGUST

- ARP – Mr Perkins*
- GPF – Mr Fitzmaurice
- JZT – Mrs Thomas

JAI – Miss Iles
AXD – Mrs Davis
ARM – Mr Mallam

100 copies

Topics Covered

Further Graphs	Induction	Vectors
Polynomials	Further Rates	Projectile Motion
Combinatorics	Further Trigonometry	Differential Equations
Binomial Theorem	Trigonometric Equations	

General Instructions:

- Write your Student Number at the top of this page and on all writing booklets
- Reading time - 10 minutes
- Working time – 2 hours
- Write using black pen
- Calculators approved by NESA may be used
- A separate reference booklet is provided
- Marks may not be awarded for careless or poorly arranged working
- Diagrams are not to scale
- For questions in Section II, show relevant mathematical reasoning and / or calculations

Total marks: 70

Section I – 10 marks (pages 2 - 5)
- Attempt Questions 1 – 10
- Allow about 15 minutes for this section

Section II – 60 marks (pages 6 - 13)
- Attempt Questions 11 - 15
- Allow about 1 hour and 45 minutes for this section
- Answer each question in a SEPARATE writing booklet.

Section I

10 marks
Attempt Questions 1 – 10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1 – 10.

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1 What is the domain and range of $y = 4\cos^{-1}\left(\frac{3x}{2}\right)$?

A. Domain: $-\frac{2}{3} \leq x \leq \frac{2}{3}$

Range: $0 \leq y \leq 4\pi$

B. Domain: $-\frac{3}{2} \leq x \leq \frac{3}{2}$

Range: $-4\pi \leq y \leq 4\pi$

C. Domain: $-\frac{2}{3} \leq x \leq \frac{2}{3}$

Range: $-4\pi \leq y \leq 4\pi$

D. Domain: $-\frac{2}{3} \leq x \leq \frac{2}{3}$

Range: $0 \leq y \leq 4$

2 Which expression is the result of the integral $\int \frac{3}{9+x^2}\,dx$?

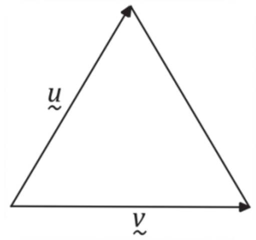
A. $\tan^{-1}\frac{x}{3} + C$

B. $\frac{1}{3}\tan^{-1}\frac{x}{3} + C$

C. $3\tan^{-1}\frac{x}{3} + C$

D. $\frac{1}{3}\tan^{-1}\frac{x}{9} + C$

3 An equilateral triangle of side 3 units is shown below. Vectors $\underset{\sim}{u}$ and $\underset{\sim}{v}$ are represented.



What is the value of $\underset{\sim}{u} \cdot \underset{\sim}{v}$?

A. 0

B. $\dfrac{9}{\sqrt{2}}$

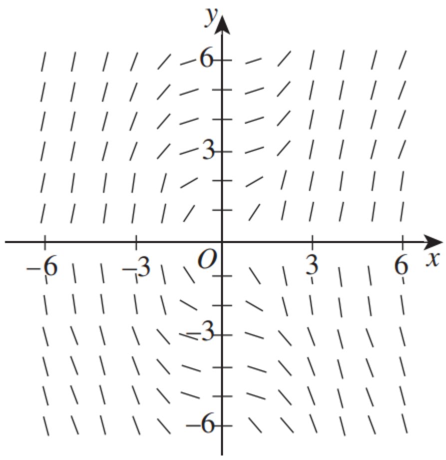
C. $\dfrac{9}{2}$

D. 9

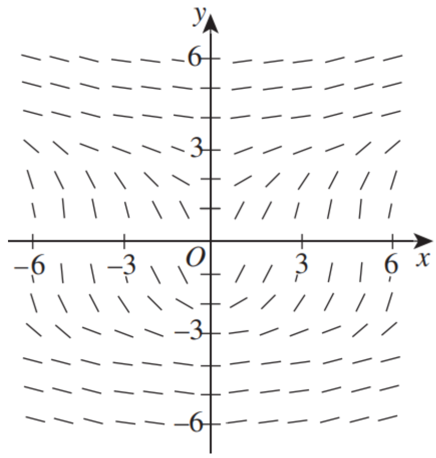
4 Which of the following direction fields best represents the differential equation

$$\frac{dy}{dx} = \frac{3x^2}{2y}?$$

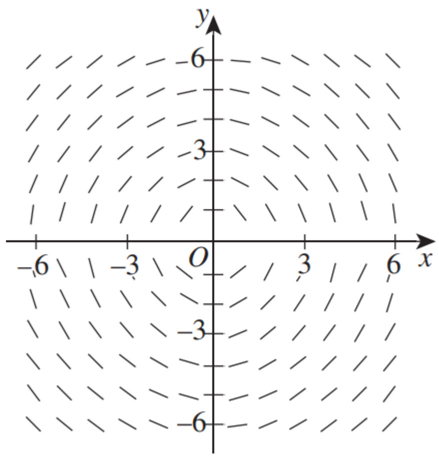
A.



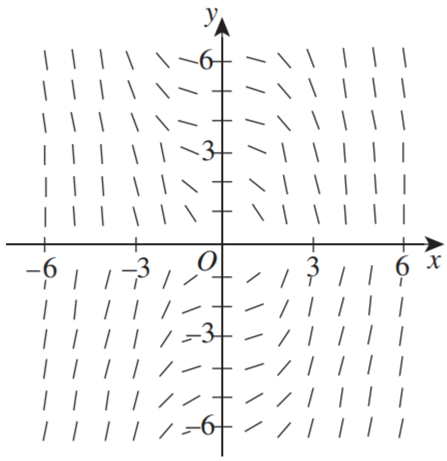
B.



C.



D.



5 The parametric equations of a circle are given below.

$$x = 2\sin t$$
$$y = 2\cos t + 1$$

What are the centre and radius of the circle?

A. The centre is $(0,1)$ and the radius is 2

B. The centre is $(0,1)$ and the radius is 4

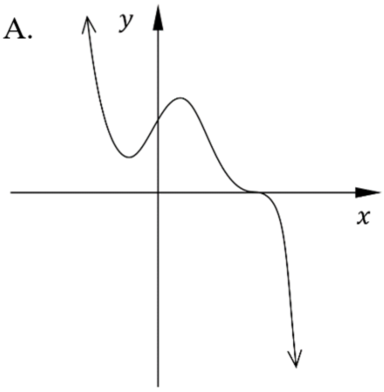
C. The centre is $(0,-1)$ and the radius is 2

D. The centre is $(0,-1)$ and the radius is 4

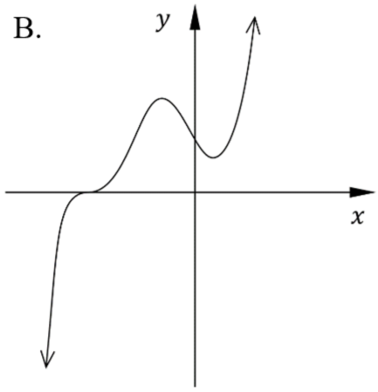
6 Consider the polynomial $P(x) = x^5 + ax^4 + bx^3 + cx^2 + dx + e$ where a, b, c, d and e are constants. $P(x)$ has one repeated root of multiplicity 3 and is divisible by $x^2 - x$.

Which of the following could be graph of $y = P(x)$?

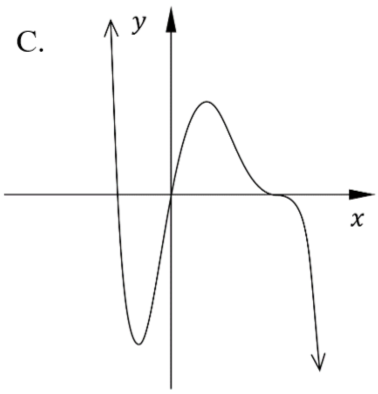
A.



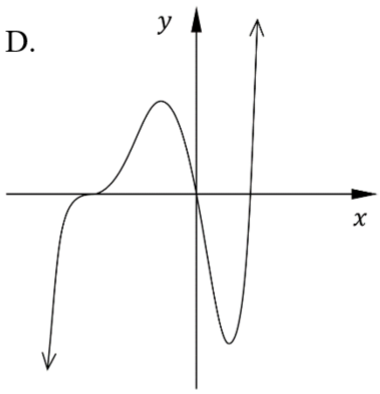
B.



C.



D.



7 Which of the following expressions is the projection of $\underset{\sim}{u} = -5\underset{\sim}{i} + 2\underset{\sim}{j}$ onto $\underset{\sim}{v} = -10\underset{\sim}{i} - 10\underset{\sim}{j}$?

A. $\frac{7}{2}\underset{\sim}{i} + \frac{7}{2}\underset{\sim}{j}$

B. $\frac{3}{20}\underset{\sim}{i} + \frac{3}{20}\underset{\sim}{j}$

C. $-\frac{3}{2}\underset{\sim}{i} - \frac{3}{2}\underset{\sim}{j}$

D. $-\frac{7}{20}\underset{\sim}{i} - \frac{7}{20}\underset{\sim}{j}$

8 A florist is selecting flowers for a bouquet. He has four yellow, three red, five violet, seven white and five blue flowers to select from.
If the florist selects the flowers randomly, what is the minimum number of flowers that he would need to select to ensure that the bouquet has four flowers of the same colour?

A. 15

B. 16

C. 19

D. 20

9 Which of the following functions is **NOT** a solution to the differential equation?
$y'' + 3y' - 4y = 0$

A. $y = e^{-4x}$

B. $y = e^{x}$

C. $y = e^{5x}$

D. $y = 0$

10 Let n be a positive integer for $n \geq 4$. What does the coefficient of x^3y^5 in the expansion of $(1 + xy + y^2)^n$ equal?

A. n

B. 2^n

C. $\binom{n}{3}\binom{n}{2}$

D. $4\binom{n}{4}$

End of Section I

Section II

60 marks
Attempt Questions 11 - 15
Allow about 1 hour and 45 minutes for this section

Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.

In Questions 11 - 15, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (12 marks) **[START A NEW BOOKLET]**

(a) Given that $\underset{\sim}{a} = \begin{pmatrix} -2 \\ 1 \end{pmatrix}$ and $\underset{\sim}{b} = \begin{pmatrix} 1 \\ 5 \end{pmatrix}$ find $\underset{\sim}{b} - \underset{\sim}{a}$. **1**

(b) The roots of $2x^3 - 4x^2 + x - 2$ are α, β and γ. **2**

What is the value of $\dfrac{1}{\alpha\beta} + \dfrac{1}{\beta\gamma} + \dfrac{1}{\gamma\alpha}$?

(c) Solve $\cos 2\theta + \sin\theta = 0$ for $[0, 2\pi]$. **2**

(d) A team of nine players is to be selected, at random, from a group of seven girls and five boys. There can be no more than four boys on the team.

(i) In how many ways can a team be formed? **2**

(ii) If three of the girls are sisters, determine the probability that all three sisters are selected for the team. **2**

(e) Use mathematical induction to prove that $4^{2n+1} + 3^{2n+1}$ is divisible by 7 for all integers $n \geq 1$. **3**

End of Question 11

Question 12 (11 marks) **[START A NEW BOOKLET]**

(a) (i) Write down the expansion of $(1-x)^5$. **1**

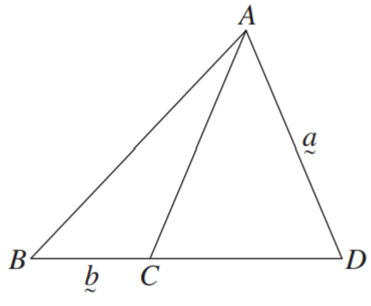
(ii) Hence, or otherwise find the term of x^2 in $(2x-3)^2\,(1-x)^5$. **2**

(b) **Answer Question 12 (b) and on the separate answer page supplied. Hand it in with your booklet for Question 12.** **2**

(c) Sand is dropping on a horizontal floor at a constant rate of 5 cm^3 per second. **3**
It forms a pile whose volume, $V\text{ cm}^3$, and height, $h\text{ cm}$, are connected by the formula
$V=-2+\sqrt{2h^3+3h+4}$.

Find the rate at which the height of the pile is increasing, when the height of the pile has reached 11 cm. Give your answer correct to 3 decimal places.

(d) The diagram shows a triangle, where $\overrightarrow{AD}=\underset{\sim}{a}$ and $\overrightarrow{BC}=\underset{\sim}{b}$. **3**



Given that $5\overrightarrow{BD}=6\overrightarrow{CD}$, prove that $\overrightarrow{AB}=\underset{\sim}{a}-6\underset{\sim}{b}$.

End of Question 12

Question 13 (13 marks) **[START A NEW BOOKLET]**

(a) Consider the function $y = 2\arcsin 2x$. **2**

Graph the function on at least $\frac{1}{3}$ of a page clearly stating domain and range.

(b) A frozen dinner has a temperature (T) of 0°C and is placed in an oven with an ambient temperature of 180°C. The rate at which the dinner warms per minute is given by this equation $\frac{dT}{dt} = k(T-180)$ where k is a constant and t is time in minutes.

(i) Using separation of variables, solve the given differential equation to show $T = 180 + Ae^{kt}$ where A is a constant. **2**

(ii) If the dinner's temperature is 25°C after 15 minutes, find the rate of change (in $^\circ\text{C}/\min$) of the dinner's temperature when it is 100°C. **2**
Give your answer to 3 d.p.

(c) (i) Express $3\sin x + 4\cos x$ in the form $A\sin(x+\alpha)$ where $0 \le \alpha \le \frac{\pi}{2}$. **2**

(ii) Hence, or otherwise, solve $3\sin x + 4\cos x = 5$ for $0 \le x \le 2\pi$. Give your answer, or answers, correct to two decimal places. **2**

(d) Use the substitution $u = x - 8$ to find $\int_{8}^{8.5} \frac{dx}{\sqrt{(7-x)(x-9)}}$. **3**

End of Question 13

Question 14 (12 marks) **[START A NEW BOOKLET]**

(a) Consider the function $f(x)=\cos^{-1}(x)+\cos^{-1}(-x)$.

(i) Show that $f(x)$ is constant by finding $f'(x)$. **1**

(ii) Find the value of the constant. **1**

(b) Consider the points $A(2,-2)$ and $B(2,6)$. Using vector methods or otherwise find the angle $\angle AOB$ to the nearest **degree** where O is the origin. **2**

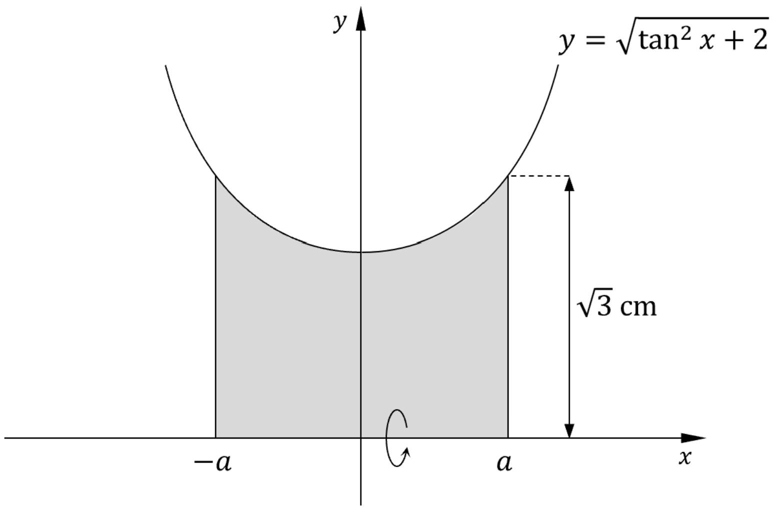
(c) (i) Solve $\dfrac{x+5}{x-1}\geq\dfrac{9}{x}$ for $x\neq 1, x\neq 0$. **3**

(ii) Hence find the set of values of x which satisfies $\dfrac{e^x+5}{e^x-1}\geq\dfrac{9}{e^x}$. **1**

(d) The shaded region below is bounded by the $x-$axis, the graph of $y=\sqrt{\tan^2 x+2}$ and the lines $x=\pm a$. **4**

Find the volume of the solid of revolution formed when the curve is rotated around the $x-$axis.

The value of a is chosen so that the outer radius is $\sqrt{3}$ centimetres as shown.



NOT TO SCALE

End of Question 14

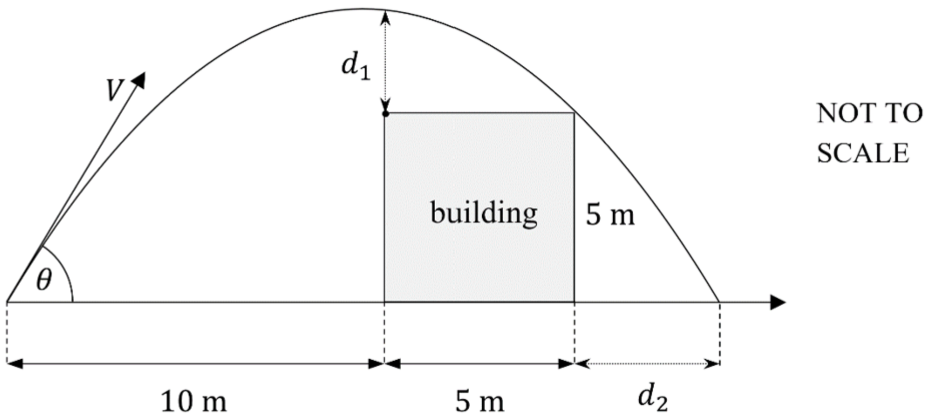
Question 15 (12 marks) **[START A NEW BOOKLET]**

(a) A student kicks a football from ground level. The ball is kicked at an angle of projection of θ, at a speed of V metres per second. **5**

The displacement vector of the ball from the student's position, t seconds after it is kicked, is given by

$\underset{\sim}{r}(t) = (Vt\cos\theta)\underset{\sim}{i} + (Vt\sin\theta - 5t^2)\underset{\sim}{j}.$ (Do NOT prove this.)

Two seconds after being kicked, the ball just clears the far side of a 5 metre high building. The building is 5 metres wide, and the closest wall of the building is 10 metres from the student.



Let d_1 be the distance by which it clears the closest wall of the building, and d_2 be the distance from the far wall of the building to the point of impact, as shown.

By firstly finding the values of d_1 and d_2, show that d_2 is within 1 metre of d_1.

Question 15 continues on the following page

Question 15 (continued)

(b) (i) Given that $t = \tan\left(\frac{\theta}{2}\right)$, show that $\cos\left(\frac{\theta}{2}\right) = \frac{1}{\sqrt{1+t^2}}$. **1**

(ii) Hence, or otherwise, solve $\sin\theta - \cos\left(\frac{\theta}{2}\right) = 0$ for $0 \le \theta \le 2\pi$. **3**

(c) α, β and γ are the roots of the equation $2x^3 + 5x^2 - 4x + 8 = 0$. Given that $\alpha^2 + \beta^2 + \gamma^2 = \frac{41}{4}$ and $\alpha^3 + \beta^3 + \gamma^3 = \frac{-341}{8}$ find the value of $\alpha^4 + \beta^4 + \gamma^4$. **3**

End of Paper

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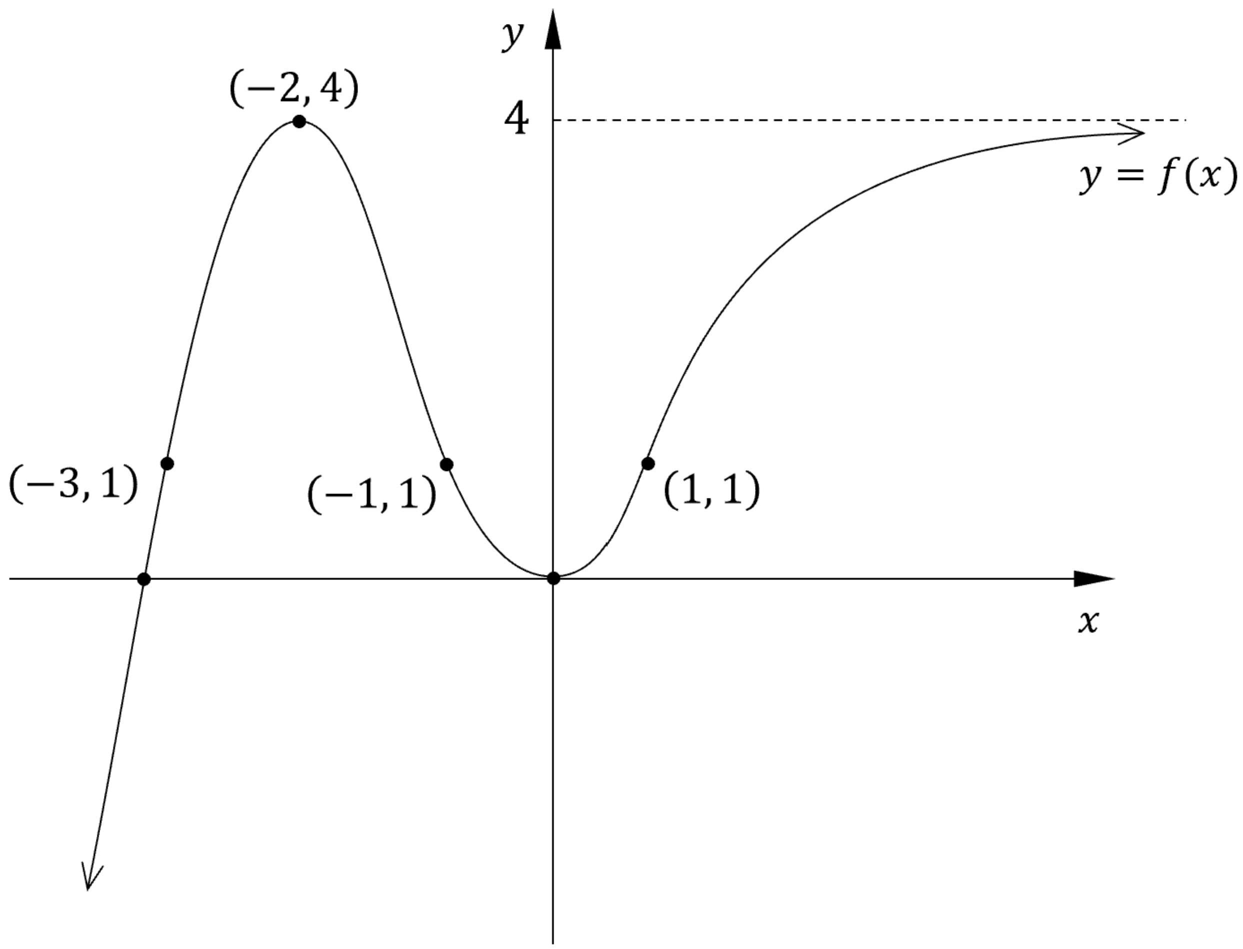
Student Number

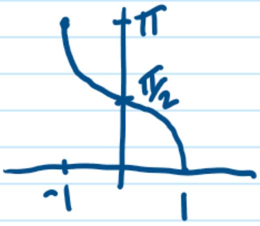
Additional Answer page for Question 12 (b)
Please place inside your Question 12 booklet.

The diagram of $y = f(x)$ is shown below. On the diagram sketch the graph of $y = \sqrt{f(x)}$ **2**
clearly showing all turning points, intercepts and asymptotes



Y12 Ext 1 Trial - student Solutions

1. $y = \cos^{-1} x$



$y \to \frac{y}{4} \quad x \to \frac{x}{2/3}$

$0 \le y \le 4\pi \quad -\frac{2}{3} \le x \le \frac{2}{3}$ (A)

2. $\int \frac{3}{9+x^2}dx = 3\int \frac{1}{3^2+x^2}dx$

$= 3 \times \frac{1}{3}\tan^{-1}\frac{x}{3} + C$

$= \tan^{-1}\frac{x}{3} + C$ (A)

3. $\underset{\sim}{u}\cdot\underset{\sim}{v} = |\underset{\sim}{u}|\cdot|\underset{\sim}{v}|\cos\theta$

$= 3 \times 3 \times \cos 60$

$= 9/2$ (C)

4. (A)

5. $x = 2\sin t \to x^2 = 4\sin^2 t$

$y - 1 = 2\cos t \to (y-1)^2 = 4\cos^2 t$

$\therefore x^2 + (y-1)^2 = 4(\sin^2 t + \cos^2 t)$

$x^2 + (y-1)^2 = 4$ (A)

6. divisible by $x^2 - x$ ie $x(x-1)$

$\therefore$ roots @ $x = 0$ & $x = 1$

choosing out of C or D

monic deg 5 $\therefore$ finishes positive $\to$ (D)

7. $\frac{\underset{\sim}{u}\cdot\underset{\sim}{v}}{\underset{\sim}{v}\cdot\underset{\sim}{v}}\underset{\sim}{v} = \frac{-5\times-10 + 2\times-10}{-10\times-10 + -10\times-10}(-10\underset{\sim}{i} - 10\underset{\sim}{j})$

$= \frac{3}{20}(-10\underset{\sim}{i} - 10\underset{\sim}{j})$

$\Rightarrow$ (C)

8. pigeonhole principle $\to$ worst case

3 yel + 3 red + 3 vio + 3 whi + 3 blu + 1

= 16 (B)

9. A: $y = e^{-4x}$
$y' = -4e^{-4x}$
$y'' = 16e^{-4x}$

B: $y = e^{x}$
$y' = e^{x}$
$y'' = e^{x}$

C: $y = e^{5x}$
$y' = 5e^{5x}$
$y'' = 25e^{5x}$

D: $y = 0$
$y' = 0$
$y'' = 0$

plug each into DE and answer $\to$ (C)

10. $(a+b)^n$ Gen term: ${}^nC_k a^{n-k}\cdot b^k$

view $(1+xy+y^2)^n$ as $(1+(xy+y^2))^n$

$\therefore$ Gen term $= {}^nC_k 1^{n-k}\cdot(xy+y^2)^k$

$= {}^nC_k \cdot y^k \cdot \underbrace{(x+y)^k}$

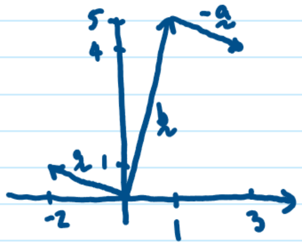
$= {}^nC_k \cdot y^k \cdot {}^kC_r x^{k-r} y^r$

$= {}^nC_k \cdot {}^kC_r \cdot x^{k-r} y^{k+r}$ $\to$ need $k+r=5$, $k-r=3$, $k=4$ $r=1$

$= {}^nC_4 \cdot {}^4C_1 \cdot x^3 y^5$

$= {}^nC_4 \times 4$ (D)

11. a)



$\underset{\sim}{b}-\underset{\sim}{a} = \begin{pmatrix}3\\4\end{pmatrix}$

b) $\frac{1}{\alpha\beta} + \frac{1}{\beta\gamma} + \frac{1}{\gamma\alpha} = \frac{\gamma+\alpha+\beta}{\alpha\beta\gamma}$

$= \frac{2}{1} = 2$

$\alpha+\beta+\gamma = -\frac{b}{a} = 2$

$\alpha\beta\gamma = -\frac{d}{a} = \frac{--2}{2} = 1$

c) $\cos 2\theta + \sin\theta = 0 \quad [0, 2\pi]$

$1 - 2\sin^2\theta + \sin\theta = 0$

$2\sin^2\theta - \sin\theta - 1 = 0$

$(2\sin\theta + 1)(\sin\theta - 1) = 0$

$\therefore \sin\theta = -\frac{1}{2}$ + $\sin\theta = 1$

$\theta = \frac{7\pi}{6}, \frac{11\pi}{6}$ + $\theta = \frac{\pi}{2}$

d) i) 4 boys: ${}^5C_4 \times {}^7C_5 = 105$

3 boys: ${}^5C_3 \times {}^7C_6 = 70$

2 boys: ${}^5C_2 \times {}^7C_7 = 10$

185

OR ${}^{12}C_9 - {}^5C_5 \times {}^7C_4 = 185$

ii) 4 boys: ${}^5C_4 \times {}^3C_3 \times {}^4C_2 = 30$

3 boys: ${}^5C_3 \times {}^3C_3 \times {}^4C_3 = 40$

2 boys: ${}^5C_2 \times {}^3C_3 \times {}^4C_4 = 10$

80

$\therefore 80/185$

OR ${}^3C_3 \times {}^9C_6 - {}^5C_5 \times {}^4C_1 = 80$

$\therefore 80/185$

e) Step 1:

Prove true for $n=1$

$4^{2\times1+1} + 3^{2\times1+1} = 64+27$

$= 91$

$= 7(13)$ $\therefore$ divisible by 7

Step 2:

Let true $n=k$

$4^{2k+1} + 3^{2k+1} = 7M$ where M is integer

$4^{2k+1} = 7M - 3^{2k+1}$

Step 3:

Prove true $n=k+1$

RTP $4^{2(k+1)+1} + 3^{2(k+1)+1} = 7L$

where L is integer

LHS $= 4^{2k+3} + 3^{2k+3}$

$= 16 \times 4^{2k+1} + 9 \times 3^{2k+1}$

$= 16(7M - 3^{2k+1}) + 9 \times 3^{2k+1}$ by step 2

$= 112M - 16\times3^{2k+1} + 9\times3^{2k+1}$

$= 112M - 7\times3^{2k+1}$

$= 7(16M - 3^{2k+1})$

$= 7L$ as M + k are integers

$=$ RHS

$\therefore$ Since true $n=1$

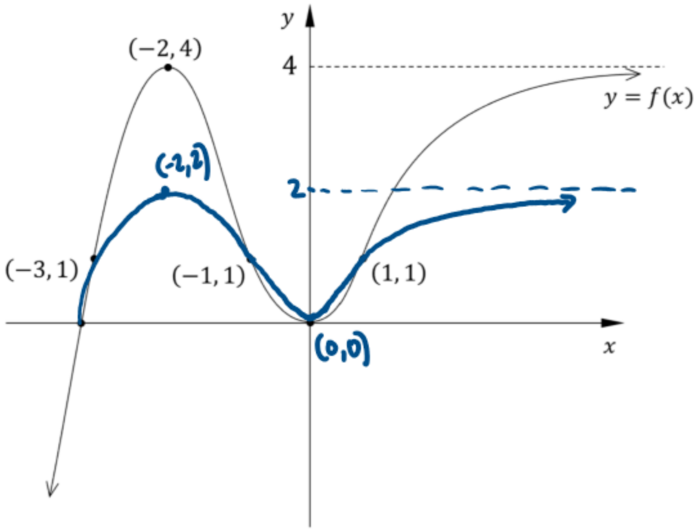
true for all by PMI

12. a) i) $(1-x)^5 = 1 - 5x + 10x^2 - 10x^3 + 5x^4 - x^5$

ii) $(2x-3)^2(1-x^5) = (4x^2 - 12x + 9)(1 - 5x + 10x^2 - 10x^3 + 5x^4 - x^5)$

x^2 terms: $4x^2 + 60x^2 + 90x^2 = 154x^2$

b) The diagram of $y = f(x)$ is shown below. On the diagram sketch the graph of $y = \sqrt{f(x)}$ clearly showing all turning points, intercepts and asymptotes **2**



c) $\frac{dV}{dt} = 5$ $\quad V = -2 + (2h^3 + 3h + 4)^{\frac{1}{2}}$

$\frac{dV}{dh} = \frac{1}{2}(2h^3 + 3h + 4)^{-\frac{1}{2}} \times (6h^2 + 3)$

$= \frac{6h^2+3}{2\sqrt{2h^3+3h+4}}$

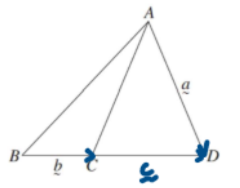
$\frac{dh}{dt} = \frac{dV}{dt} \times \frac{dh}{dV}$

$= 5 \times \frac{2\sqrt{2h^3+3h+4}}{6h^2+3}$

when $h = 11$ $\quad \frac{dh}{dt} = 0.71265$ cm/s

$= 0.713$ cm/s

d)



Given that $5\overline{BD} = 6\overline{CD}$, prove that $\overrightarrow{AB} = \underset{\sim}{a} - 6\underset{\sim}{b}$.

Let $\overrightarrow{CD} = \underset{\sim}{c}$

Given $5\overrightarrow{BD} = 6\overrightarrow{CD}$

$5(\underset{\sim}{b} + \underset{\sim}{c}) = 6\underset{\sim}{c}$

$5\underset{\sim}{b} + 5\underset{\sim}{c} = 6\underset{\sim}{c}$

$5\underset{\sim}{b} = \underset{\sim}{c}$

$\overrightarrow{AB} = \overrightarrow{AD} + \overrightarrow{DC} + \overrightarrow{CB}$

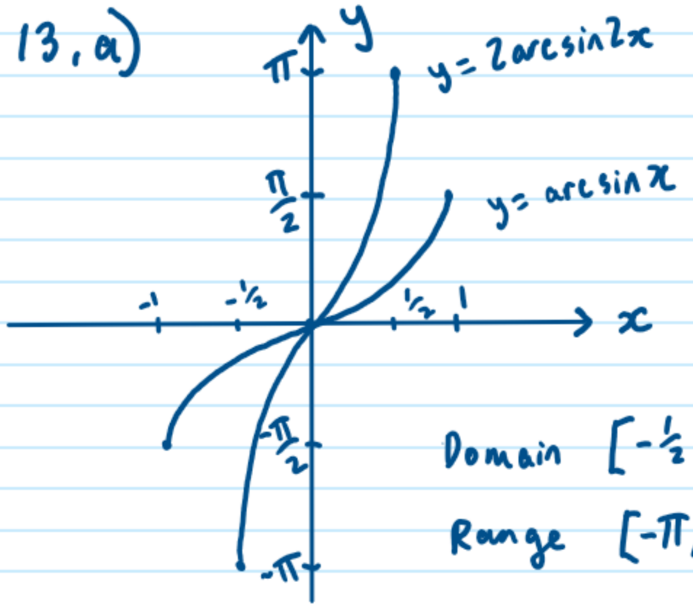
$= \underset{\sim}{a} - \underset{\sim}{c} - \underset{\sim}{b}$

$= \underset{\sim}{a} - 5\underset{\sim}{b} - \underset{\sim}{b}$

$= \underset{\sim}{a} - 6\underset{\sim}{b}$

QED

13, a)



Domain $[-\frac{1}{2}, \frac{1}{2}]$

Range $[-\pi, \pi]$

b) i) Method 1

$$\int \frac{dT}{T-180} = \int k\,dt$$

$$\ln|T-180| = kt + C$$

$$|T-180| = e^{kt+c}$$

$$T-180 = \pm e^{c} \cdot e^{kt}$$

$$T = 180 \pm e^{c} \cdot e^{kt}$$

$$T = 180 \pm A e^{kt}$$

use initial conditions to evaluate A: when $t=0$ $T=0$ $\therefore A = -180$

ie $T = 180 - 180e^{kt}$

Method 2

$$\frac{dT}{dt} = k(T-180)$$

$$\int_0^T \frac{dT}{T-180} = \int_0^t k \cdot dt$$

$$\ln|T-180| - \ln|180| = kt$$

$$\ln\frac{|T-180|}{180} = kt$$

$$|T-180| = 180e^{kt}$$

$$180 - T = 180e^{kt}$$

$$T = 180 - 180e^{kt}$$

ii) at $t=15$ $T=25$

$$\therefore 25 = 180 - 180e^{15k}$$

$$e^{15k} = {}^{31}\!/_{36}$$

$$k = \frac{\ln\left({}^{31}\!/_{36}\right)}{15}$$

at $T = 100°C$

$$\frac{dT}{dt} = \frac{\ln\left({}^{31}\!/_{36}\right)}{15}(100-180)$$

$$= 0.7975°C/min$$

$$\approx 0.798°C/min$$

$3\sin x \quad + \quad 4\cos x$

c) i) $A\sin(x+\alpha) = A\sin x\cdot\cos\alpha + A\sin\alpha\cdot\cos x$

$\therefore \begin{rcases} A\cos\alpha = 3 \\ A\sin\alpha = 4 \end{rcases} \longrightarrow A^2(\sin^2\alpha + \cos^2\alpha) = 3^2+4^2$

$A^2 = 25$

$A = 5$

$\longrightarrow \tan\alpha = 4/3$

$\alpha = 0.927$ or can be $\tan^{-1} 4/3$

$3\sin x + 4\cos x \equiv 5\sin(x+0.927)$

ii) $5\sin(x+0.927) = 5$

$\sin(x+0.927) = 1$

$x + 0.927 = \pi/2$

$x = 0.6435$

$\simeq 0.64$

$0 \leq x \leq 2\pi$

$0.927 \leq x + 0.927 \leq 2\pi + 0.927$

d) $u = x-8$ ie $x = u+8$

$du = dx$

when $x = 8.5$ $u = 0.5$

$x = 8$ $u = 0$

$$= \int_0^{0.5} \frac{du}{\sqrt{(7-(u+8))(u-1)}}$$

$$= \int_0^{0.5} \frac{du}{\sqrt{(-u-1)(u-1)}}$$

$$= \int_0^{0.5} \frac{du}{\sqrt{1-u^2}}$$

$$= \left[\sin^{-1} u\right]_0^{0.5}$$

$$= \sin^{-1}(0.5) - \sin^{-1}(0)$$

$$= \pi/6$$

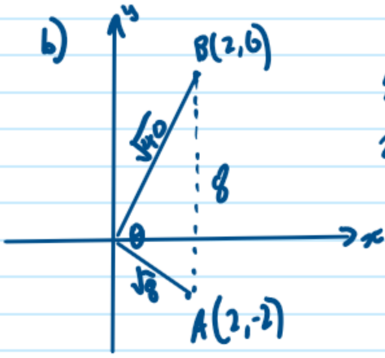
14. a) i) $f(x) = \cos^{-1}(x) + \cos^{-1}(-x)$

$f'(x) = \frac{-1}{\sqrt{1-x^2}} + \frac{1}{\sqrt{1-x^2}}$

$= 0 \quad \therefore f(x)$ is constant

ii) $\frac{\pi}{2} + \frac{\pi}{2} = \pi$

b)



Cosine rule

$8^2 = \sqrt{40}^2 + \sqrt{8}^2 - 2\sqrt{40}\sqrt{8}\cos\theta$

$2\sqrt{40}\sqrt{8}\cos\theta = 48 - 64$

$\cos\theta = \frac{-16}{2\sqrt{40}\sqrt{8}}$

$\theta = 116.565^\circ$

$= 117^\circ$

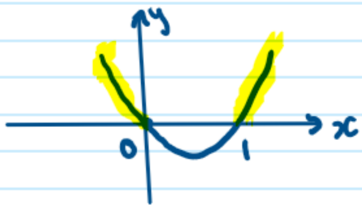
c) i) Method 1: Multiply by square of denoms.

$x^2(x+5)(x-1) \geq 9x(x-1)^2$

$x^2(x+5)(x-1) - 9x(x-1)^2 \geq 0$

$x(x-1)[x(x+5) - 9(x-1)] \geq 0$

$x(x-1)\underbrace{(x^2-4x+9)}_{\text{no roots}} \geq 0$



$x \leq 0$ & $x \geq 1$

but $x \neq 0, 1$

$\therefore x < 0$ & $x > 1$

Method 2: Critical points

$x \neq 0, 1$

$x(x+5) = 9(x-1)$

$x^2 + 5x - 9x + 9 = 0$

$x^2 - 4x + 9 = 0$

$x = \frac{4 \pm \sqrt{16 - 4\times 9}}{2}$ no solutions

$\xrightarrow[\quad 0 \qquad 1 \quad]{A \qquad B \qquad C} x$

test A

$\frac{-1+5}{-1-1} \geq \frac{9}{-1}$ ✓ $\quad \frac{0.5+5}{0.5-1} \geq \frac{9}{0.5}$ ✗ $\quad \frac{2+5}{2-1} \geq \frac{9}{2}$ ✓

$\therefore x < 0$ & $x > 1$

ii) $e^x < 0$ & $e^x > 1$

no solution $\quad x > 0$

d) when $\tan^2 x = 1$

$\tan x = \pm 1$

$x = \pm \pi/4$

set $a = \pi/4$

$\sqrt{1+2} = \sqrt{3}$

$V = 2\pi \int_0^{\pi/4} y^2\, dx$

$= 2\pi \int_0^{\pi/4} \tan^2 x + 2\, dx$

$= 2\pi \int_0^{\pi/4} \sec^2 x + 1\, dx$

$= 2\pi \left[\tan x + x\right]_0^{\pi/4}$

$= 2\pi\left(\left(1 + \frac{\pi}{4}\right) - (0+0)\right)$

$= 2\pi + \pi^2/2 \quad u^3$

15. a) $\underset{\sim}{r}(t) = Vt \cdot \cos\theta \, \underset{\sim}{i} + (Vt \cdot \sin\theta - 5t^2) \underset{\sim}{j}$

$\underset{\sim}{x}(t) = Vt \cdot \cos\theta$

$\underset{\sim}{y}(t) = Vt \cdot \sin\theta - 5t^2$

when $t=2$ $x = 15$: $15 = 2V\cos\theta$

$\frac{7.5}{\cos\theta} = V$

when $t=2$ $y=5$: $5 = 2V\sin\theta - 20$

$25 = 2V\sin\theta$

$25 = 2 \times \frac{7.5}{\cos\theta} \sin\theta$

$\frac{5}{3} = \tan\theta$

also set $y=0$: $0 = Vt\sin\theta - 5t^2$

$0 = t(V\sin\theta - 5t)$

$\therefore t=0$ or $t = \frac{V\sin\theta}{5}$

$= \frac{7.5\sin\theta}{5\cos\theta}$

$= \frac{7.5}{5} \times \frac{5}{3}$

$= 2.5$

finding d_2 set $t=2.5$: $x = \frac{7.5}{\cos\theta} \times 2.5 \times \cos\theta$

$= 18.75$

$\therefore d_2 = 3.75$

finding d_1 set $x=10$: $10 = \frac{7.5}{\cos\theta} t \times \cos\theta$

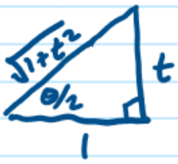
$\frac{4}{3} = t$

$y = \frac{7.5}{\cos\theta} \times \frac{4}{3} \times \sin\theta - 5 \times \left(\frac{4}{3}\right)^2$

$= 7\frac{7}{9}$

$\therefore d_1 = 2\frac{7}{9}$ or 2.77 $\longrightarrow$ $3.75 < 2.77 + 1$ QED

15.b) i)



$\therefore \cos\left(\frac{\theta}{2}\right) = \frac{1}{\sqrt{1+t^2}}$ also $\sin\theta = \frac{2t}{1+t^2}$

ii) Method 1

$0 \le \theta \le 2\pi$

$0 \le \frac{\theta}{2} \le \pi$

$\sin(2 \times \theta/2) - \cos\theta/2 = 0$

$2\sin\theta/2 \cos\theta/2 - \cos\theta/2 = 0$

$\cos\theta/2\,(2\sin\theta/2 - 1) = 0$

$\cos\theta/2 = 0$ or $\sin\theta/2 = 1/2$

$\frac{\theta}{2} = \frac{\pi}{2}$ $\qquad \theta/2 = \pi/6, \frac{5\pi}{6}$

$\theta = \pi$ $\qquad \theta = \pi/3, \frac{5\pi}{3}$

Method 2

$\sin\theta - \cos\left(\frac{\theta}{2}\right) = 0$

$\frac{2t}{1+t^2} - \frac{1}{\sqrt{1+t^2}} = 0$

$\frac{2t - \sqrt{1+t^2}}{1+t^2} = 0$

$2t = \sqrt{1+t^2}$

$4t^2 = 1+t^2$

$3t^2 = 1$

$t^2 = 1/3$

$t = \pm\frac{1}{\sqrt{3}}$

$\tan\theta/2 = \pm 1/\sqrt{3}$

$\tan\frac{\theta}{2} = \pi/6, \frac{5\pi}{6}$

$\theta = \pi/3, 5\pi/3$

BUT t-results must check if π is a solution, it is

$\therefore \theta = \frac{\pi}{3}, \pi, 5\pi/3$

Method 2

Considering $(\alpha+\beta+\gamma)^4$ = - - - - - - -

= - - - - - -

$= \alpha^4+\beta^4+\gamma^4 + 4(\alpha^3\beta + \alpha^3\gamma + \beta^3\alpha + \beta^3\gamma + \gamma^3\alpha + \gamma^3\beta)$
$+ 6(\alpha^2\beta^2 + \alpha^2\gamma^2 + \beta^2\gamma^2)$
$+ 12(\alpha^2\beta\gamma + \beta^2\alpha\gamma + \gamma^2\alpha\beta)$ ①

Also $(\alpha\beta+\alpha\gamma+\gamma\alpha)^2 = \left(\frac{-4}{2}\right)^2 = \alpha^2\beta^2 + \alpha^2\gamma^2 + \beta^2\gamma^2 + 2(\alpha^2\beta\gamma + \beta^2\alpha\gamma + \gamma^2\alpha\beta)$ ②

Also $(\alpha^3+\beta^3+\gamma^3)(\alpha+\beta+\gamma) = -\frac{341}{8} \times \frac{-5}{2} = \alpha^4+\beta^4+\gamma^4 + \alpha^3\beta + \alpha^3\gamma + \beta^3\alpha + \beta^3\gamma + \gamma^3\alpha + \gamma^3\beta$ ③

$\therefore$ ① $- 6 \times$ ② $- 4 \times$ ④ $= -3\alpha^4 - 3\beta^4 - 3\gamma^4$

$\left(-\frac{5}{2}\right)^4 - 6 \times \left(\frac{-4}{2}\right)^2 - 4 \times \left(\frac{-341}{8} \times \frac{-5}{2}\right) = -3(\alpha^4+\beta^4+\gamma^4)$

$-\frac{6579}{16} = -3(\alpha^4+\beta^4+\gamma^4)$

$\frac{2193}{16} = \alpha^4+\beta^4+\gamma^4$