
Student Number

Barker College

2025 TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Extension 1

Staff Involved:

PM TUESDAY 5TH AUGUST

Mr Fitzmaurice* Mr Chua Mr Williams
Ms Young Mr Carruthers Mr Perkins
Mr Hanlon

110 copies

Topics Covered

Further Graphs	Induction	Vectors
Polynomials	Further Rates	Projectile Motion
Combinatorics	Further Trigonometry	Differential Equations
Binomial Theorem	Trigonometric Equations	Further Calculus

General Instructions:

- Write your Student Number at the top of this page and on all writing booklets
- Reading time - 10 minutes
- Working time - 2 hours
- Write using black pen
- Calculators approved by NESA may be used
- A separate reference booklet is provided
- Marks may not be awarded for careless or poorly arranged working
- Diagrams are not to scale
- For questions in Section II, show relevant mathematical reasoning and / or calculations

Total marks: 70

Section I – 10 marks (pages 2 - 5)
- Attempt Questions 1 – 10
- Allow about 20 minutes for this section

Section II – 60 marks (pages 6 - 11)
- Attempt Questions 11 - 14
- Allow about 1 hours and 40 minutes for this section
- Answer each question in a SEPARATE writing booklet.

Section I

10 marks
Attempt Questions 1 – 10
Allow about 20 minutes for this section

Use the multiple-choice answer sheet for Questions 1 – 10.

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1 What are the domain and range of the function $f(x) = \sin^{-1}(\frac{x}{4})$?

A. Domain: $-1 \leq x \leq 1$ and range $-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$

B. Domain: $-4 \leq x \leq 4$ and range $-2\pi \leq y \leq 2\pi$

C. Domain: $-4 \leq x \leq 4$ and range $-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$

D. Domain: $-1 \leq x \leq 1$ and range $-2\pi \leq y \leq 2\pi$

2 Fifteen cards are numbered 1 to 15 and placed on a table so that their numbers are hidden.
The cards are flipped over one at a time.
What is the minimum number of cards that must be flipped to obtain at least one pair with a difference of 7?

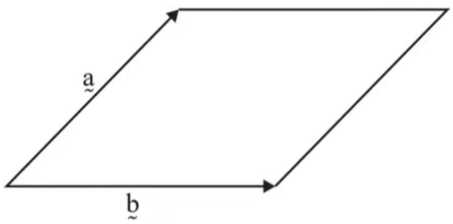
A. 7

B. 8

C. 9

D. 10

3 The diagram below shows a rhombus, spanned by the two vectors $\underset{\sim}{a}$ and $\underset{\sim}{b}$.



Which of the following statements is always true?

A. $\left(\underset{\sim}{a} + \underset{\sim}{b}\right) . \left(\underset{\sim}{a} - \underset{\sim}{b}\right) = 0$

B. $\underset{\sim}{a} = \underset{\sim}{b}$

C. $\underset{\sim}{a} . \underset{\sim}{b} = 0$

D. $\left|\underset{\sim}{a} + \underset{\sim}{b}\right| = \left|\underset{\sim}{a} - \underset{\sim}{b}\right|$

4 Which function, $f(x)$, and its inverse function, $f^{-1}(x)$, are equivalent?

A. $f(x) = \sin|x|$ for $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

B. $f(x) = x^2$ for $[0, \infty)$

C. $f(x) = |e^x|$ for $(-\infty, \infty)$

D. $f(x) = \frac{1}{x}$ for $(-\infty, 0) \cup (0, \infty)$

5 Let $P(x)$ be a polynomial of degree 6. When $P(x)$ is divided by the polynomial $Q(x)$, the remainder is $x^2 + 2$.

Which statement is true about the about the degree of $Q(x)$?

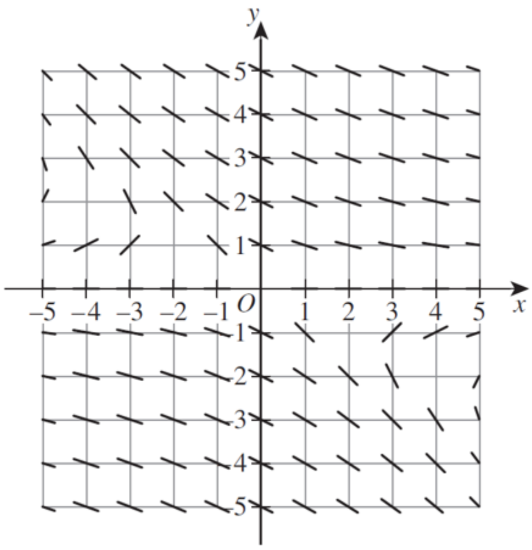
A. The degree of $Q(x)$ must be at least 2

B. The degree of $Q(x)$ must be 2

C. The degree of $Q(x)$ must be at least 3

D. The degree of $Q(x)$ must be 3

6 Consider the direction field.



Which of the following differential equations does the direction field best represent?

A. $\frac{dy}{dx} = \frac{y}{2y+x}$

B. $\frac{dy}{dx} = \frac{-y}{2y+x}$

C. $\frac{dy}{dx} = \frac{y}{-2y+x}$

D. $\frac{dy}{dx} = \frac{y}{2y-x}$

7 What is the antiderivative of $\frac{3x^2+6x^5}{4+x^6}$?

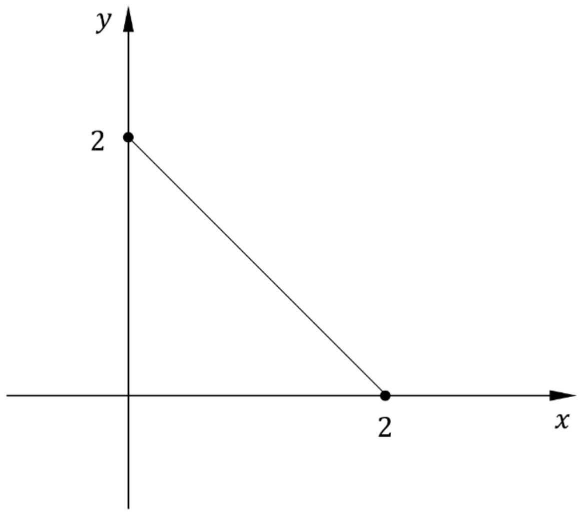
A. $\frac{1}{4}\sin^{-1}\left(\frac{x^3}{2}\right) + \ln|6x^5| + c$

B. $\frac{1}{4}\tan^{-1}\left(\frac{x^3}{4}\right) + \ln|x^6| + c$

C. $\frac{1}{2}\sin^{-1}\left(\frac{x^3}{2}\right) + \ln|4 + x^6| + c$

D. $\frac{1}{2}\tan^{-1}\left(\frac{x^3}{2}\right) + \ln|4 + x^6| + c$

8 The diagram shows the interval from (0,2) to (2,0).



Which pair of parametric equations does NOT represent the interval shown?

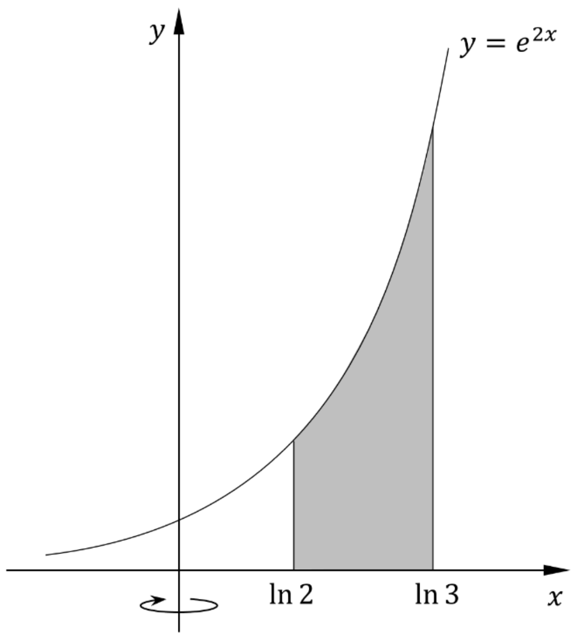
A. $\begin{cases} x = 2(1-t^2) \\ y = 2t^2 \end{cases}$ for $0 \le t \le 1$

B. $\begin{cases} x = e^t - 1 \\ y = 3 - e^t \end{cases}$ for $0 \le t \le \ln 2$

C. $\begin{cases} x = 2\sin t \\ y = 2 - 2\sqrt{1-\cos^2 t} \end{cases}$ for $0 \le t \le \frac{\pi}{2}$

D. $\begin{cases} x = 2 - \frac{2}{\pi}\cos^{-1} t \\ y = \frac{2}{\pi}\cos^{-1} t \end{cases}$ for $-1 \le t \le 1$

9 A solid of revolution is found by rotating the region bounded by the curve $y = e^{2x}$, the x-axis, the line $x = \ln 2$ and the line $x = \ln 3$ about the y-axis.



NOT TO SCALE

Which expression could be used to evaluate the volume of the solid of revolution?
(Hint: use a horizontal line to divide the shaded region into two pieces before rotating)

A. $V = 2\pi\left(\left(\ln 3\right)^2 - \left(\ln 2\right)^2\right) + \pi\int_2^3\left(\left(\ln 3\right)^2 - \left(\frac{\ln y}{2}\right)^2\right)dy$

B. $V = 2\pi\left(\ln 3 - \ln 2\right)^2 + \pi\int_2^3\left(\ln 3 - \frac{\ln y}{2}\right)^2 dy$

C. $V = 4\pi\left(\left(\ln 3\right)^2 - \left(\ln 2\right)^2\right) + \pi\int_4^9\left(\left(\ln 3\right)^2 - \left(\frac{\ln y}{2}\right)^2\right)dy$

D. $V = 4\pi\left(\ln 3 - \ln 2\right)^2 + \pi\int_4^9\left(\ln 3 - \frac{\ln y}{2}\right)^2 dy$

10 Let $ABCD$ be a parallelogram, where the diagonals intersect at P, and O is the origin. Which of the following expressions is equivalent to $\overrightarrow{OA} + \overrightarrow{OB} + \overrightarrow{OC} + \overrightarrow{OD}$?

A. $\overrightarrow{OP}$

B. $2\overrightarrow{OP}$

C. $3\overrightarrow{OP}$

D. $4\overrightarrow{OP}$

End of Section I

Section II

60 marks
Attempt Questions 11 - 14
Allow about 1 hour and 40 minutes for this section

Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.

In Questions 11 - 14, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) **[START THE QUESTION 11 WRITING BOOKLET]**

(a) Solve $\frac{2}{x-1} < x$ **3**

(b) **Answer Question 11 (b) on the additional page at the end of this booklet. Detach and hand it in with your answer booklet for Question 11.** **3**

(c) The cubic polynomial $P(x) = x^3 + x^2 - 8x - 12$ has zeroes $\alpha,\ \beta$ and γ. **2**

Find $\alpha^2 + \beta^2 + \gamma^2$.

(d) Use division of polynomials to express $P(x)$ in the form $P(x) = A(x) \times Q(x) + R(x)$, where $P(x) = 3x^3 + 14x + 11$ and $\mathrm{A}(x) = x^2 - 3x + 2$. **2**

(e) A carton of milk is removed from a fridge and placed on a table. **3**
The temperature in the fridge is 3°C, and the temperature in the room is 24°C.
After sitting on the table for five minutes, the temperature of the milk is 15°C.

The temperature (T°C) of the milk is given by $T = B + Ae^{-kt}$, where *k* is a positive constant, *t* is time in minutes and *A* and *B* are constants.

Find the total number of minutes that the milk can be left on the table before it reaches a temperature of 20°C. Give your answer correct to the nearest whole number.

(f) Show that $\sin^2 x \cos^2 x = \frac{1}{8}[1 - \cos(4x)]$. **2**

Question 12 (15 marks) **[START THE QUESTION 12 WRITING BOOKLET]**

(a) (i) In how many ways can the letters of the word SUCCESS be arranged in a line if there are no restrictions? **1**

(ii) In how many ways can the letters of the word SUCCESS be arranged in a line if the two Cs are NOT adjacent? **2**

(b) (i) Express $\sqrt{3}\cos x - \sin x$ in the form $R\cos(x+\alpha)$ where $0 \le \alpha \le \frac{\pi}{2}$. **1**

(ii) Hence, or otherwise, solve $\sqrt{3}\cos x - \sin x = 1$ for $0 \le x \le 2\pi$. **2**

(c) (i) Write down the expansion of $(1-x^2)^5$ **1**

(ii) Hence, or otherwise, find the term in x^6 in $(4x-1)^2(1-x^2)^5$. **2**

(d) Solve for n: ${}^{n}C_{3} = 3({}^{n}P_{2})$ **2**

(e) If n is an integer greater than or equal to zero, using mathematical induction, determine if $4^n - 3n - 1$ is divisible by any of the numbers 6, 7, 8, and 9. **4**

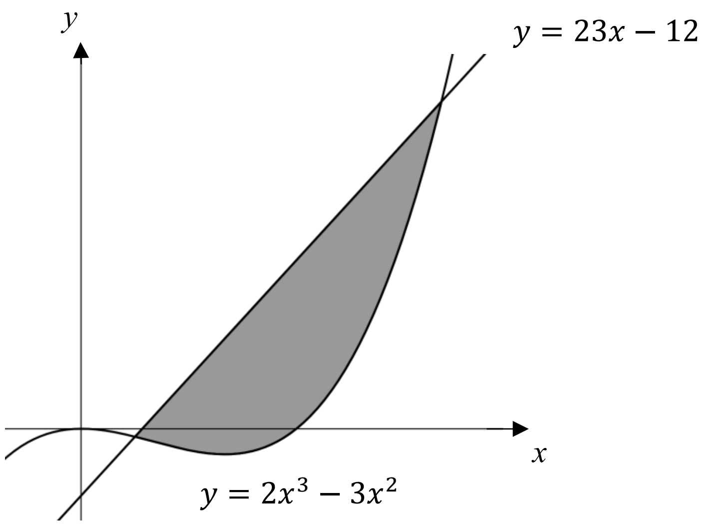
Question 13 (15 marks) **[START THE QUESTION 13 WRITING BOOKLET]**

(a) Find the *y*-intercept of the tangent to the curve $y = x^2 \arcsin x$ at the point $\left(\frac{1}{2}, \frac{\pi}{24}\right)$. **3**

(b) Use the substitution $u = \sqrt{x-2}$ to evaluate $\int_2^4 2x\sqrt{x-2}\,dx$. **3**
Give your answer as a simplified exact value.

(c) Find the curve which satisfies the differential equation $\frac{dy}{dx} = \frac{y}{2}$ **3**
and passes through the point $(0, -1)$.

(d) Find the shaded area in the diagram below: **3**



(e) A light is on the ground 20 metres from a building. A man 2 metres tall walks from the light directly toward the building at 1 m/s. How fast is the length of his shadow on the building changing when he is 14 metres from the building? **3**

Question 14 (15 marks) **[START THE QUESTION 14 WRITING BOOKLET]**

(a) A stone is thrown from the top of a 60 m cliff at an angle of 25° below the horizontal. It lands in the ocean 45 m from the base of the cliff. Assume $g = 10ms^{-2}$.

(i) Taking the origin as the point where the stone is thrown and starting with the equations $\ddot{y} = -g$, and $\ddot{x} = 0$, show that the path of the stone can be represented by $y = -x\tan 25° - \frac{5x^2}{V^2\cos^2 25°}$, where V is the initial velocity. **3**

(ii) Hence find the initial velocity of the stone, to two decimal places. **1**

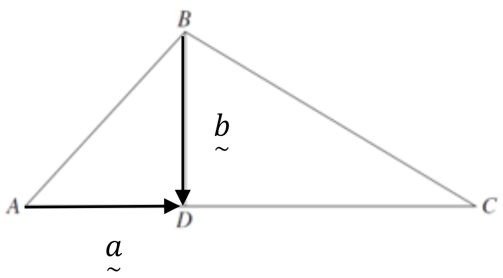
(iii) Hence find the angle the stone enters the ocean, to the nearest degree. **2**

(b) Let $f(x) = e^{-x} - 2x$.

(i) Explain why the inverse of $f(x)$ is a function. **1**

(ii) Let $g(x) = f^{-1}(x)$. Find the gradient of $g(x)$ where its graph cuts the x-axis. **2**

(c) In the triangle below, BD is perpendicular to AC and the following ratios exist:



$AD : AC$ is 1:4
$BD : AD$ is 2:1

(i) Write the simplest expression for $proj_{\underset{\sim}{a}}\overrightarrow{AB}$. **1**

(ii) Prove $\overrightarrow{BA}\cdot\overrightarrow{BC} = \left|\underset{\sim}{a}\right|^2$ **2**

(d) The digits 1, 2, 3, 4, 5 and 6 are spaced evenly around a circle. **3**

Each digit has another digit which is directly opposite it on the circle.

If the digits are placed around the circle at random, what is the probability that exactly one pair of opposite digits add to 7?

End of Paper

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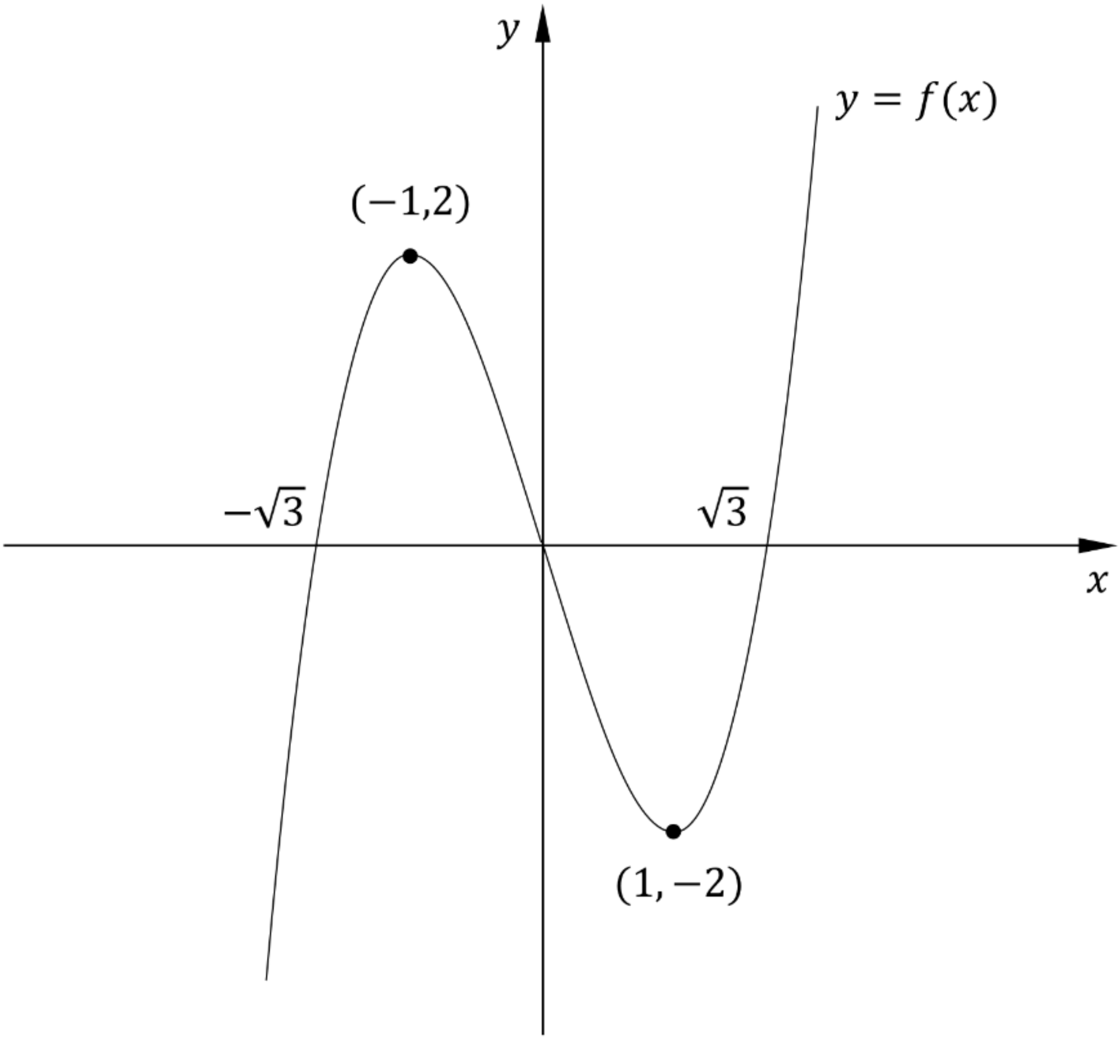
Student Number

Additional Answer page for Question 11 (b)
Detach and hand in with your Question 11 booklet.

(b) The diagram shows the graph of $y = f(x)$, where $f(x) = x^3 - 3x$. **3**



Sketch $y = \frac{1}{f(x)}$ on the diagram, clearly showing all turning points and asymptotes.

2025 Year 12 Extension 1 Trial Marking Scheme

Multiple Choice CCADCBDBCD

① Domain: $-1 \le \frac{x}{4} \le 1$

$-4 \le x \le 4$

Range: $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$ $\therefore$ C

② Worst case:

Flip: 1, 2, 3, 4, 5, 6, 7, 15

↳ so far no cards with a difference of 7.

→ the next card will give a difference of 7.

$\therefore$ 9 cards

OR

flip all odd numbered cards:

1, 3, 5, 7, 9, 11, 13, 15

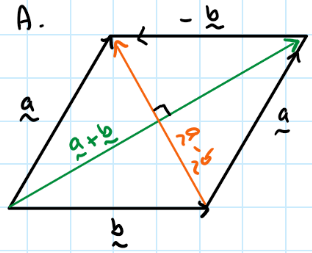
as all these differences are even.

The next card flipped will guarantee a difference of 7.

$\therefore$ 9 cards

$\therefore$ C

③ A.



$\underset{\sim}{a}+\underset{\sim}{b}$ and $\underset{\sim}{a}-\underset{\sim}{b}$ are the diagonals of the rhombus which always meet at a right-angle

$\therefore (\underset{\sim}{a}+\underset{\sim}{b})\cdot(\underset{\sim}{a}-\underset{\sim}{b}) = 0$

OR $(\underset{\sim}{a}+\underset{\sim}{b})\cdot(\underset{\sim}{a}-\underset{\sim}{b})$

$= \underset{\sim}{a}\cdot\underset{\sim}{a} - \underset{\sim}{b}\cdot\underset{\sim}{b}$

$= |\underset{\sim}{a}| - |\underset{\sim}{b}|$

$= 0$ since $|\underset{\sim}{a}| = |\underset{\sim}{b}|$ in a rhombus

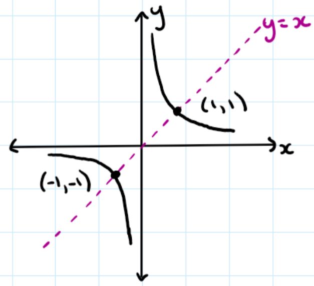
~~B~~. $|\underset{\sim}{a}| = |\underset{\sim}{b}|$ but $\underset{\sim}{a} \neq \underset{\sim}{b}$

~~C~~. $\underset{\sim}{a}$ and $\underset{\sim}{b}$ do not meet at a right angle in a rhombus (unless it is a square)

~~D~~. from above, $|\underset{\sim}{a}+\underset{\sim}{b}|$ is longer than $|\underset{\sim}{a}-\underset{\sim}{b}|$ (unless it is a square)

$\therefore$ A

④ $f(x) = \frac{1}{x}$



Inverse is a reflection over the line $y=x$ which is the same for $f(x) = \frac{1}{x}$

OR: Inverse: $x = \frac{1}{y}$

$y = \frac{1}{x}$

$f^{-1}(x) = \frac{1}{x}$ $\therefore$ D

⑤ $P(x) = D(x)\cdot Q(x) + x^2+2$

The degree of the remainder is always less than the degree of the divisor.

$\therefore$ $Q(x)$ must have a degree of 3 or more.

$\therefore C$

⑥ $\frac{dy}{dx}=0$ when $y=0$ $\rightarrow$ true for all

$\frac{dy}{dx}=-1$ when $y=-x$

A. $\frac{-x}{2(-x)+x} = \frac{-x}{-x} = 1$ ✗

B. $\frac{-(-x)}{2(-x)+x} = \frac{x}{-x} = 1$ ✓

C. $\frac{-x}{-2(-x)+x} = \frac{-x}{3x} = -\frac{1}{3}$ ✗

D. $\frac{-x}{2(-x)-x} = \frac{-x}{-3x} = \frac{1}{3}$ ✗

OR, sub in points:

$\frac{dy}{dx}$ is negative in 1st quadrant:

test $(1,1)$ $\rightarrow$ B $= \frac{-1}{2+1} = -\frac{1}{3}$ ✓

$\frac{dy}{dx}$ is positive at $(4,-1)$

test: B $= \frac{-(-1)}{2(-1)+4} = \frac{1}{2}$ ✓

etc.

$\therefore B$

⑦ $\int \frac{3x^2+6x^5}{4+x^6}\,dx$

$= \int \frac{3x^2}{4+(x^3)^2}\,dx + \int \frac{6x^5}{4+x^6}\,dx$

$= \frac{1}{2}\tan^{-1}\left(\frac{x^3}{2}\right) + \ln|4+x^6| + c$

$\therefore D$

⑧ Test end points of the domain to see if they give us $(0,2)$ and $(2,0)$

A. when $t=0$, $\left.\begin{matrix} x=2 \\ y=0 \end{matrix}\right\}(2,0)$ ✓

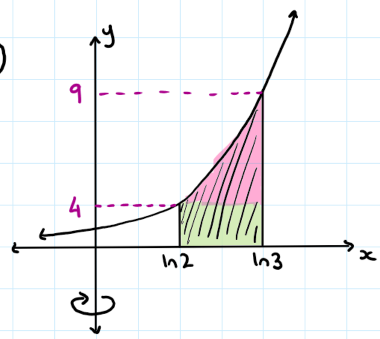
$t=1$, $\left.\begin{matrix} x=0 \\ y=2 \end{matrix}\right\}(0,2)$

B. when $t=0$, $\left.\begin{matrix} x=0 \\ y=2 \end{matrix}\right\}(0,2)$

$t=\ln 2$, $\left.\begin{matrix} x=e^{\ln 2}-1 = 2-1=1 \\ y=3-e^{\ln 2} = 3-2=1 \end{matrix}\right\}(1,1)$ ✗

$\therefore B$

⑨



$y=e^{2x} \rightarrow x=\frac{\ln y}{2}$

$x=\ln 2,\ y=e^{2\ln 2} = 4$

$x=\ln 3,\ y=e^{2\ln}$

$y=0$ to 4 | $y=4$ to 9

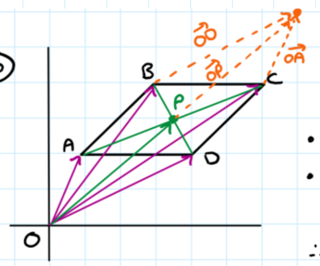
$V = \underbrace{\left(\ln 3 - \ln 2 \text{ cylinders}\right)}_{\pi r^2 h} + \left(\ln 3 \text{ cylinder} - \text{solid between curve \& y-axis}\right)$

$= \left(\pi(\ln 3)^2 4 - \pi(\ln 2)^2 4\right) + \pi\int_4^9 \left((\ln 3)^2 - \left(\frac{\ln y}{2}\right)^2\right) dy$

$= 4\pi\left((\ln 3)^2 - (\ln 2)^2\right) + \pi\int_4^9 \left((\ln 3)^2 - \left(\frac{\ln y}{2}\right)^2\right) dy$

$\therefore C$

⑩



By the parallelogram property, P is the midpoint of AC and BD:

- $\vec{OA} + \vec{OC} = 2\vec{OP}$
- $\vec{OB} + \vec{OD} = 2\vec{OP}$

$\therefore \vec{OA}+\vec{OB}+\vec{OC}+\vec{OD} = 4\vec{OP}$

$\therefore D$

Question 11

a) $\dfrac{2}{x-1} < x$

$/3$

$2(x-1) < x(x-1)^2$ ✓ multiply by $(x-1)^2$

$0 < x(x-1)^2 - 2(x-1)$

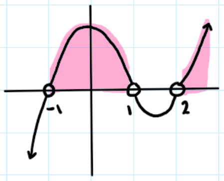
$0 < (x-1)[x(x-1) - 2]$

$0 < (x-1)(x^2 - x - 2)$

$0 < (x-1)(x+1)(x-2)$ ✓ factorise

OR

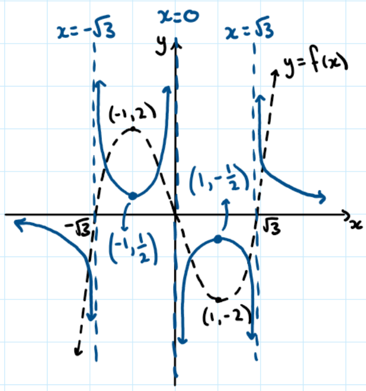
$(x-1)(x+1)(x-2) > 0$



$-1 < x < 1$ and $x > 2$ ✓ answer

b)

$/3$



✓ asymptotes
✓ turning points
✓ shape

c) $(\alpha+\beta+\gamma)^2 = (\alpha+\beta+\gamma)(\alpha+\beta+\gamma)$

$/2$

$= \alpha^2 + \alpha\beta + \alpha\gamma + \alpha\beta + \beta^2 + \beta\gamma + \alpha\gamma + \beta\gamma + \gamma^2$

$= \alpha^2 + \beta^2 + \gamma^2 + 2(\alpha\beta + \alpha\gamma + \beta\gamma)$

$\therefore \alpha^2+\beta^2+\gamma^2 = (\alpha+\beta+\gamma)^2 - 2(\alpha\beta+\alpha\gamma+\beta\gamma)$ ✓ expression for $\alpha^2+\beta^2+\gamma^2$

$= \left(\frac{-1}{1}\right)^2 - 2\left(\frac{-8}{1}\right)$

$= 17$ ✓ answer

d)

$/2$

$$\begin{array}{r|l} & 3x + 9 \\ \hline x^2 - 3x + 2 & 3x^3 + 0x^2 + 14x + 11 \\ & 3x^3 - 9x^2 + 6x \\ \hline & 9x^2 + 8x + 11 \\ & 9x^2 - 27x + 18 \\ \hline & 35x - 7 \end{array}$$

✓ some correct working

$\therefore P(x) = (x^2 - 3x + 2)(3x + 9) + 35x - 7$ ✓ answer

e) $B = 24$ when $t = 0$, $T = 3$

$/3$

$\therefore 3 = 24 + Ae^0$

$A = -21$ ✓ finding A

$\therefore T = 24 - 21e^{-kt}$

when $t = 5$, $T = 15$

$\therefore 15 = 24 - 21e^{-3k}$

$21e^{-5k} = 9$

$e^{-5k} = \frac{3}{7}$

$-5k = \ln\left(\frac{3}{7}\right)$

$k = -\frac{1}{5}\ln\left(\frac{3}{7}\right)$ ✓ finding k

$\therefore T = 24 - 21e^{\frac{1}{5}\ln\left(\frac{3}{7}\right)t}$

when $T = 20$, $20 = 24 - 21e^{-kt}$

$21e^{-kt} = 4$

$e^{-kt} = \frac{4}{21}$

$-kt = \ln\left(\frac{4}{21}\right)$

$\frac{1}{5}\ln\left(\frac{3}{7}\right)t = \ln\left(\frac{4}{21}\right)$

$t = \ln\left(\frac{4}{21}\right) \div \left(\frac{1}{5}\ln\left(\frac{3}{7}\right)\right)$

$t \approx 10$ mins ✓ answer

f) LHS $= \sin^2 x \cos^2 x$

$/2$

$= \frac{1}{2}(1 - \cos 2x) \times \frac{1}{2}(1 + \cos 2x)$ ✓ using $\sin^2 x$ and $\cos^2 x$ formulas

$= \frac{1}{4}(1 - \cos 2x)(1 + \cos 2x)$

$= \frac{1}{4}(1 - \cos^2(2x))$

$= \frac{1}{4}\left[1 - \frac{1}{2}(1 + \cos 4x)\right]$

$= \frac{1}{4}\left(1 - \frac{1}{2} - \frac{1}{2}\cos 4x\right)$

$= \frac{1}{4}\left(\frac{1}{2} - \frac{1}{2}\cos 4x\right)$

$= \frac{1}{8}(1 - \cos 4x)$ ✓ sufficient working to RHS

$=$ RHS

Question 12

a) (i) 7 letters → 3 Ss and 2 Cs

$\frac{}{1}$

$$\therefore \frac{7!}{3!\,2!} = 420$$ ✓ answer

(ii) Total − 2 Cs adjacent

$\frac{}{2}$

$$= 420 - \frac{6!}{3!}$$ ✓ $\frac{6!}{3!}$

$$= 420 - 120$$

$$= 300$$ ✓ answer

$\boxed{CC}$SUESS

→ 6 to arrange

→ 3 Ss

→ CC = CC so no need to ×2

b) (i) $\sqrt{3}\cos x - \sin x = R\cos(x+\alpha)$

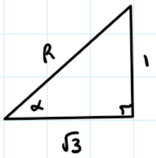
$\frac{}{1}$

$\sqrt{3}\cos x - \sin x = R\cos x\cos\alpha - R\sin x\sin\alpha$

$\therefore -R\sin\alpha = -1$

$R\sin\alpha = 1$

and $R\cos\alpha = \sqrt{3}$



$R = \sqrt{(\sqrt{3})^2 + 1^2}$ and $\tan\alpha = \frac{1}{\sqrt{3}}$

$R = 2$

$\alpha = \frac{\pi}{6}$ $\quad 0 \le \alpha \le \frac{\pi}{2}$

$\therefore \sqrt{3}\cos x - \sin x = 2\cos\left(x + \frac{\pi}{6}\right)$ ✓ answer

(ii) $2\cos\left(x+\frac{\pi}{6}\right) = 1$ $\quad 0 \le x \le 2\pi$

$\frac{}{2}$

$\cos\left(x+\frac{\pi}{6}\right) = \frac{1}{2}$ $\quad \frac{\pi}{6} \le x + \frac{\pi}{6} \le \frac{13\pi}{6}$

$x + \frac{\pi}{6} = \frac{\pi}{3}, \frac{5\pi}{3}$ ✓ one solution

$x = \frac{\pi}{6}, \frac{3\pi}{2}$ ✓ both solutions

c) (i) $(1-x^2)^5 = 1 - 5x^2 + 10x^4 - 10x^6 + 5x^8 - x^{10}$ ✓ answer

$\frac{}{1}$

(ii) $(4x-1)^2(1-x^2)^5$

$\frac{}{2}$

$= (16x^2 - 8x + 1)(1 - 5x^2 + 10x^4 - 10x^6 + 5x^8 - x^{10})$

x^6 term $= (16x^2)(10x^4) + (1)(-10x^6)$ ✓ some working towards x^6

$= 160x^6 - 10x^6$

$= 150x^6$

$\therefore$ coefficient $= 150$ ✓ answer

d) ${}^nC_3 = 3({}^nP_2)$

$\frac{}{2}$

$$\frac{n!}{3!(n-3)!} = 3 \times \frac{n!}{(n-2)!}$$ ✓ using formulas $n \ge 3$

$$\frac{1}{6\,\cancel{(n-3)!}} = \frac{3}{(n-2)\cancel{(n-3)!}}$$

$n - 2 = 18$

$n = 20$ ✓ answer

e) Show true for $n = 0$

$\frac{}{4}$

$4^0 - 3(0) - 1 = 0$

$\therefore$ divisible by 6, 7, 8 and 9. ✓ correct step 1

Let $n = k$ be an integer ≥ 0. Assume:

$4^k - 3k - 1 = AM$ where M is a positive integer, A is $\{6, 7, 8, 9\}$

$\therefore 4^k = AM + 3k + 1$ ✓ correct set up with general 'AM' or similar

Show true for $n = k+1$

RTP: $4^{k+1} - 3(k+1) - 1 = AN$ where N is a positive integer

$LHS = 4\cdot 4^k - 3k - 3 - 1$

$= 4(AM + 3k + 1) - 3k - 4$ by assumption

$= 4AM + 12k + 4 - 3k - 4$

$= 4AM + 9k$ ✓ correct simplification

If: $A = 6$, $24M + 9k \ne 6N$

$A = 7$, $28M + 9k \ne 7N$

$A = 8$, $32M + 9k \ne 8N$

$A = 9$, $36M + 9k = 9(4M + k)$

$= 9N$ since M and k are integers

$\therefore$ Proven true by induction only for $A = 9$. ✓ correct answer with reasoning

−1 mark for style e.g. missing 'integer'

−1 mark if they only show a solution for one of 6, 7, 8 or 9.

Question 13

a) $\frac{}{3}$

$v = x^2 \qquad u = \sin^{-1}x$

$v' = 2x \qquad u' = \dfrac{1}{\sqrt{1-x^2}}$

$$y' = \frac{x^2}{\sqrt{1-x^2}} + 2x\sin^{-1}x$$ ✓ differentiating

when $x = \frac{1}{2}$, gradient of the tangent:

$$y' = \frac{\sqrt{3}+\pi}{6}$$ ✓ finding the gradient in exact form

$\therefore$ equation of tangent:

$$y - \frac{\pi}{24} = \frac{\sqrt{3}+\pi}{6}\left(x - \frac{1}{2}\right)$$

$$y = \left(\frac{\sqrt{3}+\pi}{6}\right)x - \frac{(\sqrt{3}+\pi)}{12} + \frac{\pi}{24}$$

$$y\text{-int} = \frac{\pi}{24} - \frac{(2\sqrt{3}+2\pi)}{24}$$

$$= \frac{-2\sqrt{3}-\pi}{24} \quad \text{or} \quad -\frac{(2\sqrt{3}+\pi)}{24}$$ ✓ answer

b) $\frac{}{3}$

$u = \sqrt{x-2} \qquad x = u^2 + 2$

$u = (x-2)^{\frac{1}{2}}$

$\frac{du}{dx} = \frac{1}{2}(x-2)^{-\frac{1}{2}}$ when $x = 4$, $u = \sqrt{2}$

$\qquad\qquad\qquad\qquad x = 2$, $u = 0$

$\frac{du}{dx} = \frac{1}{2\sqrt{x-2}}$

$dx = 2\sqrt{x-2}\ du$ ✓ finding dx

$dx = 2u\ du$

$$\therefore \int_0^{\sqrt{2}} 2(u^2+2)u \cdot 2u\ du$$ ✓ u substitution including new limits

$$= \int_0^{\sqrt{2}} (4u^4 + 8u^2)\ du$$

$$= \left[\frac{4u^5}{5} + \frac{8u^3}{3}\right]_0^{\sqrt{2}}$$

$$= \left(\frac{4(\sqrt{2})^5}{5} + \frac{8(\sqrt{2})^3}{3}\right) - (0+0)$$

$$= \frac{16\sqrt{2}}{5} + \frac{16\sqrt{2}}{3}$$

$= 12.0679\ldots$

$= 12.07$ to 2dp ✓ answer

c) $\frac{}{3}$

$$\frac{dy}{dx} = \frac{y}{2}$$

$$\frac{1}{y}\,dy = \frac{1}{2}\,dx$$

$$\ln|y| = \frac{x}{2} + c$$ ✓ integrating both sides

sub in $(0,-1)$

$$\ln|-1| = \frac{0}{2} + c$$

$$\ln(1) = 0 + c$$

$$0 = c$$ ✓

$$\therefore \ln|y| = \frac{x}{2}$$

$$|y| = e^{\frac{x}{2}}$$

$$y = \pm e^{\frac{x}{2}}$$

But, the curve passes through $(0,-1)$

$$\therefore y = -e^{\frac{x}{2}}$$ ✓

OR

$$|y| = e^{\frac{x}{2}+c}$$

$$= e^c e^{\frac{x}{2}}$$

$$y = \pm e^c e^{\frac{x}{2}}$$

Let $A = \pm e^c$

$$\therefore y = Ae^{\frac{x}{2}}$$ ✓

Sub in $(0,-1)$

$$-1 = Ae^0$$

$$A = -1$$

$$\therefore y = -e^{\frac{x}{2}}$$ ✓

d) $\frac{}{3}$ Points of intersection:

$$2x^3 - 3x^2 = 23x - 12$$

$$2x^3 - 3x^2 - 23x + 12 = 0$$

$$(x+3)(2x-1)(x-4) = 0$$

$P(-3) = 0$

$P\left(\frac{1}{2}\right) = 0$

$3 \times -1 \times \alpha = 12$

$\alpha = -4$

OR $P(-3) = 0$

$$
\begin{array}{r|l}
 & 2x^2 - 9x + 4 \\
x+3 & 2x^3 - 3x^2 - 23x + 12 \\
 & \underline{2x^3 + 6x^2} \\
 & \quad -9x^2 - 23x \\
 & \quad \underline{-9x^2 - 27x} \\
 & \qquad 4x + 12 \\
 & \qquad \underline{4x + 12} \\
 & \qquad\quad 0
\end{array}
$$

$$\therefore (x+3)(2x^2 - 9x + 4) = 0$$

$$(x+3)(2x-1)(x-4) = 0$$ ✓ factorising

$$\therefore x = -3, \frac{1}{2}, 4$$

Use $x = \frac{1}{2}$ and 4 for shaded area

d) cont. on next page

d) cont.

$$\int_{\frac{1}{2}}^{4}\left[(23x-12)-(2x^3-3x^2)\right]dx$$

✓ integral with correct limits

$$= \left[\frac{23x^2}{2} - 12x - \frac{2x^4}{4} + \frac{3x^3}{3}\right]_{\frac{1}{2}}^{4}$$

$$= \left(\frac{23(4)^2}{2} - 12(4) - \frac{(4)^4}{2} + \frac{3(4)^3}{3}\right) - \left(\frac{23(\frac{1}{2})^2}{2} - 12\left(\frac{1}{2}\right) - \frac{(\frac{1}{2})^4}{2} + \frac{3(\frac{1}{2})^3}{3}\right)$$

$$= 72 - \left(-\frac{97}{32}\right)$$

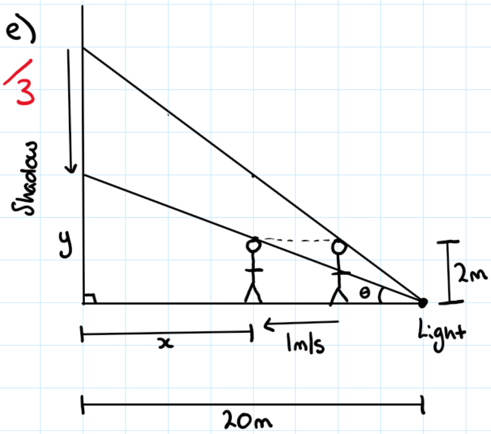
$$= \frac{2401}{32} \text{ units}^2$$

✓ answer

or 75.03125 units^2

e)

/3



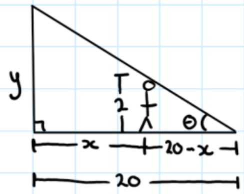
Let y = length of shadow

x = distance the man is from the wall

t = time in seconds

$$\frac{dx}{dt} = -1 \text{ m/s}$$ ✓

$$\frac{dy}{dt} = \text{rate shadow is changing}$$



Using similar triangles:

$$\frac{y}{20} = \frac{2}{20-x}$$

$$y = \frac{40}{20-x}$$ ✓ y

$$y = 40(20-x)^{-1}$$

$$\frac{dy}{dx} = \frac{40}{(20-x)^2}$$

$$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$$

$$\frac{40}{(20-x)^2} = \frac{dy}{dt} \times -1$$

$$\frac{dy}{dt} = -\frac{40}{(20-x)^2}$$

When $x = 14$, $\frac{dy}{dt} = -\frac{10}{9}$ m/s ✓ answer

or decreasing by $1.\dot{1}$ m/s

Question 14

a) (i) $\ddot{x} = 0$

3

$\dot{x} = c_1$

$= V\cos 25$

$x = Vt\cos 25 + c_2$

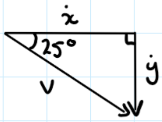
when $t=0$, $x=0$

$\therefore c_2 = 0$

✓ x

$\therefore x = Vt\cos 25$

$t = \dfrac{x}{V\cos 25}$



$\ddot{y} = -10$

$\dot{y} = -10t + c_3$

when $t=0$, $\dot{y} = -V\sin 25$

$\therefore c_3 = -V\sin 25$

$\dot{y} = -10t - V\sin 25$

$y = -5t^2 - Vt\sin 25 + c_4$

when $t=0$, $y=0$

$\therefore c_4 = 0$

$\therefore y = -5t^2 - Vt\sin 25$

✓ y

Sub t into y:

$$y = -5\left(\frac{x}{V\cos 25}\right)^2 - V\left(\frac{x}{V\cos 25}\right)\sin 25$$

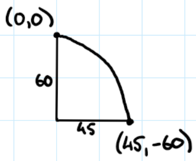
$$y = -\frac{5x^2}{V^2\cos^2 25} - x\tan 25$$

$$y = -x\tan 25 - \frac{5x^2}{V^2\cos^2 25}$$

✓ answer with sufficient working

(ii) Find V.

1



When $x = 45$, $y = -60$

$$-60 = -45\tan 25 - \frac{5(45)^2}{V^2\cos^2 25}$$

$$\frac{5(45)^2}{V^2\cos^2 25} = 60 - \tan 25$$

$$\frac{5(45)^2}{60 - 45\tan 25} = V^2\cos^2 25$$

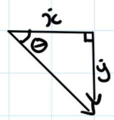
$$V^2 = \frac{5(45)^2}{\cos^2 25\,(60 - 45\tan 25)}$$

$V = 17.774\ldots$ $(V > 0)$

$V = 17.77$ m/s to 2dp ✓ answer

(iii)

2



$$\frac{dy}{dx} = -\tan 25 - \frac{10x}{17.77^2\cos^2 25}$$

when $x = 45$

$\dfrac{dy}{dx} = -2.20\ldots$ ✓ derivative

$\therefore \tan\theta = -2.20\ldots$

$\theta = \tan^{-1}(-2.20\ldots)$

$\theta = -65.5\ldots$

$\therefore 66°$ to the horizontal

✓ answer

OR

When $x = 45$

$t = \dfrac{45}{17.77\cos 25}$

$\therefore \dot{x} = 17.77\cos 25$ ✓ $\dot{y}$

$\dot{y} = -10\left(\dfrac{45}{17.77\cos 25}\right) - 17.77\sin 25$

$\tan\theta = \dfrac{\dot{y}}{\dot{x}}$

$\tan\theta = -2.20\ldots$

$\theta = \tan^{-1}(-2.20\ldots)$

$\theta = -65.5\ldots$ ✓ answer

$\therefore 66°$ to the horizontal

b) (i) $f'(x) = -e^{-x} - 2$

1

$= -(e^{-x} + 2)$

$e^{-x} > 0$

$\therefore e^{-x} + 2 > 0$

$\therefore -(e^{-x} + 2) < 0$

✓ sufficient explanation

$\therefore f(x)$ is always decreasing so it is a one-to-one function.

$\therefore f^{-1}(x)$ must also be a function

(ii) gradient of $f^{-1}(x)$ at x-axis is the reciprocal of the gradient of $f(x)$ at y-axis $(x=0)$

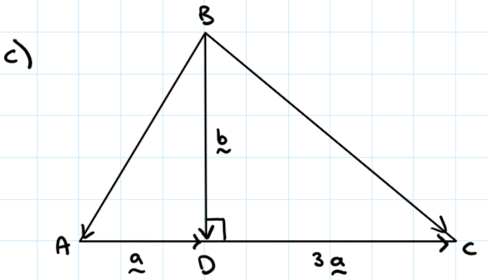
2

When $x = 0$, $f'(0) = -1 - 2$

$= -3$

✓ gradient of $f(x)$ on y-axis

$\therefore$ gradient of $g(x) = f^{-1}(x)$ at the x-axis $= -\dfrac{1}{3}$ ✓ answer c)



(i) $\text{proj}_{\underset{\sim}{a}} \overrightarrow{AB} = \underset{\sim}{a}$ ✓ by inspection

$\frac{}{1}$

(ii) $AD : AC$ is $1:4$

$\therefore \overrightarrow{DC} = 3\underset{\sim}{a}$

$\overrightarrow{BA} = \underset{\sim}{b} - \underset{\sim}{a}$

$\overrightarrow{BC} = \underset{\sim}{b} + 3\underset{\sim}{a}$

$\frac{}{2}$

$BD : AD$ is $2:1$

$\therefore |\underset{\sim}{b}| = 2|\underset{\sim}{a}|$

$|\underset{\sim}{b}|^2 = 4|\underset{\sim}{a}|^2 \quad — (1)$

✓ setting up the problem

$$
\begin{aligned}
LHS &= \overrightarrow{BA} \cdot \overrightarrow{BC} \\
&= (\underset{\sim}{b} - \underset{\sim}{a}) \cdot (\underset{\sim}{b} + 3\underset{\sim}{a}) \\
&= \underset{\sim}{b} \cdot \underset{\sim}{b} + 3\underset{\sim}{a} \cdot \underset{\sim}{b} - \underset{\sim}{a} \cdot \underset{\sim}{b} - 3\underset{\sim}{a} \cdot \underset{\sim}{a} \\
&= |\underset{\sim}{b}|^2 + 2\underset{\sim}{a} \cdot \underset{\sim}{b} - 3|\underset{\sim}{a}|^2 \\
&= 4|\underset{\sim}{a}|^2 + 2\underset{\sim}{a} \cdot \underset{\sim}{b} - 3|\underset{\sim}{a}|^2 \quad \text{from } (1) \\
&= |\underset{\sim}{a}|^2 + 2\underset{\sim}{a} \cdot \underset{\sim}{b}
\end{aligned}
$$

$\underset{\sim}{a} \perp \underset{\sim}{b}$

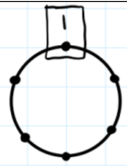
$\therefore \underset{\sim}{a} \cdot \underset{\sim}{b} = 0$

$$
\begin{aligned}
\therefore LHS &= |\underset{\sim}{a}|^2 + 0 \\
&= |\underset{\sim}{a}|^2 \\
&= RHS
\end{aligned}
$$

✓ correct answer with reasoning

d)

$\frac{}{3}$



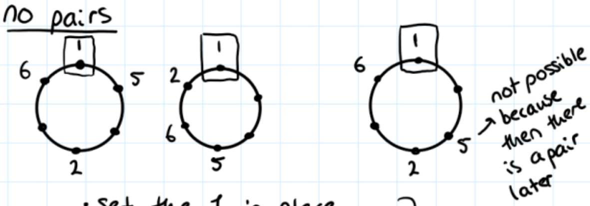
Total arrangements $= 5!$ ✓

One pair = total − no pairs − 2pairs − 3pairs add to 7

If there are 2 pairs, there must be 3 pairs

$\therefore$ = total − no pairs − 2 pairs

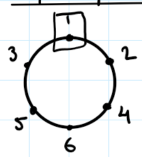
no pairs



- set the 1 in place
- 4 options for the 6
- 4 options for the 2
- 2 options for the 5
- 2 options for the 3
- 1 option for the 4

$4 \times 4 \times 2 \times 2 = 64$

2/3 pairs



✓ finding
- no pairs
or • 2/3 pairs
or • 1 pair

- Set the 1
- 1 option for the 6
- 4 options for the 2
- 1 option for the 5
- 2 options for the 3
- 1 option for the 4

$4 \times 2 = 8$

$\therefore$ one pair $= 5! - 64 - 8$
$= 48$

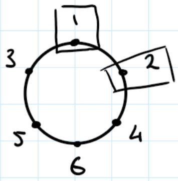
$$
\begin{aligned}
\text{Probability} &= \frac{48}{5!} \\
&= \frac{2}{5}
\end{aligned}
$$
✓

Alternate solutions on next page!

d) alternate soln

note: if there are 2 pairs there must be 3 pairs

- P(3 pairs):

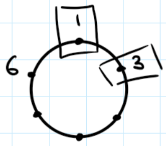


  - set the 1
    → $\frac{1}{5}$ chance the 6 is opposite
  - set the 2
    → $\frac{1}{3}$ chance the 5 is opposite
  - 3 and 4 must be opposite

$\therefore$ $P(3 \text{ pairs}) = \frac{1}{5} \times \frac{1}{3}$

$= \frac{1}{15}$ ✓

- P(0 pairs):



  - set the 1
    → $\frac{4}{5}$ chance the 6 is not opposite
  - set the 3
    → $\frac{2}{3}$ chance the 4 is not opposite
  - 2 and 5 can't be opposite

$\therefore$ $P(0 \text{ pairs}) = \frac{4}{5} \times \frac{2}{3}$

$= \frac{8}{15}$ ✓

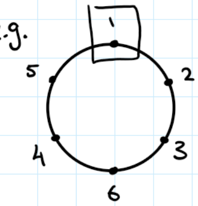
$\therefore$ $P(1 \text{ pair}) = 1 - P(3 \text{ pairs}) - P(0 \text{ pairs})$

$= \frac{2}{5}$ ✓

d) alternate soln

3 pairs: 1,6 or 2,5 or 3,4

e.g.



- set the 1
  → pair with 6 $\therefore$ 1 option for 6
- no more pairs
  $\therefore$ 4 options for 2
  2 options for 5
  2 options for 3
  1 option for 4

$\therefore$ number of ways for 1 pair

$= (4 \times 2 \times 2 \times 1) \times 3$

$= 48$ ✓

$\therefore$ probability $= \frac{48}{5!}$

$= \frac{2}{5}$ ✓