BAULKHAM HILLS HIGH SCHOOL

2022

YEAR 12 TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Extension 1

General Instructions

- Reading time – 10 minutes
- Working time – 2 hours
- Write using black pen
- Calculators approved by NESA may be used
- A reference sheet is provided at the back of this paper
- In Questions 11 – 14, show relevant mathematical reasoning and/or calculations
- Marks may be deducted for careless or badly arranged work

Total marks: 70

Section I – 10 marks (pages 2 – 6)

- Attempt Questions 1 – 10
- Allow about 15 minutes for this section

Section II – 60 marks (pages 7 – 13)

- Attempt Questions 11 – 14
- Allow about 1 hour and 45 minutes for this section

Section I

10 marks
Attempt Questions 1 – 10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1 – 10

1 The probability of success in a Bernoulli trial is 0.40. What is the variance of the trial?

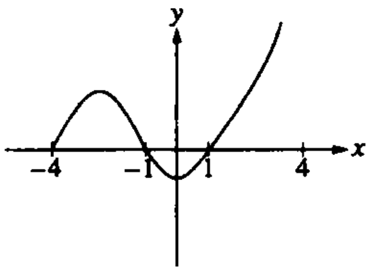
(A) 0.60

(B) 0.49

(C) 0.40

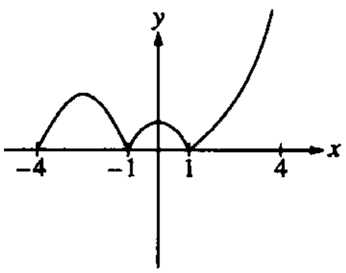
(D) 0.24

2

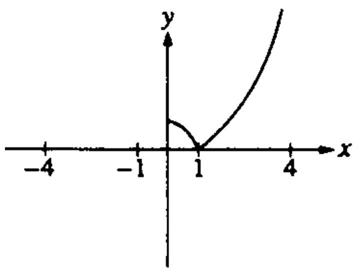


The graph of $y = f(x)$ is shown above. Which of the following could be the graph of $y = f(|x|)$?

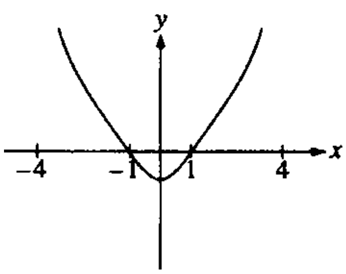
(A)



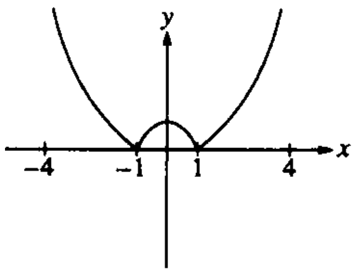
(B)



(C)



(D)



3 Given the vectors $\underset{\sim}{u} = \begin{pmatrix} -2 \\ -5 \end{pmatrix}$ and $\underset{\sim}{v} = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$, the vector projection of $\underset{\sim}{u}$ on $\underset{\sim}{v}$ is;

(A) $-\dfrac{11}{9}\begin{pmatrix} 3 \\ 1 \end{pmatrix}$

(B) $-\dfrac{11}{10}\begin{pmatrix} 3 \\ 1 \end{pmatrix}$

(C) $-\dfrac{39}{10}\begin{pmatrix} 3 \\ 1 \end{pmatrix}$

(D) $\dfrac{1}{10}\begin{pmatrix} 3 \\ 1 \end{pmatrix}$

4 A class consists of 10 girls and 14 boys. In how many ways can a committee of 2 girls and 2 boys be chosen?

(A) 136

(B) 4095

(C) 10 626

(D) 16 380

5 Which of the following is the derivative of $\cos^{-1}\left(\dfrac{4}{x}\right)$?

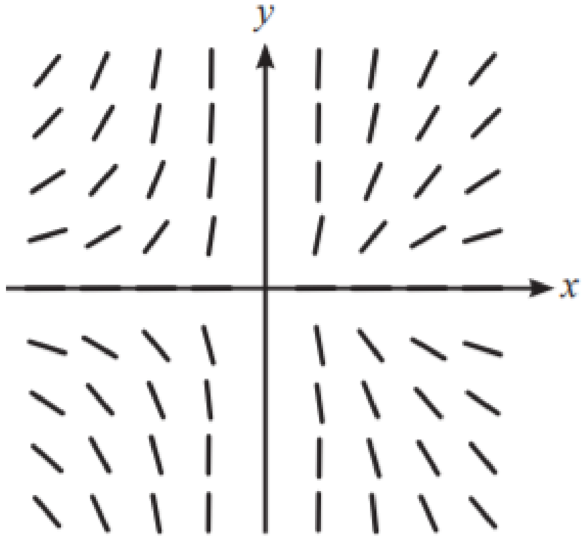
(A) $\dfrac{-4}{x\sqrt{x^2 - 16}}$

(B) $\dfrac{-4x}{\sqrt{x^2 - 16}}$

(C) $\dfrac{4}{x\sqrt{x^2 - 16}}$

(D) $\dfrac{4x}{\sqrt{x^2 - 16}}$

6 The slope field for a first order differential equation is shown.



Which of the following could be the differential equation represented?

(A) $\frac{dy}{dx} = \frac{5y}{x^2}$

(B) $\frac{dy}{dx} = \frac{5y}{x}$

(C) $\frac{dy}{dx} = \frac{5y^2}{x^2}$

(D) $\frac{dy}{dx} = \frac{5y^2}{x}$

7 Each member of a class is instructed to write down a different whole number between 1 and 49, inclusive. The teacher informs the class that there will be at least one pair of students whose numbers add up to 50.

What is the minimum number of students in the class?

(A) 24

(B) 25

(C) 26

(D) 27

8 A chemical dissolves in a pool at a rate equal to 5% of the amount of undissolved chemical. Initially the amount of undissolved chemical is 8 kg and after t hours, x kilograms has dissolved.

Which of the following differential equations would model this process?

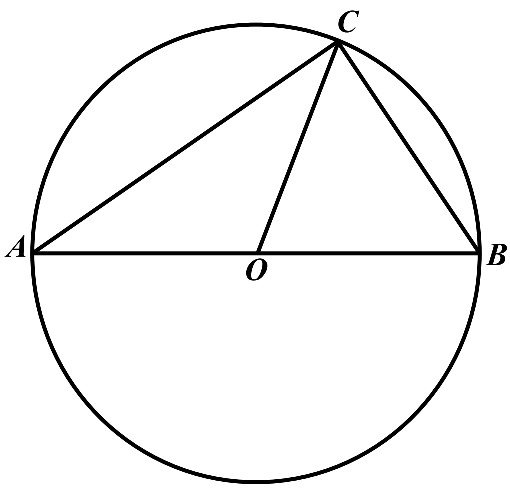
(A) $\dfrac{dx}{dt} = \dfrac{8 - x}{20}$

(B) $\dfrac{dx}{dt} = \dfrac{x - 8}{20}$

(C) $\dfrac{dx}{dt} = -\dfrac{x}{20}$

(D) $\dfrac{dx}{dt} = 8 - \dfrac{x}{20}$

9 In the diagram below, AOB is a diameter of the circle with centre O. C is a point on the circumference of the circle



If $\overrightarrow{OC} = \underset{\sim}{r}$ and $\overrightarrow{BC} = \underset{\sim}{s}$, then $\overrightarrow{AC}$ would be equal to which of the following vectors?

(A) $\underset{\sim}{r} + 2\underset{\sim}{s}$

(B) $\underset{\sim}{r} - 2\underset{\sim}{s}$

(C) $2\underset{\sim}{r} + \underset{\sim}{s}$

(D) $2\underset{\sim}{r} - \underset{\sim}{s}$

10 a and b are real numbers, and a is non-zero. When the polynomial $P(x) = x^2 - 2ax + a^4$ is divided by $x + b$, the remainder is 1.

The polynomial $Q(x) = bx^2 + x + 1$ has $ax - 1$ as a factor.

What are the possible values of b?

(A) -1 or 2

(B) -2 or 0

(C) 1 or 2

(D) 1 or 3

END OF SECTION I

Section II

60 marks
Attempt Questions 11 – 14
Allow about 1 hour and 45 minutes for this section

Answer each question on the appropriate answer sheet. Extra paper is available.

For questions in Section II, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (16 marks) Use the pages labelled Question 11 in the answer booklet

(a) Evaluate $\displaystyle\int_0^1 \frac{dx}{1+3x^2}$. *3*

(b) By expressing $2\cos x + 9\sin x$ in the form $A\cos(x - \alpha)$, solve; *4*

$$2\cos x + 9\sin x = 1 \quad , \quad 0 \le x \le 2\pi$$

Give your answer correct to two decimal places.

(c) Use the substitution $u = \sqrt{x} - 1$ to find $\displaystyle\int \frac{dx}{\sqrt{x}-1}$. *3*

(d) Find the curve that satisfies the differential equation; *3*

$$\frac{dy}{dx} = \frac{2ye^{2x}}{1+e^{2x}}$$

given that $y(0) = \pi$.

(e) Using vector techniques, calculate to the nearest degree, the acute angle between the lines *3*

$$x + y = 4 \text{ and } 3x - 4y = 2 .$$

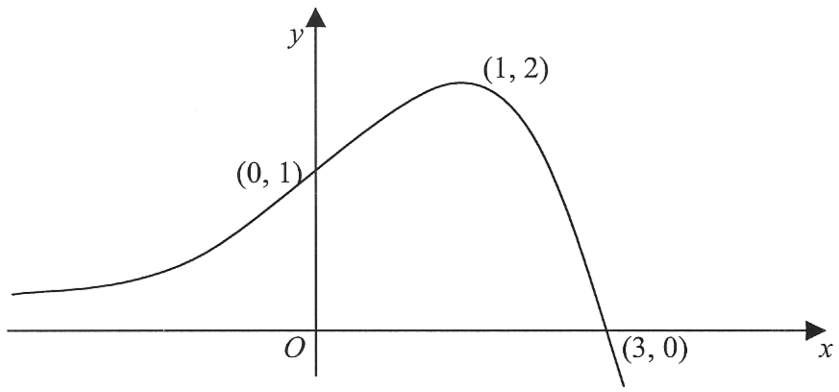
Question 12 (15 marks) Use the pages labelled Question 12 in the answer booklet

(a) There are n seats around a circular table and n people need to be arranged around the table.

(i) In how many ways can the people be arranged. *1*

(ii) Find the probability of three particular people sitting together. *2*

(b) The diagram shows the graph of $y = f(x)$. *3*



Sketch the graph of $y = \dfrac{1}{\sqrt{f(x)}}$

(c) Consider the function $f(x) = \cos^{-1}(x - 1)$.

(i) Find the domain of the function. *1*

(ii) Sketch the graph of the curve $y = f(x)$, clearly showing the coordinates of the endpoints. *2*

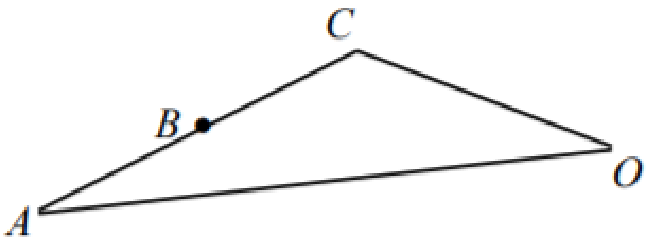
(iii) The region in the first quadrant bounded by $y = f(x)$ and the coordinate axes is rotated about the y-axis. *3*

Find the exact value of the volume of the solid of revolution.

Question 12 continues on page 9

Question 12 (continued)

(d) *3*



B is a point on the side AC of ΔOAC.

Use vectors to prove that if OB is the perpendicular bisector of AC then ΔOAC is isosceles.

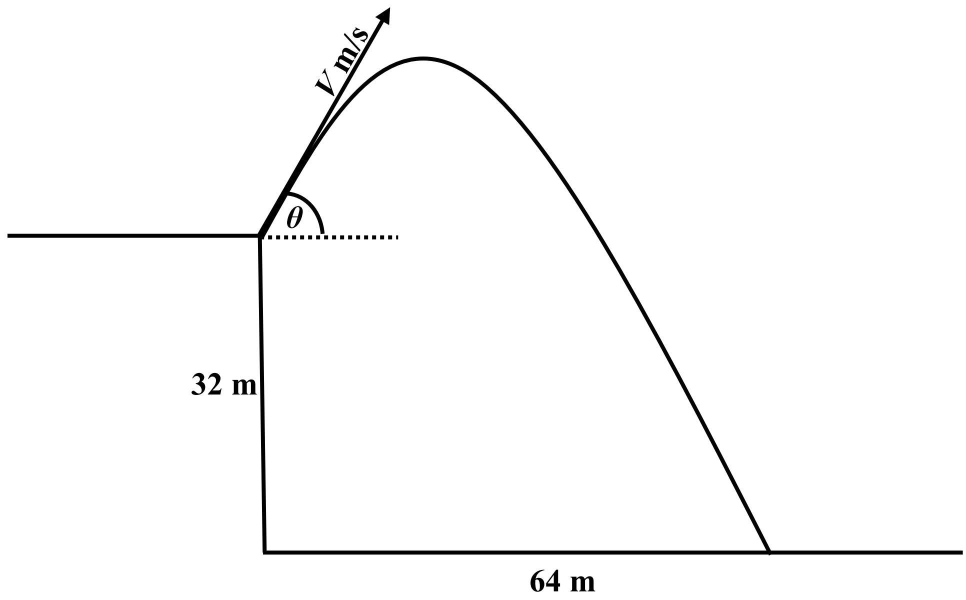
End of Question 12

Question 13 (14 marks) Use the pages labelled Question 13 in the answer booklet

(a) Use the principle of mathematical induction to show that for all integers $n \geq 1$ *3*

$$1^3 + 2^3 + 3^3 + \ldots + n^3 = \frac{1}{4}n^2(n+1)^2$$

(b) A particle is projected with an initial velocity V m/s at an angle θ to the horizontal, from a point O on the edge of a vertical cliff 32 metres above a horizontal beach.



This particle moves in a vertical plane under gravity, and its position vector, $\underset{\sim}{r}(t)$ is given by;

$$\underset{\sim}{r}(t) = \begin{pmatrix} Vt\cos\theta \\ Vt\sin\theta - 5t^2 \end{pmatrix}$$

Do NOT prove this.

The particle hits the beach after 4 seconds, 64 metres from the foot of the cliff.

(i) Find the exact value of V and find the value of θ, to the nearest degree. *3*

(ii) Find the speed of impact with the beach correct to the nearest m/s and the angle of impact with the beach correct to the nearest degree. *3*

Question 13 continues on page 11

Question 13 (continued)

(c) It is given that the angle θ satisfies the equation;

$$\sin\left(2\theta + \frac{\pi}{4}\right) = 3\cos\left(2\theta + \frac{\pi}{4}\right)$$

(i) Show that $\tan 2\theta = \frac{1}{2}$. *2*

(ii) Hence find, in surd form, the exact value of $\tan\theta$, given that θ is an obtuse angle. *3*

End of Question 13

Question 14 (15 marks) Use the pages labelled Question 14 in the answer booklet

(a) The equation $x^3 + 3ax + b = 0$ has two equal roots. *3*

Prove that $b^2 + 4a^3 = 0$.

(b) At a café, customers ordering hot drinks order either tea or coffee.

Of all customers 20% order tea and 80% order coffee. Of those who order tea, 60% take sugar and of those who order coffee, 35% take sugar.

(i) A randomly selected customer who orders a hot drink is chosen. *1*
Show that the probability that the customer takes sugar is 0.4.

(ii) A random sample of 10 customers ordering hot drinks is taken. *1*
Find the probability that exactly 4 of the customers take sugar. Give your answer correct to four decimal places.

(iii) Use the extract shown from a table giving values of $P(Z < z)$, where z has a standard normal distribution, to estimate the probability that from a sample of 150 customers ordering hot drinks, at least half of them take sugar. *3*

z	**0.00**	**0.01**	**0.02**	**0.03**	**0.04**	**0.05**	**0.06**	**0.07**	**0.08**	**0.09**
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974

Question 14 continues on page 13

Question 14 (continued)

(c) (i) By considering $(1 + x)^{n+3} \equiv (1 + x)^n(1 + x)^3$, show that; *2*

$$\binom{n+3}{k} = \binom{n}{k} + 3\binom{n}{k-1} + 3\binom{n}{k-2} + \binom{n}{k-3}$$

(ii) Between what values must k lie? *1*

(d) (i) Show that; *1*

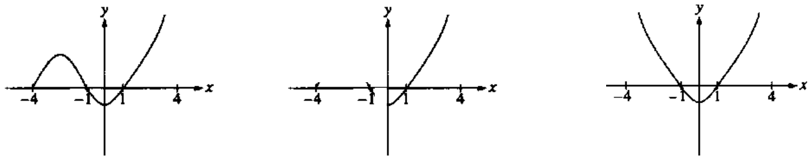
$$2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right) = \cos x + \cos y$$

(ii) Hence find all solutions to; *3*

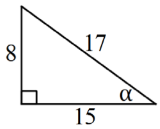
$$\cos 4\theta + \cos 3\theta + \cos 2\theta + \cos \theta = 0 \quad \text{for } 0 \le \theta \le 2\pi$$

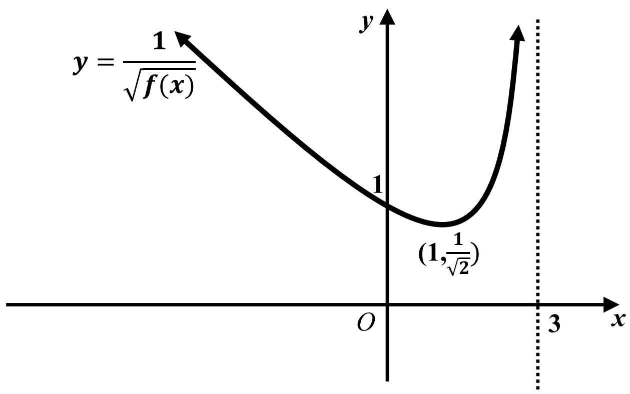
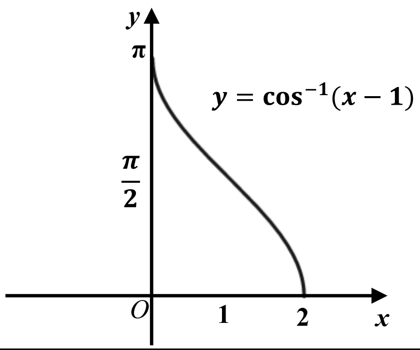
End of paper

BAULKHAM HILLS HIGH SCHOOL
YEAR 12 EXTENSION 1 TRIAL 2022 SOLUTIONS

<table>
<tr><th>Solution</th><th>Marks</th><th>Comments</th></tr>
<tr><td colspan="3">SECTION I</td></tr>
<tr><td>1. D – $\mathrm{Var}(X) = p(1-p)$
$= 0.6 \times 0.4$
$= 0.24$</td><td>1</td><td></td></tr>
<tr><td>2. C –

original graph ⇒ domain where $x < 0$ “disappears” ⇒ domain where $x > 0$ reflects in y-axis</td><td>1</td><td></td></tr>
<tr><td>3. B – $\mathrm{proj}_{\underset{\sim}{u}}\underset{\sim}{v} = \dfrac{\underset{\sim}{u}\cdot\underset{\sim}{v}}{\underset{\sim}{v}\cdot\underset{\sim}{v}}\underset{\sim}{v}$
$= \dfrac{-6-5}{3^2+1^2}\begin{pmatrix}3\\1\end{pmatrix}$
$= -\dfrac{11}{10}\begin{pmatrix}3\\1\end{pmatrix}$</td><td>1</td><td></td></tr>
<tr><td>4. B – Ways Ways $= {}^{10}\mathbf{C}_2 \times {}^{14}\mathbf{C}_2$
$= 45 \times 91$
$= 4095$</td><td>1</td><td></td></tr>
<tr><td>5. C - $f(x) = \cos^{-1}\left(\dfrac{4}{x}\right)$
$f'(x) = \dfrac{-\left(-\dfrac{4}{x^2}\right)}{\sqrt{1-\dfrac{16}{x^2}}}$
$= \dfrac{4}{x^2\sqrt{1-\dfrac{16}{x^2}}}$
$= \dfrac{4}{x\sqrt{x^2-16}}$</td><td>1</td><td></td></tr>
<tr><td>6. A - In quadrant 2; $x < 0$, $y > 0$, $\dfrac{dy}{dx} > 0$
$\therefore$ (A) or (C)
In quadrants 3 & 4; $\dfrac{dy}{dx} < 0$
$\therefore$ (C) is not possible, thus (A)</td><td>1</td><td></td></tr>
<tr><td>7. C – There are 24 possible pairings that add up to 50;
{1, 49} , {2 , 48} , {3 , 47} , … , {24 , 26}, however a students could also write down 25, so there are 25 “holes” for the k numbers to be placed in.
$\left[\dfrac{k}{25}\right] = 1$
$1 < \dfrac{k}{25} \leq 2$
$25 < k \leq 50$
$\therefore$ there are at least 26 students in the class</td><td>1</td><td></td></tr>
</table>

Solution	Marks	Comments
8. A - At time t the mass of undissolved chemical is $(8 - x)$ kilograms $\frac{dx}{dt} = 5\%\text{ of mass}$ $= \frac{1}{20} \times (8 - x)$ $= \frac{8 - x}{20}$	**1**	
9. D – $\overrightarrow{OB} = \overrightarrow{OC} + \overrightarrow{CB}$ $= \underset{\sim}{r} - \underset{\sim}{s}$ $\therefore \overrightarrow{OA} = \underset{\sim}{s} - \underset{\sim}{r}$ $\overrightarrow{AC} = \overrightarrow{OC} - \overrightarrow{OA}$ $= \underset{\sim}{r} - (\underset{\sim}{s} - \underset{\sim}{r})$ $= 2\underset{\sim}{r} - \underset{\sim}{s}$	**1**	
10. B - $P(-b) = 1$ $b^2 + 2ab + a^4 = 1$ $Q\left(\frac{1}{a}\right) = 0$ $\frac{b}{a^2} + \frac{1}{a} + 1 = 0$ $b + a + a^2 = 0$ $b = -a^2 - a$ $(-a^2 - a)^2 + 2a(-a^2 - a) + a^4 = 1$ $a^4 + 2a^3 + a^2 - 2a^3 - 2a^2 + a^4 = 1$ $2a^4 - a^2 - 1 = 0$ $(2a^2 + 1)(a^2 - 1) = 0$ $a^2 = -\frac{1}{\sqrt{2}}$ or $a^2 = 1$ no real solutions $\quad a = \pm 1$ $\therefore b = -(-1)^2 - (-1)$ or $b = -1^2 - 1$ $= 0 \quad\quad = -2$	**1**	

SECTION II		
Solution	**Marks**	**Comments**
QUESTION 11		
11(a) $\displaystyle\int_0^1 \frac{dx}{1+3x^2} = \frac{1}{\sqrt{3}}\int_0^1 \frac{\sqrt{3}\,dx}{1+(\sqrt{3}x)^2}$ $\displaystyle = \frac{1}{\sqrt{3}}\left[\tan^{-1}(\sqrt{3}x)\right]_0^1$ $\displaystyle = \frac{1}{\sqrt{3}}(\tan^{-1}\sqrt{3} - \tan^{-1} 0)$ $\displaystyle = \frac{1}{\sqrt{3}} \times \frac{\pi}{3}$ $\displaystyle = \frac{\pi}{3\sqrt{3}}$	3	**3 marks** • Correct solution **2 marks** • Correct primitive **1 mark** • Obtains a primitive of the form $k\tan^{-1}(\sqrt{3}x)$
11 (b)  $\displaystyle \tan\alpha = \frac{9}{2}$ $\displaystyle \alpha = \tan^{-1}\frac{9}{2}$ $= 1.3521\ldots$ $2\cos x + 9\sin x = 1$ $\sqrt{85}\cos(x-\alpha) = 1$ $\displaystyle \cos(x-\alpha) = \frac{1}{\sqrt{85}}$ $\displaystyle x - \tan^{-1}\frac{9}{2} = \cos^{-1}\frac{1}{\sqrt{85}}$ $x - 1.352 = 1.462\ ,\ 4.821$ $x = 2.81\ ,\ 6.17$	4	**4 marks** • Correct solution **3 marks** • Correctly writes $2\cos x + 9\sin x$ in the form $A\cos(x-\alpha)$ and finds one solution **2 marks** • Finds the values of both A and α **1 mark** • Finds A or α *Note: no penalty for rounding, if it is clear how* α *has been established*
11(c) $\displaystyle\int \frac{dx}{\sqrt{x}-1}$ $\displaystyle = 2\int \frac{\sqrt{x}}{\sqrt{x}-1} \times \frac{dx}{2\sqrt{x}}$ $\displaystyle = 2\int \frac{u+1}{u}du$ $\displaystyle = 2\int \left(1+\frac{1}{u}\right)du$ $= 2[u + \ln\|u\|] + c$ $= 2(\sqrt{x}-1) + 2\ln\|\sqrt{x}-1\| + c$ $= 2\sqrt{x} + \ln(\sqrt{x}-1)^2 + c$ $u = \sqrt{x} - 1$ $\displaystyle du = \frac{dx}{2\sqrt{x}}$ ***OR*** $u = \sqrt{x} - 1 \Rightarrow x = (u+1)^2$ $dx = 2(u+1)du$ $\displaystyle\int \frac{dx}{\sqrt{x}-1} = \int \frac{2(u+1)du}{u}$	3	**3 marks** • Correct solution **2 marks** • Obtains correct primitive function in terms of u **1 mark** • Uses the given substitution to find a correct integrand in terms of u.
11(d) $\displaystyle \frac{dy}{dx} = \frac{2ye^{2x}}{1+e^{2x}}$ $\displaystyle \int_\pi^y \frac{dy}{y} = \int_0^x \frac{2e^{2x}}{1+e^{2x}}dx$ $\displaystyle \left[\ln\|y\|\right]_\pi^y = \left[\ln\|1+e^{2x}\|\right]_0^x$ $\displaystyle \ln\left\|\frac{y}{\pi}\right\| = \ln\left\|\frac{1+e^{2x}}{2}\right\|$ $\displaystyle \frac{y}{\pi} = \frac{1+e^{2x}}{2}$ $\displaystyle y = \frac{\pi(1+e^{2x})}{2}$	3	**3 marks** • Correct solution **2 marks** • Obtains $\ln y = \ln(1+e^{2x})$ or equivalent **1 mark** • Correctly separates the variables

Solution	Marks	Comments
11 (e) $x + y = 4$, $m = -1$, direction vector $= \begin{pmatrix} 1 \\ -1 \end{pmatrix}$ $3x - 4y = 2$, $m = \dfrac{3}{4}$, direction vector $= \begin{pmatrix} 4 \\ 3 \end{pmatrix}$ $\cos\theta = \dfrac{\begin{pmatrix} 1 \\ -1 \end{pmatrix} \cdot \begin{pmatrix} 4 \\ 3 \end{pmatrix}}{\sqrt{1^2 + 1^2}\sqrt{4^2 + 3^2}}$ $= \dfrac{4 - 3}{\sqrt{2}\sqrt{25}}$ $= \dfrac{1}{5\sqrt{2}}$ $\theta = 82°$ (to nearest degree)	**3**	**3 marks** • Correct solution **2 marks** • Uses the dot product in an attempt to find $\cos\theta$ **1 mark** • Translates both slopes into direction vectors
QUESTION 12		
12 (a) (i) # of arrangements = $(n - 1)!$	**1**	**1 mark** • Correct answer
12 (a) (ii) $P(3 \text{ together}) = \dfrac{3!(n-3)!}{(n-1)!}$ $= \dfrac{6(n-3)!}{(n-1)(n-2)(n-3)!}$ $= \dfrac{6}{(n-1)(n-2)}$	**2**	**2 marks** • Correct solution **1 mark** • Calculates the number of arrangements with three particular people sitting together
12 (b) 	**3**	**3 marks** Provides correct sketch **2 marks** • Provides correct shape and asymptote • Key features correct with a concavity change in the curve **1 mark** • Provides sketch with at least two of the following key features; • Asymptote at $x = 3$ • Minimum turning point at $\left(1, \dfrac{1}{\sqrt{2}}\right)$ • Domain $x < 3$ $\lim_{x \to -\infty} = \infty$
12 (c) (i) $-1 \le x - 1 \le 1$ $0 \le x \le 2$	**1**	**1 mark** • Correct answer
12 (c) (ii) 	**2**	**2 marks** • Correct solution **1 mark** • Correct domain and range • Correct orientation of the inverse cosine graph

<table>
<tr><th>Solution</th><th>Marks</th><th>Comments</th></tr>
<tr><td>

12 (c) (iii) $y = \cos^{-1}(x-1) \Rightarrow x = \cos y + 1$

$$V = \pi\int_0^{\pi} (\cos y + 1)^2 dy$$
$$= \pi\int_0^{\pi} (\cos^2 y + 2\cos y + 1)dy$$
$$= \pi\int_0^{\pi} \left[\frac{1}{2}(1 + \cos 2y) + 2\cos y + 1\right]dy$$
$$= \pi\int_0^{\pi} \left(\frac{3}{2} + \frac{1}{2}\cos 2y + 2\cos y\right)dy$$
$$= \pi\left[\frac{3y}{2} + \frac{1}{4}\sin 2y + 2\sin y\right]_0^{\pi}$$
$$= \pi\left(\frac{3\pi}{2} + 0 + 0 - 0\right)$$
$$= \frac{3\pi^2}{2}\text{ units}^3$$

</td><td>3</td><td>

3 marks
- Correct solution

2 marks
- Finds the primitive function

1 mark
- Writes integrand in terms of y

</td></tr>
<tr><td>

12 (d) Let $\overrightarrow{OA} = \underset{\sim}{a}$ and $\overrightarrow{OC} = \underset{\sim}{c}$

$\overrightarrow{AC} = \underset{\sim}{c} - \underset{\sim}{a}$ and $\overrightarrow{OB} = \underset{\sim}{a} + \frac{1}{2}(\underset{\sim}{c} - \underset{\sim}{a})$

Since $OB \perp AC$;

$$\left[\underset{\sim}{a} + \frac{1}{2}(\underset{\sim}{c} - \underset{\sim}{a})\right]\cdot(\underset{\sim}{c} - \underset{\sim}{a}) = 0$$
$$\underset{\sim}{a}\cdot\underset{\sim}{c} - \underset{\sim}{a}\cdot\underset{\sim}{a} + \frac{1}{2}(\underset{\sim}{c} - \underset{\sim}{a})\cdot(\underset{\sim}{c} - \underset{\sim}{a}) = 0$$
$$\underset{\sim}{a}\cdot\underset{\sim}{c} - \underset{\sim}{a}\cdot\underset{\sim}{a} + \frac{1}{2}(\underset{\sim}{c}\cdot\underset{\sim}{c} - 2\underset{\sim}{a}\cdot\underset{\sim}{c} + \underset{\sim}{a}\cdot\underset{\sim}{a}) = 0$$
$$\underset{\sim}{c}\cdot\underset{\sim}{c} - \underset{\sim}{a}\cdot\underset{\sim}{a} = 0$$
$$|\underset{\sim}{c}|^2 = |\underset{\sim}{a}|^2$$
$$|\underset{\sim}{c}| = |\underset{\sim}{a}|$$
$$\therefore OC = OA$$

and thus ΔOAC is isosceles

</td><td>3</td><td>

3 marks
- Correct solution

2 marks
- Uses the fact that $OB \perp AC$ to show that the dot product equals zero

1 mark
- Attempts to express $\overrightarrow{BC}$ in terms of $\overrightarrow{OA}$ and $\overrightarrow{AC}$, or equivalent merit.

NOTE:
Simply labelling the sides as the magnitude of a vector is NOT using vectors to solve the problem

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QUESTION 13

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13 (a) When $n = 1$;

$LHS = 1^3 = 1$

$RHS = \frac{1}{4}(1)^2(1+1)^2 = \frac{1}{4} \times 1 \times 4 = 1$

$\therefore$ LHS=RHS

Hence the result is true for $n = 1$

Assume the result is true for $n = k$, where $k \in \mathbb{Z}^+$

i.e. $1^3 + 2^3 + 3^3 + \ldots + k^3 = \frac{1}{4}k^2(k+1)^2$

Prove the result is true for $n = k + 1$

i.e. $1^3 + 2^3 + 3^3 + \ldots + (k+1)^3 = \frac{1}{4}(k+1)^2(k+2)^2$

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There are 4 key parts of the induction;

1. Proving the result true for $n = 1$
2. Clearly stating the assumption and what is to be proven
3. Using the assumption in the proof
4. Correctly proving the required statement

3 marks
- Successfully does all of the 4 key parts

2 marks
- Successfully does 3 of the 4 key parts

1 mark
- Successfully does 2 of the 4 key parts

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Solution	Marks	Comments
13 (a)…*continued.* ***PROOF:*** $1^3 + 2^3 + 3^3 + \ldots + \ + k^3 + (k+1)^3$ $= \frac{1}{4}k^2(k+1)^2 + (k+1)^3$ (by assumption) $= \frac{1}{4}(k+1)^2[k^2 + 4(k+1)]$ $= \frac{1}{4}(k+1)^2(k^2 + 4k + 4)$ $= \frac{1}{4}(k+1)^2(k+2)^2$ Hence the result is true for $n = k+1$, if it is true for $n = k$ Since the result is true for $n = 1$, then it is true $\forall\ n \in \mathbb{Z}^+$ by induction.		
13 (b) (i) when $t = 4$, $x = 64$, $y = -32$ $64 = 4V\cos\theta$ $\quad$ $-32 = 4V\sin\theta - 80$ $V\cos\theta = 16$ $\quad$ $V\sin\theta = 12$ $\therefore \tan\theta = \frac{12}{16}$ $\quad$ $\Rightarrow \sin\theta = \frac{3}{5}$ $= \frac{3}{4}$ $\quad$ $\frac{3V}{5} = 12$ $\theta = 37°$ $\quad$ $V = 20$ m/s	3	**3 marks** • Correct solution **2 marks** • Finds V or θ **1 mark** • Finds two distinct equations in terms of V and θ
13 (b) (ii) $V = 20$, $\sin\theta = \frac{3}{5}$, $\cos\theta = \frac{4}{5}$ $\underset{\sim}{r}(t) = \begin{pmatrix} 16t \\ 12t - 5t^2 \end{pmatrix}$ $\underset{\sim}{v}(t) = \begin{pmatrix} 16 \\ 12 - 10t \end{pmatrix}$ when $t = 4$, $\underset{\sim}{v}(4) = \begin{pmatrix} 16 \\ -28 \end{pmatrix}$ $\lvert\underset{\sim}{v}\rvert = \sqrt{16^2 + 28^2}$ $\quad$ $\tan\alpha = \frac{\dot{y}}{\dot{x}}$ $= \sqrt{1040}$ $= 32$ m/s $\quad$ $= -\frac{28}{16}$ $\alpha = -60°$ Thus the particle impacts the beach at 32 m/s at an angle of 60° to the horizontal	3	**3 marks** • Correct solution **2 marks** • Finds speed or the angle of impact **1 mark** • Finds $\underset{\sim}{v}\ (t)$ or equivalent
13 (c) (i) $\sin\left(2\theta + \frac{\pi}{4}\right) = 3\cos\left(2\theta + \frac{\pi}{4}\right)$ $\tan\left(2\theta + \frac{\pi}{4}\right) = 3$ $\dfrac{\tan 2\theta + \tan\frac{\pi}{4}}{1 - \tan 2\theta\tan\frac{\pi}{4}} = 3$ $\dfrac{\tan 2\theta + 1}{1 - \tan 2\theta} = 3$ $\tan 2\theta + 1 = 3 - 3\tan 2\theta$ $4\tan 2\theta = 2$ $\tan 2\theta = \frac{1}{2}$	2	**2 marks** • Correct solution **1 mark** • Correct use of compound angle formula

Solution	Marks	Comments
13 (c) (ii) $\dfrac{2\tan\theta}{1-\tan^2\theta} = \dfrac{1}{2}$ $4\tan\theta = 1 - \tan^2\theta$ $\tan^2\theta + 4\tan\theta - 1 = 0$ $\tan\theta = \dfrac{-4\pm\sqrt{20}}{2}$ $= -2\pm\sqrt{5}$ However θ is obtuse, so $\tan\theta < 0$ $\therefore \tan\theta = -2-\sqrt{5}$	3	**3 marks** • Correct solution with justification **2 marks** • Finds two valid possibilities for $\tan\theta$ **1 mark** • Correct use of double angle formula • Finds two possible values using a correct method
QUESTION 14		
14 (a) $P(x) = x^3 + 3ax + b$ $P'(x) = 3x^2 + 3a$ Let the roots be α, α and β $P'(\alpha) = 0$ $3\alpha^2 + 3\alpha = 0$ $\alpha^2 = -a$ $\alpha + \alpha + \beta = 0 \quad (\Sigma\alpha)$ $\beta = -2\alpha$ $\alpha^2\beta = -b \quad (\Pi\alpha)$ $-2\alpha^3 = -b$ $4\alpha^6 = b^2$ $4(-a)^3 = b^2$ $-4a^3 = b^2$ $4a^3 + b^2 = 0$	3	**3 marks** • Correct solution **2 marks** • Creates multiple distinct valid equations linking the coefficient to the roots **1 mark** • Finds a relationship between the coefficients and the roots.
14 (b) (i) $P(\text{customer takes sugar}) = 0.6\times 0.2 + 0.35\times 0.8$ $= 0.4$	1	**1 mark** • Correct solution
14 (b) (ii) Let X = # of customers taking suger $X \sim \text{Bin}(10,0.4)$ $P(X=4) = \binom{10}{4}(0.6)^6(0.4)^4$ $= 0.2508$	1	**1 mark** • Correct expression
14 (b) (iii) $X \sim \text{Bin}(10,0.4)$ $\text{E}(X) = np$ $= 150\times 0.4$ $= 60$ $\text{Var}(X) = np(1-p)$ $= 150\times 0.4\times 0.6$ $= 36$ $X \sim \text{N}(60,36)$ $P(X \geq 75) = P\left(Z \geq \dfrac{75-60}{6}\right)$ $= P(Z \geq 2.5)$ $= 1 - \Phi(2.5)$ $= 1 - 0.9938$ $= 0.0062$	3	**3 marks** • Correct solution **2 marks** • Finds E(X) and Var(X) **1 mark** • Finds E(X) or Var(X)

Solution	Marks	Comments
14 (c) (i) $(1+x)^{n+3} = \binom{n+3}{0} + \binom{n+3}{1}x + \ldots + \binom{n+3}{k}x^k + \ldots + \binom{n+3}{n+3}x^{n+3}$ coefficient of x^k on LHS $= \binom{n+3}{k}$ $(1+x)^n = \binom{n}{0} + \binom{n}{1}x + \binom{n}{2}x^2 + \ldots + \binom{n}{n}x^n$ $(1+x)^3 = \binom{3}{0} + \binom{3}{1}x + \binom{3}{2}x^2 + \binom{3}{3}x^3$ term with x^k on RHS $= \binom{3}{0}\binom{n}{k}x^k + \binom{3}{1}x\binom{n}{k-1}x^{k-1} + \binom{3}{2}x^2\binom{n}{k-2}x^{k-2} + \binom{3}{3}x^3\binom{n}{k-3}x^{k-3}$ $\therefore$ coefficient x^k on RHS $= \binom{3}{0}\binom{n}{k} + \binom{3}{1}\binom{n}{k-1} + \binom{3}{2}\binom{n}{k-2} + \binom{3}{3}\binom{n}{k-3}$ Hence $\binom{n+3}{k} = \binom{3}{0}\binom{n}{k} + \binom{3}{1}\binom{n}{k-1} + \binom{3}{2}\binom{n}{k-2} + \binom{3}{3}\binom{n}{k-3}$ $= \binom{n}{k} + 3\binom{n}{k-1} + 3\binom{n}{k-2} + \binom{n}{k-3}$	2	**2 marks** • Correct solution **1 mark** • Correctly uses Binomial Theorem
14 (c) (ii) $3 \le k \le n$	1	**1 mark** • Correct answer
14 (d) (i) $2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$ $= 2 \times \frac{1}{2}\left[\cos\left(\frac{x+y}{2} - \frac{x-y}{2}\right) + \cos\left(\frac{x+y}{2} + \frac{x-y}{2}\right)\right]$ $= \cos\left(\frac{2y}{2}\right) + \cos\left(\frac{2x}{2}\right)$ $= \cos y + \cos x$	1	**1 mark** • Correct solution
14 (d) (ii) $\cos 4\theta + \cos 3\theta + \cos 2\theta + \cos\theta = 0$ $2\cos\frac{7\theta}{2}\cos\frac{\theta}{2} + 2\cos\frac{3\theta}{2}\cos\frac{\theta}{2} = 0$ $\cos\frac{\theta}{2}\left(\cos\frac{7\theta}{2} + \cos\frac{3\theta}{2}\right) = 0$ $\cos\frac{\theta}{2}\left(2\cos\frac{5\theta}{2}\cos\theta\right) = 0$ $\cos\frac{\theta}{2} = 0$ $\quad$ $\cos\frac{5\theta}{2} = 0$ $\quad$ $\cos\theta = 0$ $\frac{\theta}{2} = \frac{\pi}{2}$ $\quad$ $\frac{5\theta}{2} = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}, \frac{9\pi}{2}$ $\quad$ $\theta = \frac{\pi}{2}, \frac{3\pi}{2}$ $\theta = \pi$ $\quad$ $\theta = \frac{\pi}{5}, \frac{3\pi}{5}, \pi, \frac{7\pi}{5}, \frac{9\pi}{5}$ $\theta = \frac{\pi}{5}, \frac{\pi}{2}, \frac{3\pi}{5}, \pi, \frac{7\pi}{5}, \frac{3\pi}{2}, \frac{9\pi}{5}$	3	**3 marks** • Correct solution **2 marks** • Breaks the equation into three simple trig equations **1 mark** • Reduces a pair of terms into a single term