BAULKHAM HILLS
HIGH
SCHOOL

2025

YEAR 12
TRIAL
HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Extension 1

General Instructions

- Reading time – 10 minutes
- Working time – 2 hours
- Write using black pen
- Calculators approved by NESA may be used
- A reference sheet is provided at the back of this paper
- For questions in Section II, show relevant mathematical reasoning and/or calculations
- Write your NESA# and Teacher at the top of the answer booklet
- Marks may be deducted for careless or badly arranged work

Total marks: 70

Section I – 10 marks (pages 2 – 7)
- Attempt Questions 1 – 10
- Allow about 15 minutes for this section

Section II – 60 marks (pages 8 – 16)
- Attempt Questions 11 – 14
- Allow about 1 hour and 45 minutes for this section

Section I

10 marks
Attempt Questions 1 – 10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1 – 10

1 Given that $\underset{\sim}{u} = \begin{pmatrix} -2 \\ 1 \end{pmatrix}$ and $\underset{\sim}{v} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$, which of the following best describes the direction of the projection of $\underset{\sim}{u}$ onto $\underset{\sim}{v}$?

(A) the projection is in the same direction as $\underset{\sim}{u}$.

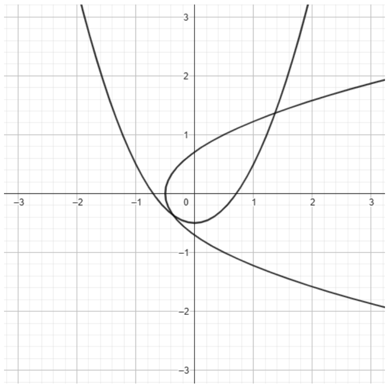
(B) the projection is in the same direction as $\underset{\sim}{v}$.

(C) the projection is in the opposite direction to $\underset{\sim}{u}$.

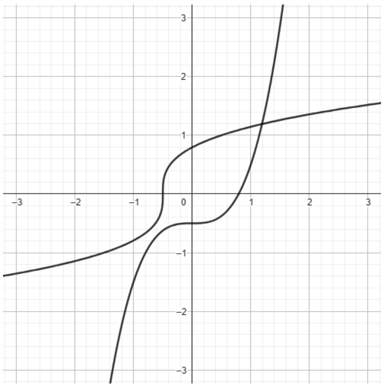
(D) the projection is in the opposite direction to $\underset{\sim}{v}$.

2 Which of the following best represents the graphs of a function and its inverse function?

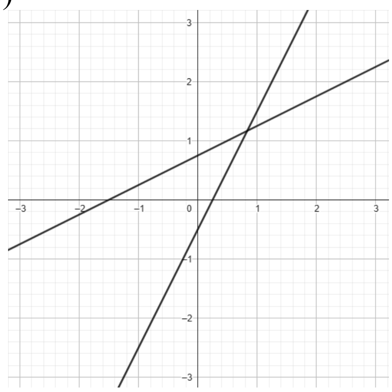
(A)



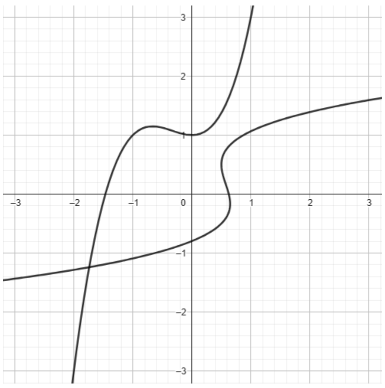
(B)



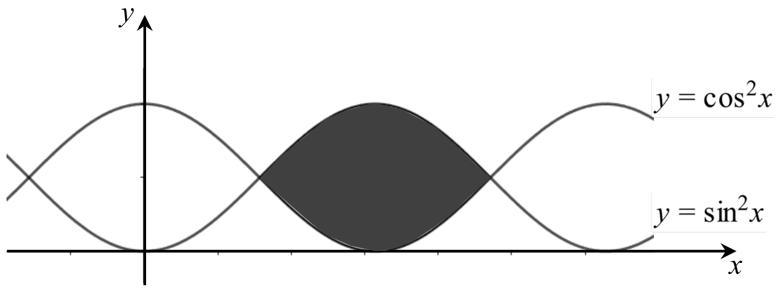
(C)



(D)



3 Consider the functions $y = \sin^2 x$ and $y = \cos^2 x$ and the region shaded in the diagram below.



What is the area of the shaded region?

(A) $\frac{1}{2}$ units^2

(B) 1 units^2

(C) $\frac{\pi}{2}$ units^2

(D) 2 units^2

4 What are the domain and range of the function $y = \sin^{-1} x + \tan^{-1} x + \cot^{-1} x$?

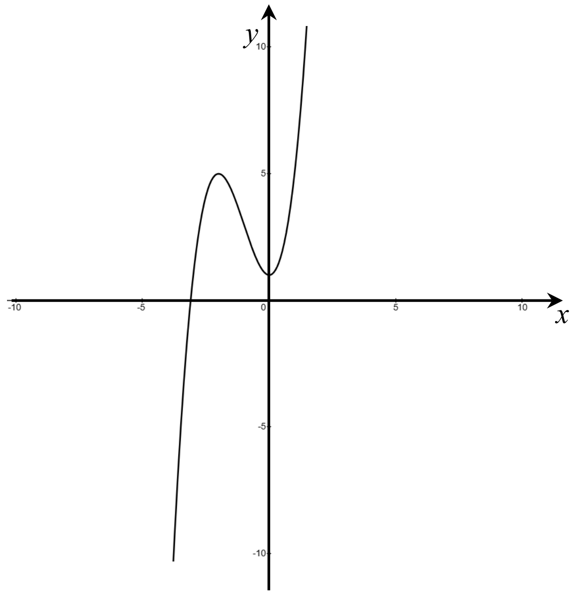
(A) Domain: $[-\infty, \infty]$ and Range: $[0, \pi]$

(B) Domain: $[-\infty, \infty]$ and Range: $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

(C) Domain: $[-1, 1]$ and Range: $[0, \pi]$

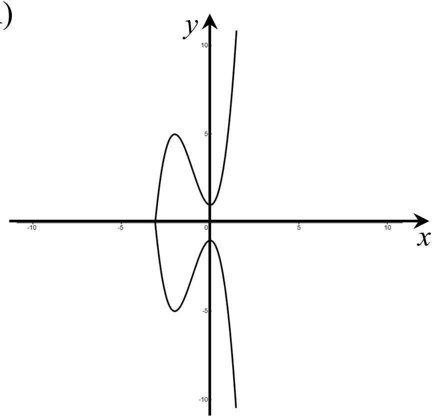
(D) Domain: $[-1, 1]$ and Range: $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

5 Consider the graph $y = f(x)$ drawn below.

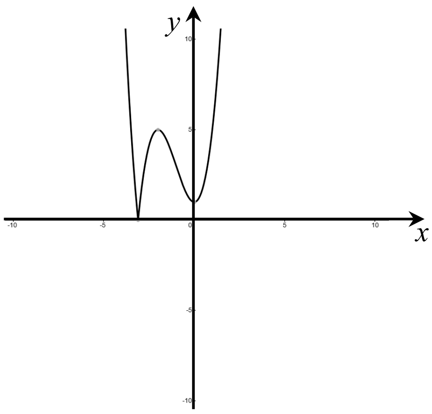


Which of the following shows the graph of $|y| = f(x)$?

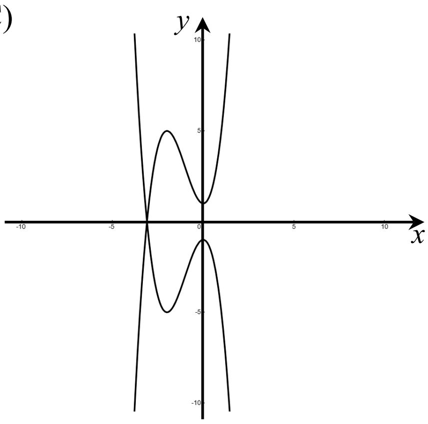
(A)



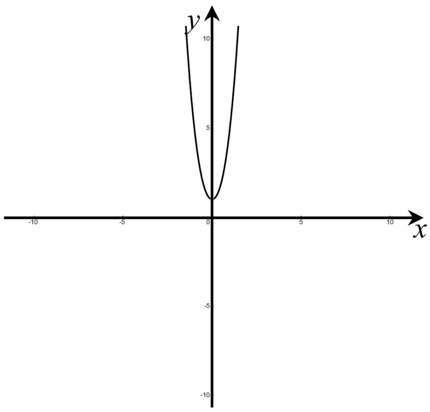
(B)



(C)

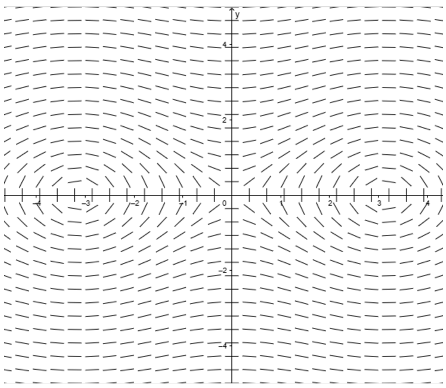


(D)

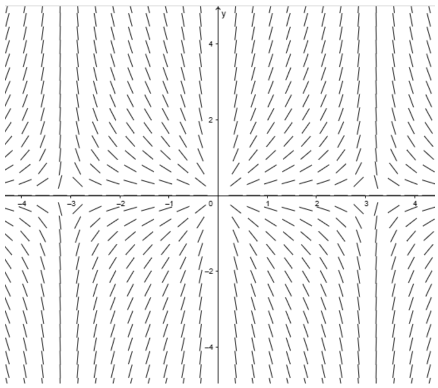


6 Which of the following best represents the direction field for the differential equation $\dfrac{dy}{dx} = \dfrac{\sin y}{x}$?

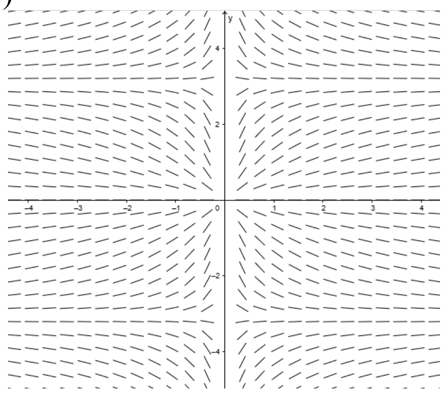
(A)



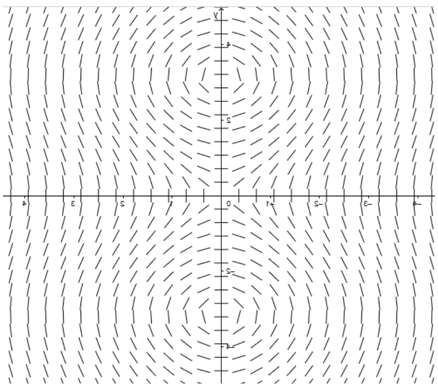
(B)



(C)



(D)



7 Five teachers and five students are to be seated around a circular table.

In how many ways can this be done if the teachers and students must alternate?

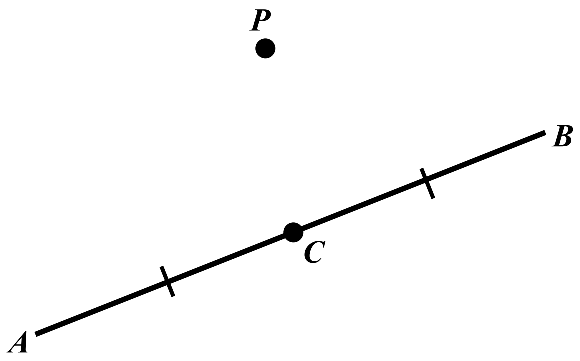
(A) $2 \times 4! \times 4!$

(B) $4! \times 4!$

(C) $4! \times 5!$

(D) $5! \times 5!$

8 The point C is the midpoint of AB, and P is any point not on the interval AB.



Which of the following statements is always correct?

(A) $\overrightarrow{PA} + \overrightarrow{PB} = 2\overrightarrow{PC}$.

(B) $\overrightarrow{PA} + \overrightarrow{PB} = \overrightarrow{PC}$.

(C) $\overrightarrow{PA} + \overrightarrow{PB} + 2\overrightarrow{PC} = 0$.

(D) $\overrightarrow{PA} + \overrightarrow{PB} + \overrightarrow{PC} = 0$.

9 Given that $\sec\theta + \tan\theta = x$, what is the value of $\tan\theta$?

(A) $\dfrac{x^2 + 1}{x}$

(B) $\dfrac{x^2 - 1}{x}$

(C) $\dfrac{x^2 + 1}{2x}$

(D) $\dfrac{x^2 - 1}{2x}$

10 Let $f(x) = (2x + 1)^3$ and $g(x) = f^{-1}(x)$.

What is the value of $g'(1)$?

(A) 6

(B) $\frac{1}{6}$

(C) $\frac{1}{27}$

(D) $\frac{1}{54}$

END OF SECTION I

Section II

60 marks
Attempt Questions 11 – 14
Allow about 1 hour and 45 minutes for this section

Answer each question on the appropriate answer sheet. Extra paper is available.

For questions in Section II, your responses should include relevant mathematical reasoning and/or calculations.

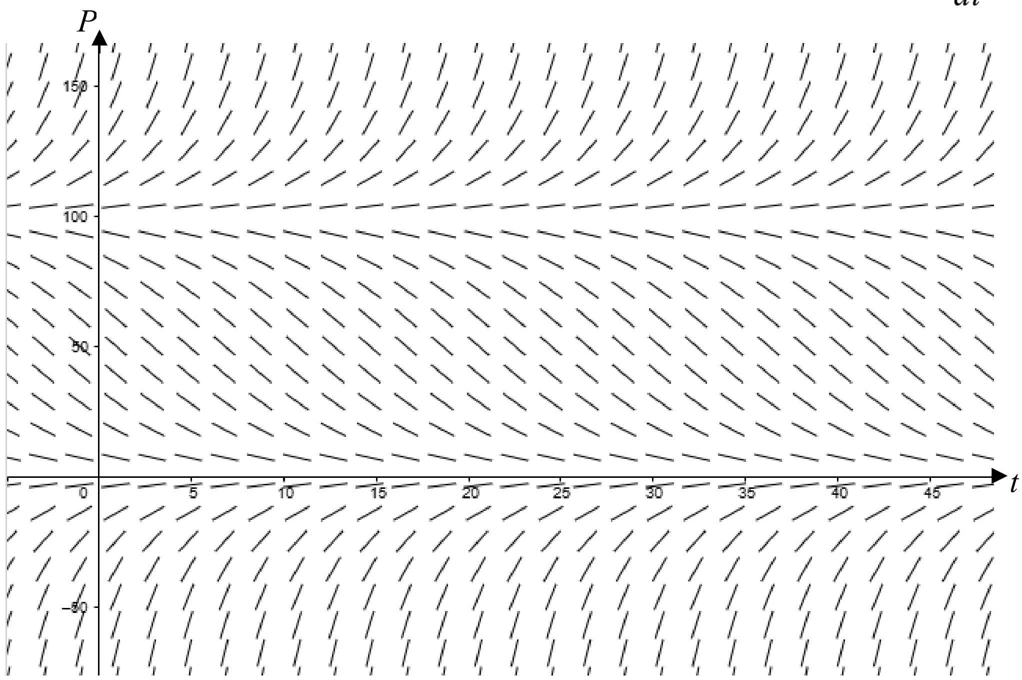
Question 11 (15 marks) Use the pages labelled Question 11 in the answer booklet

(a) Consider the vectors $\underset{\sim}{u} = 5\underset{\sim}{i} - 4\underset{\sim}{j}$ and $\underset{\sim}{v} = -2\underset{\sim}{i} + 3\underset{\sim}{j}$.

(i) Find $3\underset{\sim}{u} - 2\underset{\sim}{v}$. *1*

(ii) Calculate the angle between $\underset{\sim}{u}$ and $\underset{\sim}{v}$, correct to the nearest degree. *2*

(b) The diagram shows a direction field for a logistic differential equation $\dfrac{dP}{dt}$. *2*



Write down a possible differential equation for $\dfrac{dP}{dt}$ in terms of P.

Question 11 continues on page 9

Question 11 (continued)

(c) Find $\displaystyle\int \frac{dx}{\sqrt{25-4x^2}}$. *2*

(d) Solve $\dfrac{2x+1}{1-x} \geq 1$. *3*

(e) Show that there exists only one value of a for which the polynomial $P(x) = x^4 + 2x^3 - x^2 - 8x - a$ is divisible by $Q(x) = x^2 - 4$. *2*

(f) Solve $\sqrt{3}\sin x + \cos x = 1$ for $0 \leq x \leq 2\pi$. *3*

End of Question 11

Question 12 (14 marks) Use the pages labelled Question 12 in the answer booklet

(a) Solve the differential equation *3*

$$\frac{dy}{dx} = -2xy^2$$

(b) The parametric equations of a curve are given below *2*

$$x = t^2 + 6t - 16$$
$$y = 6\ln(t + 3)$$

Find the Cartesian equation of this curve in the form

$$y = a\ln(x + b)$$

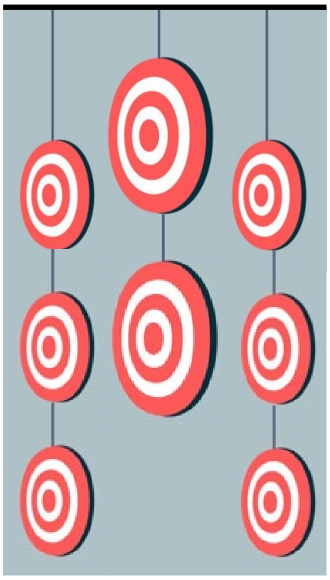
(c) Find the coefficient of x^2y^6 in the expansion of $\left(1 - \frac{y}{x}\right)(x + y)^8$. *2*

(d) Find $\int \sin x \sin 3x \; dx$. *2*

Question 12 continues on page 11

Question 12 (continued)

(e) In a shooting tournament, eight targets are arranged in two hanging columns of three and one hanging column of two, as pictured below. *2*



The shooter must break all eight targets according to the following rules:

(1) Before each shot, the shooter chooses a column from which a target is to be broken.

(2) The shooter must then break the lowest remaining target in that column.

If these rules are followed, in how many different orders can the eight targets be broken?

(f) Consider the function $f(x) = \sin^{-1}\left(\frac{1}{x}\right)$.

(i) Find the exact value of $f\left(\sqrt{2}\right)$. *1*

(ii) Find the equation of the tangent at the point where $x = \sqrt{2}$. Give your answer in the form $y = mx + c$. *2*

End of Question 12

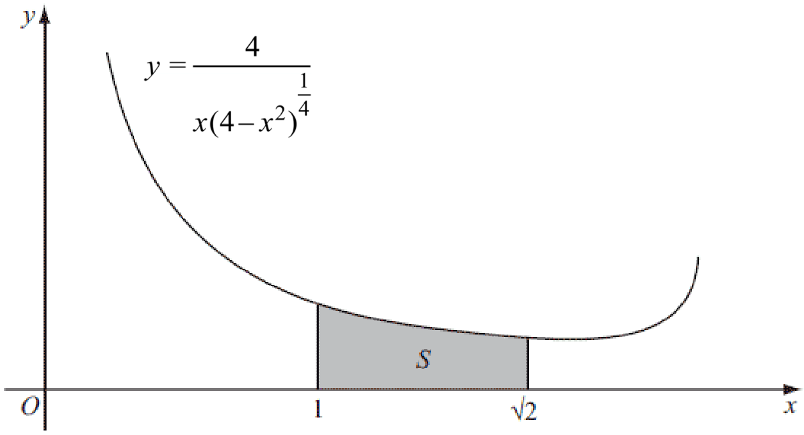
Question 13 (15 marks) Use the pages labelled Question 13 in the answer booklet

(a) A biased coin has a 0.7 chance of showing a head when tossed. *3*

What is the most likely combination of heads and tails to occur in a sequence of five tosses? Justify your answer with calculations.

(b) (i) Find the exact value of $\int_1^{\sqrt{2}} \frac{dx}{x^2\sqrt{4-x^2}}$ using the substitution $x = 2\cos u$. *3*

(ii) The region S, bounded by the curve $y = \frac{4}{x(4-x^2)^{\frac{1}{4}}}$, the lines $x = 1$ and $x = \sqrt{2}$ and the x-axis is shown. *2*



Find the exact volume of the solid of revolution when the region S is rotated about the x-axis.

(c) Use mathematical induction to prove *3*

$$1^3 + 3^3 + 5^3 + \ldots + (2n-1)^3 = n^2(2n^2-1)$$

for all integers $n \geq 1$.

Question 13 continues on page 13

Question 13 (continued)

(d) Yiwei is filling a container with capacity C litres, with water. *4*

The volume, V, in litres, of the water, t minutes after Yiwei starts filling the container can be modelled using the differential equation

$$\frac{dV}{dt} = k(C - V)$$

where k is a constant.

If one quarter of the container is filled in the first two minutes, using separation of variables, find what fraction of the container is filled in the next two minutes.

End of Question 13

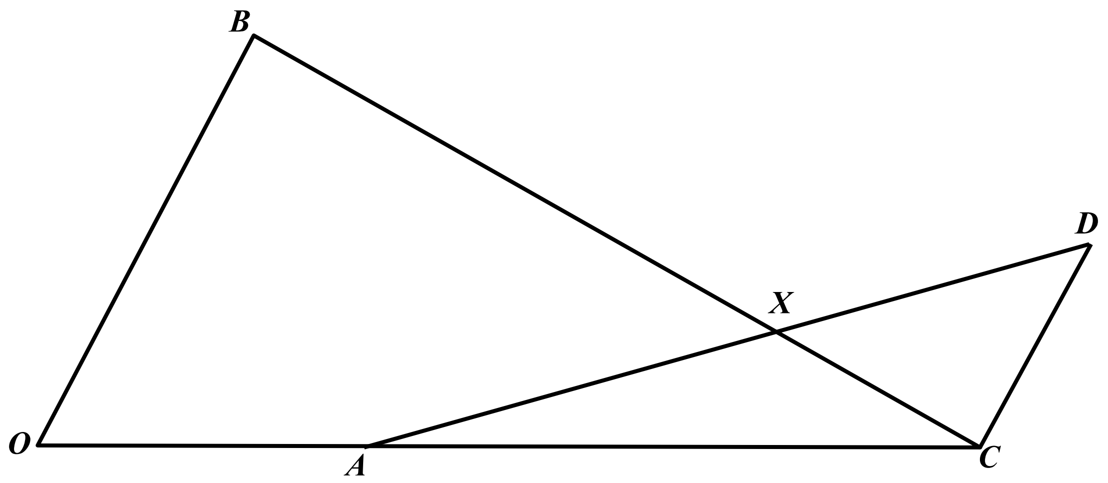
Question 14 (16 marks) Use the pages labelled Question 14 in the answer booklet

(a) The diagram shows two overlapping triangles OBC and ACD. *3*

The point X is the intersection of BC and AD.

The point A lies on AC so that $OA{:}OC = 1{:}3$.

OB is parallel to CD and $\left|\overrightarrow{OB}\right| = 2\left|\overrightarrow{CD}\right|$.



Let $\underset{\sim}{a} = \overrightarrow{OA}$ and $\underset{\sim}{b} = \overrightarrow{OB}$.

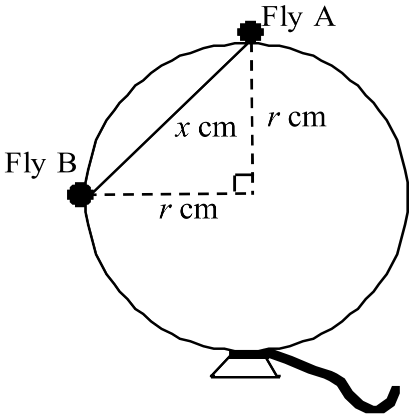
Find $\dfrac{\left|\overrightarrow{AX}\right|}{\left|\overrightarrow{AD}\right|}$.

Question 14 continues on page 15

Question 14 (continued)

(b) Two flies are sitting on a spherical balloon of radius r cm, while it is being inflated at a constant rate of 5 cm^3/s. The straight-line distance between the flies is x metres. *3*

Assume that the balloon has no air in it to begin with and that the flies are located at the North Pole and the Equator of the balloon, as illustrated.



How fast is the straight-line distance between the flies changing after 3 seconds? Give your answer correct to 2 decimal places.

(c) A lolly wholesaler has a machine that fills packets with lollies. As part of a special promotion the wholesaler announces that 1 in every 8 packets contains a prize.

The packets are packed in boxes, each containing 120 packets, before being delivered to shops.

Anika's Sweet Shop purchases 20 boxes of these sweets.

Anika selects a random sample of 50 packets from these boxes and complains to the manufacturer that she only found 5 prizes in her sample.

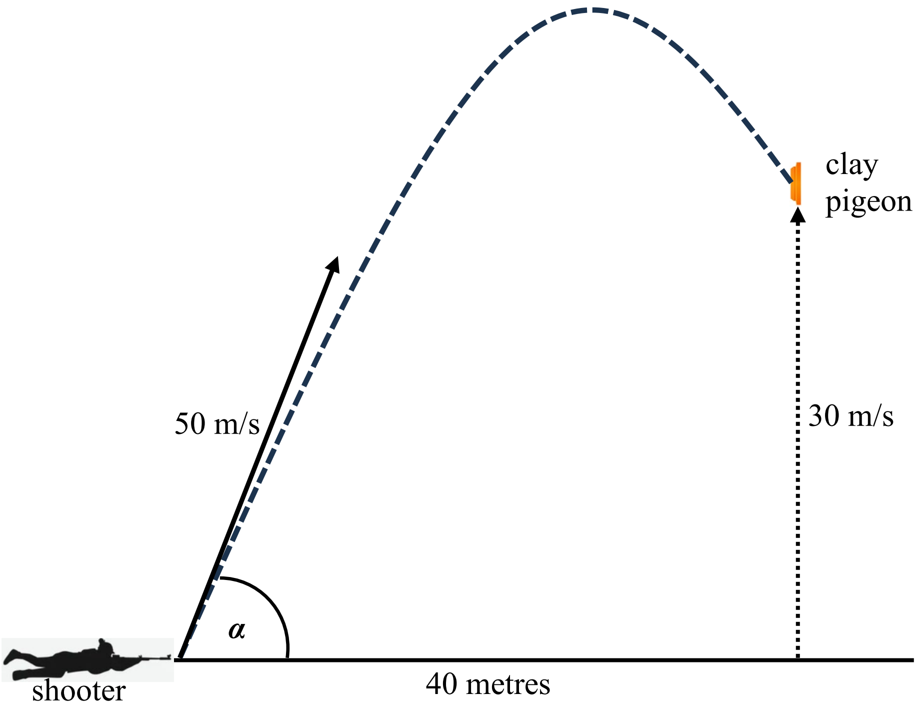
(i) Use the standard normal distribution and the information at the end of this paper to calculate the probability of Anika finding at least 5 prizes in her sample. *3*

(ii) Was it reasonable for Anika to use a normal approximation? Justify your answer with calculations. *1*

Question 14 continues on page 16

Question 14 (continued)

(d) In a clay pigeon shoot, the target is launched vertically from ground level with speed 30 m/s. After 2 seconds the competitor, who is at ground level and located 40 m away, fires a bullet at a speed of 50 m/s at angle of α degrees to the horizontal, towards the clay pigeon.



Taking the position of the shooter when the bullet is fired as the origin, the position of the bullet and the clay pigeon, at a time t seconds after the bullet is fired, are as follows

$$\vec{r}_B = \begin{pmatrix} 50t\cos\alpha \\ 50t\sin\alpha - 5t^2 \end{pmatrix} \quad \textit{bullet}$$

$$\vec{r}_C = \begin{pmatrix} 40 \\ 30(t+2) - 5(t+2)^2 \end{pmatrix} \quad \textit{clay pigeon}$$

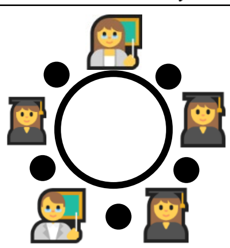
Do NOT prove these

(i) The bullet hits the clay pigeon. Show that this occurs $\frac{4}{3}$ seconds after it is fired. *3*

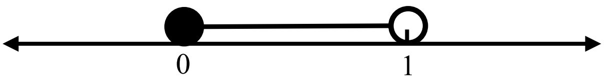
(ii) Find the impact angle, to the nearest degree, of the bullet with the clay pigeon. *3*

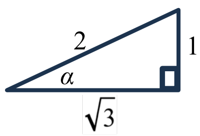
End of paper

BAULKHAM HILLS HIGH SCHOOL
YEAR 12 EXTENSION 1 TRIAL 2025 SOLUTIONS

Solution	Marks	Comments
SECTION I		
1. D – $\text{proj}_{\underset{\sim}{v}}\underset{\sim}{u} = \frac{\underset{\sim}{u}\cdot\underset{\sim}{v}}{\underset{\sim}{v}\cdot\underset{\sim}{v}}\underset{\sim}{v}$ $= \frac{-6+4}{9+16}\underset{\sim}{v}$ $= -\frac{2}{25}\underset{\sim}{v}$ $\therefore \text{proj}_{\underset{\sim}{v}}\underset{\sim}{u}$ is in the opposite direction to $\underset{\sim}{v}$	**1**	
2. B – visually a function and its inverse must • satisfy both a vertical and horizontal line test • be symmetric in the line $y = x$ Option B is the only option that satisfies both of these conditions	**1**	
3. B – $\sin^2 x = \cos^2 x$ $\tan^2 x = 1$ $\tan x = \pm 1$ $x = \frac{\pi}{4}, \frac{3\pi}{4}$ $A = \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} (\sin^2 x - \cos^2 x)dx$ $= -\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \cos 2x\, dx$ $= -\frac{1}{2}\left[\sin 2x\right]_{\frac{\pi}{4}}^{\frac{3\pi}{4}}$ $= -\frac{1}{2}(-1-1)$ $= 1$	**1**	
4. C – the domain of both $y = \tan^{-1}x$ and $y = \cot^{-1}x$ is all real x, thus the domain of $y = \sin^{-1}x + \tan^{-1}x + \cot^{-1}x$ is the same as the domain of $y = \sin^{-1}x$ $\tan^{-1}x + \cot^{-1}x = \frac{\pi}{2}$ thus the range of $y = \sin^{-1}x + \tan^{-1}x + \cot^{-1}x$ is the same as the domain of $y = \sin^{-1}x + \frac{\pi}{2}$ Domain: $[-1,1]$ $-\frac{\pi}{2} \le \sin^{-1}x \le \frac{\pi}{2}$ $0 \le \sin^{-1}x + \frac{\pi}{2} \le \pi$ Range: $[0,\pi]$	**1**	
5. A - $\lvert y\rvert = f(x) \Leftrightarrow \pm y = f(x)$ This means that the graphs of both $y = f(x)$ and $y = -f(x)$ should appear on the graph. Option A is the graph that shows this. *NOTE:* Option B is $y = f(\lvert x\rvert)$ Option C is $\lvert y\rvert = \lvert f(x)\rvert$ and Option D is $y = \lvert f(x)\rvert$	**1**	
6. C - $\frac{dy}{dx} = \frac{\sin y}{x}$ $x = 0 \Rightarrow \frac{dy}{dx}$ is undefined, this eliminates Option A and Option D $\sin y = 0$ $\Rightarrow$ rows of horizontal lines at these y values $y = 0, \pm\pi, \ldots$ Option C is the only option that satisfies both of these conditions *NOTE:* Option A is $\frac{dy}{dx} = \frac{\sin x}{y}$ Option B is $\frac{dy}{dx} = \frac{y}{\sin x}$ and Option D is $\frac{dy}{dx} = \frac{x}{\sin y}$	**1**	
7. C – The teachers sit down making sure they leave an empty chair either side, this can be done in 4! ways. This creates five spots for the students to be inserted into.  $\therefore$ #Ways $= 4! \times 5!$	**1**	

Solution	Marks	Comments
8. A - $\overrightarrow{PC} = \overrightarrow{PA} + \overrightarrow{AC}$ $\overrightarrow{PC} = \overrightarrow{PB} + \overrightarrow{BC}$ $= \overrightarrow{PB} - \overrightarrow{AC} \qquad (\overrightarrow{AC} = \overrightarrow{CB})$ $\therefore 2\overrightarrow{PC} = \overrightarrow{PA} + \overrightarrow{PB}$	**1**	
9. D – $\sec^2\theta - \tan^2\theta = 1$ $(\sec\theta + \tan\theta)(\sec\theta - \tan\theta) = 1$ $x(\sec\theta - \tan\theta) = 1$ $\sec\theta - \tan\theta = \frac{1}{x}$ $\sec\theta + \tan\theta = x$ $2\tan\theta = x - \frac{1}{x}$ $= \frac{x^2 - 1}{x}$ $\tan\theta = \frac{x^2-1}{2x}$	**1**	
10. B – as $f(x)$ and $g(x)$ are inverse functions, the gradient of $g(x)$ at $x = 1$ will be the reciprocal of the gradient of $f(x)$ at $f(x) = 1$. $f(x) = (2x+1)^3$ when $f(x) = 1$, $(2x+1)^3 = 1$ $f'(x) = 3(2x+1)^2(2)$ $2x + 1 = 1$ $= 6(2x+1)^2$ $x = 0$ $\therefore g'(1) = \frac{1}{f'(0)}$ $= \frac{1}{6(1)^2}$ $= \frac{1}{6}$	**1**	

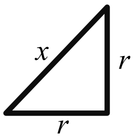
SECTION II		
Solution	**Marks**	**Comments**
QUESTION 11		
11(a) (i) $3\underset{\sim}{u}-2\underset{\sim}{v}=3\left(5\underset{\sim}{i}-4\underset{\sim}{j}\right)-2\left(-2\underset{\sim}{i}+3\underset{\sim}{j}\right)$ $=15\underset{\sim}{i}-12\underset{\sim}{j}+4\underset{\sim}{i}-6\underset{\sim}{j}$ $=19\underset{\sim}{i}-18\underset{\sim}{j}$	1	**1 mark** • Correct answer
11 (a) (ii) $\cos\theta=\dfrac{\underset{\sim}{u}\cdot\underset{\sim}{v}}{\lvert\underset{\sim}{u}\rvert\lvert\underset{\sim}{v}\rvert}$ $=\dfrac{-10-12}{\sqrt{5^2+4^2}\sqrt{2^2+3^2}}$ $=-\dfrac{22}{\sqrt{533}}$ $\theta=162.349...$ $=162°$ (to nearest degree)	2	**2 marks** • Correct solution **1 mark** • Finds $\lvert\underset{\sim}{u}\rvert$ and $\lvert\underset{\sim}{v}\rvert$ • Finds $\underset{\sim}{u}\cdot\underset{\sim}{v}$
11 (b) trivial solutions are $P=0$ and $P=100$ and between the trivial solutions the curve is deceasing $\therefore$ any solution of the form $\dfrac{dP}{dt}=kP(100-P)$ where $k<0$ is an acceptable answer. *NOTE:* $\dfrac{dP}{dt}=-0.0025P(100-P)$ is the equation for the given direction field.	2	**2 marks** • Correct solution **1 mark** • Any equation in the form $\dfrac{dP}{dt}=kP(100-P)$ • Identifies the two trivial solutions
11 (c) $\displaystyle\int\frac{dx}{\sqrt{25-4x^2}}=\frac{1}{2}\int\frac{2dx}{\sqrt{25-4x^2}}$ $=\dfrac{1}{2}\sin^{-1}\left(\dfrac{2x}{5}\right)+c$	2	**2 marks** • Correct solution **1 mark** • Obtains a primitive in the form $k\sin^{-1}\left(\dfrac{2x}{5}\right)$ or $k\cos^{-1}\left(\dfrac{2x}{5}\right)$
11 (d) $\dfrac{2x+1}{1-x}\geq 1$ $1-x\neq 0$ $2x+1=1-x$ $x\neq 1$ $3x=0$ $x=0$  $0\leq x<1$	3	**3 marks** • Correct graphical solution on number line or algebraic solution, with correct working **2 marks** • Bald answer • Identifies the correct critical points via a correct method • Correct conclusion to their critical points obtained using a correct method **1 mark** • Uses a correct method • Acknowledges a problem with the denominator.

Solution	Marks	Comments
11 (e) If $P(x)$ is divisible by x^2-4 , then it is divisible by both $(x-2)$ and $(x+2)$ $P(2)=2^4+2(2)^3-2^2-8(2)-a$ $P(-2)=(-2)^4+2(-2)^3-(-2)^2-8(-2)-a$ $0=12-a$ $0=12-a$ $a=12$ $a=12$ ∴ there is only one value of a, that being 12. *OR* $$\begin{array}{r} x^2+2x+3 \\ x^2-4 \overline{\smash{\big)}\, x^4+2x^3-x^2-8x-a} \\ \underline{x^4 \qquad -4x^2 \qquad\qquad} \\ 2x^3+3x^2-8x-a \\ \underline{2x^3 \qquad\quad -8x \qquad} \\ 3x^2 \qquad -a \\ \underline{3x^2 \qquad -12} \\ -a+12 \end{array}$$ if x^2-4 , is a factor then $R(x)=0$ $-a+12=0$ $a=12$ ∴ there is only one value of a, that being 12.	2	**2 marks** • Correct solution **1 mark** • Uses the factor theorem to show either $(x-2)$ or $(x+2)$ is a factor • Shows $R(x)=-a+12$
11 (f) $\sqrt{3}\sin x+\cos x=1$ for $0\le x\le 2\pi$ $2\sin\left(x+\frac{\pi}{6}\right)=1$ $\sin\left(x+\frac{\pi}{6}\right)=\frac{1}{2}$ $x+\frac{\pi}{6}=\frac{\pi}{6},\frac{5\pi}{6}$ $x=0,\frac{2\pi}{3},2\pi$  $\tan\alpha=\frac{1}{\sqrt{3}}$ $\alpha=\frac{\pi}{6}$ *OR* $\sqrt{3}\sin x+\cos x=1$ for $0\le x\le 2\pi$ $\frac{2\sqrt{3}}{1+t^2}+\frac{1-t^2}{1+t^2}=1$ let $t=\tan\left(\frac{x}{2}\right)$ $2\sqrt{3}t+1-t^2=1+t^2$ $2t^2-2\sqrt{3}t=0$ $2t\left(t-\sqrt{3}\right)=0$ $t=0$ or $t=\sqrt{3}$ $\tan\frac{x}{2}=0$ $\tan\frac{x}{2}=\sqrt{3}$ $0\le\frac{x}{2}\le\pi$ $\frac{x}{2}=0,\pi$ $\frac{x}{2}=\frac{\pi}{3}$ $x=0,2\pi$ $x=\frac{2\pi}{3}$ $\therefore x=0,\frac{2\pi}{3},2\pi$	3	**3 marks** • Correct solution **2 marks** • Obtains an equation or graph involving a single trig function • Obtains a quadratic equation in terms of t, and attempts to solve it. **1 mark** • Attempts to write the sum of two trig functions as a single trig function • Uses the t-results
QUESTION 12		
12 (a) $\frac{dy}{dx}=-2xy^2$ $y=0$ is a trivial solution $\int\frac{dy}{y^2}=-\int 2x\,dx$ $-\frac{1}{y}=-x^2+c$ $y=\frac{1}{x^2-c}$ $\therefore\ y=\frac{1}{x^2-c}$ or $y=0$	3	**3 marks** • Correct solution **2 marks** • Finds the non-trivial solution **1 mark** • Correctly separates the variables • Finds the trivial solution

Solution	Marks	Comments
12 (b) $x = t^2 + 6t - 16$ $t^2 + 6t - (16 + x) = 0$ $t = \dfrac{-6 \pm \sqrt{36 + 4(16 + x)}}{2}$ $= \dfrac{-6 \pm \sqrt{100 + 4x}}{2}$ $= -3 \pm \sqrt{25 + x}$ $= -3 + \sqrt{25 + x} \quad (t > -3)$ $y = 6\ln\left(-3 + \sqrt{25 + x} + 3\right)$ $= 6\ln\sqrt{25 + x}$ $= 3\ln(25 + x)$	2	**2 marks** • Correct solution **1 mark** • Obtains t in terms of x
12 (c) $\left(1 - \dfrac{y}{x}\right)(x + y)^8 = (x + y)^8 - \dfrac{y}{x}(1 + x)^8$ $(x + y)^8 = \displaystyle\sum_{k=0}^{8} \binom{8}{k} x^{8-k} y^k$ term with $x^2y^6 \Rightarrow k = 6$ coefficient= $\binom{8}{6}$ $\dfrac{y}{x}(x + y)^8 = \displaystyle\sum_{k=0}^{8} \binom{8}{k} x^{8-k} y^k \times \dfrac{y}{x}$ $\displaystyle\sum_{k=0}^{8} \binom{8}{k} x^{7-k} y^{k+1}$ term with $x^2y^6 \Rightarrow k = 5$ coefficient= $\binom{8}{5}$ $\therefore$ coefficient of x^2y^6 in $\left(1 - \dfrac{y}{x}\right)(x + y)^8$ $= \binom{8}{6} - \binom{8}{5}$ $= -28$	2	**2 marks** • Correct solution **1 mark** • Uses the binomial theorem to find the general term of $(x + y)^8$ or equivalent merit
12 (d) $\displaystyle\int \sin x \sin 3x \, dx = \frac{1}{2}\int (\cos(-2x) - \cos 4x)dx$ $= \dfrac{1}{2}\displaystyle\int (\cos 2x - \cos 4x)dx$ $= \dfrac{1}{2}\left(\dfrac{1}{2}\sin 2x - \dfrac{1}{4}\sin 4x\right) + c$ $= \dfrac{1}{4}\sin 2x - \dfrac{1}{8}\sin 4x + c$	2	**2 marks** • Correct solution **1 mark** • Uses product to sum result or equivalent merit
12 (e) This is equivalent to asking: "In how many ways can all the letters in the word AAABBCCC be arranged?" # orders $= \dfrac{8!}{3!2!3!}$ $= 560$	2	**2 marks** • Correct solution **1 mark** • Divides 8! by a relevant number or equivalent merit.
12 (f) (i) $f(x) = \sin^{-1}\left(\dfrac{1}{x}\right)$ $f\left(\sqrt{2}\right) = \sin^{-1}\left(\dfrac{1}{\sqrt{2}}\right)$ $= \dfrac{\pi}{4}$	1	**1 mark** • Correct answer

Solution	Marks	Comments
12 (f) (ii) $f(x) = \sin^{-1}\left(\frac{1}{x}\right)$ $f'(x) = \dfrac{-\frac{1}{x^2}}{\sqrt{1-\frac{1}{x^2}}}$ $= -\dfrac{1}{x\sqrt{x^2-1}}$ when $x = \sqrt{2}$, $f'\left(\sqrt{2}\right) = -\dfrac{1}{\sqrt{2}\sqrt{2-1}}$ $= -\dfrac{1}{\sqrt{2}}$ $y - \frac{\pi}{4} = -\frac{1}{\sqrt{2}}\left(x - \sqrt{2}\right)$ $y = -\frac{x}{\sqrt{2}} + 1 + \frac{\pi}{4}$ $NOTE\text{: } f'(x) = \dfrac{-\lvert x\rvert}{x^2\sqrt{x^2-1}} = \begin{cases} -\dfrac{1}{x\sqrt{x^2-1}} & x > 1 \\ \dfrac{1}{x\sqrt{x^2-1}} & x < -1 \end{cases}$	**2**	**2 marks** • Correct solution **1 mark** • Attempts to find the derivative of $\sin^{-1}\left(\frac{1}{x}\right)$ or equivalent merit.
QUESTION 13		
13 (a) $X = \#$ of heads tossed $X \sim \text{Bin}(0.7,5)$ $\text{E}(X) = 0.7 \times 5$ $= 3.5$ as it is impossible to toss 3.5 heads, the most likely result will be either 3 heads or 4 heads. $P(X = 3) = \binom{5}{3}(0.3)^2(0.7)^3$ $= 0.3087$ $P(X = 4) = \binom{5}{4}(0.3)^1(0.7)^4$ $= 0.36015$ $\therefore$ the most likely combination is 4 heads and 1 tail.	**3**	**3 marks** • Correct solution **2 marks** • Uses binomial probability in an attempt to find a solution **1 mark** • Finds E(X)
13 (b) (i) $\displaystyle\int_1^{\sqrt{2}} \frac{dx}{x^2\sqrt{4-x^2}}$ $x = 2\cos u$ $dx = -2\sin u\, du$ when $x = 1$, $u = \frac{\pi}{3}$ when $x = \sqrt{2}$, $u = \frac{\pi}{4}$ $\displaystyle = \int_{\frac{\pi}{3}}^{\frac{\pi}{4}} \frac{-2\sin u\, du}{4\cos^2 u\sqrt{4-4\cos^2 u}}$ $\displaystyle = -\frac{1}{2}\int_{\frac{\pi}{3}}^{\frac{\pi}{4}} \frac{\sin u\, du}{\cos^2 u(2\sin u)}$ $\displaystyle = \frac{1}{4}\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{du}{\cos^2 u}$ $\displaystyle = \frac{1}{4}\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \sec^2 u\, du$ $= \frac{1}{4}\Big[\tan u\Big]_{\frac{\pi}{4}}^{\frac{\pi}{3}}$ $= \frac{1}{4}\left(\sqrt{3} - 1\right)$	**3**	**3 marks** • Correct solution **2 marks** • Obtains correct primitive in terms of u, or equivalent merit **1 mark** • Obtains correct integrand in terms of u, or equivalent merit

Solution	Marks	Comments
13 (b) (ii) $V = \pi \int y^2 dx$ $= \pi \int_1^{\sqrt{2}} \frac{16}{x^2\sqrt{4-x^2}} dx$ $= 16\pi \int_1^{\sqrt{2}} \frac{dx}{x^2\sqrt{4-x^2}}$ $= 16\pi \times \frac{1}{4}\left(\sqrt{3}-1\right)$ $= 4\pi\left(\sqrt{3}-1\right)$ units^3	2	**2 marks** • Correct solution **1 mark** • Finds an integral expression for the volume of the solid in terms of x
13 (c) $1^3 + 3^3 + 5^3 + ... + (2n-1)^3 = n^2(2n^2-1)$ When $n = 1$; LHS $= [2(1)-1]^3$ $= 1^3$ $= 1$ RHS $= 1^2\left[2(1)^2-1\right]$ $= 1 \times 1$ $= 1$ $\therefore$ LHS=RHS Hence the result is true for $n = 1$ Assume the result is true for $n = k$, where $k \in \mathbb{Z}^+$ $1^3 + 3^3 + 5^3 + ... + (2k-1)^3 = k^2(2k^2-1)$ Prove the result is true for $n = k + 1$ i.e. $1^3 + 2^3 + 5^3 + ... + (2k+1)^3 = (k+1)^2\left[2(k+1)^2-1\right]$ $= (k+1)^2(2k^2+4k+1)$ ***PROOF:*** $1^3 + 2^3 + 5^3 + ... + (2k-1)^3 + (2k+1)^3$ $= k^2(2k^2-1) + (2k+1)^3$ (by assumption) $= 2k^4 - k^2 + 8k^3 + 12k^2 + 6k + 1$ $= 2k^4 + 8k^3 + 11k^2 + 6k + 1$ $= k^2(2k^2+4k+1) + 2k(2k^2+4k+1) + (2k^2+4k+1)$ $= (k^2+2k+1)(2k^2+4k+1)$ $= (k+1)^2(2k^2+4k+1)$ Hence the result is true for $n = k + 1$, if it is true for $n = k$ Since the result is true for $n = 1$, then it is true $\forall\ n \in \mathbb{Z}^+$ by induction.	3	There are 4 key parts of the induction; 1. Proving the result true for $n = 1$ 2. Clearly stating the assumption and what is to be proven 3. Using the assumption in the proof 4. Correctly proving the required statement **3 marks** • Successfully does all of the 4 key parts **2 marks** • Successfully does 3 of the 4 key parts **1 mark** • Successfully does 2 of the 4 key parts
13 (d) $\frac{dV}{dt} = k(C-V)$ $\int_0^V \frac{dV}{C-V} = k\int_0^t dt$ $-\left[\ln(C-V)\right]_0^V = kt$ $(V < C \therefore \lvert C-V \rvert = C-V)$ $\ln\left(\frac{C-V}{C}\right) = -kt$ $\frac{C-V}{C} = e^{-kt}$ $C - V = Ce^{-kt}$ $V = C - Ce^{-kt}$ when $t = 2$, $V = \frac{C}{4} \Rightarrow \frac{C}{4} = C - Ce^{-2k}$ $e^{-2k} = \frac{3}{4}$ when $t = 4$, $V = C - Ce^{-4k}$ $= C - C(e^{-2k})^2$ $= C - \frac{9C}{16}$ $= \frac{7C}{16}$ $\therefore$ in the next two months the volume has increased by $\frac{7C}{16} - \frac{C}{4} = \frac{3C}{16}$. i.e. $\frac{3}{16}$ of the container is filled in the next two minutes.	4	**4 marks** • Correct solution **3 marks** • Finds e^{2k} **2 marks** • Integrates to find v as a function of t involving an exponential function **1 mark** • Separates the variables in the differential equation

Solution	Marks	Comments
QUESTION 14		
14 (a) $\overrightarrow{AD} = \overrightarrow{AC} + \overrightarrow{CD}$ $= \overrightarrow{OA} + \overrightarrow{CD}$ $= 2\underset{\sim}{a} + \frac{1}{2}\underset{\sim}{b}$ $\overrightarrow{BX} = \mu\overrightarrow{BC}$ $= \mu(3\underset{\sim}{a} - \underset{\sim}{b})$ $\overrightarrow{AX} = \lambda\overrightarrow{AD}$ $= \lambda\left(2\underset{\sim}{a} + \frac{1}{2}\underset{\sim}{b}\right)$ $\overrightarrow{AX} = \overrightarrow{AO} + \overrightarrow{OB} + \overrightarrow{BX}$ $= -\underset{\sim}{a} + \underset{\sim}{b} + \mu(3\underset{\sim}{a} - \underset{\sim}{b})$ $= (3\mu - 1)\underset{\sim}{a} + (1-\mu)\underset{\sim}{b}$ equating components $1 - \mu = \frac{\lambda}{2}$ $\quad 3\mu - 1 = 2\lambda$ $\quad \therefore \frac{\lvert\overrightarrow{AX}\rvert}{\lvert\overrightarrow{AD}\rvert} = \frac{4}{7}$ $\mu = 1 - \frac{\lambda}{2}$ $\quad 3\left(1 - \frac{\lambda}{2}\right) - 1 = 2\lambda$ $\frac{7\lambda}{2} = 2$ $\lambda = \frac{4}{7}$	3	**3 marks** • Correct solution **2 marks** • Finds two ways of expressing $\overrightarrow{AX}$ in terms of $\underset{\sim}{a}$ and $\underset{\sim}{b}$ or equivalent merit **1 mark** • Finds $\overrightarrow{AX}$ or $\overrightarrow{BX}$ in terms of $\underset{\sim}{a}$ and $\underset{\sim}{b}$ or equivalent merit
14 (b)  $x^2 = r^2 + r^2$ $= 2r^2$ $x = \sqrt{2}r$ $V = \frac{4}{3}\pi r^3$ $\frac{dV}{dr} = 4\pi r^2$ $\frac{dx}{dt} = \frac{dV}{dt} \times \frac{dr}{dV} \times \frac{dx}{dr}$ $= 5 \times \frac{1}{4\pi r^2} \times \sqrt{2}$ $= \frac{5\sqrt{2}}{4\pi r^2}$ $\frac{dV}{dt} = 5$ $\int_0^V dV = 5\int_0^3 dt$ $V = \left[3t\right]_0^5$ $= 15 - 0$ $= 15$ $\frac{4}{3}\pi r^3 = 15$ $r^3 = \frac{45}{4\pi}$ $r = \sqrt[3]{\frac{45}{4\pi}}$ when $t = 3$, $\frac{dx}{dt} = \frac{5\sqrt{2}}{4\pi \times \left(\sqrt[3]{\frac{45}{4\pi}}\right)^2}$ $= 0.2404...$ $= 0.24$ cm/s (to 2 decimal places)	3	**3 marks** • Correct solution **2 marks** • Finds an expression for $\frac{dx}{dt}$ in terms of r • Finds the radius and volume of the balloon after 3 seconds **1 mark** • Finds the direct straight line distance between the flies in terms of r
14 (c) (i) $X \sim \text{Bin}(50, 0.125)$ $\mu = 50 \times 0.125$ $= 6.25$ $\sigma^2 = 50 \times 0.125 \times 0.875$ $= 5.46875$ $X \sim \text{N}(6.25, 5.46875) \Leftrightarrow Z \sim N(0,1)$ $P(X \geq 5) = 1 - P(X \leq 5)$ $= 1 - P\left(Z \leq \frac{5 - 6.25}{\sqrt{5.46875}}\right)$ $= 1 - \Phi(-0.53)$ $= \Phi(1.53)$ $= 0.7019$ *NOTE: if continuity correction used* $P(Y \geq 4.5) = 1 - P(Y \leq 4.5)$ $= 1 - P\left(Z \leq \frac{4.5 - 6.25}{\sqrt{5.46875}}\right)$ $= 1 - \Phi(-0.75)$ $= \Phi(0.75)$ $= 0.7734$	3	**3 marks** • Correct solution **2 marks** • Finds the correct z score **1 mark** • Finds the mean and variance of a binomial distribution
14 (c) (ii) $np = 50 \times 0.125$ $= 6.25$ $nq = 50 \times 0.875$ $= 43.75$ as $np > 5$ and $nq > 5$ it was reasonable to use a normal approximation to the binomial distribution.	1	**1 mark** • Correct solution

Solution	Marks	Comments
14 (d) (i) collison occurs when $\vec{\mathbf{r}}_B = \vec{\mathbf{r}}_C$ $50t\cos\alpha = 40$ $50t\sin\alpha - 5t^2 = 30(t+2) - 5(t+2)^2$ $\cos\alpha = \frac{4}{5t}$ $= 30t + 60 - 5t^2 - 20t - 20$ $= -5t + 10t + 40$ $50t\sin\alpha = 10t + 40$ $\sin\alpha = \frac{t+4}{5t}$ $\sin^2\theta + \cos^2\theta = 1$ $\left(\frac{t+4}{5t}\right)^2 + \left(\frac{4}{5t}\right)^2 = 1$ $t^2 + 8t + 16 + 16 = 5t^2$ $24t^2 - 8t - 32 = 0$ $3t^2 - t - 4 = 0$ $(3t-4)(t+1) = 0$ $t = \frac{4}{3}$ or $t = -1$ However $t > 0$, thus the bullet hits the clay pigeon $\frac{4}{3}$ seconds after it is fired.	3	**3 marks** • Correct solution **2 marks** • Equates both components to obtain a quadratic in terms of t **1 mark** • Equates either the horizontal or vertical components to establish a relationship between t and α
14 (d) (ii) $\vec{\mathbf{v}}_B = \begin{pmatrix} 50\cos\alpha \\ 50\sin\alpha - 10t \end{pmatrix}$ $\vec{\mathbf{v}}_C = \begin{pmatrix} 0 \\ 30 - 10(t+2) \end{pmatrix}$ When $t = \frac{4}{3}$, $\cos\alpha = \frac{4}{5t}$ $\therefore \sin\alpha = \frac{4}{5}$ $= \frac{4}{5} \times \frac{3}{4}$ $= \frac{3}{5}$ $\vec{\mathbf{v}}_B = \begin{pmatrix} 50 \times \frac{3}{5} \\ 50 \times \frac{4}{5} - 10 \times \frac{4}{3} \end{pmatrix}$ $= \begin{pmatrix} 30 \\ 40 - \frac{40}{3} \end{pmatrix}$ $= \begin{pmatrix} 30 \\ \frac{80}{3} \end{pmatrix}$ as the clay pigeon is moving vertically $\cos\theta = \frac{\vec{\mathbf{v}}_B \cdot \vec{\mathbf{j}}}{\lvert\vec{\mathbf{v}}_B\rvert}$ $= \frac{\begin{pmatrix} 30 \\ \frac{80}{3} \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 1 \end{pmatrix}}{\sqrt{30^2 + \left(\frac{80}{3}\right)^2}}$ $= \frac{80}{3} \times \frac{3}{10\sqrt{145}}$ $= \frac{8}{\sqrt{145}}$ $\theta = 48.366...$ $= 48°$ (to nearest degree) $\therefore$ the bullet impacts the clay pigeon at an angle of 48°	3	**3 marks** • Correct solution **2 marks** • Finds the velocity vector of the bullet at the time of collision **1 mark** • Finds the velocity vector of the bullet