Blacktown Boys' High School

2025

HSC Trial Examination

Mathematics Extension 1

General Instructions

- Reading time – 10 minutes
- Working time – 2 hours
- Write using black pen
- Calculators approved by NESA may be used
- A reference sheet is provided for this paper
- All diagrams are not drawn to scale
- In Questions 11 – 14, show relevant mathematical reasoning and/or calculations

Total marks: 70

Section I – 10 marks (pages 3 – 6)

- Attempt Questions 1 – 10
- Allow about 15 minutes for this section

Section II – 60 marks (pages 7 – 12)

- Attempt Questions 11 – 14
- Allow about 1 hour and 45 minutes for this section

Assessor: X. Chirgwin

Student Name: ______________________________________

Teacher Name: ______________________________________

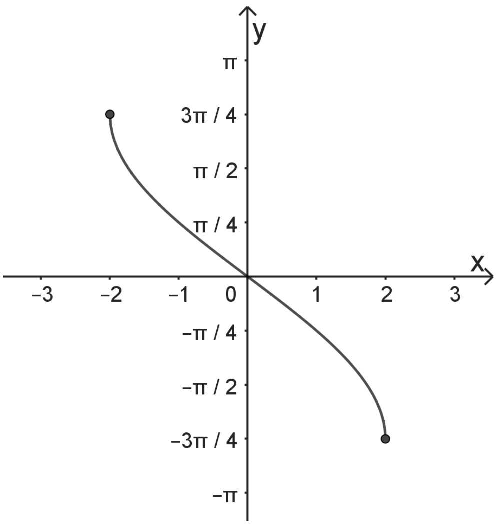
Students are advised that this is a trial examination only and cannot in any way guarantee the content or format of the 2025 Higher School Certificate Examination.

Section I

10 marks
Attempt Questions 1–10
Allow about 15 minutes for this section

Use the multiple choice answer sheet for Questions 1–10.

Q1. Which function best describes the following graph?



A. $y = \frac{3}{4}\sin^{-1}\left(-\frac{x}{2}\right)$

B. $y = \frac{3}{2}\sin^{-1}\left(-\frac{x}{2}\right)$

C. $y = \frac{3}{4}\sin^{-1}(-2x)$

D. $y = \frac{3}{2}\sin^{-1}(-2x)$

Q2. Which of the following is the projection of $\boldsymbol{a} = -2\boldsymbol{i} - 3\boldsymbol{j}$ onto $\boldsymbol{b} = 5\boldsymbol{i} - 4\boldsymbol{j}$?

A. $\frac{10}{13}\boldsymbol{i} - \frac{8}{13}\boldsymbol{j}$

B. $-\frac{110}{13}\boldsymbol{i} + \frac{88}{13}\boldsymbol{j}$

C. $\frac{10}{41}\boldsymbol{i} - \frac{8}{41}\boldsymbol{j}$

D. $-\frac{110}{41}\boldsymbol{i} + \frac{88}{41}\boldsymbol{j}$

Q3. Which of the following is equivalent to $\int \left(\sin^2\left(\frac{x}{2}\right)+1\right)dx$?

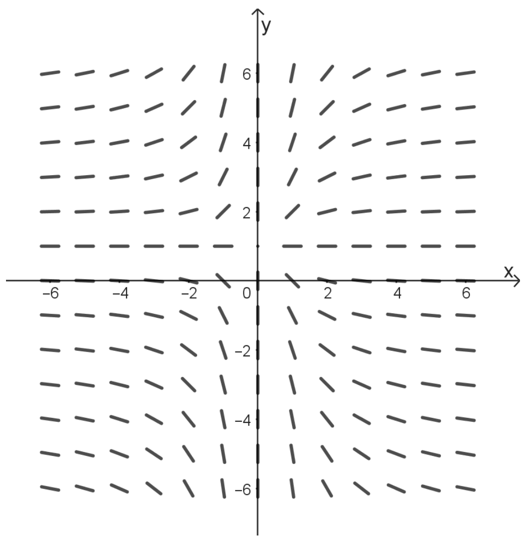
A. $-\frac{x}{2}-\frac{1}{2}\cos x + c$

B. $-\frac{x}{2}+\frac{1}{2}\cos x + c$

C. $\frac{3x}{2}-\frac{1}{2}\sin x + c$

D. $\frac{3x}{2}+\frac{1}{2}\sin x + c$

Q4. The direction field shown below best represents the differential equation



A. $\frac{dy}{dx}=\frac{y-1}{x}$

B. $\frac{dy}{dx}=\frac{y-1}{x^2}$

C. $\frac{dy}{dx}=\frac{x}{y-1}$

D. $\frac{dy}{dx}=\frac{x^2}{y-1}$

Q5. Given that $P(x) = x^3 + x^2 - 8x - 12$ has a double root at $x = -2$. Which of the following is the solution to $P(x) < 0$?

A. $x < -2$

B. $x < 3$

C. $-2 < x < 3$

D. $x < -2, -2 < x < 3$

Q6. A decorator is selecting ribbons. She has three blue, five red, six purple, eight green, four orange, and seven yellow ribbons to select from. If the decorator selects the ribbons randomly, what is the minimum number of ribbons that she would need to select to ensure that she has three ribbons of the same colour?

A. 12

B. 13

C. 14

D. 15

Q7. The displacement x of a particle at time t is given by $x = 20\sin 2t + 21\cos 2t$. What is the maximum speed of the particle?

A. 58

B. 54

C. 42

D. 29

Q8. In how many ways can 8 people from a group of 21 people be chosen and then arranged in a circle?

A. $\frac{20!}{12!}$

B. $\frac{20!}{12!\,8}$

C. $\frac{21!}{13!}$

D. $\frac{21!}{13!\,8}$

Q9. For the population of individuals who own a smart phone, it is know that the proportion of the population that have a particular app is 0.25.
A technology company wishes to survey n number of people about this. Let $\hat{p}$ be the random variable representing the proportion of people surveyed that have this app. What is the smallest sample size, n, for which the standard deviation of $\hat{p}$ is less than 0.035?

A. 51

B. 52

C. 153

D. 154

Q10. The differential equation $\left(\frac{dy}{dx}\right)^2 + k\frac{dy}{dx} + 1 = 0$ has exactly one solution, where $k > 0$ and $y = 1$ when $x = 1$. A solution to the differential equation is

A. $x + y = 2$

B. $2x - y = 1$

C. $y = e^{x-1}$

D. $y = 1 + \log_e x$

End of Section I

Section II

60 Marks
Attempt Questions 11–14
Allow about 1 hour and 45 minutes for this section

Answer each question on a SEPARATE writing booklet. Extra writing booklets are available.

In Questions 11–14, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Use the Question 11 Writing Booklet.

(a) (i) Given that $f(x) = e^x \cos^{-1}(2x)$, find $f'(x)$. **2**

(ii) Find the exact gradient of the tangent of $y = f(x)$ at the origin. **1**

(b) Solve $\dfrac{6}{1-2x} \leq 5$. **3**

(c) (i) In how many different ways can all the letters of the word PINEAPPLE be arranged in a line? **1**

(ii) What is the probability that a random rearrangement of the letters of PINEAPPLE in a line has three Ps together? **2**

(d) A customer service centre records every call they receive. It is found that 30% of all calls made to this centre are complaints.
A sample of 2100 calls is selected. The number of calls in the sample which are complaints is denoted by the random variable X.

(i) Find the expected value $E(X)$. **1**

(ii) Find the standard deviation of X. **1**

(iii) By considering a normal distribution, find the approximate probability that $588 \leq X \leq 672$. **1**

(e) Using the substitution $u = 2 - x$, find $\displaystyle\int 3x(2-x)dx$. **3**

End of Questions 11

Question 12 (15 marks) Use the Question 12 Writing Booklet.

(a) (i) Show that ${}^{n}C_2 = \dfrac{n(n-1)}{2}$. **1**

(ii) Solve for n where $2 \times {}^{n}C_4 = 51 \times {}^{n}C_2$. **2**

(b) Prove by mathematical induction that $3^{2n} + 16n - 1$ is divisible by 8 for all positive integers n. **3**

(c) (i) Onirban throws a ball at a target 15 times. Each time Onirban throws the ball, the probability of the ball hitting the target is 0.48. Find the probability that Onirban hits the target exactly 3 times, round to 4 decimal places. **2**

(ii) Onirban now throws the ball at the target 250 times. Use a normal approximation and the information on page 12 to calculate the probability that he will hit the target more than 110 times. **3**

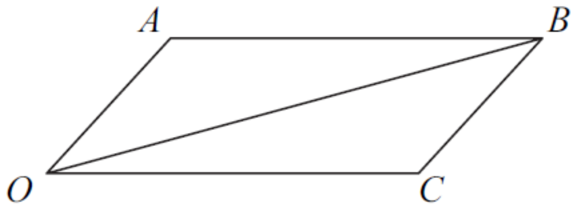
(d) (i) Express $\sin 2x - \sqrt{3}\cos 2x$ in the form $A\sin(2x - \alpha)$, where $A > 0$ and $0 < \alpha < \dfrac{\pi}{2}$. **2**

(ii) Hence solve $\sin 2x - \sqrt{3}\cos 2x + \sqrt{2} = 0$, for $0 \le x \le 2\pi$. **2**

End of Questions 12

Question 13 (15 marks) Use the Question 13 Writing Booklet.

(a) Given that $OABC$ is a parallelogram, where $\overrightarrow{OA} = \binom{1}{2}, \overrightarrow{OC} = \binom{4}{0}$.



(i) Find the size of angle OAB, to the nearest minute. **2**

(ii) Hence find the exact area of the parallelogram $OABC$. **2**

(iii) X is a point on OB such that $OX = kOB$, where $0 < k < 1$. Use vectors to determine the value of k for which $\overrightarrow{AX} = \overrightarrow{XC}$. **3**

(iv) Hence show that the diagonals of the parallelogram $OABC$ bisect each other. **1**

(b) Find the particular solution to the differential equation **4**

$$\frac{dy}{dx} = \frac{\sqrt{6y-5}}{x^2}, \quad x \neq 0, y > \frac{5}{6}$$

that passes through the point $(1, 1)$. Express y in terms of x.

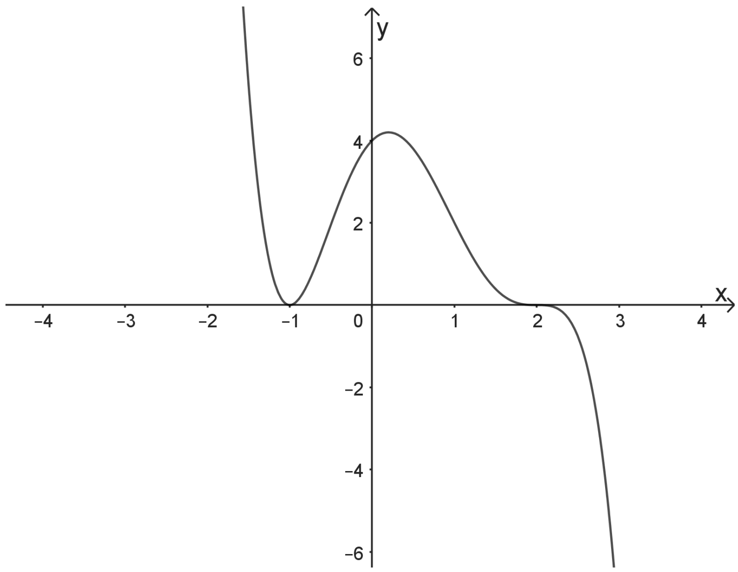
(c) Use the substitution $u = (2x+1)^2$ to find the exact value of **3**

$$\int_0^1 \left(\frac{6x+3}{1+(2x+1)^4} + \frac{2x+1}{1+(2x+1)^2}\right) dx$$

End of Questions 13

Question 14 (15 marks) Use the Question 14 Writing Booklet.

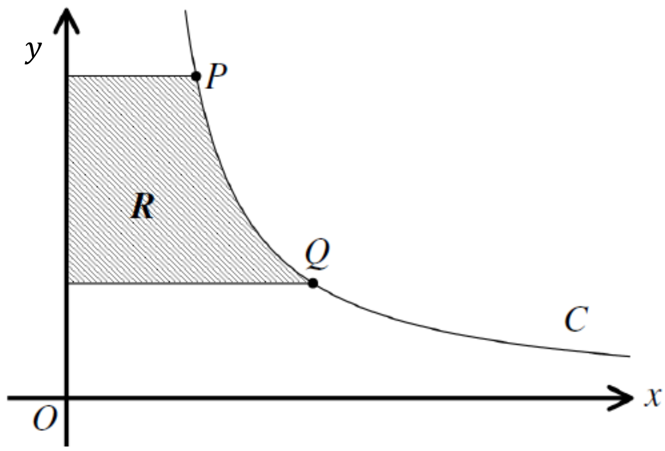
(a) Consider the graph of $y = f(x)$ below. **2**



Sketch the graph of $y = \dfrac{1}{\sqrt{f(x)}}$.

(b) The figure below shows the graph of the curve C with equation **3**

$$y = \frac{12}{x-3}, \qquad x \neq 3.$$



The points P and Q lie on C where $x = 4$ and $x = 9$ respectively.

The shaded region R is bounded by the curve and two horizontal lines passing through the points P and Q. Find the exact volume of the solid of revolution when the region R is rotated about the y-axis.

Question 14 continues on next page

Question 14 (continued)

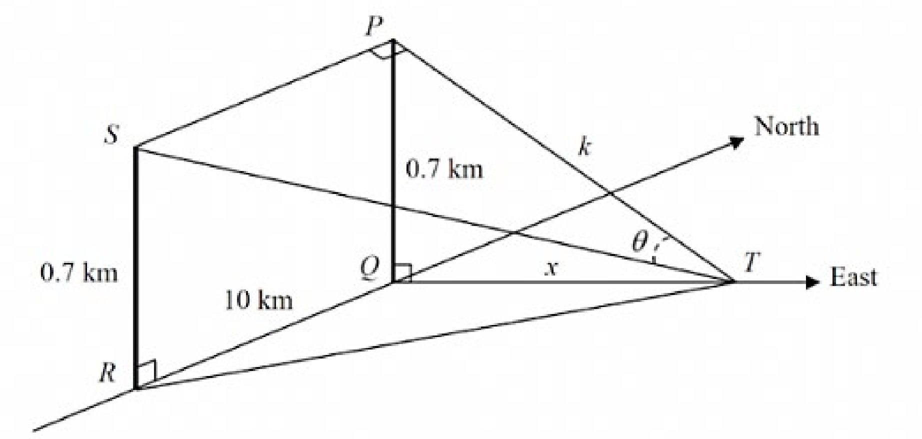
(c) Points scored in the long jump event of a competition can be calculated using a solution of the differential equation **4**

$$(m-220)\frac{dP}{dm} = 1.5P, \quad m > 220$$

where m is the distance jumped in centimetres and P the points scored.

Given that a jump of 544 centimetres scored 756 points, find an expression for P in terms of m and determine the centimetres jumped to have a score of 1792 points.

(d) PQ and SR are two towers of height 0.7 km. Q is 10 km due North of R. A vehicle at T is travelling due East away from Q at a constant speed of 10 km/h. Let the distance QT be x, the distance PT be k and $\angle PTS = \theta$.



(i) Show that **2**

$$\frac{dk}{dt} = \frac{10x}{\sqrt{x^2+0.49}}$$

(ii) Show that **2**

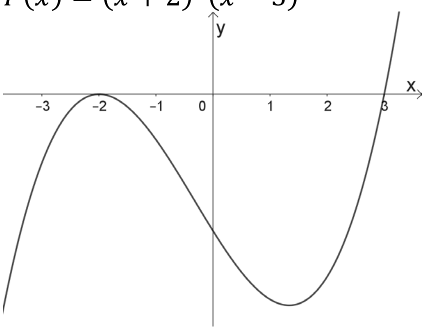
$$\frac{dk}{d\theta} = -10\,\text{cosec}^2\,\theta$$

(iii) Find the exact rate at which θ is changing when the vehicle is 2.4 km from Q. **2**

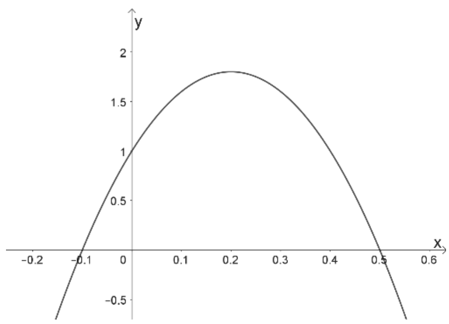
End of Paper

2025 Mathematics Extension 1 AT4 Trial Solutions

Section 1

Q1	**B** $D: [-2, 2]$ $R: \left[-\frac{3\pi}{4}, \frac{3\pi}{4}\right]$	1 Mark
Q2	**C** $proj_{\boldsymbol{b}}\boldsymbol{a} = \frac{\boldsymbol{a}\cdot\boldsymbol{b}}{\lvert\boldsymbol{b}\rvert^2}\boldsymbol{b}$ $= \frac{-2\times 5 + (-3)\times(-4)}{\left(\sqrt{5^2+(-4)^2}\right)^2}(5\boldsymbol{i} - 4\boldsymbol{j})$ $= \frac{10}{41}\boldsymbol{i} - \frac{8}{41}\boldsymbol{j}$	1 Mark
Q3	**C** $\int \left(\sin^2\left(\frac{x}{2}\right) + 1\right) dx$ $= \int \left(\frac{1}{2} + \frac{1}{2}\cos x + 1\right) dx$ $= \int \left(\frac{3}{2} + \frac{1}{2}\cos x\right) dx$ $= \frac{3x}{2} - \frac{1}{2}\sin x + c$	1 Mark
Q4	**B** Vertical gradient at $x = 0$ and horizontal gradient at $y = 1$, this eliminates C and D. Second quadrant $x < 0$ and $y > 1$, the gradient is positive, this indicates B.	1 Mark
Q5	**D** $P(x) = x^3 + x^2 - 8x - 12$ $P(x) = (x+2)^2(x-3)$  $P(x) < 0$ $x < -2, -2 < x < 3$	1 Mark
Q6	**B** Using the pigeonhole principle, for three ribbons of the same colour to be selected, it is assumed that two ribbons of each colour are selected. Then the next ribbon selected must be the third ribbon of the one colour. $2 \times 6 + 1 = 13$ ribbons.	1 Mark
Q7	**A** $x = 20\sin 2t + 21\cos 2t = 29\sin\left(2t + \tan^{-1}\left(\frac{21}{20}\right)\right)$ $v = 58\cos\left(2t + \tan^{-1}\left(\frac{21}{20}\right)\right)$ Maximum speed is 58.	1 Mark

Q8	**D** ${}^{21}C_8 \times 7!$ $= \dfrac{21! \times 7!}{8! \times 13!}$ $= \dfrac{21!}{13!\,8}$	1 Mark
Q9	**D** $p = 0.25$ $Var(\hat{p}) = \dfrac{p(1-p)}{n}$ $\sqrt{\dfrac{p(1-p)}{n}} < 0.035$ $\sqrt{\dfrac{0.25(1-0.25)}{n}} < 0.035$ $\sqrt{\dfrac{0.1875}{n}} < 0.035$ $\dfrac{0.1875}{n} < 0.001225$ $n > \dfrac{0.1875}{0.001225}$ $n > 153.06 \ldots$ $n = 154$	1 Mark
Q10	**A** $\left(\dfrac{dy}{dx}\right)^2 + k\dfrac{dy}{dx} + 1 = 0$ The quadratic equation has exactly one solution, then $\Delta = 0$. $k^2 - 4 \times 1 \times 1 = 0$ $k = 2$ since $k > 0$ $\left(\dfrac{dy}{dx}\right)^2 + 2\dfrac{dy}{dx} + 1 = 0$ $\left(\dfrac{dy}{dx} + 1\right)^2 = 0$ $\dfrac{dy}{dx} = -1$ $y = -x + c$ Since it goes through $(1,1)$ $1 = -1 + c$ $c = 2$ $y = -x + 2$ $x + y = 2$	1 Mark

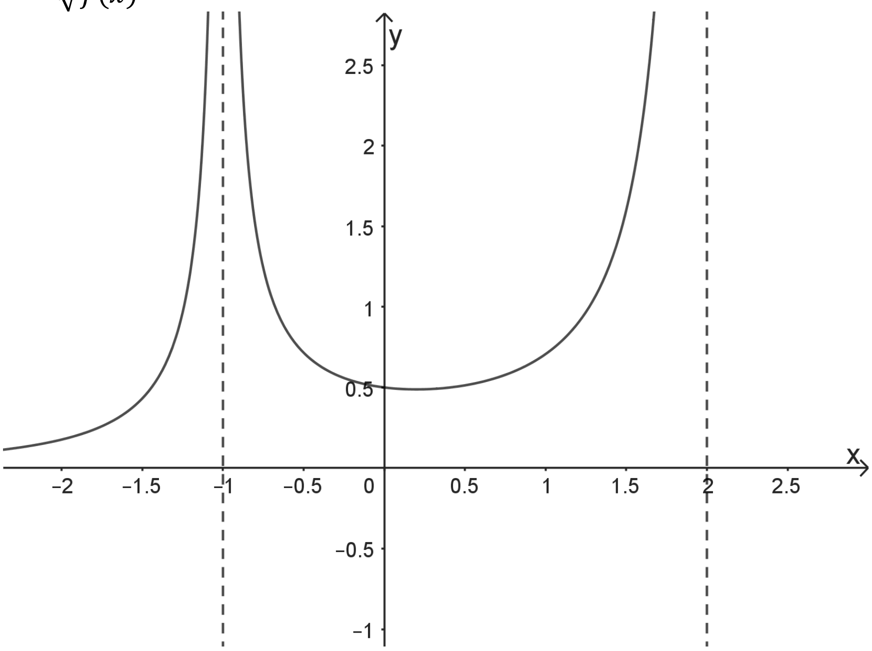
Section 2		
Q11ai	$\frac{d}{dx}(e^x \cos^{-1}(2x))$ $= e^x \times -\frac{2}{\sqrt{1-4x^2}} + e^x \cos^{-1}(2x)$ $= e^x \left(\cos^{-1}(2x) - \frac{2}{\sqrt{1-4x^2}}\right)$	2 Marks Correct solution 1 Mark Correct differentiation of $\cos^{-1}(2x)$
Q11aii	At the origin $m_T = e^0 \left(\cos^{-1}(2 \times 0) - \frac{2}{\sqrt{1-4\times 0^2}}\right)$ $m_T = \frac{\pi}{2} - 2$	1 Mark Correct solution
Q11b	$\frac{6}{1-2x} \leq 5, \quad x \neq \frac{1}{2}$ $6(1-2x) \leq 5(1-2x)^2$ $6(1-2x) - 5(1-2x)^2 \leq 0$ $(1-2x)\big(6-5(1-2x)\big) \leq 0$ $(1-2x)(6-5+10x) \leq 0$ $(1-2x)(1+10x) \leq 0$ $x \leq -\frac{1}{10}, x > \frac{1}{2}$ 	3 Marks Correct solution 2 Marks Determines both key values 1 Mark Multiplies by sq. of denominator and makes progress
Q11ci	9 letters altogether for the word PINEAPPLE with 3 Ps and 2 Es. $\frac{9!}{3!\,2!}$ $= 30240$	1 Mark Correct solution
Q11cii	Number of arrangements with 3 Ps together $\frac{7!}{2!}$ $= 2520$ Probability is $\frac{2520}{30240} = \frac{1}{12}$	2 Marks Correct solution 1 Mark Finds the number of arrangements of 3 Ps together
Q11di	$E(X) = np$ $E(X) = 2100 \times 0.3$ $E(X) = 630$	1 Mark Correct solution
Q11dii	$\sigma = \sqrt{npq}$ $\sigma = \sqrt{2100 \times 0.3 \times 0.7}$ $\sigma = 21$	1 Mark Correct solution
Q11diii	$630 - 2 \times 21 = 588$ and $630 + 2 \times 21 = 672$ This represents 2 standard deviations from the mean $P(588 \leq X \leq 672) \approx 95\%$	1 Mark Correct solution
Q11e	$u = 2 - x$ $du = -dx$ $\int 3x(2-x)dx$ $= \int 3(2-u)u \times -du$	3 Marks Correct solution 2 Marks Correct integration in terms of u

	$= \int (3u^2 - 6u)du$ $= \frac{3u^3}{3} - \frac{6u^2}{2} + c$ $= u^3 - 3u^2 + c$ $= (2-x)^3 - 3(2-x)^2 + c$	1 Mark Correct substitution of u
Q12ai	${}^nC_2 = \frac{n!}{2!\,(n-2)!}$ ${}^nC_2 = \frac{n \times (n-1) \times (n-2)!}{2!\,(n-2)!}$ ${}^nC_2 = \frac{n(n-1)}{2}$	1 Mark Correct solution
Q12aii	$2 \times {}^nC_4 = 51 \times {}^nC_2$ $2 \times \frac{(n-2)(n-3)}{4!} = 51 \times \frac{n(n-1)}{2}$ $\frac{(n-2)(n-3)}{12} = \frac{51}{2}$ $(n-2)(n-3) = 306$ $n^2 - 5n + 6 = 306$ $n^2 - 5n - 300 = 0$ $(n-20)(n+15) = 0$ $n = 20, n > 0$	2 Marks Correct solution 1 Mark Progresses to $n^2 - 5n - 300 = 0$
Q12b	RTP: $3^{2n} + 16n - 1$ is divisible by 8 1. Prove statement is true for $n = 1$ $3^{2\times1} + 16 \times 1 - 1$ $= 24 = 8 \times 3$ Statement is true for $n = 1$ 2. Assume statement is true for $n = k, k \in \mathbb{Z}^+$ $3^{2k} + 16k - 1 = 8P$ 3. Prove statement is true for $n = k + 1$ i.e. $3^{2(k+1)} + 16(k+1) - 1 = 8Q$ $LHS = 3^{2(k+1)} + 16(k+1) - 1$ $LHS = 3^{2k+2} + 16k + 16 - 1$ $LHS = 3^2 \times 3^{2k} + 16k + 15$ $LHS = 3^2 \times (8P - 16k + 1) + 16k + 15$ (from assumption) $LHS = 8P \times 9 - 9 \times 16k + 9 + 16k + 15$ $LHS = 8P \times 9 - 8 \times 16k + 24$ $LHS = 8(9P - 16k + 3)$ $LHS = 8Q$, where $Q = 9P - 16k + 3$ $\therefore$ Statement is true for all positive integers n.	3 Marks Correct solution 2 Marks Make significant progress 1 Mark Proves initial case
Q12ci	${}^{15}C_3 0.48^3 (1 - 0.48)^{12}$ $= {}^{15}C_3 0.48^3 \times 0.52^{12}$ $= 0.019668 \ldots$ $= 0.0197$ (4 d.p.)	2 Marks Correct solution 1 Mark Finds $0.48^3 \times 0.52^{12}$

Q12cii	$E(X) = np = 250 \times 0.48 = 120$ $Var(X) = npq = 120 \times 0.52 = 62.4$ With continuity correction: $P(X > 110) = P(X \geq 110.5)$ $z = \dfrac{110.5 - 120}{\sqrt{62.4}}$ $z = -1.2026 \ldots \approx -1.20$ $P(z > -1.20) = P(z < 1.20)$ $P(z > -1.20) \approx 0.8849$ Without continuity correction: $P(X > 110) = P(X \geq 111)$ $z = \dfrac{111 - 120}{\sqrt{62.4}}$ $z = -1.139 \ldots \approx -1.14$ $P(z > -1.14) = P(z < 1.14)$ $P(z > -1.14) \approx 0.8729$	3 Marks Correct solution 2 Marks Finds z 1 Mark Finds $E(X)$ or $Var(X)$
Q12di	$\sin 2x - \sqrt{3}\cos 2x = A\sin(2x - \alpha)$ $\sin 2x - \sqrt{3}\cos 2x = A\sin 2x\cos\alpha - A\cos 2x\sin\alpha$ $A\cos\alpha = 1 \ldots\ldots (1)$ $A\sin\alpha = \sqrt{3} \ldots\ldots (2)$ $(1)^2 + (2)^2$ $A^2\cos^2\alpha + A^2\sin^2\alpha = 1^2 + \left(\sqrt{3}\right)^2$ $A^2 = 4$ $A = 2$ $(2) \div (1)$ $\dfrac{A\sin\alpha}{A\cos\alpha} = \dfrac{\sqrt{3}}{1}$ $\tan\alpha = \sqrt{3}$ $\alpha = \dfrac{\pi}{3}$ $\therefore \sin 2x - \sqrt{3}\cos 2x = 2\sin\left(2x - \dfrac{\pi}{3}\right)$	2 Marks Correct solution 1 Mark Correct A or α
Q12dii	$\sin 2x - \sqrt{3}\cos 2x + \sqrt{2} = 0, \qquad 0 \leq x \leq 2\pi$ $2\sin\left(2x - \dfrac{\pi}{3}\right) = -\sqrt{2}, \qquad 0 \leq 2x \leq 4\pi$ $\sin\left(2x - \dfrac{\pi}{3}\right) = -\dfrac{\sqrt{2}}{2}, \qquad -\dfrac{\pi}{3} \leq 2x - \dfrac{\pi}{3} \leq \dfrac{11\pi}{3}$ $2x - \dfrac{\pi}{3} = -\dfrac{\pi}{4}, \pi + \dfrac{\pi}{4}, 2\pi - \dfrac{\pi}{4}, 3\pi + \dfrac{\pi}{4}$ $2x - \dfrac{\pi}{3} = -\dfrac{\pi}{4}, \dfrac{5\pi}{4}, \dfrac{7\pi}{4}, \dfrac{13\pi}{4}$ $2x = \dfrac{\pi}{12}, \dfrac{19\pi}{12}, \dfrac{25\pi}{12}, \dfrac{43\pi}{12}$ $x = \dfrac{\pi}{24}, \dfrac{19\pi}{24}, \dfrac{25\pi}{24}, \dfrac{43\pi}{24}$	2 Marks Correct solution 1 Mark Makes significant progress and obtains at least one correct answer

Q13ai	Let $\angle OAB = \theta$ $\cos\theta = \dfrac{\overrightarrow{AO}\cdot\overrightarrow{AB}}{\lvert\overrightarrow{AO}\rvert\lvert\overrightarrow{AB}\rvert} = \dfrac{\overrightarrow{AO}\cdot\overrightarrow{OC}}{\lvert\overrightarrow{AO}\rvert\lvert\overrightarrow{OC}\rvert}$ $\cos\theta = \dfrac{(-1)\times 4 + (-2)\times 0}{\sqrt{(-1)^2+(-2)^2}\times\sqrt{4^2+0^2}}$ $\cos\theta = -\dfrac{4}{4\sqrt{5}} = -\dfrac{1}{\sqrt{5}}$ $\theta = 116°33'54.18''$ $\angle OAB = 116°34'$ (nearest minute)	2 Marks Correct solution 1 Mark Correct sub into formula and makes progress
Q13aii	Area of triangle OAB $A_{OAB} = \dfrac{1}{2}\lvert\overrightarrow{OA}\rvert\lvert\overrightarrow{AB}\rvert\sin\theta$ Using Pythagoras' theorem, $\cos\theta = -\dfrac{1}{\sqrt{5}},\quad \sin\theta = \dfrac{2}{\sqrt{5}}$ Area of parallelogram $OABC$ is twice the area of triangle OAB $A_{OABC} = 2\times\dfrac{1}{2}\lvert\overrightarrow{OA}\rvert\lvert\overrightarrow{AB}\rvert\sin\theta$ $A_{OABC} = 4\sqrt{5}\times\dfrac{2}{\sqrt{5}}$ $A_{OABC} = 8\, units^2$	2 Marks Correct solution 1 Mark Finds the area of triangle OAB
Q13aiii	$\overrightarrow{OX} = k\overrightarrow{OB} = k(\overrightarrow{OA}+\overrightarrow{AB})$ $\overrightarrow{OX} = k\left(\binom{1}{2}+\binom{4}{0}\right) = k\binom{5}{2}$ $\overrightarrow{OX} = \overrightarrow{OA}+\overrightarrow{AX}$ $\overrightarrow{AX} = \overrightarrow{OX}-\overrightarrow{OA}$ $\overrightarrow{AX} = k\binom{5}{2}-\binom{1}{2}$ $\overrightarrow{XC} = \overrightarrow{XO}+\overrightarrow{OC}$ $\overrightarrow{XC} = -k\binom{5}{2}+\binom{4}{0}$ $\overrightarrow{AX} = \overrightarrow{XC}$ $k\binom{5}{2}-\binom{1}{2} = -k\binom{5}{2}+\binom{4}{0}$ $5k-1 = -5k+4$ $10k = 5$ $k = 0.5$	3 Marks Correct solution 2 Marks Finds $\overrightarrow{AX}$ and $\overrightarrow{XC}$ 1 Mark Finds $\overrightarrow{OX}$
Q13aiv	Since $k=\frac{1}{2}$, then X is the midpoint of $\overrightarrow{OB}$. $\overrightarrow{AX} = \dfrac{1}{2}\binom{5}{2}-\binom{1}{2} = \binom{2.5-1}{1-2}$ $\overrightarrow{AX} = \binom{1.5}{-1}$ $\overrightarrow{XC} = -\dfrac{1}{2}\binom{5}{2}+\binom{4}{0} = \binom{-2.5+4}{-1+0}$ $\overrightarrow{XC} = \binom{1.5}{-1}$ Same point, so X is the midpoint of AC. $\therefore$ The diagonals of the parallelogram bisect each other.	1 Mark Correct solution

Q12b	$\frac{dy}{dx}=\frac{\sqrt{6y-5}}{x^2},\ \ x\neq 0, y>\frac{5}{6}$ $\int \frac{dy}{\sqrt{6y-5}}=\int \frac{dx}{x^2}$ $\int (6y-5)^{-\frac{1}{2}}dy=\int x^{-2}dx$ $\frac{(6y-5)^{\frac{1}{2}}}{6\times\frac{1}{2}}=\frac{x^{-1}}{-1}+c$ $\frac{\sqrt{6y-5}}{3}=-\frac{1}{x}+c$ Sub $(1,1)$ $\frac{\sqrt{6\times 1-5}}{3}=-\frac{1}{1}+c$ $\frac{1}{3}=-1+c$ $c=\frac{4}{3}$ $\frac{\sqrt{6y-5}}{3}=-\frac{1}{x}+\frac{4}{3}$ $\sqrt{6y-5}=-\frac{3}{x}+4$ $6y-5=\left(4-\frac{3}{x}\right)^2$ $y=\frac{1}{6}\left(\left(4-\frac{3}{x}\right)^2+5\right)$	4 Marks Correct solution 3 Marks Makes significant progress to find $\frac{\sqrt{6y-5}}{3}=-\frac{1}{x}+\frac{4}{3}$ 2 Marks Correct integration to find $\frac{\sqrt{6y-5}}{3}=-\frac{1}{x}+c$ 1 Mark Correct split of variables and makes progress
Q12c	$I=\int_0^1\left(\frac{6x+3}{1+(2x+1)^4}+\frac{2x+1}{1+(2x+1)^2}\right)dx$ $I=\int_0^1\frac{6x+3}{1+(2x+1)^4}dx+\int_0^1\frac{2x+1}{1+(2x+1)^2}dx$ $u=(2x+1)^2$ $du=4(2x+1)dx$ $x=1, u=9$ $x=0, u=1$ $I=\frac{3}{4}\int_1^9\frac{du}{1+u^2}+\frac{1}{4}\int_1^9\frac{du}{1+u}$	3 Marks Correct solution 2 Marks Correct integration 1 Mark Correct substitution in terms of u

	$I = \frac{3}{4}[\tan^{-1} u]_1^9 + \frac{1}{4}[\ln\lvert 1+u\rvert]_1^9$ $I = \frac{3}{4}[\tan^{-1} 9 - \tan^{-1} 1] + \frac{1}{4}[\ln 10 - \ln 2]$ $I = \frac{3}{4}\left[\tan^{-1} 9 - \frac{\pi}{4}\right] + \frac{1}{4}\ln 5$	
Q14a	$y = \frac{1}{\sqrt{f(x)}}$ 	2 Marks Correct solution 1 Mark Correct graph with most key features
Q14b	$y = \frac{12}{x-3} \qquad x = 4, y = 12, \qquad x = 9, y = 2$ $x - 3 = \frac{12}{y}$ $x = \frac{12}{y} + 3$ $x^2 = \left(\frac{12}{y} + 3\right)^2$ $x^2 = \frac{144}{y^2} + \frac{72}{y} + 9$ $V = \pi \int_2^{12} \left(\frac{144}{y^2} + \frac{72}{y} + 9\right) dy$ $V = \pi \left[-\frac{144}{y} + 72\ln\lvert y\rvert + 9y\right]_2^{12}$ $V = \pi \left[\left(-\frac{144}{12} + 72\ln\lvert 12\rvert + 9 \times 12\right) - \left(-\frac{144}{2} + 72\ln\lvert 2\rvert + 9 \times 2\right)\right]$ $V = \pi[(96 + 72\ln 12) - (-54 + 72\ln 2)]$ $V = \pi(150 + 72\ln 6)\ units^3$	3 Marks Correct solution 2 Marks Correct integration 1 Mark Correct volume expression

Q14c	$(m-220)\dfrac{dP}{dm}=1.5P$ $\int \dfrac{1}{P}\,dP = 1.5\int \dfrac{1}{m-220}\,dm$ $\ln\lvert P\rvert = 1.5\ln\lvert m-220\rvert + c$ $P = Ae^{1.5\ln\lvert m-220\rvert}$ $P = Ae^{\ln(m-220)^{1.5}}, \quad m-220>0$ $P = A(m-220)^{1.5}$ $P = 756, m = 544$ $756 = A\times(544-220)^{1.5}$ $A = \dfrac{756}{5832}$ $A = \dfrac{7}{54}$ $P = \dfrac{7}{54}(m-220)^{1.5}$ $1792 = \dfrac{7}{54}(m-220)^{1.5}$ $1792\times\dfrac{54}{7} = (m-220)^{1.5}$ $13824 = (m-220)^{\frac{3}{2}}$ $13824^{\frac{2}{3}} = m-220$ $m = 576+220$ $m = 796$ cm	4 Marks Correct solution 3 Marks Determines $P=\dfrac{7}{54}(m-220)^{1.5}$ 2 Marks Finds $P = Ae^{1.5\ln\lvert m-220\rvert}$ 1 Mark Correct split of variables and some correct integration
Q14di	$\dfrac{dx}{dt}=10$ $0.7^2+x^2=k^2$ $k=\sqrt{x^2+0.49}=(x^2+0.49)^{\frac{1}{2}}$ $\dfrac{dk}{dx}=\dfrac{1}{2}\times 2x\times(x^2+0.49)^{-\frac{1}{2}}$ $\dfrac{dk}{dx}=\dfrac{x}{\sqrt{x^2+0.49}}$ $\dfrac{dk}{dt}=\dfrac{dk}{dx}\times\dfrac{dx}{dt}$	2 Marks Correct solution 1 Mark Correct $\dfrac{dk}{dx}$

	$\frac{dk}{dt} = \frac{x}{\sqrt{x^2+0.49}} \times 10$ $\frac{dk}{dt} = \frac{10x}{\sqrt{x^2+0.49}}$	
Q14dii	$\tan\theta = \frac{10}{k}$ $k = \frac{10}{\tan\theta} = 10(\tan\theta)^{-1}$ $\frac{dk}{d\theta} = -1 \times 10 \times \sec^2\theta \times (\tan\theta)^{-2}$ $\frac{dk}{d\theta} = -10 \times \frac{1}{\cos^2\theta} \times \frac{\cos^2\theta}{\sin^2\theta}$ $\frac{dk}{d\theta} = \frac{-10}{\sin^2\theta}$ $\frac{dk}{d\theta} = -10\,\mathrm{cosec}^2\,\theta$	2 Marks Correct solution 1 Mark Correct differentiation
Q14diii	$x = 2.4$ $k = \sqrt{0.7^2 + 2.4^2}$ $k = \frac{5}{2}$ $\tan\theta = \frac{10}{k} \quad \rightarrow \quad \theta = \tan^{-1}\frac{10}{k}$ $\theta = \tan^{-1}\frac{10}{\frac{5}{2}} = \tan^{-1} 4$ $\frac{d\theta}{dt} = \frac{d\theta}{dk} \times \frac{dk}{dt}$ $\frac{d\theta}{dt} = \frac{1}{-10\,\mathrm{cosec}^2\,\theta} \times \frac{10x}{\sqrt{x^2+0.49}}$ $\frac{d\theta}{dt} = \frac{1}{-10\,\mathrm{cosec}^2(\tan^{-1} 4)} \times \frac{10 \times 2.4}{\sqrt{2.4^2+0.49}}$ $\frac{d\theta}{dt} = \frac{1}{-10 \times \frac{17}{16}} \times \frac{24}{\frac{5}{2}}$ $\frac{d\theta}{dt} = -\frac{384}{425}\ rad/h$	2 Marks Correct solution 1 Mark Makes significant progress