Caringbah High School

Year 12 2025
Mathematics Extension 1
HSC Course
Assessment Task 4 Trial Examination

General Instructions
- Reading time – 10 minutes
- Working time – 2 hours
- Write using black pen
- NESA-approved calculators may be used
- A reference sheet is provided
- In Questions 11-14, show relevant mathematical reasoning and/or calculations
- Marks may not be awarded for partial or incomplete answers

Total marks – 70

Section I **10 marks**
Attempt Questions 1-10
Mark your answers on the answer sheet provided. You may detach the sheet and write your name on it.

Section II **60 marks**
Attempt Questions 11-14
Write your answers in the answer booklets provided. Ensure your name or student number is clearly visible.

Student Number: ______________________________ **Class:** ____________

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<tr><td colspan="7">Marker’s Use Only</td></tr>
<tr><td>Section I</td><td colspan="4">Section II</td><td colspan="2" rowspan="2">Total</td></tr>
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Section I

10 marks
Attempt Questions 1-10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1–10

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1. What is the solution of the inequality $x(x-4) < 5$?

(A) $(0, 4)$

(B) $(-1, 5)$

(C) $(-\infty, -1) \cup (5, \infty)$

(D) $(-\infty, 0) \cup (4, \infty)$

2. What are the domain and range of the function $f(x) = \cos^{-1}\left(\dfrac{x}{4}\right)$?

(A) Domain: $-1 \leq x \leq 1$, Range: $0 \leq y \leq \pi$

(B) Domain: $-4 \leq x \leq 4$, Range: $0 \leq y \leq 4\pi$

(C) Domain: $-4 \leq x \leq 4$, Range: $0 \leq y \leq \pi$

(D) Domain: $-1 \leq x \leq 1$, Range: $0 \leq y < 4\pi$

3. $(x-1)$ is a factor of the polynomial $P(x) = x^3 + kx^2 + 3x - 2$, for some constant k.

What is the remainder when $P'(x)$ is divided by $(x-1)$?

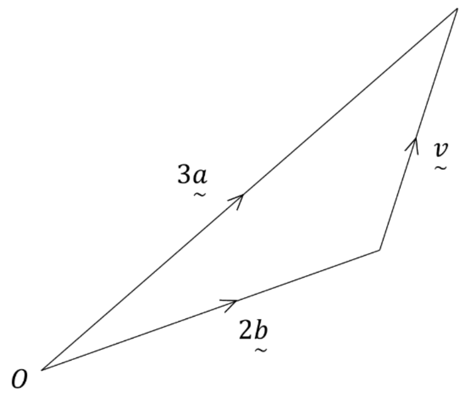
(A) -2

(B) -1

(C) 1

(D) 2

4. It is given that $\underset{\sim}{a} = \overrightarrow{OA} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$ and $\underset{\sim}{b} = \overrightarrow{OB} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$, and $\underset{\sim}{v}$ starts at the tip of $2\underset{\sim}{b}$ and finishes at the tip of $3\underset{\sim}{a}$ as shown.



Which vector below is equal to $\underset{\sim}{v}$?

(A) $\begin{pmatrix} -2 \\ 3 \end{pmatrix}$

(B) $\begin{pmatrix} 2 \\ -3 \end{pmatrix}$

(C) $\begin{pmatrix} 4 \\ 4 \end{pmatrix}$

(D) $\begin{pmatrix} 10 \\ 9 \end{pmatrix}$

5. Given that $\cos x - 2\sin x + 2 = 0$. What are the values of t, if $t = \tan\left(\frac{x}{2}\right)$?

(A) $t = 1$ or $t = 3$

(B) $t = 1$ or $t = -2$

(C) $t = 1$ or $t = 2$

(D) $t = 1$ or $t = -3$

6. Fifteen cards are numbered 1 to 15 and placed on a table so that their numbers are hidden. The cards are flipped over one at a time. What is the minimum number of cards that must be flipped to obtain at least one pair with a difference of 7?

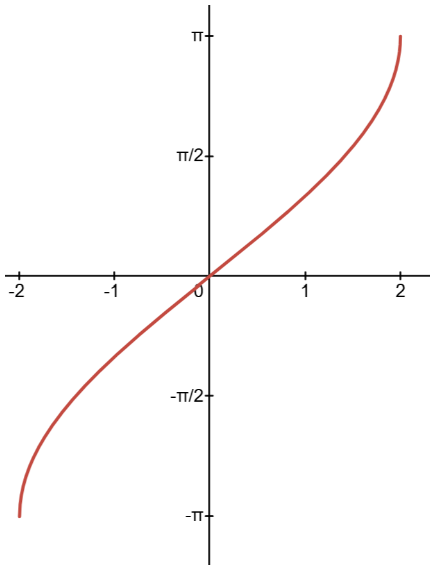
(A) 7

(B) 8

(C) 9

(D) 10

7. The diagram shows the graph of a function. Which function does the graph represent?



(A) $y = 2\sin^{-1}(2x)$

(B) $y = -2\cos^{-1}(2x)$

(C) $y = 2\sin^{-1}\left(\dfrac{x}{2}\right)$

(D) $y = 2\cos^{-1}\left(\dfrac{x}{2}\right)$

8. What is the primitive of $\dfrac{3 + 2x}{4 + x^2}$?

(A) $\dfrac{3}{2}\tan^{-1}(x) + \ln(4 + x^2) + c$

(B) $3\tan^{-1}\left(\dfrac{x}{2}\right) + \ln(4 + x^2) + c$

(C) $\dfrac{3}{2}\tan^{-1}(x) + \ln(x^2) + c$

(D) $\dfrac{3}{2}\tan^{-1}\left(\dfrac{x}{2}\right) + \ln(4 + x^2) + c$

9. The coefficient of x^2 in the expansion of $\left(5x + \dfrac{2}{x}\right)^8$ is:

(A) 112 000

(B) 1 400 000

(C) 700 000

(D) 224 000

Section I continues next page

10. In a game of basketball, Sam has a scoring accuracy of 70%. What is the ratio of Sam scoring at least one of three attempts compared to scoring exactly twice out of three attempts?

(A) 3: 2

(B) 973: 189

(C) 49: 3

(D) 139: 63

End of Section I

Section II

60 marks
Attempt Questions 11–14
Allow about 1 hour 45 minutes for this section

Answer each question in a SEPARATE writing booklet.

Your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Use a SEPARATE writing booklet.

(a) In how many ways can the letters of the word SUCCESS be arranged in a line if:

(i) there are no restrictions? **1**

(ii) the two Cs are NOT adjacent? **2**

(b) Use division of polynomials to express $P(x)$ in the form **2**

$P(x) = A(x)Q(x) + R(x)$, where $P(x) = 3x^3 + 14x + 11$ and $A(x) = x^2 - 3x + 2$.

(c) Use mathematical induction to prove that $8^n - 1$ is divisible by 7, for all integers $n \geq 1$. **3**

(d) $\underset{\sim}{a}$ and $\underset{\sim}{b}$ are two non-zero vectors. Prove that the vectors $\underset{\sim}{u}$ and $\underset{\sim}{v}$ below are perpendicular to each other. **2**

$$\underset{\sim}{u} = |\underset{\sim}{b}|\underset{\sim}{a} + |\underset{\sim}{a}|\underset{\sim}{b}$$
$$\underset{\sim}{v} = |\underset{\sim}{b}|\underset{\sim}{a} - |\underset{\sim}{a}|\underset{\sim}{b}$$

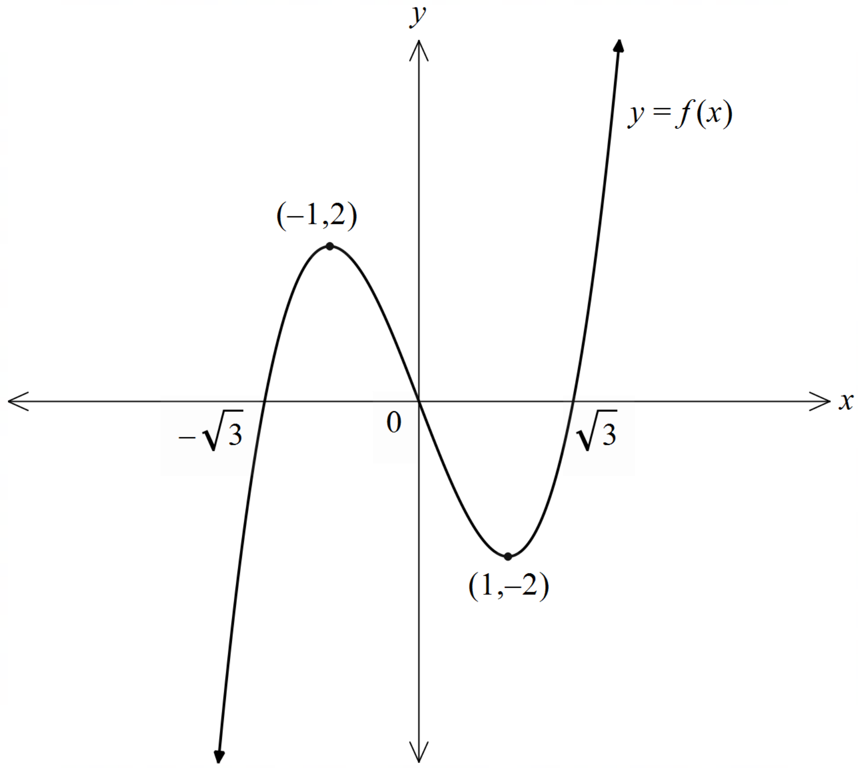
(e) Using the substitution $u = x - 1$, or otherwise, find **2**

$$\int (x+3)(x-1)^5 \, dx$$

Question 11 continues next page

(f) The diagram below shows the graph of $y = f(x)$. **3**

Sketch $y = \dfrac{1}{f(x)}$ **over the graph in template provided**, showing all turning points, relevant coordinates, and asymptotes.



End of Question 11

Question 12 (15 marks) Use a SEPARATE writing booklet.

(a) Given $\underset{\sim}{u} = 4\underset{\sim}{i} + 3\underset{\sim}{j}$ and $\underset{\sim}{v} = 3\underset{\sim}{i} + 5\underset{\sim}{j}$. Find

(i) $\underset{\sim}{u} \cdot \underset{\sim}{v}$ **1**

(ii) the angle between $\underset{\sim}{u}$ and $\underset{\sim}{v}$ to the nearest degree. **2**

(b) **(i)** Given $\cos x + \sqrt{3}\,\sin x = A\cos(x - \alpha)$, find A and α, where $0 \le \alpha \le 2\pi$ and $A > 0$. **2**

(ii) Hence, or otherwise, solve $\cos x + \sqrt{3}\,\sin x = 2,\ 0 \le x \le 2\pi$. **2**

(c) A spherical bubble is moving up through a liquid. As it rises, the bubble gets bigger, and its radius increases at a rate of 0.3 mm/s. **2**
At what rate is the volume of bubble increasing when its radius reaches 0.7 mm, give your answer correct to one decimal place.

(d) The region bounded by the curve $y = \sqrt{x}$, the y-axis and the line $y = 2$ is rotated about the y-axis. Calculate the exact volume of the solid of revolution. **2**

(e) A factory produces computer chips. Each chip has a 2% chance of being faulty and is produced independently. The factory's quality control department tests a random sample of 100 chips.

(i) Calculate the mean and variance of the random variable **2**

$$X \sim \text{Bin}\,(100, 0.02).$$

(ii) Using a binomial distribution, calculate the probability that a maximum of three chips is faulty. Give your answer correct to four decimal places. **2**

End of Question 12

Question 13 (15 marks) Use a SEPARATE writing booklet.

(a) A four-person team is to be chosen at random from nine women and seven men.

(i) In how many ways can the team be chosen. **1**

(ii) What is the probability that the team will consist of 4 men. **1**

(b) Evaluate **3**

$$\int_0^{\frac{\pi}{2}} \sin^2(3x)\, dx$$

(c) The unit vectors $\underset{\sim}{p}$ and $\underset{\sim}{q}$ make an angle of $\frac{\pi}{3}$ with each other. **3**

Find the magnitude of $\underset{\sim}{p} - \frac{\underset{\sim}{q}}{2}$.

(d) Use Mathematical Induction to prove that **3**

$$\cos x + \cos(3x) + \cos(5x) + \ldots + \cos((2n-1)x) = \frac{\sin(2nx)}{2\sin x},$$

for all positive integers $n \geq 1$

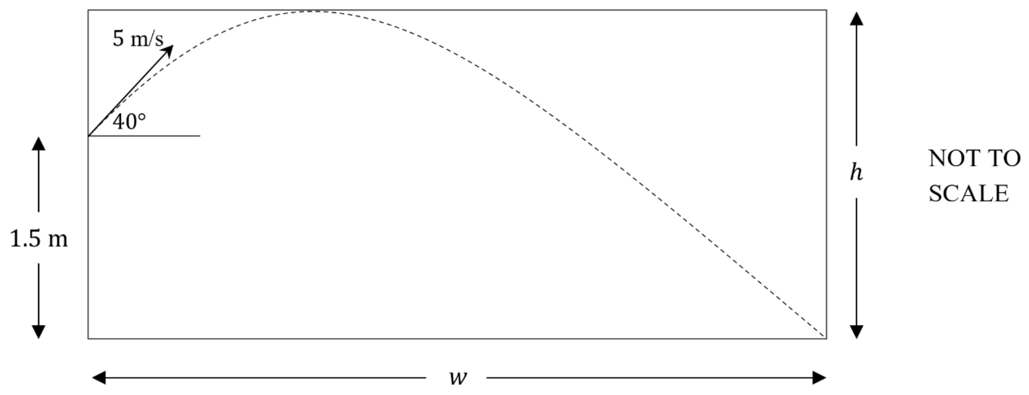
Question 13 continues next page

(e) A student is standing in the doorway to a classroom, which has a height of h metres and a width of w metres. He throws a ball from a height of 1.5 metres, at a speed of 5 m/s with an angle of projection of 40°.
The ball just touches the ceiling before landing at the junction of the floor and the wall, as shown in figure below.
The horizontal and vertical displacement t seconds after projection are:

$$x = 5t\cos(40°)$$

$$y = 5t\sin(40°) - 5t^2 + 1.5$$

(DO NOT PROVE THESE)



Find the width and height of the room in metres, correct to 1 decimal place. **4**

End of Question 13

Question 14 (15 marks) Use a SEPARATE writing booklet.

(a) Given that k is a non-zero constant and n is a positive integer, then **3**

$$(1 + kx)^n = 1 + 40x + 120k^2 x^2 + \ldots$$

Find the values of k and n.

(b) It is known that $\cos(4\theta) = 8\cos^4\theta - 8\cos^2\theta + 1$ (DO NOT PROVE THIS)

(i) Find the exact values of the solutions to the equation **2**

$8x^4 - 8x^2 + 1 = 0$, for $0 \le \theta \le \pi$

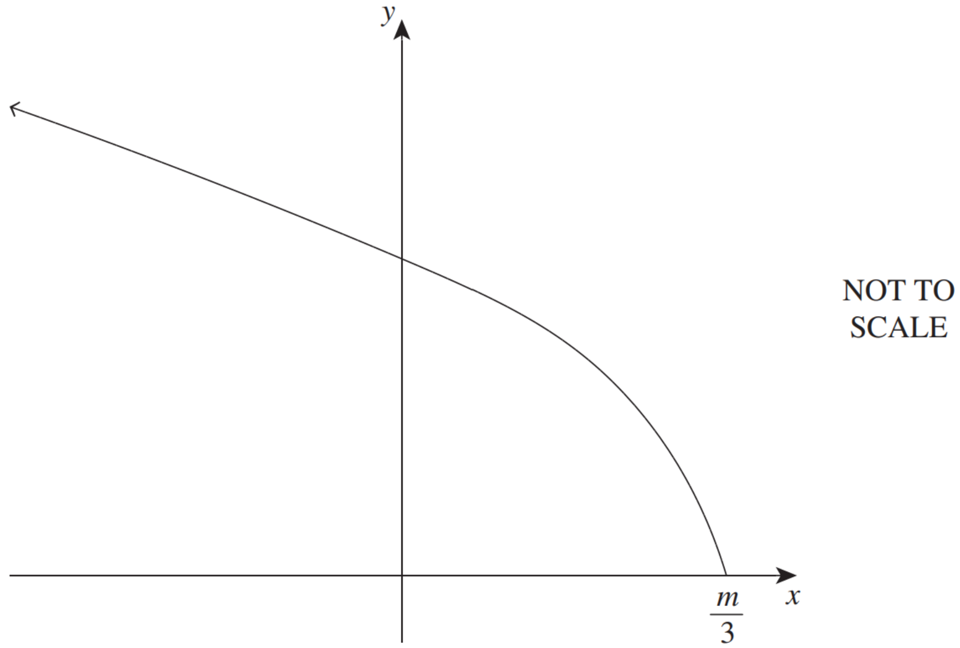
(ii) Hence, or otherwise, find the exact value of $\cos\frac{\pi}{8}\cos\frac{3\pi}{8}$. **2**

(c) Use the substitution of $x = \tan^2(\theta)$, $0 \le \theta < \frac{\pi}{2}$, to evaluate: **4**

$$\int_0^1 \frac{1-x}{\sqrt{x}\,(1+x)^2}\,dx$$

Question 14 continues next page

(d) Let $f(x) = \sqrt{m - 3x}$ for $x \le \frac{m}{3}$. The graph of $y = f(x)$ is shown below. **4**



The area enclosed by the graph $y = f(x)$, the x-axis and the y-axis is rotated about the y-axis.

Find the value of m such that the volume of the solid formed is $\frac{5000\pi}{27}$ cubic units.

End of Examination

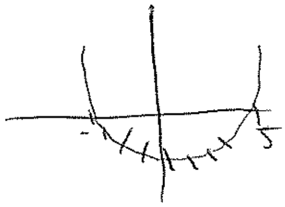
Multiple Choice

1) $x(x-4)<5$

$x^2-4x-5<0$

$(x-5)(x+1)<0$

$-1<x<5$ – B



2) $-1\leq \frac{x}{4}\leq 1$

$-4\leq x\leq 4$

Range $= 0\leq y\leq \pi$ – C

3) $P(1)=0$

$1+k+3-2=0$

$k=-2$

$P'(x)=3x^2+2kx+3$

$P'(x)=3x^2-4x+3$

$P'(1)=3-4+3=2$

– D

4) $\underset{\sim}{v}=-2\underset{\sim}{b}+3\underset{\sim}{a}$

$=-2\begin{pmatrix}2\\3\end{pmatrix}+3\begin{pmatrix}2\\1\end{pmatrix}$

$=\begin{bmatrix}-4 & +6\\-6 & +3\end{bmatrix}$

$=\begin{bmatrix}2\\-3\end{bmatrix}$ – B

5) $\frac{1-t^2}{1+t^2}-2\left(\frac{2t}{1+t^2}\right)+2=0$

$1-t^2 \quad -\cancel{2}4t+2(1+t^2)=0$

$t^2-4t+3=0$

$(t-3)(t-1)=0$

$t=3$ or $t=1$ – A

6) $(1,8), (2,9), (3,10), (4,11), \ldots$

By PGP $\underline{\underline{9}}$ $\Rightarrow$ C

7) $\sin^{-1}x$ dilated by a factor of

- 2 Horizontally
- 2 Vertically

$$\Rightarrow \frac{y}{2} = \sin^{-1}\left(\frac{x}{2}\right)$$

$$y = 2\sin^{-1}\left(\frac{x}{2}\right) \Rightarrow \text{C}$$

8) $$\int \frac{3+2x}{4+x^2}$$

$$= \int \frac{3}{4+x^2} + \int \frac{2x}{4+x^2}$$

$$= \frac{3}{2}\tan^{-1}\left(\frac{x}{2}\right) + \ln(4+x^2) + C$$

$$\Rightarrow \text{D}$$

9) $${}^8C_r (5x)^r \left(\frac{2}{x}\right)^{8-r}$$

$$= {}^8C_r\, 5^r x^r \times 2^{8-r} x^{r-8}$$

$$= {}^8C_r \times 5^r \times 2^{8-r} x^{2r-8}$$

$2r-8=2$

$2r=10$

$r=5$

$$\Rightarrow {}^8C_5 \times 5^5 \times 2^{8-5}$$

$$= 1400\,000$$

$$\Rightarrow \text{B}$$

(16) $X \sim (3, 0.1)$

- $P(X \geq 1) = 1 - P(X = 0)$

$$= 1 - {}^3C_0 (0.7)^0 (0.3)^3$$

$$= 1 - 0.027$$

$$= 0.973$$

- $P(X = 2) = {}^3C_2 (0.7)^2 (0.3)$

$$= 0.441$$

- $\dfrac{0.973}{0.441} = \dfrac{139}{63} \Rightarrow$ D

Question 11

a) i) $\dfrac{7!}{3! \times 2!} = 420$ — 1

ii) $420 -$ (Cs are adjacent)

$= 420 - \left(\dfrac{6!}{3!}\right)$ — 1

$= 300$ — 1

b)

$$\begin{array}{r|l} & 3x + 9 \\ \hline x^2 - 3x + 2 & 3x^3 + 0x^2 + 14x + 11 \\ & -(3x^3 - 9x^2 + 6x) \\ \hline & 9x^2 + 8x + 11 \\ & -(9x^2 - 27x + 18) \\ \hline & 35x - 7 \end{array}$$

— 1 (quotient $3x+9$)

— 1 (remainder $35x-7$)

$P(x) = (x^2 - 3x + 2)(3x + 3) + 17x + 5$

c) $\underline{n=1}$: $8^1 - 1 = 7$ is divisible by 7 — 1

$\Longrightarrow$ true for $n = 1$

* Suppose true for $n = k$, $k \geq 1$, that is

$8^k - 1 = 7m$, for some integer m

To prove true for $n = k+1$, that is

$8^{k+1} - 1 = 7p$, for some integer p — 1

$$\begin{aligned} 8^{k+1} - 1 &= 8^k . 8 - 1 \\ &= (7m+1) \times 8 - 1 \\ &= 56m + 8 - 1 \\ &= 56m + 7 = 7(8m+1) = 7p \end{aligned}$$ — 1

So by mathematical induction $8^n - 1$ is divisible by 7, for all integers $n \geq 1$

d) $$\underset{\sim}{u} \cdot \underset{\sim}{v} = \left(|\underset{\sim}{b}|\underset{\sim}{a} + |\underset{\sim}{a}|\underset{\sim}{b}\right)\left(|\underset{\sim}{b}|\underset{\sim}{a} - |\underset{\sim}{a}|\underset{\sim}{b}\right) \quad — 1$$

$$= |\underset{\sim}{b}|^2 \underset{\sim}{a} \cdot \underset{\sim}{a} - \cancel{|\underset{\sim}{a}||\underset{\sim}{b}|\underset{\sim}{a} \cdot \underset{\sim}{b}} + \cancel{|\underset{\sim}{a}||\underset{\sim}{b}|\underset{\sim}{b} \cdot \underset{\sim}{a}}$$

$$- |\underset{\sim}{a}|^2 \underset{\sim}{b} \cdot \underset{\sim}{b}$$

$$= |\underset{\sim}{b}|^2 |\underset{\sim}{a}|^2 - |\underset{\sim}{a}|^2 |\underset{\sim}{b}|^2$$

$$= 0 \quad — 1$$

$\Longrightarrow$ $\underset{\sim}{u}$ and $\underset{\sim}{v}$ are perpendicular to each other

e) $u = x - 1$
$du = dx$

$$\int (x+3)(x-1)^5 \, dx$$

$$= \int (u+1+3)(u)^5 \, du \quad — 1$$

$$= \int (u+4) u^5 \, du$$

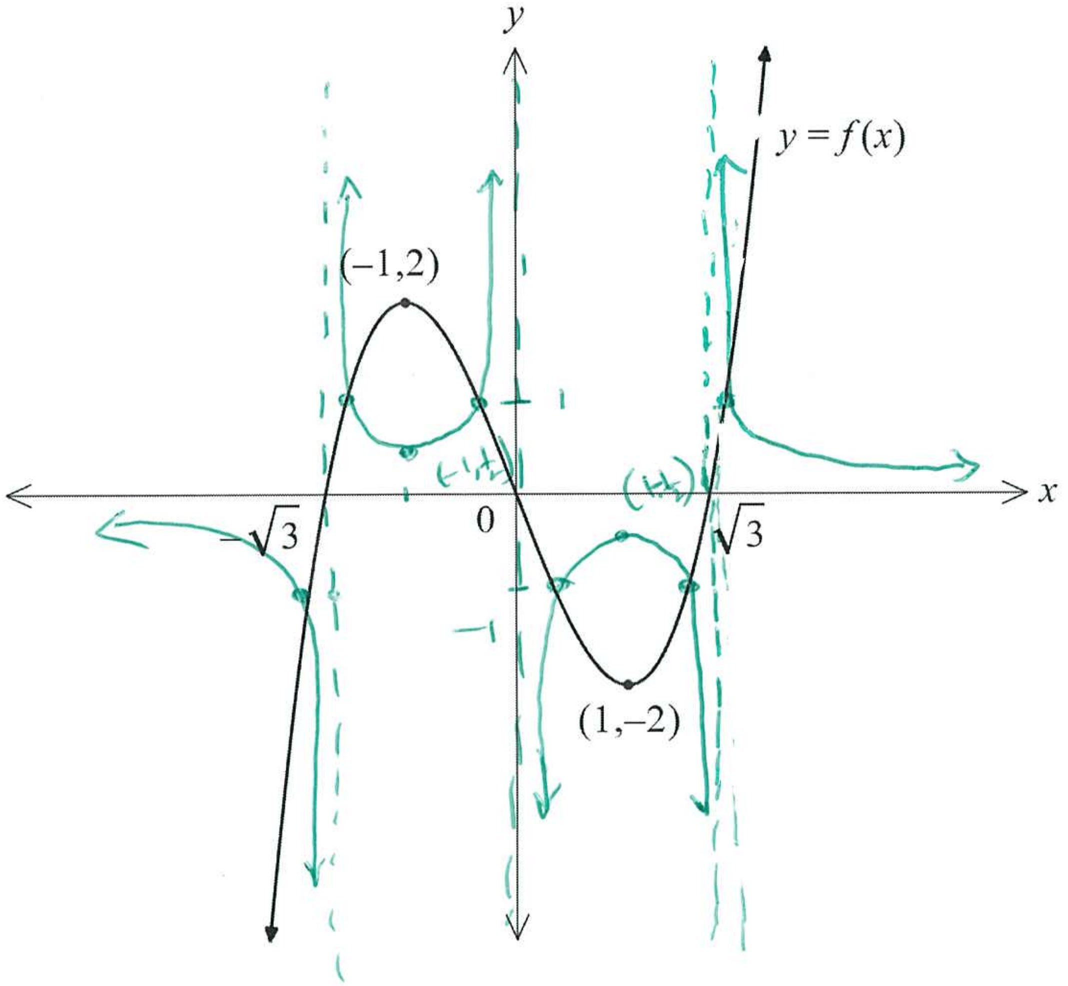
$$= \int u^6 + 4u^5 \, du$$

$$= \frac{u^7}{7} + \underbrace{\frac{4u^6}{6}}_{\text{or } \frac{2}{3}u^6} + c$$

$$= \frac{(x-1)^7}{7} + \frac{4(x-1)^6}{6} + c \quad — 1$$

Student Number/Name: ____________________

Question 11 (f) Template



Asymptote − ①
min / max clear coordinate − ①
intersection with original at − ①
1 , −1

Question 12:

a) i) $\underset{\sim}{u} \cdot \underset{\sim}{v} = 4 \times 3 + 3 \times 5$

$= 12 + 15$

$= 27$ — 1

ii) $|\underset{\sim}{u}| = \sqrt{16+9} = 5$

$|\underset{\sim}{v}| = \sqrt{9+25} = \sqrt{34}$ — 1

$\cos\theta = \dfrac{\underset{\sim}{u} \cdot \underset{\sim}{v}}{|\underset{\sim}{u}||\underset{\sim}{v}|}$

$\cos\theta = \dfrac{27}{5\sqrt{34}}$

$\boxed{\theta \simeq 22^\circ}$ — 1

b) i) $\cos x + \sqrt{3}\sin x = A(\cos x \cos\alpha + \sin x \sin\alpha)$

$\begin{cases} A\cos\alpha = 1 \\ A\sin\alpha = \sqrt{3} \end{cases}$

$\sin\alpha = \dfrac{\sqrt{3}}{2} > 0$

$\cos\alpha = \dfrac{1}{2} > 0$

$\Rightarrow \alpha$ in Q1

$\Rightarrow \boxed{\alpha = \dfrac{\pi}{3}}$ — 1

$A = \sqrt{1+3}$

$A = \sqrt{4}$

$\boxed{A = 2}$ — 1

ii) $2\cos\left(x - \dfrac{\pi}{3}\right) = 2$ — 1

$\cos\left(x - \frac{\pi}{3}\right) = 1$

$u = x - \frac{\pi}{3}$ $\qquad 0 \leq x \leq 2\pi$

$\cos u = 1$ $\qquad \frac{\pi}{3} \leq u \leq \frac{5\pi}{3}$

$u = 0, \cancel{2\pi}$

$\Rightarrow x - \frac{\pi}{3} = 0$

$\boxed{x = \frac{\pi}{3}}$ — 1

c) $\frac{dr}{dt} = 0.3$ mm/s

$\frac{dv}{dt} = ??$

$\frac{dv}{dt} = \frac{dv}{dr} \times \frac{dr}{dt}$

$= 4\pi r^2 \times 0.3$

$V = \frac{4}{3}\pi r^3$

$\frac{dv}{dr} = \frac{4}{3} \times 3\pi r^2$

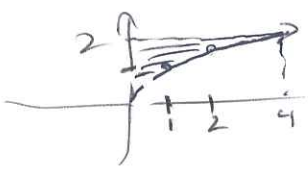
$\frac{dv}{dr} = 4\pi r^2$ — 1

$r = 0.7$

$\Rightarrow \frac{dv}{dt} = 4\pi \times (0.7)^2 \times 0.3$

$= 1.8$ mm³/s — 1

d) $\begin{cases} y = \sqrt{x} \\ x = y^2 \end{cases}$ 1 —



$V = \pi \int_0^2 [f(y)]^2 \, dy$

$= \pi \int_0^2 y^4 \, dy$

$= \frac{\pi}{5}[2^5 - 0]$

$= \frac{32\pi}{5}$ — 1

e) i) $X \sim Bin(100, 0.02)$

$$\mu = np$$
$$= 100(0.02)$$
$$= 2 \quad - 1$$

$$Var(X) = npq$$
$$= 100(0.02)(0.98)$$
$$= 1.96 \quad - 1$$

ii) $P(X \leq 3)$

$$= P(X=0) + P(X=1) + P(X=2) + P(X=3)$$
$$1 - \;= {}^{100}C_0 (0.02)^0 (0.98)^{100} + {}^{100}C_1 (0.02)^1 (0.98)^{99}$$
$$+ {}^{100}C_2 (0.02)^2 (0.98)^{98} + {}^{100}C_3 (0.02)^3 (0.98)^{97}$$

$$= 0.8590 \quad - 1$$

Question 13

a) i) ${}^{16}C_4 = 1820$ — 1

ii) ${}^{7}C_4 = 35$ — 1

b) $$\int_0^{\frac{\pi}{2}} \sin^2(3x)\,dx$$

$$= \int_0^{\frac{\pi}{2}} \frac{1-\cos 2(3x)}{2}$$

$$= \int_0^{\frac{\pi}{2}} \frac{1-\cos 6x}{2} \quad — 1$$

$$= \left[\frac{1}{2}x - \frac{\sin 6x}{12}\right]_0^{\frac{\pi}{2}} \quad — 1$$

$$= \left(\frac{\pi}{4} - \frac{\sin 3\pi}{12}\right) - (0-0)$$

$$= \frac{\pi}{4} \quad — 1$$

c) $|\underset{\sim}{p}| = |\underset{\sim}{q}| = 1$

$\underset{\sim}{p} \cdot \underset{\sim}{q} = |\underset{\sim}{p}||\underset{\sim}{q}| \cos \frac{\pi}{3}$

$\underset{\sim}{p} \cdot \underset{\sim}{q} = \frac{1}{2}$ — 1

$$\left|\underset{\sim}{p} - \frac{\underset{\sim}{q}}{2}\right|^2 = \left(\underset{\sim}{p} - \frac{\underset{\sim}{q}}{2}\right)\cdot\left(\underset{\sim}{p} - \frac{\underset{\sim}{q}}{2}\right)$$

$$= |\underset{\sim}{p}|^2 - \frac{1}{2}\underset{\sim}{p}\cdot\underset{\sim}{q} - \frac{1}{2}\underset{\sim}{q}\cdot\underset{\sim}{p} + \frac{|\underset{\sim}{q}|^2}{4} \quad — 1$$

$$= 1 - \underset{\sim}{p}\cdot\underset{\sim}{q} + \frac{1}{4}$$

$$= 1 - \frac{1}{2} + \frac{1}{4} = \frac{3}{4}$$

$$\Rightarrow \left|\underset{\sim}{p} - \frac{\underset{\sim}{q}}{2}\right| = \frac{\sqrt{3}}{2} \quad — 1$$

(d) $\underline{n=1}$ $\quad RHS = \frac{\sin 2x}{2\sin x} = \frac{2\sin x\cos x}{2\sin x} = \cos x = LHS$

Hence true for $n=1$ $\quad — 1$

* Suppose true for $n=k$, $k \geq 1$, so

$$\cos x + \cos 3x + \cos 5x + \dots + \cos((2k-1)x) = \frac{\sin 2kx}{2\sin x}$$

* To prove true for $n=k+1$

$$\cos x + \cos 3x + \cos 5x + \dots + \cos((2k-1)x) + \cos(((2(k+1)-1)x) = \frac{\sin 2(k+1)x}{2\sin x}$$

* $LHS = \underbrace{\cos x + \cos 3x + \dots + \cos((2k-1)x)} + \cos((2k+1)x)$

$$= \frac{\sin 2kx}{2\sin x} + \cos((2k+1)x)$$

$$= \frac{\sin 2kx + 2\sin x\cos((2k+1)x)}{2\sin x} \quad — 1$$

$$= \frac{\sin 2kx + \sin((2k+1)x + x) - \sin((2k+1)x - x)}{2\sin x}$$

$$= \frac{\cancel{\sin 2kx} + \sin(2kx + 2x) - \cancel{\sin(2kx)}}{2\sin x}$$

$$= \frac{\sin(2(n+1)x)}{2\sin x} \quad — 1$$

$$= RHS$$

$\Rightarrow$ By mathematical induction, statement true for all $n \geq 1$.

(e) $\begin{cases} x = 5t\cos 40 \\ y = 5t\sin 40 - 5t^2 + 1.5 \end{cases}$

$\Rightarrow \begin{cases} \dot{x} = 5\cos 40 \\ \dot{y} = 5\sin 40 - 10t \end{cases}$

- $\dot{y} = 0$

$\Rightarrow 5\sin 40 - 10t = 0$

$10t = 5\sin 40$ — 1

$t = \frac{\sin 40}{2}$

* $h = y_{t=\frac{\sin 40}{2}} = 5\left(\frac{\sin 40}{2}\right)\sin 40 - 5\left(\frac{\sin^2 40}{2}\right) + 1.5$ — 1

$\simeq 2.0$

- $y = 0$

$5t\sin 40 - 5t^2 + 1.5 = 0$

$5t^2 - 5t\sin 40 - 1.5 = 0$

$t = \frac{5\sin 40 + \sqrt{(5\sin 40)^2 - 4(5)(-1.5)}}{2(5)}$

$$t = \frac{5\sin 40 + \sqrt{25\sin^2 40 + 30}}{10} \quad — \ 1$$

(or decimal roundy)

$$* \quad x = 5(\,*\,) \times \cos 40$$

$$\boxed{x \simeq 3.7} \quad — \ 1$$

Question 14

a)
$$(1+kx)^n = {}^nC_0 + {}^nC_1 kx + {}^nC_2 (kx)^2 + \dots$$
$$= 1 + nkx + \frac{n!}{2!(n-2)!}(kx)^2 + \dots$$
$$= 1 + nkx + \frac{n(n-1)}{2}(kx)^2 + \dots \quad (A)$$
— 1

Given $(1+kx)^n = 1 + 40x + 120k^2x^2$ (B)

$\Rightarrow$ (A) = (B) — 1

$nk = 40$ & $\frac{n(n-1)}{2} = 120$

$n^2 - n - 240 = 0$

$(n+15)(n-16) = 0$

$\boxed{n = 16}$ — 1

sub,

$k = \frac{40}{16}$

$\boxed{k = \frac{5}{2}}$

b) (i) Let $x = \cos\theta$

$\cos 4\theta = 0$

$4\theta = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}$ — 1

$\theta = \frac{\pi}{8}, \frac{3\pi}{8}, \frac{5\pi}{8}, \frac{7\pi}{8}$

So $x = \cos\frac{\pi}{8}, \cos\frac{3\pi}{8}, \cos\frac{5\pi}{8}, \cos\frac{7\pi}{8}$ — 1

(ii) $\alpha\beta\gamma\delta = \frac{e}{a}$

$$\cos\frac{\pi}{8}\cos\frac{3\pi}{8}\cos\frac{5\pi}{8}\cos\frac{7\pi}{8} = \frac{1}{8} \quad — 1$$

$$\cos\frac{\pi}{8}\cos\frac{3\pi}{8}\cos\left(\pi-\frac{3\pi}{8}\right)\cos\left(\pi-\frac{\pi}{8}\right) = \frac{1}{8}$$

$$-\cos^2\left(\frac{\pi}{8}\right)\left(-\cos^2\left(\frac{3\pi}{8}\right)\right) = \frac{1}{8}$$

$$\left(\cos\left(\frac{\pi}{8}\right)\cos\left(\frac{3\pi}{8}\right)\right)^2 = \frac{1}{8}$$

$$\cos\frac{\pi}{8}\cos\frac{3\pi}{8} = \frac{1}{\sqrt{8}} \text{ or } \frac{1}{2\sqrt{2}} \quad — 1$$

e) $x = \tan^2\theta$

$$\left.\begin{array}{l} \frac{dx}{d\theta} = 2\sec^2\theta\tan\theta \\ dx = 2\sec^2\theta\tan\theta \end{array}\right\} — 1$$

$\tan^2\theta = 0$

$\theta = 0$

$\tan^2\theta = 1$

$\theta = \frac{\pi}{4}$

$$\int_0^{\frac{\pi}{4}} \frac{1-\tan^2\theta}{\cancel{\tan\theta}(1+\tan^2\theta)} \times (2\cancel{\sec^2\theta}\cancel{\tan\theta}\,d\theta)$$

$$= 2\int_0^{\frac{\pi}{4}} \frac{1-\tan^2\theta}{1+\tan^2\theta}\,d\theta \quad — 1$$

$$= 2\int_0^{\frac{\pi}{4}} \frac{1 \quad \tan^2\theta}{\sec^2\theta}\,d\theta$$

$$= 2\int_0^{\frac{\pi}{4}} \left(\frac{1}{\sec^2\theta} - \frac{\tan^2\theta}{\sec^2\theta}\right) d\theta$$

$$= 2\int_0^{\frac{\pi}{4}} \left(\cos^2\theta - \frac{\sin^2\theta}{\cancel{\cos^2\theta}} \times \cancel{\cos^2\theta} \right)$$

$$= 2\int_0^{\frac{\pi}{4}} \cos^2\theta - \sin^2\theta \quad — 1$$

$$= 2\int_0^{\frac{\pi}{4}} \cos 2\theta$$

$$= 2\left[\frac{\sin 2\theta}{2}\right]_0^{\frac{\pi}{4}}$$

$$= 2\left[\frac{1}{2} - 0\right]$$

$$= 1 \quad — 1$$

d) $y^2 = m - 3x$

$y^2 - m = -3x$

$\boxed{x = \frac{m - y^2}{3}}$ — 1

$$V = \pi \int_0^{\sqrt{m}} \left(\frac{m - y^2}{3}\right)^2 dy \quad — 1$$

$$= \frac{\pi}{9} \int_0^{\sqrt{m}} m^2 - 2my^2 - y^4 \, dy$$

$$= \frac{\pi}{9} \left[m^2 y - \frac{2my^3}{3} + \frac{y^5}{5}\right]_0^{\sqrt{m}}$$

$$= \frac{\pi}{9} \left[m^2\sqrt{m} - \frac{2m^2\sqrt{m}}{3} + \frac{m^2\sqrt{m}}{5} - 0\right] \quad — 1$$

$$= \frac{\pi}{9} \times \frac{8m^2\sqrt{m}}{15} = \frac{5000}{27}\pi$$

$$\frac{8m^2\sqrt{m}}{135} = \frac{5000}{27}$$

$$m^{5/2} = 3125$$

$$\boxed{m = 25} \quad - (1)$$