



Student Number:

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Cheltenham Girls High School

2022

TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Extension 1

General Instructions

- Reading time – 10 minutes
- Working time – 2 hours
- Write using black pen
- Calculators approved by NESA may be used
- A NESA reference sheet is provided
- Use the multiple-choice answer sheet attached for Question 1 - 10
- For Questions in Section II, show mathematical reasoning and/or calculations to gain maximum marks
- Write your Student Number on the front of each Writing Booklet for Questions 11, 12, 13 and 14

Total Marks: 70

Section I – 10 marks (pages 2 – 6)

- Attempt Questions 1 – 10
- Allow about 15 minutes for this section

Section II – 60 marks (pages 7 – 15)

- Attempt Questions 11 – 14
- Allow about 1 hour and 45 minutes for this section

Section I

10 marks

Attempt Questions 1 – 10.

Allow about 15 minutes for this section.

Use the multiple-choice answer sheet for Questions 1 – 10.

1 What is the derivative of $\tan^{-1}(x^4)$ with respect to x ?

A. $\frac{1}{1+x^8}$

B. $\frac{4x^3}{1+x^8}$

C. $\frac{4x^3}{1+x^4}$

D. $\frac{x^4}{1+x^8}$

2 Which of the following polynomials has a factor of $(x+1)$?

A. $P(x) = x^3 - 5x^2 + 4$

B. $P(x) = x^3 - 5x^2 + 6$

C. $P(x) = x^4 - x^2 + 2$

D. $P(x) = x^4 + x^2$

Section 1 is continued over the page

Section 1 continued

3 What is the domain and range of the function $y = 6 \cos^{-1}(3x)$?

A. Domain $\left[-\frac{1}{3}, \frac{1}{3}\right]$; Range $[0, 6\pi]$.

B. Domain $\left[-\frac{1}{3}, \frac{1}{3}\right]$; Range $[0, 3\pi]$.

C. Domain $[0, 6\pi]$; Range $\left[-\frac{1}{3}, \frac{1}{3}\right]$.

D. Domain $[0, 3\pi]$; Range $\left[-\frac{1}{3}, \frac{1}{3}\right]$.

4 What is the value of $\int_1^{\sqrt{3}} \frac{dx}{\sqrt{3-x^2}}$, correct to 3 significant figures?

A. -1.05

B. -0.955

C. 0.955

D. 1.05

5 Find the volume generated when $y = \sin x$ between $x = 0$ and $x = \frac{\pi}{3}$ is rotated around the x -axis.

A. $\frac{\pi^2}{6} - \frac{3}{8}$ cubic units.

B. $\frac{\pi^2}{6} - \frac{\sqrt{3}\pi}{8}$ cubic units.

C. $\frac{\pi^2}{3} - \frac{3\pi}{4}$ cubic units.

D. $\frac{\pi^2}{6} - \frac{\sqrt{3}\pi}{4}$ cubic units.

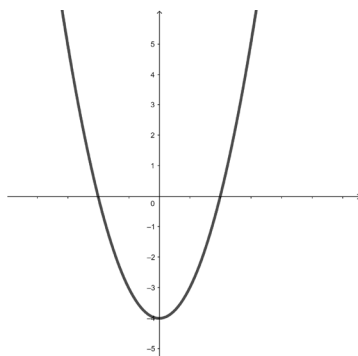
Section 1 is continued over the page

Section 1 continued

6 Which of the following is equivalent to $\frac{2 \tan \theta}{1 + \tan^2 \theta}$?

- A. $\cos 2\theta$
- B. $\sin 2\theta$
- C. $\tan 2\theta$
- D. $\cot 2\theta$

7 The graph of $y = f(x)$ is shown below. The diagram is not drawn to scale.



NOT TO
SCALE

How many real solutions does the equation $[f(x)]^2 = x$ have?

- A. 1
- B. 2
- C. 3
- D. 4

Section 1 is continued over the page

Section 1 continued

- 8 Hasti draws a vector from the origin to the point $X(-3, 7)$. She then draws a vector, $-2\mathbf{i} + 4\mathbf{j}$ from X , ending at point Y .

How far is point Y from the origin?

- A. $\sqrt{10}$
- B. $\sqrt{34}$
- C. $\sqrt{122}$
- D. $\sqrt{146}$
- 9 Which set of values for a and b that satisfy the following equation?

$$\begin{pmatrix} 3a - 4b \\ 2a + 3b \end{pmatrix} = \begin{pmatrix} -16 \\ 12 \end{pmatrix}$$

- A. $a = 0$ and $b = 4$
- B. $a = 1$ and $b = \frac{19}{4}$
- C. $a = \frac{9}{2}$ and $b = 1$
- D. $a = 4$ and $b = 0$

Section 1 is continued over the page

Section 1 continued

- 10** Which of the following differential equations **CANNOT** be solved by separation of the variables?

A. $\frac{dy}{dx} = xy$

B. $\frac{dy}{dx} = 3x^2y$

C. $\frac{dy}{dx} = \sin x + xy$

D. $\frac{dy}{dx} = \frac{x}{y}$

~ End of Section I ~

Section II

60 marks

Attempt Questions 11 – 14.

Allow about 1 hour and 45 minutes for this section.

Answer each question in the appropriate writing booklet. Extra writing booklets are available.

In Questions 11 – 14, your responses should include relevant mathematical reasoning and/or calculations to gain maximum marks

Question 11 (15 marks) START A WRITING BOOKLET.	Marks
(a) The equation $x^3 - 2x^2 + 4x - 3 = 0$ has roots $x = \alpha$, $x = \beta$ and $x = \gamma$. Find the value of $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma}$.	2
(b) The vectors $\vec{p} = \begin{pmatrix} 6 \\ a \end{pmatrix}$ and $\vec{q} = \begin{pmatrix} a+1 \\ -9 \end{pmatrix}$ are perpendicular. Find the value of a .	2
(c) A school has twenty prefects. Vionna and Chloe are two of those prefects. A committee of six prefects is to be formed to organise a function. What is the <u>probability</u> that the committee contains Vionna but does not contain Chloe?	2
(d) Using the substitution $u = 2x - 1$, find $\int 2x (2x - 1)^3 dx$.	3

Question 11 continues over the page

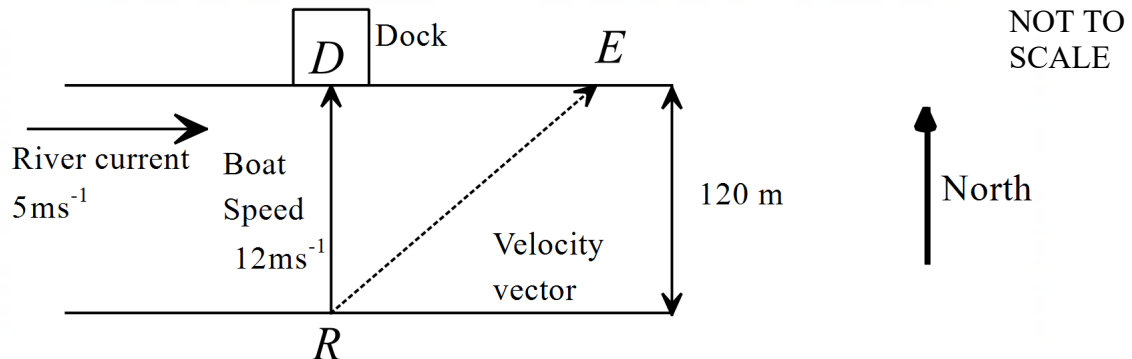
Question 11 (continued)**Marks**

- (e) A function is defined by $f(x) = 1 + \frac{3}{x-2}$ for $x > 2$.
- (i) Find the vertical and horizontal asymptotes for $y = f(x)$. **2**
- (ii) Find the inverse function $y = f^{-1}(x)$. **1**
- (iii) State the domain of $y = f^{-1}(x)$. **1**
- (f) Solve $\frac{x}{2-x} > 1$ **2**

End of Question 11

- (a) Rylie has a boat which moves at a top speed of 12 ms^{-1} in still water.

From point R , he wants to go due north to point D on the opposite side of the river, as shown in the diagram below.



Today the current in the river is flowing at 5 ms^{-1} . From R , he steers the boat due north toward D at top speed. Due to the current he drifts down the river and arrives at point E .

- (i) Taking R as the origin, write down Rylie's velocity vector in the form $x\mathbf{i} + y\mathbf{j}$ and find the magnitude of this vector. 2
- (ii) What is the bearing of Rylie's velocity vector and how far does he travel from R to E ? 2
- (iii) On what bearing should Rylie have pointed the boat, so that he arrived at the D , with the boat travelling at its top speed? 2

- (b) Write $\sqrt{3} \sin x - \cos x$ in the form $R \sin (x - \alpha)$ where $R > 0$ and

$0 \leq \alpha \leq \frac{\pi}{2}$ and hence, solve $\sqrt{3} \sin \theta - \cos \theta - 1 = 0$ for $[0, 2\pi]$.

3

Question 12 is continued over the page

Question 12 (continued)**Marks**

- (c) Solve the differential equation below, to give an equation for y in terms of x , given that $y(0) = 0$. **3**

$$\frac{dy}{dx} = e^x \cos^2 y$$

- (d) Prove by mathematical induction that $n^3 + 2n$ is divisible by 3 for all positive integers n . **3**

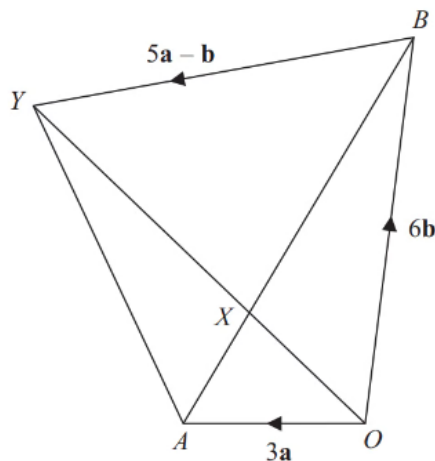
End of Question 12

Question 13 (15 marks) START A NEW WRITING BOOKLET**Marks**

(a) Determine the coefficient of x^4 in the expansion $\left(\frac{2}{x} + 3x\right)^6$

2

(b) In the diagram below, $OAYB$ is a quadrilateral. $\vec{OA} = 3\mathbf{a}$ and $\vec{OB} = 6\mathbf{b}$

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(i) Express $\vec{AB}$ in terms of $\mathbf{a}$ and $\mathbf{b}$.

1

(ii) X is the point on AB such that $AX : XB = 1 : 2$ and $\vec{BY} = 5\mathbf{a} - \mathbf{b}$

3

Prove that $\vec{OX} = \frac{2}{5}\vec{OY}$

(c) Given $\underline{a} = \begin{pmatrix} 5 \\ -2 \end{pmatrix}$ and $\underline{b} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$, find $|\text{proj}_{\underline{b}} \underline{a}|$

2

(d) A family has 2 adults and 2 children. Their car has 2 seats in the front and 3 seats in the back.

1

In how many ways can they be arranged for a trip in the car if only the adults are seated on any of the front seats and only the children are seated on any of the back seats?

Question 13 is continued over the page

Question 13 (continued)**Marks**

- (e) Megan has taken a frozen quiche from the freezer with an initial temperature $T = 0^\circ \text{C}$ and placed it in an oven set to 200°C . After ten minutes she checks the quiche and now it has a temperature of $T = 20^\circ \text{C}$. The rate that the temperature of the quiche increases is proportional to the difference in temperature between the oven and the quiche.
- (i) Write a differential equation to model this scenario and use it to show that $T = 200 + Ae^{-kt}$. **2**
- (ii) Find the value of k in exact form. **2**
- (iii) The quiche is ready when the internal temperature reaches 60°C . How much longer after Megan checked the quiche will it be ready? Answer to the nearest minute. **2**

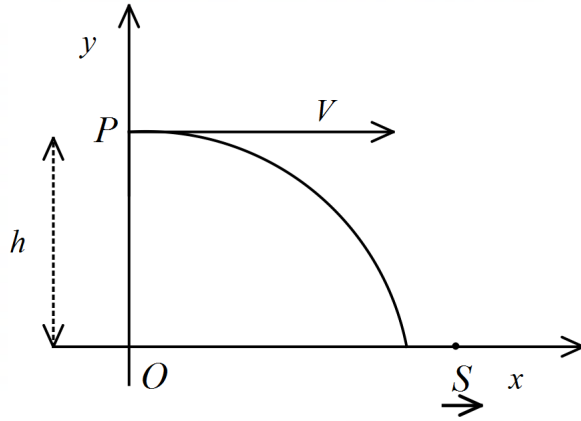
End of Question 13

Question 14 (15 marks) START A NEW WRITING BOOKLET**Marks**

- (a) Evaluate $\sin\left(2\sin^{-1}\frac{1}{7}\right)$ in the form $\frac{a}{b}\sqrt{c}$ where a, b and c are rational. **2**
- (b) Seven people sit around a circular table. Three of the people, Chandana, Simar and Lara, are to sit next to each other and Lara wants to sit in the middle. In how many ways can the seven people be seated around the table? **1**
- (c) Solve $\sqrt{2}\cos\left(x+\frac{\pi}{4}\right)+\sqrt{2}\cos\left(x-\frac{\pi}{4}\right)=1$ for $-\pi\leq x\leq\pi$ **3**

Question 14 is continued over the page

- (d) A particle is projected horizontally from a point P , h metres above O , with a velocity of V metres per second. The only acceleration that the particle undergoes is the acceleration due to gravity ($-g$) vertically in a downward direction. There is no horizontal acceleration.



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- (i) Find $\vec{s}$ the displacement vector of the particle at time t . 2
- (ii) A package of supplies is dropped from a plane to a disabled sailboat (S) in the ocean. 2
 The plane is travelling at a constant velocity of 252 km/h and is 150 m above sea level and directly due west of the disabled sailboat.
 How long will it take the package to hit the water? (Take $g = -10 \text{ m/s}^2$)
- (iii) A current is causing the sailboat to drift at a speed of 1.8 km/h in the same direction as 2
 the plane is travelling. Between what two values (distances) can the plane drop the package so that it lands at most 40 m from the disabled sailboat?

Question 14 is continued over the page

Question 14 (continued)**Marks**

- | | | | |
|-----|------|---|----------|
| (e) | (i) | Write out the expansion for $(1 + x)^n$ | 1 |
| | (ii) | Hence, prove that ${}^nC_1 + 2 \cdot {}^nC_2 + 3 \cdot {}^nC_3 + 4 \cdot {}^nC_4 + \dots + n \cdot {}^nC_n = n \cdot 2^{n-1}$ | 2 |

End of paper

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Multiple Choice

Question 1

What is the derivative of $\tan^{-1}(x^4)$ with respect to x ?

A. $\frac{1}{1+x^8}$

B. $\frac{4x^3}{1+x^8}$

C. $\frac{4x^3}{1+x^4}$

D. $\frac{x^4}{1+x^8}$

$\frac{d}{dx}(\tan^{-1} x^4)$ $= \frac{1}{1+(x^4)^2} \times 4x^3$ $= \frac{4x^3}{1+x^8}$	Using $y' = \frac{f'(x)}{1+[f(x)]^2}$ where $f(x) = x^4$ and $f'(x) = 4x^3$	B
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Question 2

Which of the following polynomials has a factor of $(x+1)$?

A. $P(x) = x^3 - 5x^2 + 4$

B. $P(x) = x^3 - 5x^2 + 6$

C. $P(x) = x^4 - x^2 + 2$

D. $P(x) = x^4 + x^2$

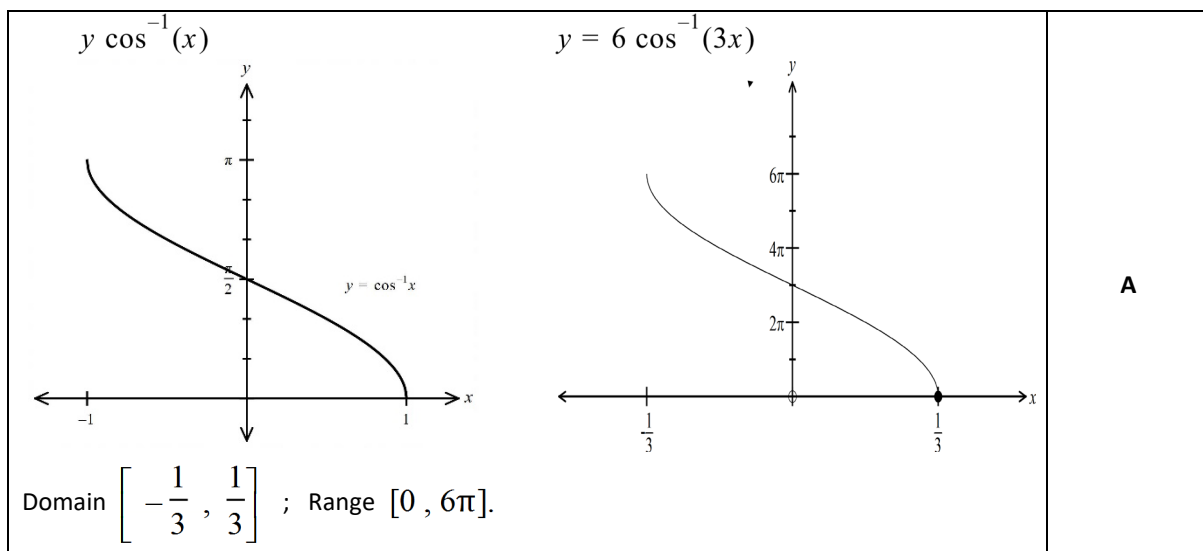
$P(-1) = (-1)^3 - 5(-1)^2 + 6 = 0$	B	1
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Question 3

What is the domain and range of the function $y = 6 \cos^{-1}(3x)$?

- A. Domain $\left[-\frac{1}{3}, \frac{1}{3}\right]$; Range $[0, 6\pi]$.
- B. Domain $\left[-\frac{1}{3}, \frac{1}{3}\right]$; Range $[0, 3\pi]$.
- C. Domain $[0, 6\pi]$; Range $\left[-\frac{1}{3}, \frac{1}{3}\right]$.
- D. Domain $[0, 3\pi]$; Range $\left[-\frac{1}{3}, \frac{1}{3}\right]$.



Question 4

What is the value of $\int_1^{\sqrt{3}} \frac{dx}{\sqrt{3-x^2}}$, correct to 3 significant figures?

- A. -1.05
- B. -0.955
- C. 0.955
- D. 1.05

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The integral is of the form $\int \frac{f'(x)}{\sqrt{a^2 - [f(x)]^2}} dx$ where $a = 3$ and $f(x) = x$ $\int_1^{\sqrt{3}} \frac{dx}{\sqrt{3 - x^2}} = \left[\sin^{-1} \frac{x}{\sqrt{3}} \right]_{\sqrt{3}}^1$ $= \sin^{-1} \frac{\sqrt{3}}{\sqrt{3}} - \sin^{-1} \frac{1}{\sqrt{3}}$ $= 1.5707963 - 0.6154797$ $= 0.9553166$ $= 0.955 \text{ (3 sig fig)}$	c
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Question 5

Find the volume generated when $y = \sin x$ between $x = 0$ and $x = \frac{\pi}{3}$ is rotated around the x -axis.

- A. $\frac{\pi^2}{6} - \frac{3}{8}$ cubic units.
- B. $\frac{\pi^2}{6} - \frac{\sqrt{3}\pi}{8}$ cubic units.
- C. $\frac{\pi^2}{3} - \frac{3\pi}{4}$ cubic units.
- D. $\frac{\pi^2}{6} - \frac{\sqrt{3}\pi}{4}$ cubic units.

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$V = \pi \int_0^{\frac{\pi}{3}} \sin^2 x \, dx$ $= \frac{\pi}{2} \int_0^{\frac{\pi}{3}} 1 - \cos 2x \, dx$ $= \frac{\pi}{2} \left[x - \frac{1}{2} \sin 2x \right]_0^{\frac{\pi}{3}}$ $= \frac{\pi}{2} \left[\left(\frac{\pi}{3} - \frac{\sqrt{3}}{4} \right) - (0) \right]$ $= \frac{\pi^2}{6} - \frac{\sqrt{3}\pi}{8}$	B
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Question 6

Which of the following is equivalent to $\frac{2 \tan \theta}{1 + \tan^2 \theta}$?

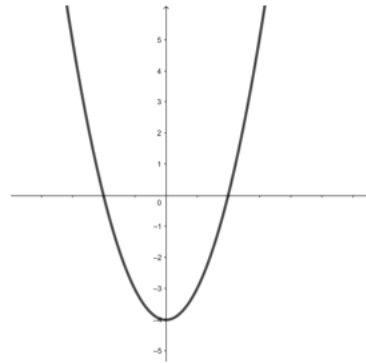
- A. $\cos 2\theta$
- B. $\sin 2\theta$
- C. $\tan 2\theta$
- D. $\cot 2\theta$

Let $t = \tan \theta$ then $\frac{2 \tan \theta}{1 + \tan^2 \theta} = \frac{2t}{1 + t^2}$ $= \sin 2\theta$	B
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Question 7

The graph of $y = f(x)$ is shown below. The diagram is not drawn to scale.

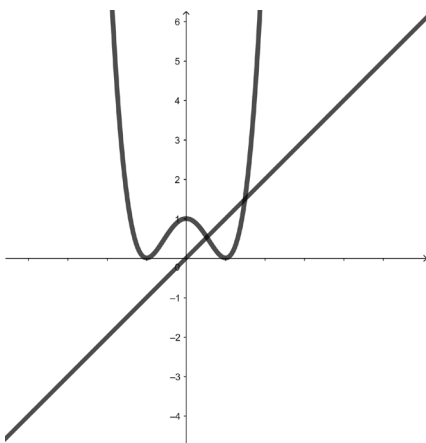


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How many real solutions does the equation $[f(x)]^2 = x$ have?

- A. 1
- B. 2
- C. 3
- D. 4

From graph below 2 real solutions	B	1
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Question 8

Hasti draws a vector from the origin to the point $X(-3, 7)$. She then draws a vector, $-2\mathbf{i} + 4\mathbf{j}$ from X , ending at point Y .

How far is point Y from the origin?

A. $\sqrt{10}$

B. $\sqrt{34}$

C. $\sqrt{122}$

D. $\sqrt{146}$

$$\begin{aligned}\begin{pmatrix} -3 \\ 7 \end{pmatrix} + \begin{pmatrix} -2 \\ 4 \end{pmatrix} &= \begin{pmatrix} -3-2 \\ 7+4 \end{pmatrix} \\ &= \begin{pmatrix} -5 \\ 11 \end{pmatrix} \\ \sqrt{(-5)^2 + (11)^2} &= \sqrt{25 + 121} \\ &= \sqrt{146}\end{aligned}$$

Answer: **D**

Question 9

Which set of values for a and b that satisfy the following equation?

$$\begin{pmatrix} 3a - 4b \\ 2a + 3b \end{pmatrix} = \begin{pmatrix} -16 \\ 12 \end{pmatrix}$$

A. $a = 0$ and $b = 4$

B. $a = 1$ and $b = \frac{19}{4}$

C. $a = \frac{9}{2}$ and $b = 1$

D. $a = 4$ and $b = 0$

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$3a - 4b = -16 \quad (1)$ $2a + 3b = 12 \quad (2)$ $6a - 8b = -32 \quad (3) \quad (1) \times 2$ $6a + 9b = 36 \quad (4) \quad (2) \times 3$ $-17b = -68 \quad (3) - (4)$ $b = 4 \quad (5)$ sub (5) into (1) $3a - 4 \times 4 = -16$ $a = 0$ $\therefore a = 0$ and $b = 4$ Or could be done by substituting the Multiple-Choice values of a and b into the equation.	A
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Question 10

Which of the following differential equations **CANNOT** be solved by separation of the variables?

- A. $\frac{dy}{dx} = xy$
- B. $\frac{dy}{dx} = 3x^2y$
- C. $\frac{dy}{dx} = \sin x + xy$
- D. $\frac{dy}{dx} = \frac{x}{y}$

Option C. Cannot split the x pronumerals of $\sin x$ and x from y , so it is not possible to separate the variables distinctly

Answer: C

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Section II

Question 11 (15 marks) Use the Question 11 writing booklet.

- (a) The equation $x^3 - 2x^2 + 4x - 3 = 0$ has roots $x = \alpha$, $x = \beta$ and $x = \gamma$.

Find the value of $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma}$.

Correctly evaluating the expression $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma}$	2 marks
Calculating a minimum of one required value correctly involving α , β and γ	1 mark

$$a = 1, b = -2, c = 4 \text{ and } d = -3$$

$$\alpha + \beta + \gamma = -\frac{b}{a} = -\frac{-2}{1} = 2$$

$$\alpha\beta + \alpha\gamma + \beta\gamma = \frac{c}{a} = \frac{4}{1} = 4$$

$$\alpha\beta\gamma = -\frac{d}{a} = -\frac{-3}{1} = 3$$

$$\begin{aligned} \frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma} &= \frac{\alpha\beta + \alpha\gamma + \beta\gamma}{\alpha\beta\gamma} \\ &= \frac{4}{3} \end{aligned}$$

- (b) The vectors $\vec{p} = \begin{pmatrix} 6 \\ a \end{pmatrix}$ and $\vec{q} = \begin{pmatrix} a+1 \\ -9 \end{pmatrix}$ are perpendicular.

Find the value of a .

Provides correct answer for a	2 marks
Recognises that dot product is zero.	1 mark

$$\text{Perpendicular} \Rightarrow \begin{pmatrix} 6 \\ a \end{pmatrix} \bullet \begin{pmatrix} a+1 \\ -9 \end{pmatrix} = 0$$

$$6a + 6 - 9a = 0$$

$$a = 2$$

- (c) A school has twenty prefects. Vionna and Chloe are two of those prefects.

A committee of six prefects is to be formed to organise a function.

What is the probability that the committee contains Vionna but does not contain Chloe?

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Calculates the correct answer in notation form or as a fraction	2 marks
Recognises ${}^{18}C_5$ as part of the calculation but does not find the probability	1 mark

$$\text{Number of different committees} = {}^{20}C_6 = 38760$$

Vionna is chosen. 5 other prefects chosen in ${}^{18}C_5$ ways (given Chloe not chosen)

$$\text{Prob} = \frac{{}^{18}C_5}{{}^{20}C_6} = \frac{21}{95}$$

(d) Using the substitution $u = 2x - 1$, find $\int 2x (2x - 1)^3 dx$.

Obtains correct solution	3 marks
for correct substitution and reasonable progress using the pronumeral u only	2 marks
For correct substitution but did not re-substitute x for u or mistakes in the integration	1 mark

$$u = 2x - 1 \Rightarrow 2x = u + 1$$

$$du = 2 dx \Rightarrow dx = \frac{1}{2} du$$

$$\int 2x(2x - 1)^3 dx = \frac{1}{2} \int (u + 1) u^3 du$$

$$= \frac{1}{2} \int u^4 + u^3 du$$

$$= \frac{1}{2} \left[\frac{u^5}{5} + \frac{u^4}{4} \right] + c$$

$$= \frac{1}{2} \left[\frac{(2x - 1)^5}{5} + \frac{(2x - 1)^4}{4} + c \right]$$

$$= \frac{(2x - 1)^5}{10} + \frac{(2x - 1)^4}{8} + c$$

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(e) A function is defined by $f(x) = 1 + \frac{3}{x-2}$ for $x > 2$.

(i) Find the vertical and horizontal asymptotes for $y = f(x)$.

(ii) Find the inverse function $y = f^{-1}(x)$.

(iii) State the domain of $y = f^{-1}(x)$.

(e) (i)

For correct solution, both asymptotes identified	2 marks
For identifying only one of the asymptotes or one solution incorrect	1 mark

Vertical asymptote at $x = 2$

$$y = 1 + \frac{3}{x-2}$$

$$y - 1 = \frac{3}{x-2}$$

$$x - 2 = \frac{3}{y-1}$$

$\therefore$ Horizontal asymptote at $y = 1$

(e) (ii)

Obtains correct solution	1 mark
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$$x = 2 + \frac{3}{y-1}$$

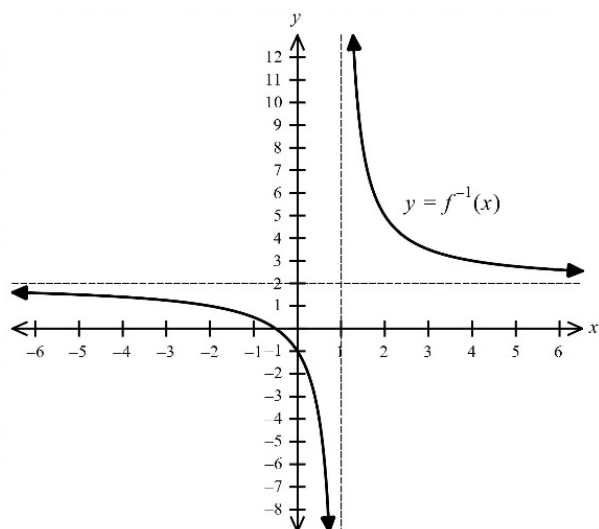
$$f^{-1}(x) = 2 + \frac{3}{x-1}$$

(e) (iii)

Obtains correct domain (in either format shown below)	1 mark
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Domain : all real x , $x > 1$

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Alternative notation 1 $\Rightarrow 1 < x < \infty$

Alternative notation 2 $\Rightarrow (1, \infty)$

(f) Solve $\frac{x}{2-x} > 1$

Finds the correct solution, represented by the inequality shown in the solution	2 marks
Makes some progress using a correct technique towards solving the quadratic inequality	1 mark

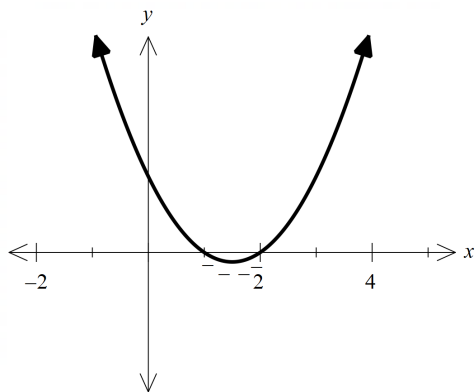
$$\begin{aligned} \frac{x}{2-x} &> 1 \\ (2-x)^2 \times \frac{x}{2-x} &> 1 \times (2-x)^2 \\ (2-x)^2 \times \frac{x}{2-x} &> 1 \times (2-x)^2 \\ (2-x) \times x &> (2-x)^2 \\ 2x - x^2 &> 4 - 4x + x^2 \\ 0 &> 4 - 6x + 2x^2 \\ 0 &> 2(2 - 3x + x^2) \\ 0 &> (x-2)(x-1) \end{aligned}$$

In the diagram below, the solution lies $x > 1$ and $x < 2$ so $1 < x < 2$

where $2 - x \neq 0$ so $x \neq 2$

Graph shown over the page...

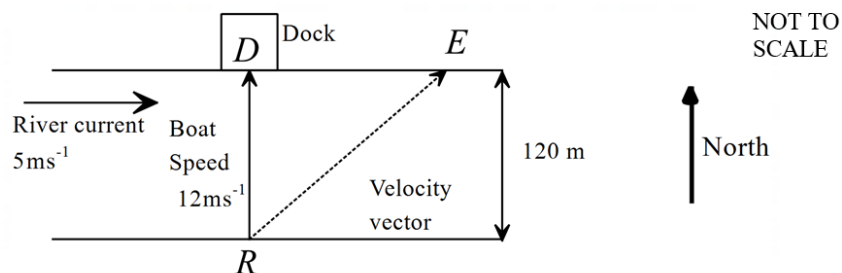
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Question 12

- (a) Rylie has a boat which moves at a top speed of 12 ms^{-1} in still water.

From point R , he wants to go due north to point D on the opposite side of the river, as shown in the diagram below.

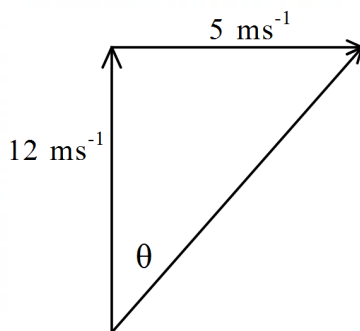


Today the current in the river is flowing at 5 ms^{-1} . From R , he steers the boat due north toward D at top speed. Due to the current he drifts down the river and arrives at point E .

- (i) Taking R as the origin, write down Rylie's velocity vector in the form $x\mathbf{i} + y\mathbf{j}$ and find the magnitude of this vector.

For correct solution giving both component vector and magnitude	2 marks
For either the component vector or magnitude correct	1 mark

$$\begin{aligned} \mathbf{v} &= 5\mathbf{i} + 12\mathbf{j} \\ |\mathbf{v}| &= \sqrt{5^2 + 12^2} \\ &= 13 \text{ ms}^{-1} \end{aligned}$$



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- (ii) What is the bearing of Rylie's velocity vector and how far does he travel from R to E ?

For correct solution giving both the bearing and the distance	2 marks
For either correct bearing or distance	1 mark

$$\tan \theta = \frac{5}{12}$$

$$\theta = 22^{\circ}37'11.51''$$

$$= 023^{\circ}T$$

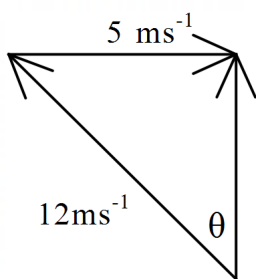
$$\frac{x}{13} = \frac{120}{12}$$

$$x = \frac{120 \times 13}{12}$$

$$x = 130\text{m}$$

- (iii) On what bearing should Rylie have pointed the boat, so that he arrived at the D , with the boat travelling at its top speed?

For correct solution giving the bearing	2 marks
For the correct angle but not converting to a bearing	1 mark



$$\sin \theta = \frac{5}{12}$$

$$\theta = 24^{\circ}37'$$

$$\begin{aligned} \therefore \text{Bearing} &= 360^{\circ} - 24^{\circ}37' \\ &= 335^{\circ} \text{ (nearest degree)} \end{aligned}$$

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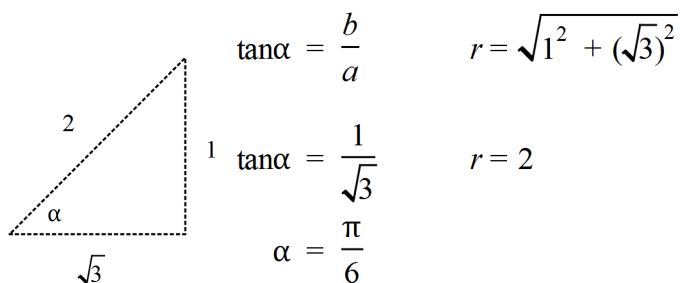
- (b) Write $\sqrt{3}\sin x - \cos x$ in the form $R \sin(x + \alpha)$ where $R > 0$ and $0 \leq \alpha \leq \frac{\pi}{2}$ and hence, solve $\sqrt{3} \sin \theta - \cos \theta - 1 = 0$ for $[0, 2\pi]$.

Excludes solution not in domain to obtain $\theta = \frac{\pi}{3}, \pi$	3 marks
Find solutions $x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{13\pi}{6}$	2 marks
Changes the form of the problem using the auxiliary equation and finds r and α	1 mark

$$\sqrt{3} \sin \theta - \cos \theta - 1 = 0 \quad [0, 2\pi]$$

$$\sqrt{3} \sin \theta - \cos \theta = 1$$

$$a \sin x - b \cos x = r \sin(x - \alpha)$$



$$\therefore \sqrt{3} \sin \theta - \cos \theta = 2 \sin \left(\theta - \frac{\pi}{6} \right)$$

$$\sqrt{3} \sin \theta - \cos \theta = 1 \quad [0, 2\pi]$$

$$2 \sin \left(\theta - \frac{\pi}{6} \right) = 1$$

$$\sin \left(\theta - \frac{\pi}{6} \right) = \frac{1}{2}$$

$$\sin x = \frac{1}{2}$$

$$x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{13\pi}{6}$$

$$\theta - \frac{\pi}{6} = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{13\pi}{6}$$

$$\theta = \frac{\pi}{3}, \pi, \frac{14\pi}{6} \text{ (not in domain)}$$

$$\therefore \theta = \frac{\pi}{3}, \pi$$

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- (c) Solve the differential equation below, to give an equation for y in terms of x , given that $y(0) = 0$.

$$\frac{dy}{dx} = e^x \cos^2 y$$

For correct solution	3 marks
Integrates both sides correctly and finds c	2 marks
Integrates and change $dy/\cos^2 y$ to $\sec^2 y$	1 mark

$$\frac{dy}{dx} = e^x \cos^2 y$$

$$\frac{dy}{\cos^2 y} = e^x dx$$

$$\sec^2 y dy = e^x dx$$

$$\int \sec^2 y dy = \int e^x dx$$

$$\tan(y) = e^x + c$$

Initial condition $y(0) = 0$

$$\tan(0) = e^0 + c$$

$$0 = 1 + c$$

$$c = -1$$

$$\therefore \tan(y) = e^x - 1$$

$$\therefore y = \tan^{-1}(e^x - 1)$$

- (d) Prove by mathematical induction that $n^3 + 2n$ is divisible by 3 for all positive integers n .

Obtains correct solution including final statement	3 marks
Progress towards proving for $n = k+1$	2 marks
Showing steps 1 & 2 (for $n = 1$ and assume...) <u>and</u> substituting $n = k+1$	1 mark

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Required to prove that

$$n^3 + 2n \text{ divisible by 3 for } n > 0$$

Step 1 - Show true for $n = 1$

$$\begin{aligned} n^3 + 2n &= 1^3 + 2 \times 1 \\ &= 3 \text{ which is divisible by 3} \end{aligned}$$

$\therefore$ true for $n = 1$

Step 2 - Assume true for $n = k$

$$k^3 + 2k = 3p \text{ where } p \text{ is an integer}$$

Step 3 - Prove true for $n = k+1$

i.e prove that $(k+1)^3 + 2(k+1) = 3q$ where q is an integer

$$\begin{aligned} (k+1)^3 + 2(k+1) &= k^3 + 3k^2 + 3k + 1 + 2k + 2 \\ &= (k^3 + 2k) + 3k^2 + 3k + 3 \\ &= 3p + 3k^2 + 3k + 3 \\ &= 3(p + k^2 + k + 1) \\ &= 3q \text{ where } q = p + k^2 + k + 1 \end{aligned}$$

$\therefore (k+1)^3 + 2(k+1)$ is divisible by 3

$\therefore$ true for $n = 1$ and if true for $n = k$, then true for $n = k + 1$

$\therefore$ true for all positive integers

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Question 13 (15 marks) START A NEW WRITING BOOKLET

- (a) Determine the coefficient of x^4 in the expansion $\left(\frac{2}{x} + 3x\right)^6$

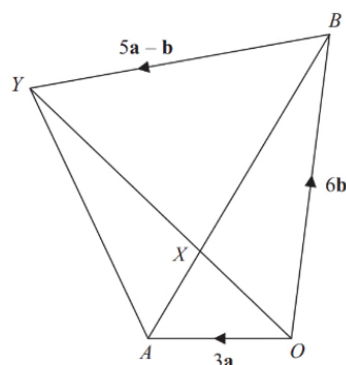
Correctly finds the value of 2916	2 marks
Shows correct terms in binomial expansion but does not find the correct answer	1 mark

$$\left(\frac{2}{x} + 3x\right)^6 = {}^6C_0\left(\frac{2}{x}\right)^6(3x)^0 + {}^6C_1\left(\frac{2}{x}\right)^5(3x)^1 + {}^6C_2\left(\frac{2}{x}\right)^4(3x)^2 + {}^6C_3\left(\frac{2}{x}\right)^3(3x)^3 + {}^6C_4\left(\frac{2}{x}\right)^2(3x)^4 + {}^6C_5\left(\frac{2}{x}\right)^1(3x)^5 + \dots$$

Find the coefficient of ${}^6C_5\left(\frac{2}{x}\right)^1(3x)^5$

$${}^6C_5 \times 2 \times 3^5 = 2916$$

- (b) In the diagram below, $OAYB$ is a quadrilateral. $\vec{OA} = 3\mathbf{a}$ and $\vec{OB} = 6\mathbf{b}$



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- (i) Express $\vec{AB}$ in terms of $\mathbf{a}$ and $\mathbf{b}$.

Correctly finds the solution $6\mathbf{b} - 3\mathbf{a}$	1 mark
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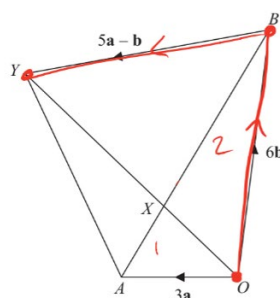


Diagram NOT accurately drawn

- (a) Express $\vec{AB}$ in terms of $\mathbf{a}$ and $\mathbf{b}$.

$$\begin{aligned}\vec{AB} &= \vec{AO} + \vec{OB} \\ &= -3\mathbf{a} + 6\mathbf{b}\end{aligned}$$

$OAYB$ is a quadrilateral.

$$\vec{OA} = 3\mathbf{a}$$

$$\vec{OB} = 6\mathbf{b}$$

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- (ii) X is the point on AB such that $\overline{AX} : \overline{XB} = 1 : 2$ and $\overrightarrow{BY} = 5\mathbf{a} - \mathbf{b}$

Prove that $\overrightarrow{OX} = \frac{2}{5} \overrightarrow{OY}$

Correctly proves that $\overrightarrow{OX} = \frac{2}{5} \overrightarrow{OY}$ with connections explicitly written out as shown below	3 marks
Correctly finds either $\overrightarrow{OX}$ or $\overrightarrow{OY}$	2 marks
Working towards calculating <u>only one</u> vector, either $\overrightarrow{OX}$ or $\overrightarrow{OY}$ showing clear working out with a small error	1 mark

X is the point on AB such that $AX : XB = 1 : 2$

and $\overrightarrow{BY} = 5\mathbf{a} - \mathbf{b}$

*(b) Prove that $\overrightarrow{OX} = \frac{2}{5} \overrightarrow{OY}$

$$\begin{aligned}
 \overrightarrow{OX} &= \overrightarrow{OA} + \overrightarrow{AX} \\
 &= 3\mathbf{a} + \frac{1}{3}(-3\mathbf{a} + 6\mathbf{b}) \\
 &= 3\mathbf{a} - \mathbf{a} + 2\mathbf{b} \\
 &= 2\mathbf{a} + 2\mathbf{b} \\
 &= \underline{\underline{2(\mathbf{a} + \mathbf{b})}}
 \end{aligned}$$

$$\overrightarrow{AX} = \frac{1}{3} \times \overrightarrow{AB}$$

$$\begin{aligned}
 \overrightarrow{OY} &= \overrightarrow{OB} + \overrightarrow{BY} \\
 &= 6\mathbf{b} + 5\mathbf{a} - \mathbf{b} \\
 &= 5\mathbf{a} + 5\mathbf{b} \\
 &= \underline{\underline{5(\mathbf{a} + \mathbf{b})}}
 \end{aligned}$$

$$\text{so } \underline{\underline{\overrightarrow{OX} = \frac{2}{5} \overrightarrow{OY}}}$$

- (c) Given $\mathbf{a} = \begin{pmatrix} 5 \\ -2 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$, find $|\text{proj}_{\mathbf{b}} \mathbf{a}|$

Correctly finds the answer $ \text{proj}_{\mathbf{b}} \mathbf{a} $	2 marks
Determines $\text{proj}_{\mathbf{b}} \mathbf{a}$	1 mark

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$$\begin{aligned}
 |proj_{\vec{b}} \vec{a}| &= \left| \frac{\vec{a} \cdot \vec{b}}{|\vec{b}|^2} \vec{b} \right| \\
 &= \frac{5 \times 3 - 2 \times 4}{25} (3\vec{i} + 4\vec{j}) \\
 &= \sqrt{\frac{7}{25}} (3^2 + 4^2) \\
 &= \frac{\sqrt{7}}{\sqrt{25}} \times \sqrt{3^2 + 4^2} \\
 &= \sqrt{7}
 \end{aligned}$$

- (d) A family has 2 adults and 2 children. Their car has 2 seats in the front and 3 seats in the back.

In how many ways can they be arranged for a trip in the car if only the adults are seated on any of the front seats and only the children are seated on any of the back seats?

Calculates the correct answer	1 mark
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Front adults
 Back children
 $2 \times 3 \times 2 = 12$

- (e) Megan has taken a frozen quiche from the freezer with an initial temperature $T = 0^\circ \text{C}$ and placed it in an oven set to 200°C . After ten minutes she checks the quiche and now it has a temperature of $T = 20^\circ \text{C}$. The rate that the temperature of the quiche increases is proportional to the difference in temperature between the oven and the quiche.

- (i) Write a differential equation to model this scenario and use it to show that $T = 200 + Ae^{-kt}$.

Obtains correct solution	2 marks
Makes progress towards solving the differential equation by placing the correct variables on each side of the equation and trying to integrate	1 mark

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$$\frac{dT}{dt} = k(200 - T)$$

$$\frac{dT}{200 - T} = k dt$$

$$\int \frac{dT}{200 - T} = \int k dt$$

$$-\ln|200 - T| = kt + c$$

$$\ln|200 - T| = -kt - c$$

$$200 - T = \pm e^{-c} e^{-kt}$$

$$T = 200 \pm e^{-c} e^{-kt} \quad \text{Let } A = \pm e^{-c}$$

$$\therefore T = 200 + A e^{-kt}$$

(ii) Find the value of k in exact form.

Finds correct solution for k	2 marks
Correctly finds A and makes some progress towards finding k	1 mark

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When $t = 0$ $T = 0^{\circ}\text{C}$

$$0 = 200 + A$$

$$A = -200$$

$$\therefore T = 200 - 200 e^{-kt}$$

When $t = 10$ $T = 20^{\circ}\text{C}$

$$20 = 200 - 200 e^{-10k}$$

$$-\frac{180}{-200} = e^{-10k}$$

$$\frac{9}{10} = e^{-10k}$$

$$\frac{9}{10} = \frac{1}{e^{10k}}$$

$$\frac{10}{9} = e^{10k}$$

$$\ln\left(\frac{10}{9}\right) = 10k$$

$$k = \frac{1}{10} \ln\left(\frac{10}{9}\right)$$

- (iii) The quiche is ready when the internal temperature reaches 60°C . How much longer after Megan checked the quiche will it be ready? Answer to the nearest minute.

Obtains correct solution. Students can change k to a decimal and as long as working out is shown fully, no penalty should be made for rounding errors	2 marks
Correctly substitutes and makes some progress towards the answer	1 mark

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When $T = 60^{\circ}\text{C}$

$$T = 200 - 200 e^{-\left(\frac{\ln\left(\frac{10}{9}\right)}{10}\right)t}$$

$$60 = 200 - 200 e^{-\left(\frac{\ln\left(\frac{10}{9}\right)}{10}\right)t}$$

$$-140 = -200 e^{-\left(\frac{\ln\left(\frac{10}{9}\right)}{10}\right)t}$$

$$\frac{-140}{-200} = e^{-\left(\frac{\ln\left(\frac{10}{9}\right)}{10}\right)t}$$

$$\frac{7}{10} = e^{-\left(\frac{\ln\left(\frac{10}{9}\right)}{10}\right)t}$$

$$\ln\left(\frac{7}{10}\right) = -\left(\frac{\ln\left(\frac{10}{9}\right)}{10}\right)t$$

$$t = \frac{\ln\left(\frac{7}{10}\right)}{-\left(\frac{\ln\left(\frac{10}{9}\right)}{10}\right)t}$$

$$t = \frac{10\left(\ln\left(\frac{7}{10}\right)\right)}{-\left(\ln\left(\frac{10}{9}\right)\right)}$$

$$t = 33.85 \text{ mins}$$

$$t = 34 \text{ mins}$$

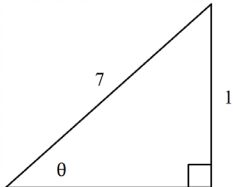
$\therefore$ It will take $34 - 10 = 24$ mins longer

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Question 14 (15 marks) START A NEW WRITING BOOKLET

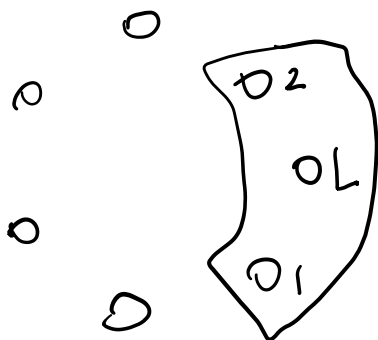
- (a) Evaluate $\sin\left(2\sin^{-1}\frac{1}{7}\right)$ in the form $\frac{a}{b}\sqrt{c}$ where a, b and c are rational.

Finds the correct solution, with working out shown	2 marks
Uses Pythagoras theorem for $\theta = \sin^{-1}\frac{1}{7}$ to find the 3 rd side	1 mark

Let $\theta = \sin^{-1}\frac{1}{7}$ Find the unknown side in the triangle, by Pythagoras' theorem $\begin{aligned}\sqrt{7^2 - 1^2} &= \sqrt{49 - 1} \\ &= \sqrt{48} \\ &= 4\sqrt{3}\end{aligned}$ Since $\sin 2\theta = 2\sin\theta\cos\theta$ $\begin{aligned}&= 2 \times \frac{1}{7} \times \frac{4\sqrt{3}}{7} \\ &= \frac{8\sqrt{3}}{49}\end{aligned}$	
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- (b) Seven people sit around a circular table. Three of the people, Chandana, Simar and Lara, are to sit next to each other and Lara wants to sit in the middle. In how many ways can the seven people be seated around the table?

Finds the correct solution	1 mark
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fix the group

$$\begin{aligned}4! \times 2 &= 24 \times 2 \\ &= 48\end{aligned}$$

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(c) Solve $\sqrt{2} \cos\left(x + \frac{\pi}{4}\right) + \sqrt{2} \cos\left(x - \frac{\pi}{4}\right) = 1$ for $-\pi \leq x \leq \pi$

Correctly solves the equation for the given domain.	3 marks
Finds one correct solution.	2 marks
Correctly expands the expression or equivalent progress.	1 mark

$$\sqrt{2} \cos\left(x + \frac{\pi}{4}\right) + \sqrt{2} \cos\left(x - \frac{\pi}{4}\right) = 1$$

$$\sqrt{2} \left[\left(\cos x \cos \frac{\pi}{4} - \sin x \sin \frac{\pi}{4} \right) + \left(\cos x \cos \frac{\pi}{4} + \sin x \sin \frac{\pi}{4} \right) \right] = 1$$

$$2\sqrt{2} \cos x \cos \frac{\pi}{4} = 1$$

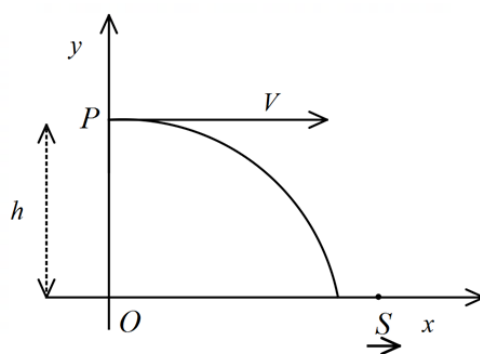
$$2\sqrt{2} \cos x \frac{\sqrt{2}}{2} = 1$$

$$\cos x = \frac{1}{2}$$

$$x = \pm \frac{\pi}{3}$$

Question 14 (continued)

- (d) A particle is projected horizontally from a point P , h metres above O , with a velocity of V metres per second. The only acceleration that the particle undergoes is the acceleration due to gravity ($-g$) vertically in a downward direction. There is no horizontal acceleration.



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- (i) Find $\underline{s}$ the displacement vector of the particle at time t .

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Finds the correct displacement vector. Students may have also substituted in the value of θ as 0. This is also an acceptable response and should receive full marks	2 marks
Finds the correct velocity vector only	1 mark

$$\underline{a} = -g \underline{j}$$

$$\underline{v} = \int \underline{a} \, dt$$

$$= \int -g \underline{j} \, dt$$

$$= -gt \underline{j} + c$$

$$\text{When } t=0, \underline{v} = V \cos \theta \underline{i} + V \sin \theta \underline{j}$$

$$V \cos \theta \underline{i} + V \sin \theta \underline{j} = \underline{j} + c$$

$$V \cos \theta \underline{i} + V \sin \theta \underline{j} = c$$

$$\underline{v} = -gt \underline{j} + V \cos \theta \underline{i} + V \sin \theta \underline{j}$$

$$\therefore \underline{v} = V \cos \theta \underline{i} + (-gt + V \sin \theta) \underline{j}$$

$$\underline{s} = \int \underline{v} \, dt$$

$$= \int V \cos \theta \underline{i} + (-gt + V \sin \theta) \underline{j} \, dt$$

$$= Vt \cos \theta \underline{i} + \left(\frac{-gt^2}{2} + [V \sin \theta]t \right) \underline{j} + c_1$$

$$= Vt \cos \theta \underline{i} + \left(\frac{-gt^2}{2} + Vt \sin \theta \right) \underline{j} + c_1$$

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The particle starts at $(0, h)$

$$\text{When } t = 0 \quad \underline{s} = 0\underline{i} + h\underline{j}$$

$$0\underline{i} + h\underline{j} = V(0) \cos \theta \underline{i} + \left[\frac{-g(0^2)}{2} + V(0) \sin \theta \right] \underline{j} + c_1$$

$$h\underline{j} = c_1$$

$$\underline{s} = Vt \cos \theta \underline{i} + \left[\frac{-gt^2}{2} + Vt \sin \theta \right] \underline{j} + h\underline{j}$$

$$\underline{s} = Vt \cos \theta \underline{i} + \left[\frac{-gt^2}{2} + Vt \sin \theta + h \right] \underline{j}$$

Or

$$\underline{s} = Vt \underline{i} + \left[\frac{-gt^2}{2} + h \right] \underline{j}$$

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- (ii) A package of supplies is dropped from a plane to a disabled sailboat (S) in the ocean. The plane is travelling at a constant velocity of 252 km/h and is 150 m above sea level and directly due west of the disabled sailboat.

How long will it take the package to hit the water? (Take $g = -10 \text{ m/s}^2$)

Writes correct solution	2 marks
Writes correct process but student did not change km/h to m/s	1 mark

$$V = 252 \text{ km/h}$$

$$= \frac{252 \times 1000}{60 \times 60}$$

$$= 70 \text{ m/s}$$

$$\text{When } h = 150 \text{ m} \quad g = 10 \text{ m/s}^2 \quad V = 70 \text{ m/s} \quad \theta = 0^\circ$$

The package hits the water when the y component of the displacement is 0.

$$\left[\frac{-gt^2}{2} + Vt \sin \theta + h \right] j = 0$$

$$\frac{-10 \times t^2}{2} + 70t \sin(0) + 150 = 0$$

$$-5t^2 + 150 = 0$$

$$t^2 = 30$$

$$t = \sqrt{30} \quad \text{as } t > 0$$

$$t \approx 5.5 \text{ s}$$

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- (iii) A current is causing the sailboat to drift at a speed of 1.8 km/h in the same direction as the plane is travelling. Between what two values (distances) can the plane drop the package so that it lands at most 40 m from the disabled sailboat?

Obtains a correct solution	2 marks
Finds the horizontal distance with some progress towards allowing for the drift	1 mark

In $\sqrt{30}$ s the package will travel horizontally

$$x = (V t \cos \theta) i$$

$$= 70 \times \sqrt{30}$$

$$= 383.4 \text{ m}$$

Boat is drifting at 1.8 km/h

$$1.8 \text{ km/h} = \frac{1800}{3600}$$

$$= 0.5 \text{ m/s}$$

$\therefore$ In $\sqrt{30}$ s the boat will drift $\sqrt{30} \times 0.5$ m

If we let D be the horizontal distance when the package is dropped. The distance to the disabled sailboat from O will be:

$$(D + (\sqrt{30} \times 0.5))\text{m}$$

To land at most 40 m from the sailboat

$$383.4 - 40 \leq (D + (\sqrt{30} \times 0.5)) \leq 383.4 + 40$$

$$343.4 \leq (D + (\sqrt{30} \times 0.5)) \leq 423.4$$

$$340.7 \leq D \leq 420.7$$

$\therefore$ package should be dropped between

340.7 m and 420.7 m away from the sailboat.

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Question 14 (continued)

- (e) (i) Write out the expansion for $(1 + x)^n$

Write correct expansion with at least 4 terms listed	1 mark
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$$(1+x)^n = {}^nC_0 + {}^nC_1x + {}^nC_2x^2 + {}^nC_3x^3 + \dots + {}^nC_{n-1}x^{n-1} + {}^nC_nx^n$$

- (ii) Hence, prove that ${}^nC_1 + 2 \cdot {}^nC_2 + 3 \cdot {}^nC_3 + 4 \cdot {}^nC_4 + \dots + n \cdot {}^nC_n = n \cdot 2^{n-1}$

Shows the substitution $x = 1$ and clearly demonstrates working to prove the required line with sequential working out shown	2 marks
Differentiates as shown on the line (*) below	1 mark

$$(1+x)^n = {}^nC_0 + {}^nC_1x + {}^nC_2x^2 + {}^nC_3x^3 + \dots + {}^nC_{n-1}x^{n-1} + {}^nC_nx^n$$

By differentiating both sides,

$$n(1+x)^{n-1} = {}^nC_1 + 2 \times {}^nC_2x + 3 \times {}^nC_3x^2 + 4 \times {}^nC_4x^3 + \dots + (n-1) {}^nC_{n-1}x^{n-2} + n \times {}^nC_nx^{n-1}$$

Substitute $x = 1$ into both sides of the equation

$$n(1+1)^{n-1} = {}^nC_1 + 2 \times {}^nC_2 + 3 \times {}^nC_3 + 4 \times {}^nC_4 + \dots + (n-1) {}^nC_{n-1} + n \times {}^nC_n$$

$$n(2)^{n-1} = {}^nC_1 + 2 \times {}^nC_2 + 3 \times {}^nC_3 + 4 \times {}^nC_4 + \dots + (n-1) {}^nC_{n-1} + n \times {}^nC_n$$

$$n(2)^{n-1} = {}^nC_1 + 2 {}^nC_2 + 3 {}^nC_3 + 4 {}^nC_4 + \dots + (n-1) {}^nC_{n-1} + n {}^nC_n$$

as required