Student Number: | | | | | | | | |

Cheltenham Girls High School

2023 TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Extension 1

General Instructions

- Reading time – 10 minutes
- Working time – 2 hours
- Write using black pen
- Calculators approved by NESA may be used
- A NESA reference sheet is provided
- Use the multiple-choice answer sheet attached for Question 1 - 10
- For Questions in Section II, show mathematical reasoning and/or calculations to gain maximum marks
- Write your Student Number on the front of each Writing Booklet for Questions 11, 12, 13 and 14
- **Start a new writing booklet for each of Questions 11, 12, 13 and 14**

Total Marks: 70

Section I – 10 marks

- Attempt Questions 1 – 10
- Allow about 15 minutes for this section

Section II – 60 marks

- Attempt Questions 11 – 14
- Allow about 1 hour and 45 minutes for this section

Section I

10 marks

Attempt Questions 1 – 10.

Allow about 15 minutes for this section.

Use the multiple-choice answer sheet for Questions 1 – 10.

1 The graph of $y = g(x)$ is shown below.

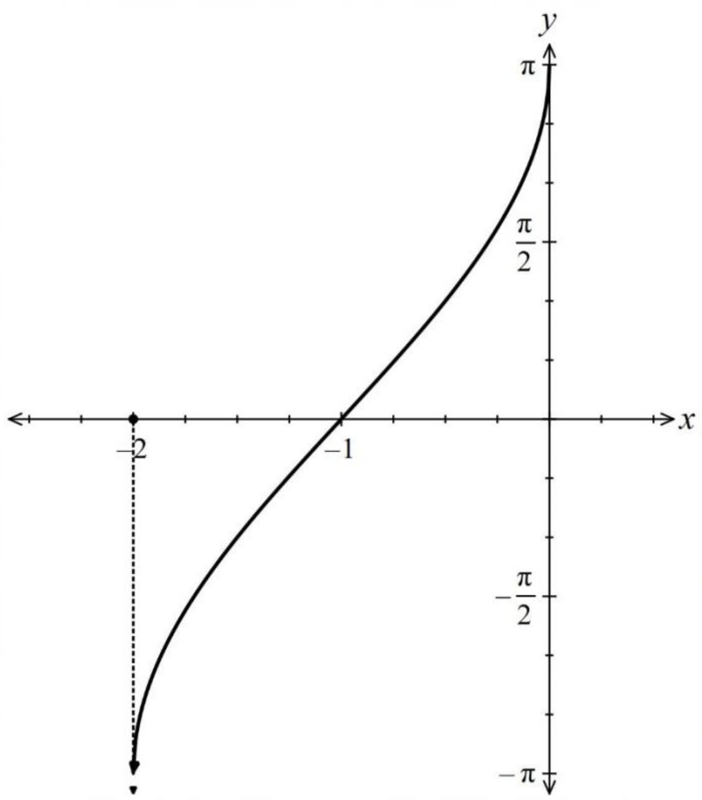


DIAGRAM NOT TO SCALE

Which equation could describe $g(x)$?

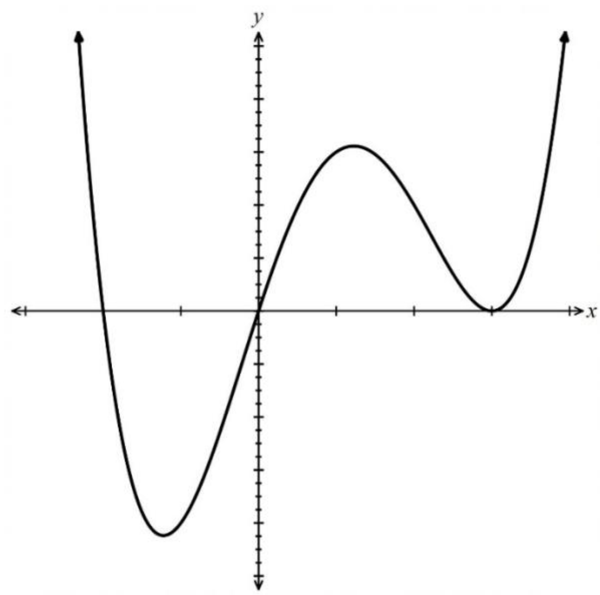
A. $g(x) = \frac{1}{2}\cos^{-1}(x-1)$

B. $g(x) = 2\cos^{-1}(x+1)$

C. $g(x) = \frac{1}{2}\sin^{-1}(x-1)$

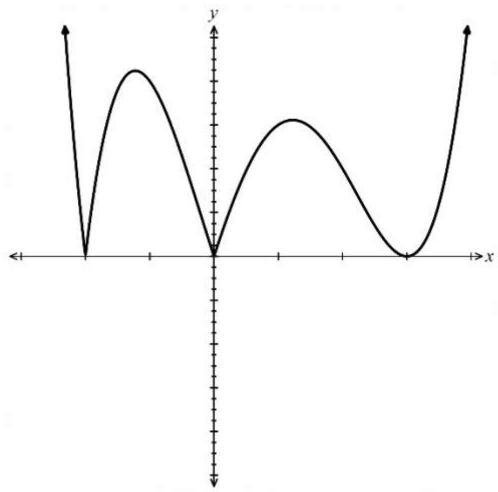
D. $g(x) = 2\sin^{-1}(x+1)$

2 The graph of $y = f(x)$ is shown below. Diagrams are not drawn to scale.

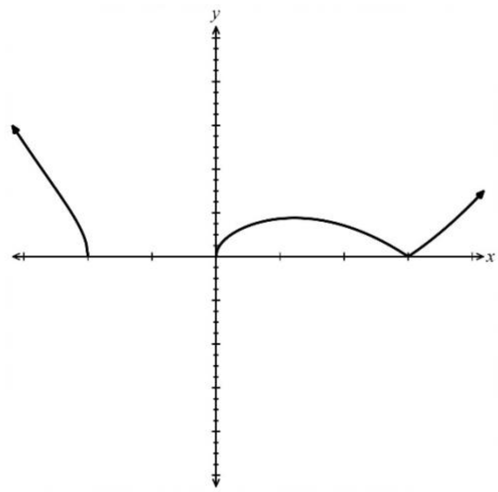


Which graph shows $y^2 = f(x)$?

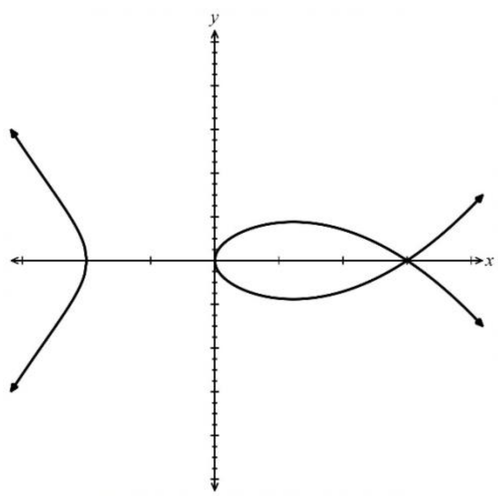
A.



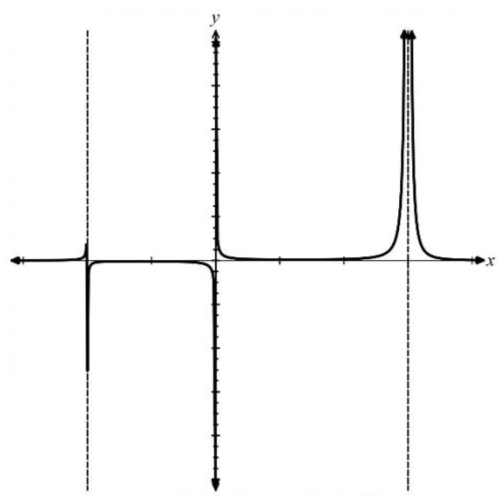
B.



C.



D.



3 A partially completed polynomial division is shown below.

$$
\begin{array}{r}
x^2 + x + 1 \\
x^2+2x \overline{\big) x^4+3x^3+3x^2-4x+5} \\
\underline{x^4+2x^3} \qquad\qquad\qquad \\
x^3+3x^2 \qquad\qquad \\
\underline{x^3+2x^2} \qquad\qquad \\
x^2-4x \qquad \\
\underline{x^2+2x} \qquad
\end{array}
$$

Which of the following expressions is equivalent to : $\frac{x^4+3x^3+3x^2-4x+5}{x^2+2x}$?

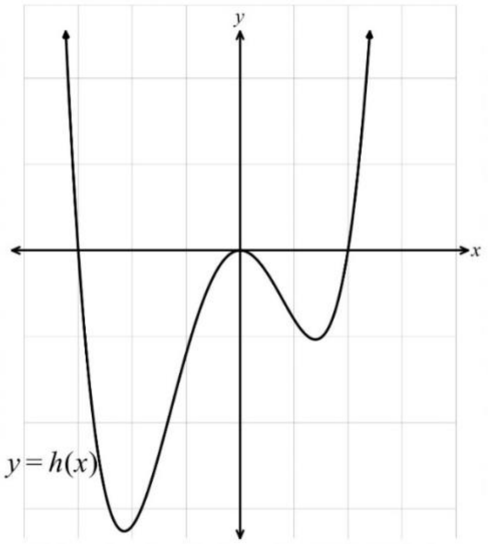
A. $x^2+x+1+\frac{5-6x}{x^2+2x}$

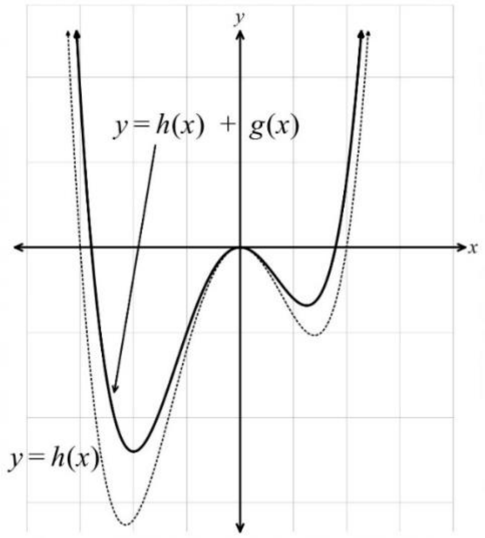
B. $x^2+x+1+\frac{6x-5}{x^2+2x}$

C. $x^2+2x+\frac{5-6x}{x^2+2x}$

D. $x^2+2x+\frac{6x-5}{x^2+2x}$

4 The graph on the left shows $y = h(x)$ and that on the right shows $y = h(x)$ and also $y = h(x) + g(x)$.





Which equation could describe $y = g(x)$?

A. $y = x$

B. $y = x^2$

C. $y = x^2 + 6$

D. $y = x^3$

5 Find the Cartesian equation of the curve represented by the parametric equations:

$$x = 4 - t$$
$$y = 3t^3$$

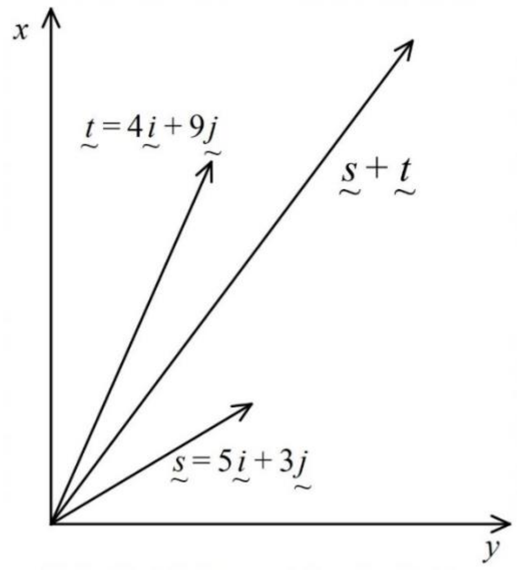
A. $y = 64 - 48x + 12x^2 - x^3$

B. $y = 64 + 48x + 12x^2 + x^3$

C. $y = 192 + 144x + 36x^2 + 3x^3$

D. $y = 192 - 144x + 36x^2 - 3x^3$

6 The diagram below shows the vectors $\underset{\sim}{s}$ and $\underset{\sim}{t}$ along with the vector $\underset{\sim}{s} + \underset{\sim}{t}$.



What is the value of $|\underset{\sim}{s} + \underset{\sim}{t}|$?

A. 12

B. 15

C. 16

D. 20

7 Find the term independent of *x* in the expansion of $\left(x - \frac{2}{x^2}\right)^6$.

A. 12

B. 30

C. 60

D. 160

8 Of the vectors $\underset{\sim}{u} = (-2, 6)$, $\underset{\sim}{v} = (9, 3)$ and $\underset{\sim}{w} = (3, 2)$,which pairs are perpendicular?

A. Only $\underset{\sim}{u}$ and $\underset{\sim}{v}$ are perpendicular.

B. Only $\underset{\sim}{u}$ and $\underset{\sim}{w}$ are perpendicular.

C. Only $\underset{\sim}{v}$ and $\underset{\sim}{w}$ are perpendicular.

D. There are no pairs of vectors which are perpendicular.

9 Which of the following is equivalent to $\int \cos^2 7x \; dx$?

A. $\frac{x}{2} + \frac{1}{14} \cos 7x + C$

B. $\frac{x}{2} - \frac{1}{28} \cos 14x + C$

C. $\frac{x}{2} + \frac{1}{14} \sin 7x + C$

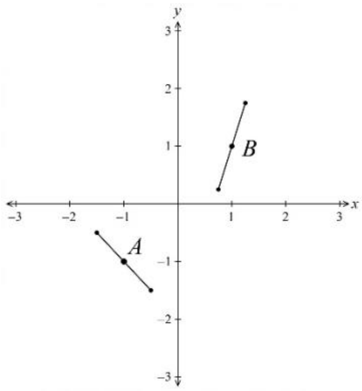
D. $\frac{x}{2} + \frac{1}{28} \sin 14x + C$

10 A direction field is to be drawn for the differential equation:

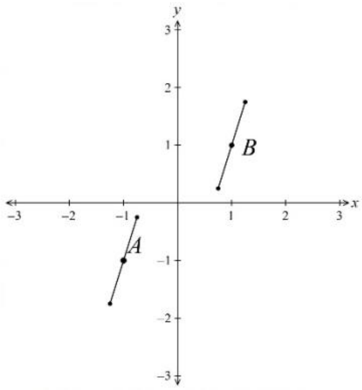
$$\frac{dy}{dx} = 2x + \frac{y}{x}.$$

Which of the graphs below shows the correct slope lines at the points A (–1, –1) and B (1, 1) ?

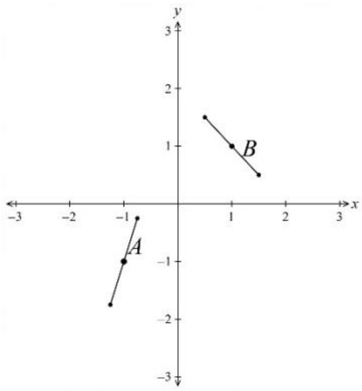
A.



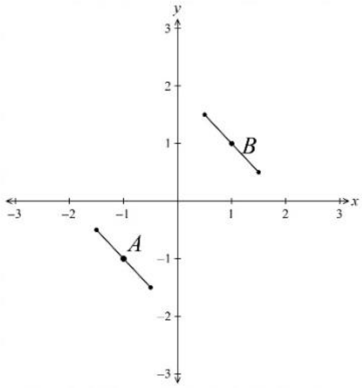
B.



C.



D.



Section II

60 marks
Attempt Questions 11 – 14.
Allow about 1 hour and 45 minutes for this section.

Answer each question in the appropriate writing booklet. Extra writing booklets are available.

In Questions 11 – 14, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) **Start a NEW writing booklet**

(a) Solve the inequality $\frac{3}{x-2} \leq 4$. **3**

(b) Use the substitution $u = 3x^3 - 2x^2$, to find $\int \frac{9x^2-4x}{\sqrt{3x^3-2x^2}}\ dx$ **3**

(c) Harry sets up a playlist which has ten tracks on the media player on his phone. **2**

Three of the tracks are by ***G Flip*** and two are by ***D'Arcy***.

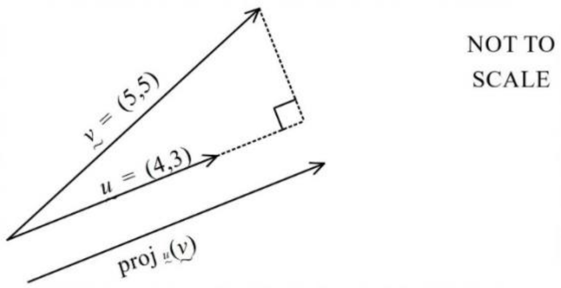
He sets the ten tracks to play in a random order.

What is the probability that the three tracks by ***G Flip*** are played together and the two by ***D'Arcy*** are also played together? (The five tracks do not need to be together)

Question 11 continues on next page

Question 11 continued …

(d) The diagram shows the vectors $\underset{\sim}{u}$ and $\underset{\sim}{v}$ and the projection of $\underset{\sim}{v}$ onto $\underset{\sim}{u}$ denoted by $\text{proj}_{\underset{\sim}{u}}(\underset{\sim}{v})$. **3**



Determine the ordered pair for the projection vector, using the formula:

$$\text{proj}_{\underset{\sim}{u}}\,(\underset{\sim}{v}) = \frac{\underset{\sim}{u}.\underset{\sim}{v}}{|\underset{\sim}{u}|^2}\,\underset{\sim}{u}.$$

(e) Find the exact value of $2\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) - \cos^{-1}\left(\frac{1}{2}\right) + \tan^{-1}(-1)$. **2**

(f) Calculate, correct to the nearest minute, the angle between the vectors $\underset{\sim}{s} = (7,\ -4)$ and $\underset{\sim}{t} = (3,\ 2)$. **2**

End of Question 11

Question 12 (16 marks) **Start a NEW writing booklet**

(a) Given that $f(x) = \tan^{-1}(e^{2x-1})$, find the value of $f'\left(\frac{1}{2}\right)$. **2**

(b) Express $9\sin x + 40\cos x$ in the form $A\sin(x + \alpha)$ where $0 \le \alpha \le \frac{\pi}{2}$, and hence or otherwise solve $9\sin x + 40\cos x = 6$ for $0 \le x \le \pi$. **3**

(x and α in radians, correct to three significant figures).

(c) (i) Find the value of the coefficient of x^4 in the expansion of $(1+x)^3(1+x)^6$. **1**

(ii) Using the expansion of $(1+x)^3(1+x)^6$ show that: **2**

$$\binom{3}{0}\binom{6}{6} + \binom{3}{1}\binom{6}{5} + \binom{3}{2}\binom{6}{4} + \binom{3}{3}\binom{6}{3} = \binom{9}{6}.$$

Question 12 continues on next page

Question 12 continued …

(d) A bottle of juice is removed from a refrigerator, where its temperature was 4° C and placed on a bench in a room which has a constant temperature of 28° C.

The juice warms at a rate proportional to the difference between the temperature of the room and the temperature (*T*) of the juice.

That is, *T* satisfies the differential equation $\frac{dT}{dt} = -k(T - 28)$.

(i) Show that $T = 28 + Ae^{-kt}$ satisfies this differential equation. **1**

(ii) If the temperature of the juice after ten minutes is 8° C, find its temperature after 40 minutes. (Answer to the nearest degree.) **2**

(e) Use the principle of mathematical induction to show that for all integers $n \geq 1$, **3**

$$1 + 5 + 25 + \ldots\ldots + 5^{n-1} = \frac{1}{4}(5^n - 1)$$

(f) Show that $tan^{-1}\,1 + tan^{-1}\,2 + tan^{-1}\,3 = \pi$. **2**

End of Question 12

Question 13 (14 marks) **Start a NEW writing booklet**

(a) Find in the form $y = f(x)$ the solution of the differential equation $\frac{dy}{dx} = \frac{2e^{-\frac{1}{2}y}}{\cos^2 x}$ given that $y = \ln 3$ when $x = \frac{\pi}{3}$. **3**

(b) The area in the first quadrant enclosed by the curve $y = x^2 + 2x + 1$, the line $y = 4$ and the y-axis is shaded in the diagram below. **3**

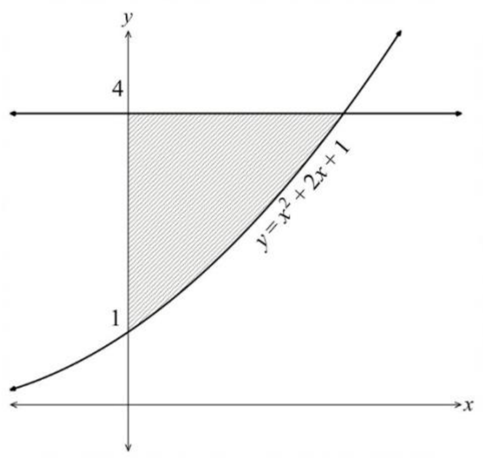


DIAGRAM NOT TO SCALE

Find the volume of the solid formed when the shaded area is rotated about the y-axis.

(c) (i) Show that $\cos 30° \cos 15° = \frac{1}{2}\left[\cos 15° + \frac{1}{\sqrt{2}}\right]$. **1**

(ii) Hence, or otherwise, find the exact value of $\cos 15°$ **2**

Question 13 continues on next page

Question 13 continued …

(d) The polynomial $P(x) = 2x^3 + mx^2 + nx - 20$ has a double root and $P(2) = P'(2) = 0$. **3**

Find the values of m and n and find the other root of $P(x)$.

(e) At a film festival there are nine feature films. On the opening night there are three sessions, at 6 pm, 8 pm and 10 pm. Every feature film is shown at each of the three sessions. **2**

There are 200 film critics at the festival and each one attends three different films on the opening night and writes a review of each.

Use the pigeonhole principle to show that at least one combination of three films will be reviewed by at least three different critics.

End of Question 13

Question 14 (15 marks) **Start a NEW writing booklet**

(a) A vessel is being filled with water at a constant rate of 2 cm^3/s. The volume of water in it at depth h cm is given by $V = \frac{\pi}{4}(2h - \sin 2h)$. **3**

Calculate the rate at which the water is rising, when the depth is $\frac{\pi}{4}$ cm.

(b) A drone, which is hovering at a height of 5 km, releases a target object which falls under gravity. At the same time, a jet, which is at a height of 3.6 km and is 5km west of the drone, fires a projectile at a speed of 400 m/s at an angle of θ° to the horizontal toward the target.

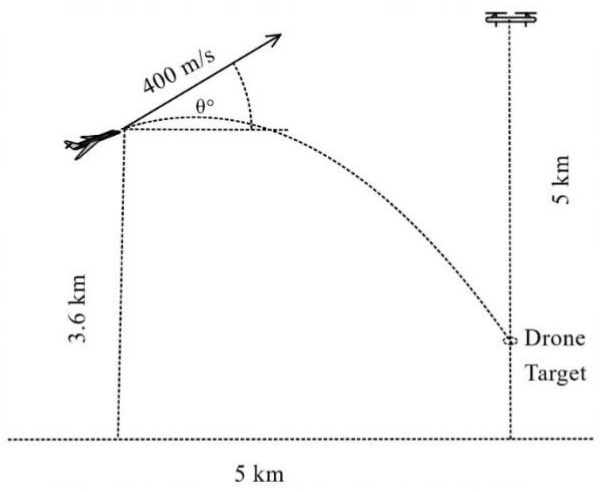


DIAGRAM NOT TO SCALE

Using a point on the ground directly below the jet as the origin, the positions of the projectile and target at time t seconds after the projectile is launched are as follows:

Projectile

$$\underset{\sim}{p}(t) = \begin{pmatrix} 400t\cos\theta \\ 3600 + 400t\sin\theta - 5t^2 \end{pmatrix}$$

Drone Target

$$\underset{\sim}{d}(t) = \begin{pmatrix} 5000 \\ 5000 - 5t^2 \end{pmatrix}$$

(i) Calculate the size of angle θ, if the projectile is to hit the target, to the nearest minute. **2**

(ii) Determine how many seconds after the projectile is fired that it hits the target, the height of the target at that time, and the speed at which the target is falling when it is struck by the projectile. **3**

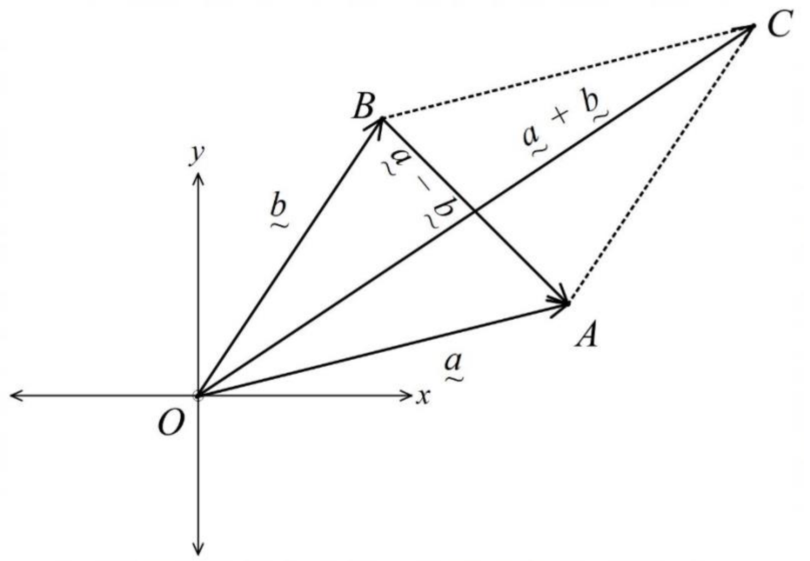
Question 14 continues on next page

Question 14 continued …

(c) (i) For a vector $\underset{\sim}{a} = (x_1,\ y_1)$ show that $\underset{\sim}{a} \cdot \underset{\sim}{a} = |\underset{\sim}{a}|^2$. **1**

(ii) The vectors $\underset{\sim}{a}$ and $\underset{\sim}{b}$ form two adjacent sides of a parallelogram. **2**

The diagonals of the parallelogram are the vectors $\underset{\sim}{a} + \underset{\sim}{b}$ and $\underset{\sim}{a} - \underset{\sim}{b}$, as shown.



Show that for the parallelogram, $(\underset{\sim}{a} + \underset{\sim}{b}).(\underset{\sim}{a} - \underset{\sim}{b}) = |\underset{\sim}{a}|^2 - |\underset{\sim}{b}|^2$ and hence show that if OBCA is a rhombus, the diagonals would be perpendicular.

(d) If the roots of the quadratic equation $8x^2 - 5x + k = 0$ are $\sin\theta$ and $\cos 2\theta$ for some angle θ, find the possible values of k. **4**

End of examination

2023 CGHS Trial HSC Examination
Mathematics Extension 1 Course

Student Number ______________________ Teacher ________________

Section I – Multiple Choice Answer Sheet

Allow about 15 minutes for this section

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample: 2 + 4 = (A) 2 (B) 6 (C) 8 (D) 9

A ○ B ● C ○ D ○

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ● B ☒ C ○ D ○

If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word **correct** and drawing an arrow as follows.

A ☒ B ☒ correct C ○ D ○

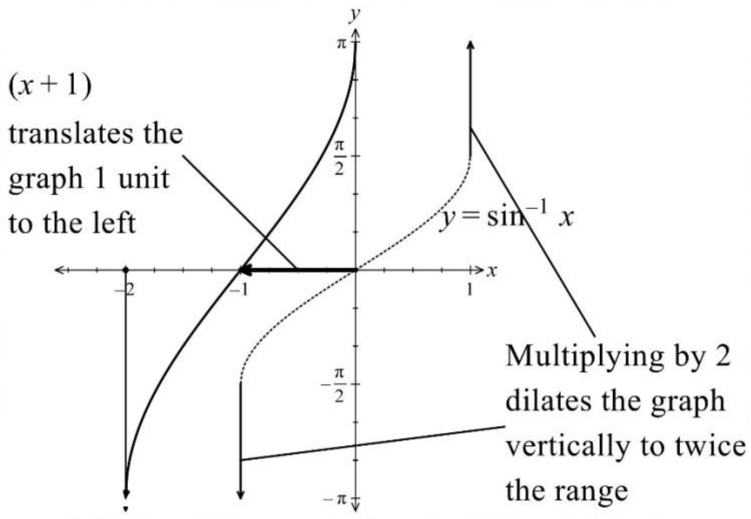
1. A ○ B ○ C ○ D ○
2. A ○ B ○ C ○ D ○
3. A ○ B ○ C ○ D ○
4. A ○ B ○ C ○ D ○
5. A ○ B ○ C ○ D ○
6. A ○ B ○ C ○ D ○
7. A ○ B ○ C ○ D ○
8. A ○ B ○ C ○ D ○
9. A ○ B ○ C ○ D ○
10. A ○ B ○ C ○ D ○

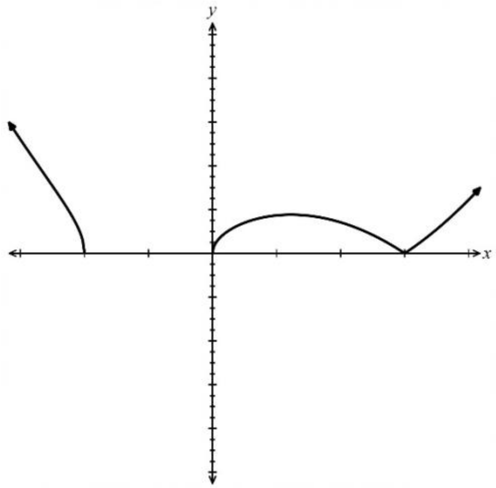
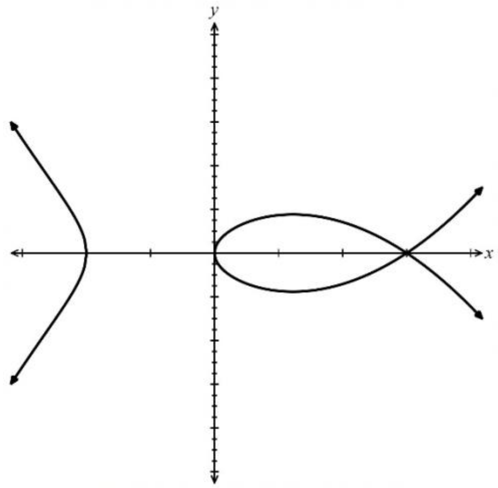
CGHS Mathematics Ext1 Trial Exams 2023

Mathematics
Extension 1

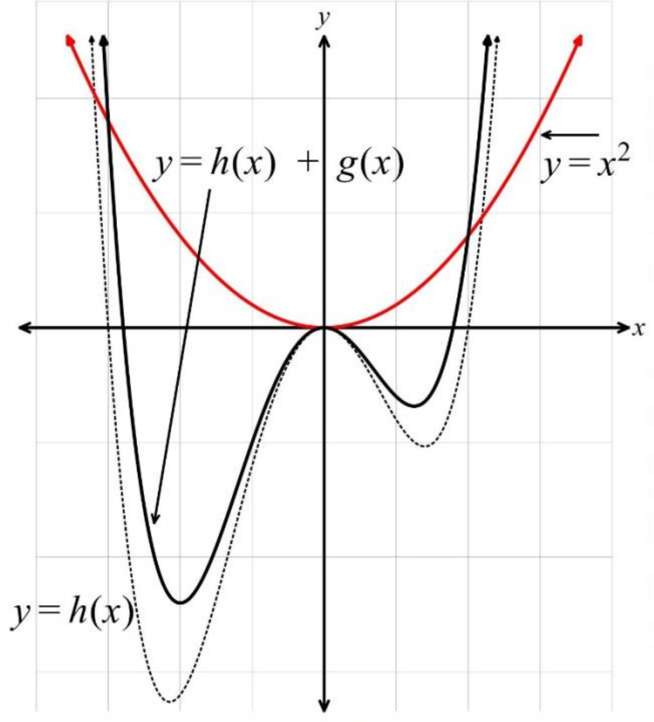
SOLUTIONS

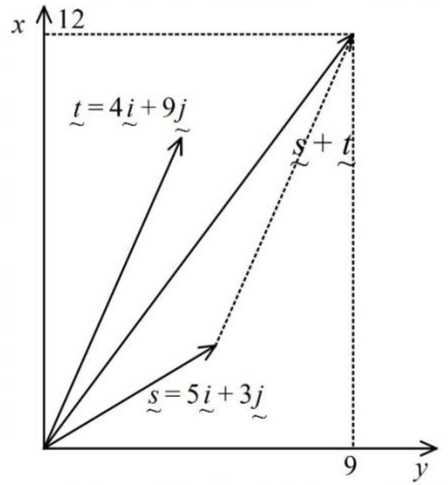
CGHS [Status] E1 Multiple Choice Worked Solutions

No	Working	Answer
1	 So equation is $g(x) = 2\sin^{-1}(x+1)$	D

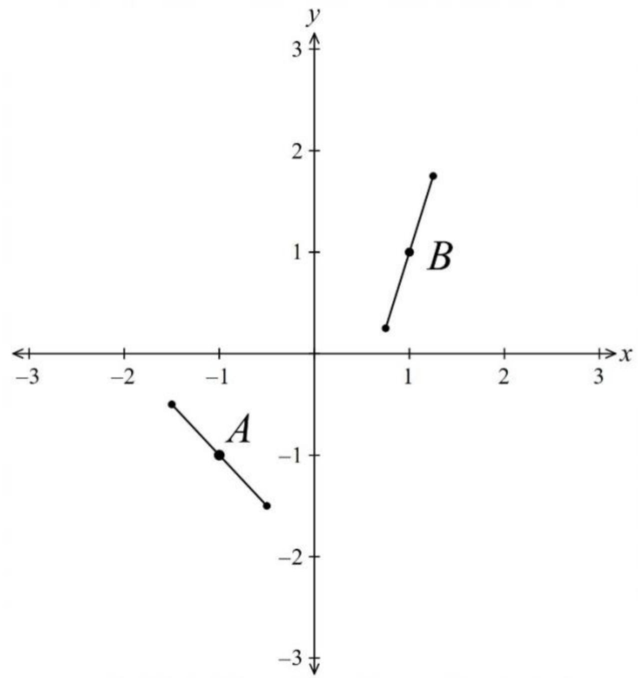
2	Graph B is $y = \sqrt{f(x)}$ since y values only exist for positive x and y values are only positive and are lower than $f(x)$ Graph C is $y^2 = f(x)$ since it is equivalent to $y = \pm\sqrt{f(x)}$ so same graph as B with a reflection in x axis added. 	**C**

3	$$\begin{array}{r} x^2 + x + 1 \\ x^2+2x \overline{) x^4+3x^3+3x^2-4x+5} \\ x^4+2x^3 \\ x^3+3x^2 \\ x^3+2x^2 \\ x^2-4x \\ x^2+2x \\ -6x+5 \end{array}$$ So the division can be written as $$\frac{x^4+3x^3+3x^2-4x+5}{x^2+2x} = x^2+x+1+\frac{5-6x}{x^2+2x}$$	**A**

4	Since the graphs of $y = h(x)$ and $y = h(x) + g(x)$ both pass through the origin, $y = g(x)$ must also pass through the origin so C is eliminated. The graph of $y = h(x) + g(x)$ is above that of $y = h(x) + g(x)$ except at the origin, so $y = g(x)$ must always have positive y values, so that when added to $y = h(x)$, the result is greater than $y = h(x)$. Only B meets this criteria, as A and D have negative y values to the left of the origin, so the graph of $y = h(x) + g(x)$ would be below $y = h(x)$ in this region. 	**B**
5	$x = 4 - t$ ....① $y = 3t^3$ ......② $t = 4 - x$ ③ from rearranging ① $y = 3(4 - x)^3$ $= 3(1 \times 4^3 + 3 \times 4^2 \times (-x) + 3 \times 4 \times (-x)^2 + 1 \times (-x)^3)$ $= 3(64 - 48x + 12x^2 - x^3)$ $y = 192 - 144x + 36x^2 - 3x^3$	**D**

<table>
<tr><td>6</td><td>Graphically


$|\underset{\sim}{s}+\underset{\sim}{t}| = \sqrt{9^2+12^2}$
$= \sqrt{225}$
$= 15$</td><td>Algebraically
$\underset{\sim}{s} + \underset{\sim}{t} = 5\underset{\sim}{i} + 4\underset{\sim}{i} + 3\underset{\sim}{j} + 9\underset{\sim}{j}$
$= 9\underset{\sim}{i} + 12\underset{\sim}{j}$
$|\underset{\sim}{s} + \underset{\sim}{t}| = \sqrt{9^2+12^2}$
$= \sqrt{225}$
$= 15$</td><td>B</td></tr>
<tr><td>7</td><td colspan="2">We want the coefficient of x^0 in $\left(x - \frac{2}{x^2}\right)^6$
$T_{k+1} = {}^6C_k\,(x)^{6-k}\left(-\frac{2}{x^2}\right)^k$
$= {}^6C_k\,(x)^{6-k}\left(-2^k\right)\left(x^{-2k}\right)$
$= {}^6C_k\,\left(-2^k\right)(x)^{6-k}\left(x^{-2k}\right)$
$= {}^6C_k\,\left(-2^k\right)(x)^{6-3k}$
For term independent of x
$6 - 3k = 0$
$3k = 6$
$k = 2$
$T_3 = {}^6C_2\,(x)^{6-2}\left(\frac{2}{x^2}\right)^2$
$= 15.\ x^4 .\ \frac{4}{x^4}$
$= 60$</td><td>C</td></tr>
<tr><td>8</td><td colspan="2">$\underset{\sim}{u} = (-2,\ 6)$, $\underset{\sim}{v} = (9,\ 3)$ and $\underset{\sim}{w} = (3,\ 2)$
$\underset{\sim}{u}.\underset{\sim}{v} = -2\times 9 + 6\times 3 = -18 + 18 = 0\ \therefore \perp$
$\underset{\sim}{u}.\underset{\sim}{w} = -2\times 3 + 6\times 2 = -6 + 12 = 6\ \therefore$ not $\perp$
$\underset{\sim}{v}.\underset{\sim}{w}. = 9\times 3 + 3\times 2 = 27 + 6 = 33\ \therefore$ not $\perp$
Only $\underset{\sim}{u}$ and $\underset{\sim}{v}$ are perpendicular</td><td>A</td></tr>
</table>

9	$\int \cos^2 7x \ dx = \frac{1}{2}\int 1 + \cos 14x \ \ dx$ $= \frac{1}{2}\left(x + \frac{1}{14}\sin 14x\right) + C$ $= \frac{x}{2} + \frac{1}{28}\sin 14x + C$	**D**
10	$\frac{dy}{dx} = 2x + \frac{y}{x}$ At $(-1,\ -1)$ $\frac{dy}{dx} = 2(-1) + \frac{-1}{-1} = -2+1 = -1$ At $(-1,\ -1)$ $m = -1$ At $(1,\ 1)$ $\frac{dy}{dx} = 2(1) + \frac{1}{1} = 2+1 = 3$ At $(1,\ 1)$ $m = 3$ 	**A**

CGHS Trial HSC Examination

Mathematics Extension 1

Name ______________________________ Teacher ______________________

Section I – Multiple Choice Answer Sheet

Allow about 15 minutes for this section
Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample:	2 + 4 =	(A) 2	(B) 6	(C) 8	(D) 9
		A ○	B ●	C ○	D ○

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ● B ● C ○ D ○

If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word **correct** and drawing an arrow as follows.

correct

A ● B ● C ○ D ○

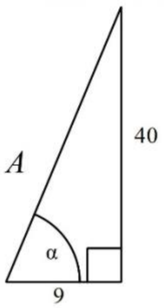
1.	A ○	B ○	C ○	D ●
2.	A ○	B ○	C ●	D ○
3.	A ●	B ○	C ○	D ○
4.	A ○	B ●	C ○	D ○
5.	A ○	B ○	C ○	D ●
6.	A ○	B ●	C ○	D ○
7.	A ○	B ○	C ●	D ○
8.	A ●	B ○	C ○	D ○
9.	A ○	B ○	C ○	D ●
10.	A ●	B ○	C ○	D ○

11		CGHS Ext 1 HSC [Status] **Question 11 Worked Solutions**	Marks	**Allocation and Comments**
(a)		$(x-2)^2 \times \dfrac{3}{(x-2)} \le 4 \times (x-2)^2$ $(x-2)3 \le 4(x-2)^2 \quad x \ne 2$ $(x-2)(3-4x+8) \le 0$ $(x-2)(11-4x) \le 0$ $x < 2$ and $x \ge \dfrac{11}{4}$	3	3 marks for correct solution 2 marks for correct solution but with $x \le 2$ as one of the solutions 1 mark for correct some progress
(b)		Using $u = 3x^3 - 2x^2$, find $\int \dfrac{9x^2-4x}{\sqrt{3x^3-2x^2}}\, dx$ $u = 3x^3 - 2x^2$ $\dfrac{du}{dx} = 9x^2 - 4x$ $du = (9x^2 - 4x)dx$ $\int \dfrac{9x^2-4x}{\sqrt{3x^3-2x^2}}\, dx = \int \dfrac{du}{\sqrt{u}}$ $= \int u^{-\frac{1}{2}}\, du$ $= 2\, u^{\frac{1}{2}} + C$ $= 2(3x^3-2x^2)^{\frac{1}{2}} + C$ $= 2\sqrt{3x^3-2x^2} + C$	**3**	3 marks for correct answer 2 marks for obtaining correct term to integrate with minor error in integration or equivalent merit 1 mark for correct expression for *du* or equivalent merit

11		CGHS Ext 1 HSC [Status] **Question 11 Worked Solutions**	Marks	**Allocation and Comments**
(c)		There are ${}^{10}\mathbf{P}_{10}$ (10!) $=$ 3 628 800 ways that 10 tracks can be arranged in the playlist. If the tracks by D′Arcy are together and those by G Flip are together, we can treat them as two single tracks, So there are ${}^{7}\mathbf{P}_{7}$ (7!) ways that the 7 tracks ($D + G + 5$ others) can be arranged. But the two tracks by D′Arcy can be arranged in ${}^{2}\mathbf{P}_{2}$(2!) ways and the three tracks by G Flip can be arranged in ${}^{3}\mathbf{P}_{3}$(3!) ways. So the number of ways of having the three tracks together $= 7! \times 3! \times 2! = 60\,480$ ways Probability $= \dfrac{7! \times 3! \times 2!}{10!}$ $= \dfrac{60\,480}{3\,628\,800} = \dfrac{1}{60} = 0.0166666667$	**2**	2 marks for correct answer in any of the formats shown. 1 mark for correct number of arrangements of 10 tracks or equivalent merit
(d)		$\underset{\sim}{u}.\underset{\sim}{v} = 4 \times 5 + 3 \times 5 = 20 + 15 = 35$ $\lvert\underset{\sim}{u}\rvert = \sqrt{4^2 + 3^2} = \sqrt{25} = 5$ $proj_{\underset{\sim}{u}}\,(\underset{\sim}{v}) = \dfrac{\underset{\sim}{u}.\underset{\sim}{v}}{\lvert\underset{\sim}{u}\rvert^2}\,\underset{\sim}{u}$ $= \dfrac{35}{5^2}\,(4,3)$ $= \dfrac{35}{25}(4,3)$ $= 1.4(4,3)$ $= (5.6,\ 4.2)$	**3**	3 marks for correct answer 2 marks for correct values of $\underset{\sim}{u}.\underset{\sim}{v}$ and $\lvert\underset{\sim}{u}\rvert$ or equivalent merit 1 mark for correct value of $\underset{\sim}{u}.\underset{\sim}{v}$ or $\lvert\underset{\sim}{u}\rvert$ or equivalent merit

11	CGHS Ext 1 HSC [Status] **Question 11 Worked Solutions**	Marks	**Allocation and Comments**
(e)	We want the exact value of $2\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) - \cos^{-1}\left(\frac{1}{2}\right) + \tan^{-1}(-1)$ Taking each part separately $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{3}$ $\cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3}$ $\tan^{-1}(-1) = -\frac{\pi}{4}$ $2\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) - \cos^{-1}\left(\frac{1}{2}\right) + \tan^{-1}(-1) = 2\times\frac{\pi}{3} - \frac{\pi}{3} - \frac{\pi}{4}$ $= \frac{\pi}{3} - \frac{\pi}{4}$ $= \frac{\pi}{12}$	2	2 marks for correct answer 1 mark for correct value of at least two of the inverse trig identities needed in the calculation
(f)	$\underset{\sim}{s} = (7, -4)$ and $\underset{\sim}{t} = (3, 2)$ $\underset{\sim}{s}.\underset{\sim}{t} = 7\times 3 + -4\times 2 = 21 - 8 = 13$ $\lvert\underset{\sim}{s}\rvert = \sqrt{49+16} = \sqrt{65}$ $\lvert\underset{\sim}{t}\rvert = \sqrt{9+4} = \sqrt{13}$ $\cos\theta = \frac{\underset{\sim}{s}.\underset{\sim}{t}}{\lvert\underset{\sim}{s}\rvert\lvert\underset{\sim}{t}\rvert}$ $= \frac{13}{\sqrt{65}\times\sqrt{13}}$ $= 0.4472136$ $\theta = \arccos(0.4472136)$ $= 63.43494853$ $= 63°26'$	**2**	2 marks for correct answer 1 mark for using the correct formula and correct value of $\underset{\sim}{s}.\underset{\sim}{t}$, or of $\lvert\underset{\sim}{s}\rvert$ or of $\lvert\underset{\sim}{t}\rvert$ or equivalent merit

12		CGHS Ext 1 HSC [Status] **Question 12 Worked Solutions**	Marks	**Allocation and Comments**
(a)		$f'(x) = \frac{d}{dx}\left(\tan^{-1}(e^{2x-1})\right)$ $= \frac{\frac{d}{dx}(e^{2x-1})}{1+(e^{2x-1})^2}$ $= \frac{2e^{2x-1}}{1+e^{4x-2}}$ $f'\left(\frac{1}{2}\right) = \frac{2e^{2\left(\frac{1}{2}\right)-1}}{1+e^{4\left(\frac{1}{2}\right)-2}}$ $= \frac{2e^0}{1+e^0}$ $= \frac{2}{2} = 1$	**2**	2 marks for correct answer 1 mark for correct expression for $f'(x)$ or equivalent merit

12		**CGHS Ext 1 HSC** [Status] **Question 12 Worked Solutions**	Marks	**Allocation and Comments**
(b)		 $A\sin(x+\alpha) = A\sin x\cos\alpha + A\cos x\sin\alpha$ $= 9\sin x + 40\cos x$ $A\cos\alpha = 9$ and $A\sin\alpha = 40$ $\dfrac{A\sin\alpha}{A\cos\alpha} = \dfrac{40}{9}$ $\tan\alpha = \dfrac{40}{9}$ $A^2 = 9^2 + 40^2$ $= 1\,681$ $A = \sqrt{1\,681} = 41$ $\alpha = \tan^{-1}\left(\dfrac{40}{9}\right) = 1.35$ (3 sf) $9\sin x + 40\cos x = 6$ $41\sin(x + 1.35) = 6$ $\sin(x + 1.35) = \dfrac{6}{41}$ $x + 1.35 = 0.147,\ 2.99$ for $(0 \le x + 1.35 \le 2\pi)$ $x = 1.64$ for $0 \le x \le \pi$	**3**	3 marks for correct answer 2 marks for correct values of α and A or equivalent merit 1 mark for correct value of α or equivalent merit

12		**CGHS Ext 1 HSC** [Status] **Question 12 Worked Solutions**	Marks	**Allocation and Comments**
(c)	(i)	$(1+x)^3(1+x)^6 = \left[\binom{3}{0}+\binom{3}{1}x+\binom{3}{2}x^2+\binom{3}{3}x^3\right]$ $\times\left[\binom{6}{0}+\binom{6}{1}x+\binom{6}{2}x^2+\binom{6}{3}x^3+\binom{6}{4}x^4+\binom{6}{5}x^5+\binom{6}{6}x^6\right]$ Coefficient of $x^4 = \binom{3}{0}\times\binom{6}{4}+\binom{3}{1}\times\binom{6}{3}+\binom{3}{2}\times\binom{6}{2}+\binom{3}{3}\times\binom{6}{1}$ $= 1\times 15+3\times 20+3\times 15+1\times 6 = 126$ Alternative: $(1+x)^3(1+x)^6 = (1+x)^9$ $= \left[\binom{9}{0}+\binom{9}{1}x+\ldots+\binom{9}{4}x^4+\ldots\ldots\ldots+\binom{9}{8}x^8+\binom{9}{9}x^9\right]$ Coefficient of $x^4 = \binom{9}{4} = 126$	**1**	1 mark for correct answer
	(ii)	$(1+x)^3(1+x)^6 = (1+x)^9$ $\left[\binom{3}{0}+\binom{3}{1}x+\binom{3}{2}x^2+\binom{3}{3}x^3\right]*\left[\binom{6}{0}+\binom{6}{1}x+\binom{6}{2}x^2+\binom{6}{3}x^3+\binom{6}{4}x^4+\binom{6}{5}x^5+\binom{6}{6}x^6\right]$ $= \left[\binom{9}{0}+\binom{9}{1}x+\ldots+\binom{9}{6}x^6+\ldots\ldots\ldots+\binom{9}{8}x^8+\binom{9}{9}x^9\right]$ Equating coefficients of $x^6 = \binom{3}{0}\times\binom{6}{6}+\binom{3}{1}\times\binom{6}{5}+\binom{3}{2}\times\binom{6}{4}+\binom{3}{3}\times\binom{6}{3} = \binom{9}{6}$	**2**	2 marks for expansions and equating the correct coefficients. 1 mark for correct expansion with error in or failure to equate coefficients
(d)	(i)	$\frac{dT}{dt} = -k(T-28)$ $T = 28 + Ae^{-kt}$ $\therefore\ T - 28 = Ae^{-kt}$ $\frac{dT}{dt} = -kAe^{-kt}$ $\frac{dT}{dt} = -k(T-28)$	**1**	1 mark for correctly differentiating and rearranging to show required result

12		**CGHS Ext 1 HSC** [Status] **Question 12 Worked Solutions**	Marks	**Allocation and Comments**
	(ii)	$T = 28 + Ae^{-kt}$ When $t = 0,\ T = 4$ $4 = 28 + A(e^0)$ $A = 4 - 28 = -24$ When $t = 10,\ T = 8$ $8 = 28 - 24e^{-10k}$ $24e^{-10k} = 20$ $e^{-10k} = \frac{20}{24}$ $-10k = \ln\left(\frac{20}{24}\right)$ $k = \frac{\ln\left(\frac{20}{24}\right)}{-10} = 0.01823216$ $T = 28 - 24e^{-0.0182t}$ When $t = 40$ $T = 28 - 24e^{-0.0182\times 40}$ $= 16.41102946$ The temperature will be 16° (to the nearest degree)	**2**	2 marks for correct answer 1 mark for correctly finding the values of A and k or equivalent merit

12	**CGHS Ext 1 HSC** [Status] **Question 12 Worked Solutions**	Marks	**Allocation and Comments**
(e)	Prove that: $1+5+25+\ldots\ldots+5^{n-1} = \frac{1}{4}(5^n-1)$ Show the statement is true for $n=1$ LHS $= 5^{1-1} = 5^0 = 1$ RHS $= \frac{1}{4}(5^1-1) = \frac{1}{4}(4) = 1$ $\therefore$ true for $n=1$ Assume that statement is true for $n = k$ $1+5+25+125+\ldots\ldots+5^{k-1} = \frac{1}{4}(5^k-1)$ Show that it is true for $n = k+1$ i.e. that $1+5+25+125+\ldots\ldots+5^{k-1}+5^k = \frac{1}{4}(5^{k+1}-1)$ LHS $= 1+5+25+125+\ldots\ldots+5^{k-1}+5^k$ $= \frac{1}{4}(5^k-1) + 5^k$ $= \frac{1}{4}(5^k-1) + \frac{4\times 5^k}{4}$ $= \frac{1}{4}(5^k-1+4\times 5^k)$ $= \frac{1}{4}(5\times 5^k-1)$ $= \frac{1}{4}(5^{k+1}-1)$ $\therefore$ if true for $n=k$, is also true for $n = k+1$ Since it is true for $n=1$, by induction it is also true for all $n \geq 1$ So $1+5+25+\ldots\ldots+5^{n-1} = \frac{1}{4}(5^n-1)$	**3**	3 marks for correct proof with three steps: 1. Show the statement is true for $n=1$ 2. Assume that statement is true for $n = k$ and show that it is true for $n = k+1$ 3. Conclude result by induction (Order of steps 1 and 2 can be reversed) 2 marks if third step is left out or error in 2nd step or equivalent merit 1 mark for 1st step correct, along with statement of assumption or equivalent merit

12		CGHS Ext 1 HSC [Status] **Question 12 Worked Solutions**	Marks	**Allocation and Comments**
(f)		$\tan^{-1} 1 + \tan^{-1} 2 + \tan^{-1} 3 = \pi$ $\tan^{-1} 2 + \tan^{-1} 3 = \dfrac{3\pi}{4}$ Let $\alpha = \tan^{-1} 2$ and $\beta = \tan^{-1} 3$ So $\tan\alpha = 2$ and $\tan\beta = 3$ $\tan(\alpha+\beta) = \dfrac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$ $= \dfrac{2+3}{1-6} = -1$ $\alpha + \beta = \tan^{-1}(-1)$ $\therefore \alpha + \beta = \pi - \dfrac{\pi}{4} = \dfrac{3\pi}{4}$ $\tan^{-1} 1 + \tan^{-1} 2 + \tan^{-1} 3 = \pi$	**2**	2 marks for providing correct working and answer. 1 mark for assigning $\tan\alpha = 2$ and $\tan\beta = 3$ or equivalent progress.

13		CGHS Ext 1 HSC [Status] **Question 13 Worked Solutions**	Marks	**Allocation and Comments**
(a)		$\frac{dy}{dx} = \frac{2e^{-\frac{1}{2}y}}{\cos^2 x}$ $\int \frac{1}{2} e^{\frac{1}{2}y} dy = \int \sec^2 x \, dx$ $e^{\frac{1}{2}y} = \tan x + C$ $y = \ln 3\,, x = \frac{\pi}{3} \implies e^{\frac{1}{2}\ln 3} = \sqrt{3} + C$ $e^{\ln\sqrt{3}} = \sqrt{3} + C$ $\sqrt{3} = \sqrt{3} + C$ $\therefore C = 0$ $\therefore e^{\frac{1}{2}y} = \tan x$ $\frac{1}{2}y = \ln(\tan x)$ $y = 2\ln(\tan x)$	**3**	1 Mark - Some progress e.g. separates variables and obtains one of the two anti-derivatives 2 Marks - Substantial progress e.g. correct process to find $e^{\frac{1}{2}y}$ as a function of x 3 Marks – Correct answer with full worked solutions

13		**CGHS Ext 1 HSC** [Status] **Question 13 Worked Solutions**	Marks	**Allocation and Comments**
(b)		$y = x^2 + 2x + 1$ $y = (x+1)^2$ $x + 1 = y^{\frac{1}{2}}$ (positive square root since in $1st$ quad) $x = y^{\frac{1}{2}} - 1$ $x^2 = \left(y^{\frac{1}{2}} - 1\right)^2 \quad = y - 2y^{\frac{1}{2}} + 1$ $\text{Volume} = \pi \int_1^4 x^2 \ dy$ $= \pi \int_1^4 y - 2y^{\frac{1}{2}} + 1 \ dy$ $= \pi \left[\frac{y^2}{2} - \frac{4}{3} y^{\frac{3}{2}} + y\right]_1^4$ $= \pi\left(\frac{4^2}{2} - \frac{4}{3} \times (4)^{\frac{3}{2}} + 4\right) - \left(\frac{1^2}{2} - \frac{4}{3} \times (1)^{\frac{3}{2}} + 1\right)$ $= \frac{4\pi}{3} - \frac{\pi}{6}$ $= \frac{7\pi}{6}$ cubic units	**3**	3 marks for correct answer 2 marks for determining correct integral and minor error in evaluating or equivalent merit 1 mark for some correct working relating to integration to find volume
(c)	(i)	$\cos A \cos B = \frac{1}{2}[\cos(A-B) + \cos(A+B)]$ $\cos 30° \cos 15° = \frac{1}{2}[\cos(30-15)° + \cos(30+15)]$ $= \frac{1}{2}[\cos 15° + \cos 45°]$ $= \frac{1}{2}\left[\cos 15° + \frac{1}{\sqrt{2}}\right]$	**1**	1 mark for substitution to obtain correct result

13		CGHS Ext 1 HSC [Status] **Question 13 Worked Solutions**	Marks	**Allocation and Comments**
	(ii)	$\frac{\sqrt{3}}{2}\cos 15° = \frac{1}{2}\left[\cos 15° + \frac{1}{\sqrt{2}}\right]$ $\sqrt{3}\cos 15° = \cos 15° + \frac{1}{\sqrt{2}}$ $\sqrt{3}\cos 15° - \cos 15° = \frac{1}{\sqrt{2}}$ $\cos 15°\left(\sqrt{3} - 1\right) = \frac{1}{\sqrt{2}}$ $\cos 15° = \frac{1}{\sqrt{2}\left(\sqrt{3}-1\right)}$ $= \frac{1}{\left(\sqrt{6}-\sqrt{2}\right)}$ $= \frac{\sqrt{6}+\sqrt{2}}{(6-2)}$ $= \frac{\sqrt{6}+\sqrt{2}}{4}$	**2**	2 marks for correct result or equivalent expression 1 mark for correct substitution of exact value into LHS and an attempt to rearrange to required result

13		**CGHS Ext 1 HSC** [Status] **Question 13 Worked Solutions**	Marks	**Allocation and Comments**
(d)		$P(x) = 2x^3 + mx^2 + nx - 20$ has a double root, so let roots be $\alpha,\ \beta,\ \beta$ Since $P(2) = P'(2) = 0$, the double root is $x = \beta = 2$ $P(2) = 16 + 4\text{m} + 2\text{n} - 20 = 0$ $4m + 2n = 4 \quad \ldots.1$ $P'(2) = 24 + 4m + n = 0$ $4m + n = -24\ldots..2$ $2n - n = 4 -- 24\ldots..1 - 2$ $n = 28$ $4m + 2(28) = 4$ $4m + 56 = 4$ $4m = -52$ $m = \frac{-52}{4} = -13$ Roots of $P(x) = 2x^3 - 13x^2 + 28x - 20 = (x-2)^2(2x - \alpha) = 0$ can then be found by inspection or as follows: For $ax^3 + bx^2 + cx + d$ with roots α, 2 and 2 $\alpha + 2 + 2 = -\frac{b}{a} \Rightarrow \alpha + 4 = -\frac{(-13)}{2}$ $2\alpha + 8 = 13$ $2\alpha = 5$ $\alpha = \frac{5}{2}$ **So $\boldsymbol{m = -13}$ and $\boldsymbol{n = 28}$, and the roots are $\boldsymbol{\alpha = \frac{5}{2}}$ and a double root $\boldsymbol{\beta = 2}$**	**3**	3 marks for finding the values of *m* and *n* and the value of the 3rd root (on the bolded line) 2 marks for the values of *m* and *n* or equivalent merit 1 mark for value of *m* or *n* or for some relevant algebraic manipulation or equivalent merit

13	**CGHS Ext 1 HSC** [Status] **Question 13 Worked Solutions**	Marks	**Allocation and Comments**
(e)	There are ${}^{9}\mathbf{C}_3 = 84$ combinations of three feature films. Dividing the 200 critics between the 84 choices $= \dfrac{200}{84} = 2.38095238$ So if every critic chose a different combination, all combinations would be taken up after 84 critics and every combination would be taken up twice after 168 critics. The next critic would need to choose the same combination as two earlier critics. So at least one combination will be chosen by three critics, and as there are 31 more critics it is possible it will be chosen by more than three. So at least one combination of three films will be reviewed by at least three different critics .	**2**	2 marks for complete explanation. 1 mark for finding ${}^{9}\mathbf{C}_3 = 84$ or equivalent merit.

14		CGHS Ext 1 HSC [Status] **Question 14 Worked Solutions**	Marks	**Allocation and Comments**
(a)		$V = \frac{\pi}{4}(2h - sin\,2h)$ $\frac{dV}{dh} = \frac{\pi}{4}(2 - 2cos\,2h) = \frac{\pi}{2}(1 - cos\,2h)$ $\frac{dh}{dV} = \frac{2}{\pi(1-\cos 2h)}$ Given $\frac{dV}{dt} = 2$ $\frac{dh}{dt} = \frac{dV}{dt} \times \frac{dh}{dV} = \frac{2}{\pi(1-\cos 2h)} \times 2 = \frac{4}{\pi(1-\cos 2h)}$ When $h = \frac{\pi}{4}$ $\frac{dh}{dt} = \frac{4}{\pi(1-\cos\frac{\pi}{2})} = \frac{4}{\pi}$ cm/s	**3**	1 Mark - Finding $\frac{dV}{dh}$ or establish the correct chain and substitutes correctly 2 Marks - Finding $\frac{dV}{dh}$ and establish the chain rule for $\frac{dh}{dt}$ 3 Marks – Correct working and answer

14		CGHS Ext 1 HSC [Status] **Question 14 Worked Solutions**	Marks	**Allocation and Comments**
(b)	(i)	Drone $\underset{\sim}{d}(t) = \begin{pmatrix} 5000 \\ 5000 - 5t^2 \end{pmatrix}$ Projectile $\underset{\sim}{p}(t) = \begin{pmatrix} 400t\cos\theta \\ 3600 + 400t\sin\theta - 5t^2 \end{pmatrix}$ Equating x values $5000 = 400t\cos\theta$ $t\cos\theta = \frac{5000}{400} = 12.5$ Equating y values $5000 - 5t^2 = 3600 + 400t\sin\theta - 5t^2$ $1400 = 400t\sin\theta$ $t\sin\theta = \frac{1400}{400} = 3.5$ $\frac{t\sin\theta}{t\cos\theta} = \frac{3.5}{12.5} = 0.28$ $\tan\theta = 0.28$ $\theta = \arctan(0.28) = 15.64224646 = 15°39'$	**2**	2 marks for correct answer 1 mark for equating x and y values or equivalent merit
	(ii)	Find t by substituting back $\cos(15°39')t = 12.5$ $t = \frac{12.5}{0.962964} = 12.98075525 = 13.0$ seconds (to 1 d.p) Find height by substituting back into $y = 5000 - 5 \times (12.98075525)^2$ $= 4157.4999657$ For target $= -10t$ $= -10 \times (12.98075525)$ $= -129.807$ $= 130$ m/s in a downward direction	**3**	3 marks for time, height and speed. 2 marks for the time and either of the height or speed or equivalent merit 1 mark for the time only or equivalent merit

14		CGHS Ext 1 HSC [Status] **Question 14 Worked Solutions**	Marks	**Allocation and Comments**
(c)	(i)	$\underset{\sim}{a} = (x_1, y_1)$ $\lvert\underset{\sim}{a}\rvert = \sqrt{{x_1}^2 + {y_1}^2}$ $\underset{\sim}{a} \cdot \underset{\sim}{a} = x_1 \times x_1 + y_1 \times y_1$ $= {x_1}^2 + {y_1}^2$ $= \lvert\underset{\sim}{a}\rvert^2$	**1**	1 mark for valid demonstration
	(ii)	$(\underset{\sim}{a} + \underset{\sim}{b}).(\underset{\sim}{a} - \underset{\sim}{b}) = \underset{\sim}{a} \cdot \underset{\sim}{a} - \underset{\sim}{a} \cdot \underset{\sim}{b} + \underset{\sim}{b} \cdot \underset{\sim}{a} - \underset{\sim}{b} \cdot \underset{\sim}{b}$ $= \underset{\sim}{a} \cdot \underset{\sim}{a} - \underset{\sim}{a} \cdot \underset{\sim}{b} + \underset{\sim}{a} \cdot \underset{\sim}{b} - \underset{\sim}{b} \cdot \underset{\sim}{b}$ $= \underset{\sim}{a}.\underset{\sim}{a} - \underset{\sim}{b}.\underset{\sim}{b}$ $= \lvert\underset{\sim}{a}\rvert^2 - \lvert\underset{\sim}{b}\rvert^2$ Now if $AOBC$ is a rhombus, the adjacent sides are equal in length $\therefore \quad \lvert\underset{\sim}{a}\rvert = \lvert\underset{\sim}{b}\rvert$ So $(\underset{\sim}{a} + \underset{\sim}{b}).(\underset{\sim}{a} - \underset{\sim}{b}) = \lvert\underset{\sim}{a}\rvert^2 - \lvert\underset{\sim}{b}\rvert^2$ $= \lvert\underset{\sim}{a}\rvert^2 - \lvert\underset{\sim}{a}\rvert^2$ $= 0$ Now if the dot product of two vectors is zero, they are perpendicular So $(\underset{\sim}{a} + \underset{\sim}{b}) \perp (\underset{\sim}{a} - \underset{\sim}{b})$ $\therefore$ the diagonals of a rhombus are perpendicular	**2**	2 marks for showing both results correctly 1 mark for only showing one result correctly

14		CGHS Ext 1 HSC [Status] **Question 14 Worked Solutions**	Marks	**Allocation and Comments**
(d)		Roots of $8x^2 - 5x + k = 0$ are $\sin\theta$ and $\cos 2\theta$ $\sin\theta + \cos 2\theta = \frac{5}{8}$ (from sum of roots) $8\sin\theta + 8\cos 2\theta - 5 = 0$ $8\sin\theta + 8(1 - 2\sin^2\theta) - 5 = 0$ $-16\sin^2\theta + 8\sin\theta + 3 = 0$ $16\sin^2\theta - 8\sin\theta - 3 = 0$ $(4\sin\theta - 3)(4\sin\theta + 1) = 0$ $\sin\theta = \dfrac{3}{4}$ or $-\dfrac{1}{4}$ $\cos 2\theta = -\frac{1}{8}$ or $\frac{7}{8}$ $\sin\theta\cos 2\theta = \dfrac{k}{8}$ (from product of roots) $\dfrac{3}{4} \times -\dfrac{1}{8} = \dfrac{k}{8}$ or $-\dfrac{1}{4} \times \dfrac{7}{8} = \dfrac{k}{8}$ $k = -\dfrac{3}{4}$ or $k = -\dfrac{7}{4}$	**4**	1 Mark – able to provide sum of roots, or attempts to solve simultaneous equations created by substitution of roots 2 Marks – able to come out with a quadratic equation and attempt to solve 3 Marks – able to use product of roots 4 Marks – correct solutions and correct answer *Other solutions or working are possible.*