Student Number: | | | | | | | | |

Teacher: ____________________

Cheltenham Girls High School

2024 TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Extension 1

Assessment Task 4

General Instructions

- Reading time – 10 minutes
- Working time – 2 hours
- Write using black pen
- Calculators approved by NESA may be used
- A NESA reference sheet is provided at the back of this paper
- For Section I: Multiple choice questions, a multiple choice answer sheet is provided
- For questions in Section II:
  - Start a new writing booklet for Questions 11, 12, 13 and 14
  - Show relevant mathematical reasoning and/or calculations to gain maximum marks

Total Marks: 70

Section I – 10 marks (pages 2 – 8)

- Attempt Questions 1 – 10
- Allow about 15 minutes (maximum) for this section

Section II – 60 marks (pages 9 – 15 + page 17)

- Attempt Questions 11 – 14
- Allow about 1 hour and 45 minutes for this section

Section I

10 marks

Attempt Questions 1 – 10.

Allow about 15 minutes for this section.

Use the multiple-choice answer sheet for Questions 1 – 10.

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1 Evaluate $\int_0^{\frac{1}{2}} \frac{dx}{\sqrt{1-4x^2}}$

A. $\frac{\pi}{2}$ B. $\frac{\pi}{3}$ C. $\frac{\pi}{4}$ D. $\frac{\pi}{6}$

2 Consider the three statements below:

Statement 1: In a group of thirteen people, at least two will have their birthday in the same month.

Statement 2: A room which has five windows in its four walls must have at least one wall which has at least two windows.

Statement 3: A drawer contains ten socks which are identical except that they are a mixture of seven different colours. If six socks are drawn from the drawer they must include at least one pair of socks of matching colours.

Which statements are correct applications of the Pigeonhole Principle?

A. Statements 1 and 2 only

B. Statements 1 and 3 only

C. Statements 2 and 3 only

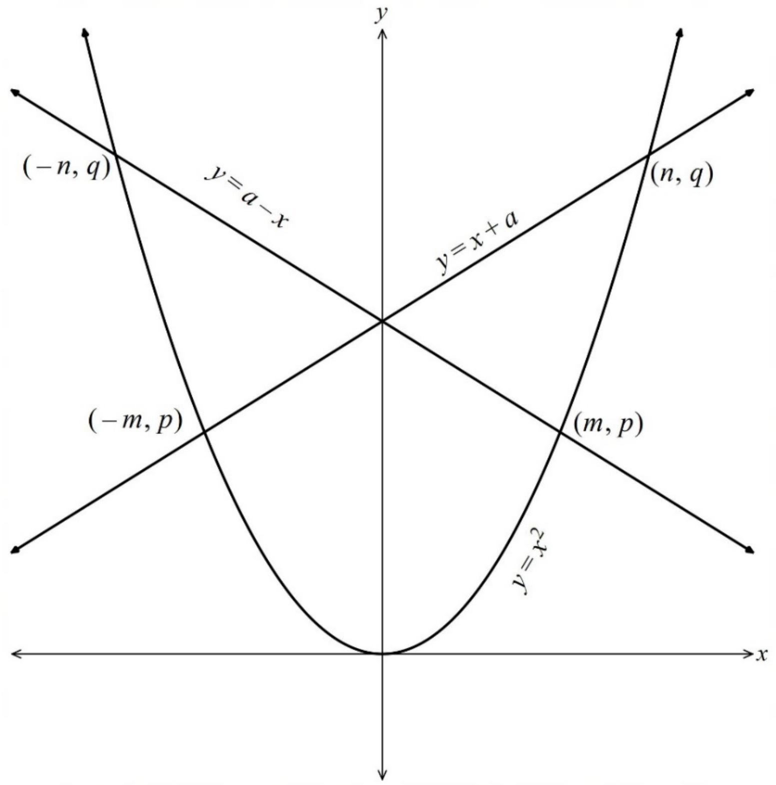
D. All three statements

Multiple choice questions continued over the page…

3 The graph below shows the curve $y = x^2$ and the lines $y = x + a$ and $y = a - x$, along with the points of intersection between the curve and the lines.

(The terms a, m, n, p and q are all positive constants.)

NOT TO SCALE



What is the solution to $x^2 + x - a < 0$?

A. $-n < x < -m$

B. $-n < x < m$

C. $-m < x < m$

D. $-m < x < n$

4 sin 2A cos2A =

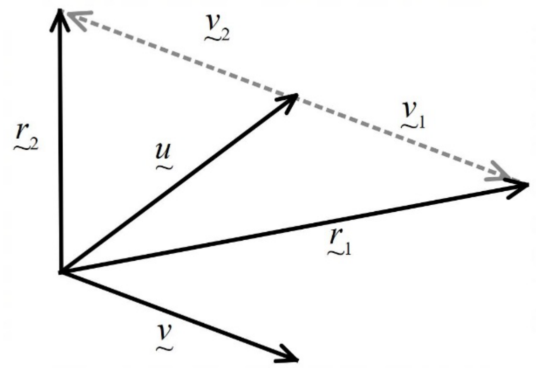
A. 4 sin A cos A   B. $\frac{1}{2}$ sin 2A   C. 2 sin 2A   D. $\frac{1}{2}$ sin 4A

Multiple choice questions continued over the page…

5 The diagram shows two vectors $\underset{\sim}{u}$ and $\underset{\sim}{v}$.

Two resultant vectors, $\underset{\sim}{r}_1$ and $\underset{\sim}{r}_2$, are constructed using $\underset{\sim}{v}_1$ and $\underset{\sim}{v}_2$ which are parallel to, and equal in length to $\underset{\sim}{v}$.

NOT TO SCALE



Which statement is true?

A. $\underset{\sim}{r}_1 = \underset{\sim}{u} - \underset{\sim}{v}$ and $\underset{\sim}{r}_2 = \underset{\sim}{u} + \underset{\sim}{v}$

B. $\underset{\sim}{r}_1 = \underset{\sim}{v} + \underset{\sim}{u}$ and $\underset{\sim}{r}_2 = \underset{\sim}{v} - \underset{\sim}{u}$

C. $\underset{\sim}{r}_1 = \underset{\sim}{u} + \underset{\sim}{v}$ and $\underset{\sim}{r}_2 = \underset{\sim}{u} - \underset{\sim}{v}$

D. $\underset{\sim}{r}_1 = \underset{\sim}{v} - \underset{\sim}{u}$ and $\underset{\sim}{r}_2 = \underset{\sim}{u} - \underset{\sim}{v}$

6 How many distinct arrangements are possible from the letters of the word NECESSITIES?

A. 50 400

B. 554 400

C. 1 663 200

D. 39 916 800

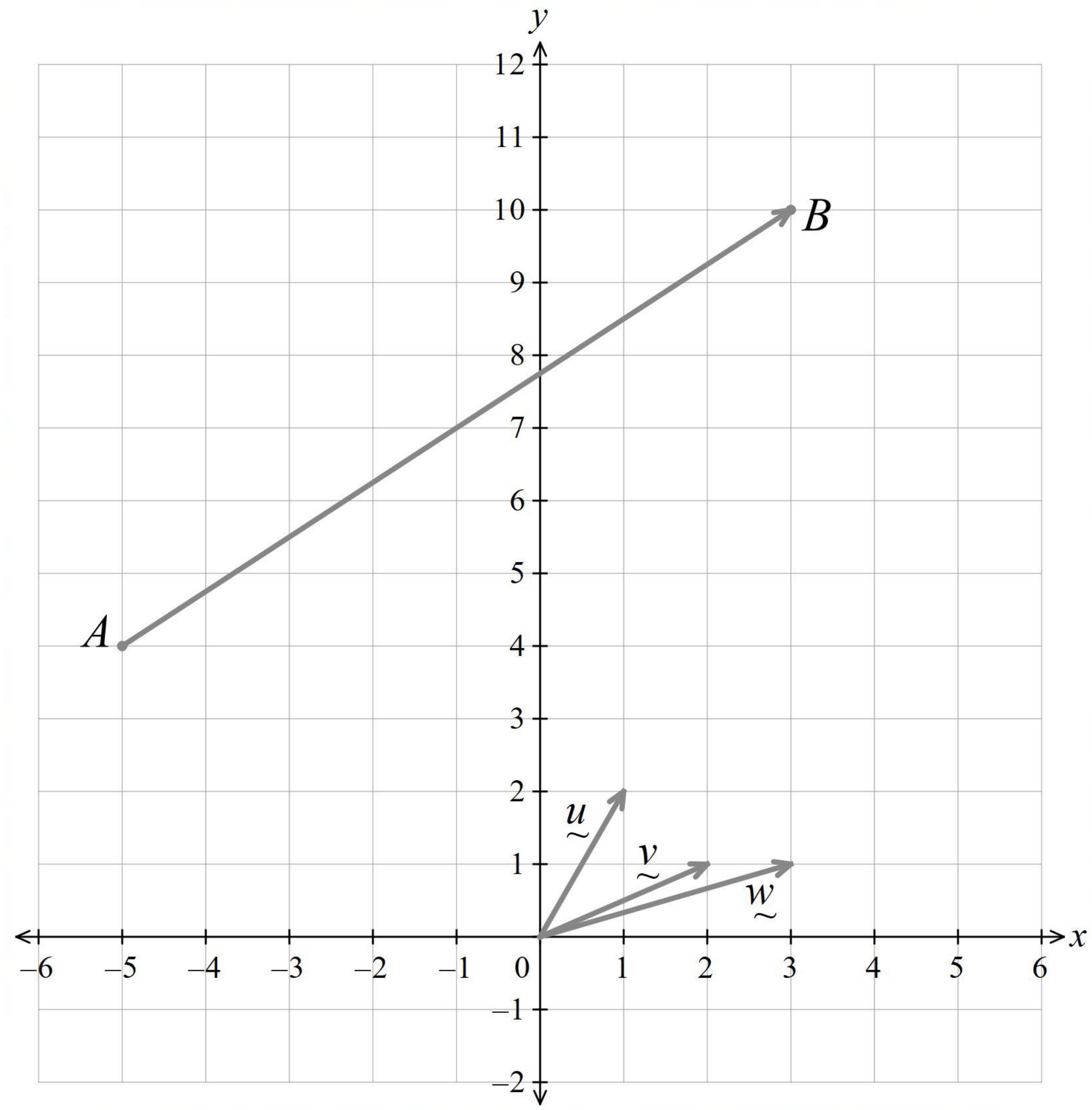
Multiple choice questions continued over the page…

7 On the diagram below, the vector $\overrightarrow{AB}$ is shown joining $A(-5,4)$ and $B(3,10)$.
There are three other vectors shown on the diagram:

$\underset{\sim}{u} = \underset{\sim}{i} + 2\underset{\sim}{j}$

$\underset{\sim}{v} = 2\underset{\sim}{i} + \underset{\sim}{j}$

$\underset{\sim}{w} = 3\underset{\sim}{i} + \underset{\sim}{j}$



Which of the following vectors is parallel to $\overrightarrow{AB}$?

A. $\underset{\sim}{u} - \underset{\sim}{v}$

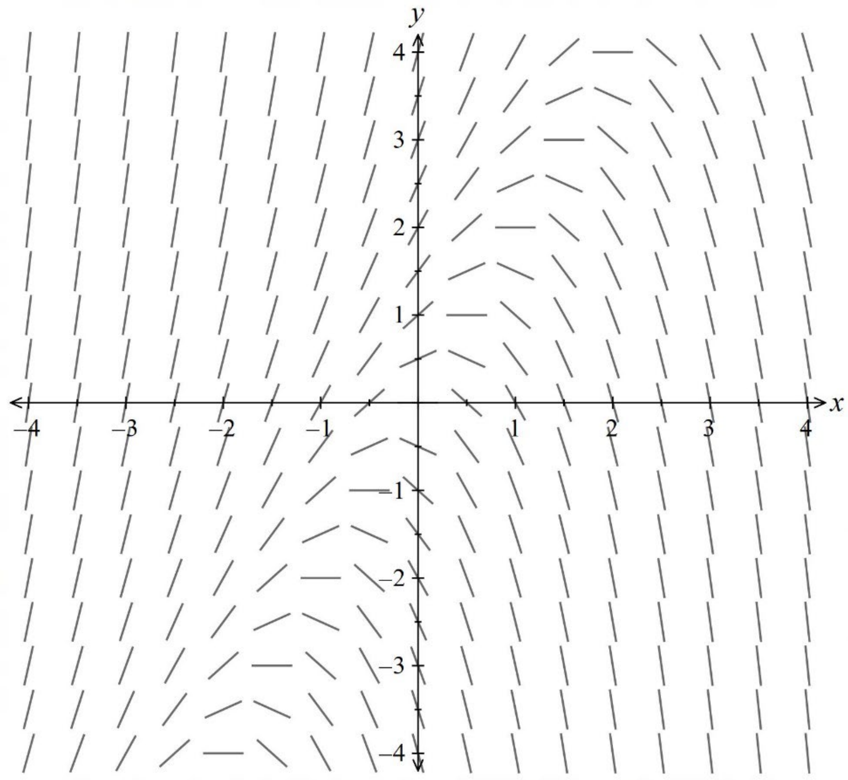
B. $\underset{\sim}{u} + \underset{\sim}{v}$

C. $\underset{\sim}{w} + \underset{\sim}{v}$

D. $\underset{\sim}{w} + \underset{\sim}{u}$

Multiple choice questions continued over the page…

8 The direction field for a differential equation is shown below.



Considering the point (1, 1) determine which of the differential equations below could give this direction field?

A. $y' = (y - 2x)$

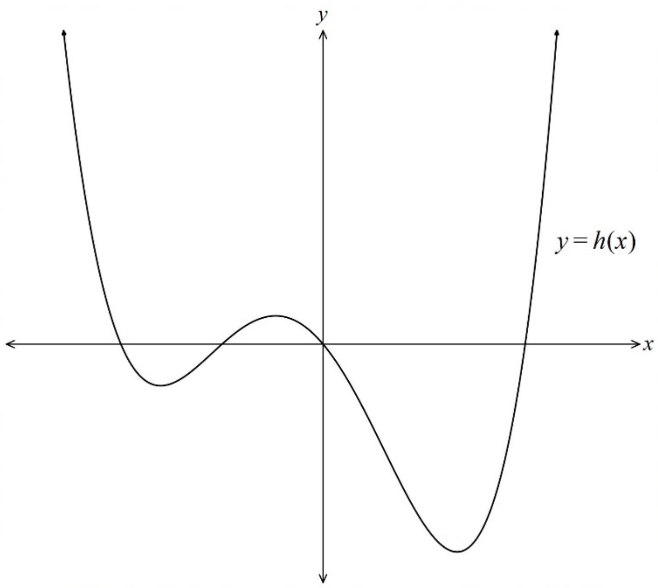
B. $y' = (y + 2x)$

C. $y' = (y + x)$

D. $y' = (x - y)$

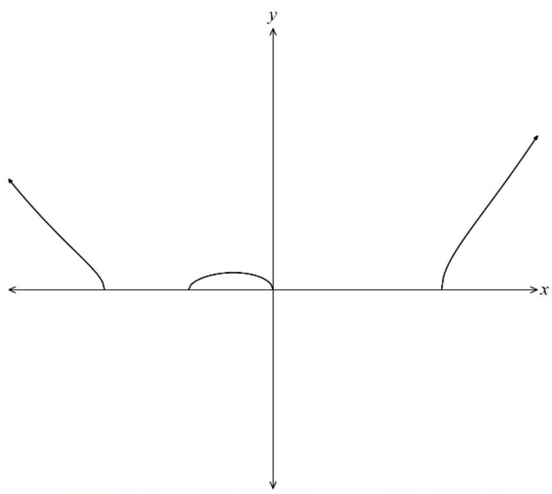
Multiple choice questions continued over the page…

9 The graph of $y = h(x)$ is shown below. Diagrams are not drawn to scale.

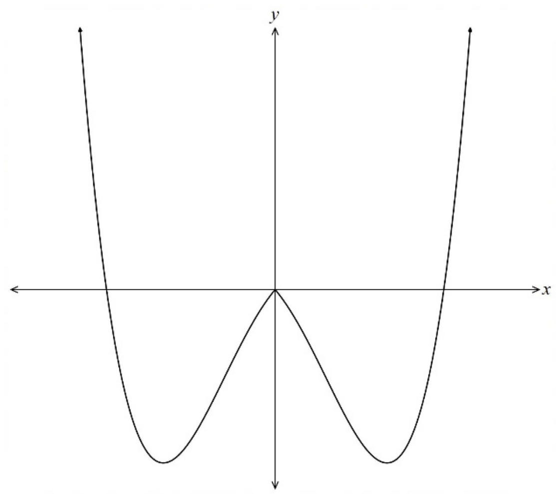


Which graph shows $y = |h(x)|$?

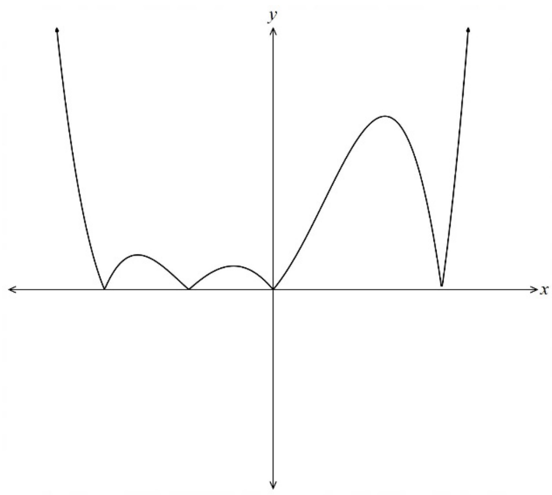
A.



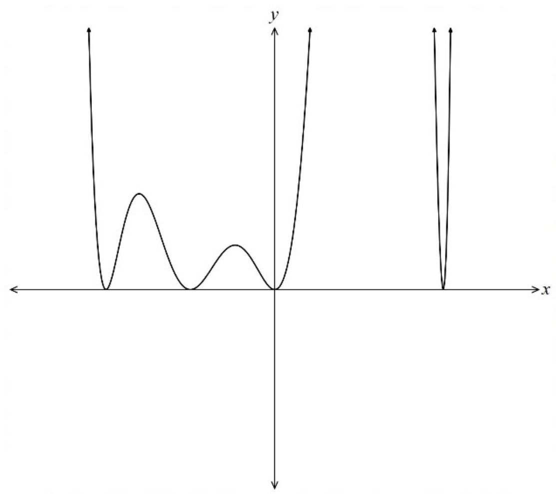
B.



C.

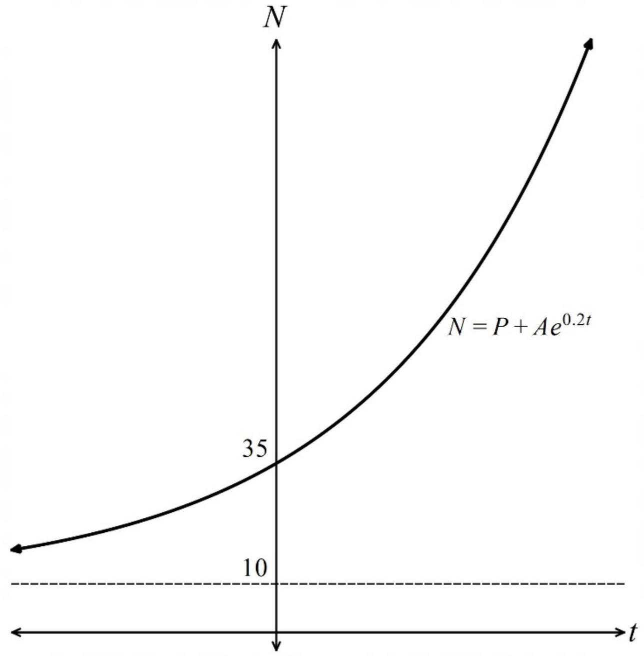


D.



Multiple choice questions continued over the page…

10 The quantity N increases exponentially over time according to the equation $N = P + A\,e^{0.2t}$. The graph below shows the change in N over time (t).



NOT TO SCALE

Which equation describes the rate of change of N with respect to t?

A. $\frac{dN}{dt} = 0.2(N - 10)$

B. $\frac{dN}{dt} = 0.2(N - 25)$

C. $\frac{dN}{dt} = 10(N - 0.2)$

D. $\frac{dN}{dt} = 25(N - 10)$

~ End of Multiple Choice Questions ~

Student Number:

Section II

Teacher: ______________________

60 marks
Attempt Questions 11 – 14.
Allow about 1 hour and 45 minutes for this section.

Answer each question in the appropriate writing booklet. Extra writing booklets are available.

In Questions 11 – 14, your responses should include relevant mathematical reasoning and/or calculations.

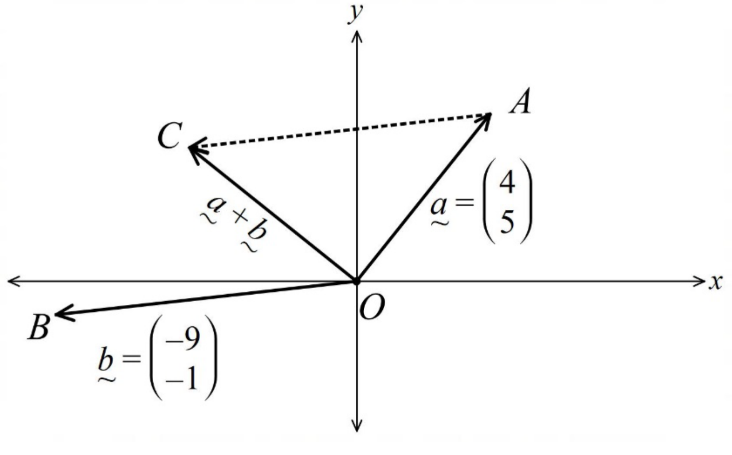
Question 11 (15 marks) START A NEW WRITING BOOKLET.

(a) Show that the vectors $\underset{\sim}{s} = \begin{pmatrix} 5 \\ 10 \end{pmatrix}$ and $\underset{\sim}{t} = \begin{pmatrix} -4 \\ 2 \end{pmatrix}$ are perpendicular. **2**

(b) Use the substitution $u = \tan\theta$ to evaluate $\int_0^{\frac{\pi}{3}} \tan\theta \sec^2\theta \, d\theta$ **2**

(c) Find the coefficient of x^9 in the expansion of $\left(\frac{3x^3}{2} + \frac{1}{5}\right)^6$ **2**

(d) The diagram shows the vectors $\underset{\sim}{a} = \begin{pmatrix} 4 \\ 5 \end{pmatrix}$, $\underset{\sim}{b} = \begin{pmatrix} -9 \\ -1 \end{pmatrix}$ and the resultant vector $\underset{\sim}{a} + \underset{\sim}{b}$. **3**



NOT TO SCALE

Show that ΔOAC, formed by the vectors, is an isosceles triangle and find the size of $\angle OAC$.

Question 11 continues on page 10

Question 11 continued…

(e) Find the exact value of the integral below: **4**

$$\int_0^1 \frac{2x^2+1}{25+(2x^3+3x)^2}\,dx$$

(f) Given that $P(x) = x^4 - 9x^3 + 25x^2 - 27x + 10$, has a double root, factorise $P(x)$ into linear factors. **2**

End of Question 11

Question 12 (14 marks) START A NEW WRITING BOOKLET.

(a) (i) A function is defined as $f(x) = (x-1)^3 - 4; x \geq 0$. **2**

Use the separate page provided at the end of the paper to answer the following question:

On the diagram given, on page 17, the graph of $y = f(x)$ has been drawn.

On the same diagram, on page 17, draw the graph of $y = f^{-1}(x)$.

(ii) Write the equation that describes $y = f^{-1}(x)$ and state its domain. **2**

(b) Express $\sqrt{6}\sin x + \sqrt{2}\cos x$ in the form $R\sin(x+\alpha)$ and hence or otherwise, solve: **3**

$$\sqrt{6}\sin x + \sqrt{2}\cos x = 2, \qquad [0, 2\pi].$$

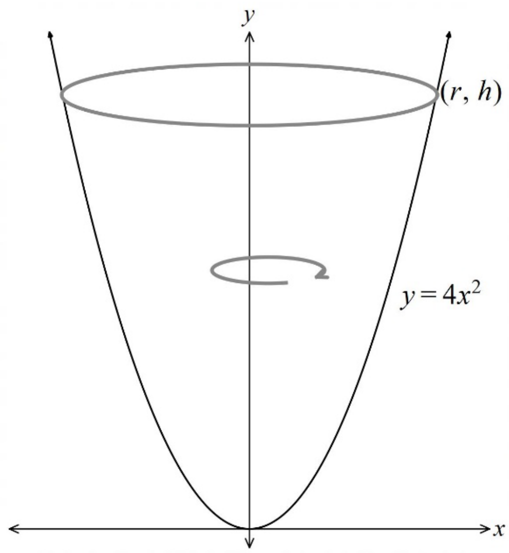
(c) Given that ${}^{n}\mathbf{C}_4 = p$ and ${}^{n}\mathbf{C}_5 = q$, use the Pascals Triangle relationships between coefficients to find an expression, in terms of p and q, for ${}^{n+1}\mathbf{C}_{n-4} + {}^{n}\mathbf{C}_{n-4}$. **2**

Question 12 continues on page 12

Question 12 continued…

(d) (i) The internal surface of a water reservoir is created by rotating the parabola $y = 4x^2$ about the y-axis between $y = 0$ and $y = h$. **2**

NOT TO SCALE



Show that the volume of the water in the reservoir is given by $V = 2\pi r^4$.

(ii) The variable h represents the depth of the water in the reservoir at time t and r is the radius of the upper surface of the water at that time. All measurements are in metres. **1**

The radius of the surface is reducing at a constant rate such that $\frac{dr}{dt} = -2$.
(Time in hours.)

Show that the rate of change of the depth (h) is given by $\frac{dh}{dt} = -16r$.

(iii) Find the rate of change of the volume (V) when the radius is 5 metres. **2**

End of Question 12

Question 13 (16 marks) START A NEW WRITING BOOKLET.

(a) Evaluate: $\cos\left[2\sin^{-1}\frac{3}{8}\right]$. Write your answer in simplest, exact form. **2**

(b) Seven girls are to sit around a circular table. Answer the questions below.

(i) In how many different ways can they be seated if there are no restrictions? **1**

(ii) If three of the girls, Mahua, Inara and Lily must sit next to each other and Lily wants to sit in the middle, in how many ways can these seven girls be seated around the table? **1**

(c) Use the method of separation of variables to solve the differential equation: $\frac{dy}{dx} = \frac{3y}{x}$, given that $\frac{dy}{dx} = 6$ when $x = 2$. **4**

(d) Given that $\tan^{-1}\left(\frac{p}{q}\right) = \alpha$, find an expression for $\cos\left(\frac{\pi}{2} - \alpha\right)$ in terms of p and q. **2**

(e) The polynomial equation $x^3 - 5x^2 + 2x - 12 = 0$ has roots α, β and γ. Find:

(i) $\alpha + \beta + \gamma$. **1**

(ii) $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma}$ **2**

(f) Use mathematical induction to prove that $9^n - 4^n$ is divisible by 5 for all integers $n \geq 1$. **3**

End of Question 13

Question 14 (15 marks) START A NEW WRITING BOOKLET.

(a) Using the substitution $u = \sqrt{x}$, find $\int \frac{dx}{\sqrt{x}\, e^{\sqrt{x}}}$. **2**

(b) Solve: $\frac{x}{x+1} \geq 0$ **2**

(c) Evaluate: $\int_{\frac{\pi}{12}}^{\frac{\pi}{4}} 2\sin^2 4x \, dx$ **3**

Write your answer in simplest, exact form.

(d) By considering the binomial expansion $(1 + x)^n$, prove that: **3**

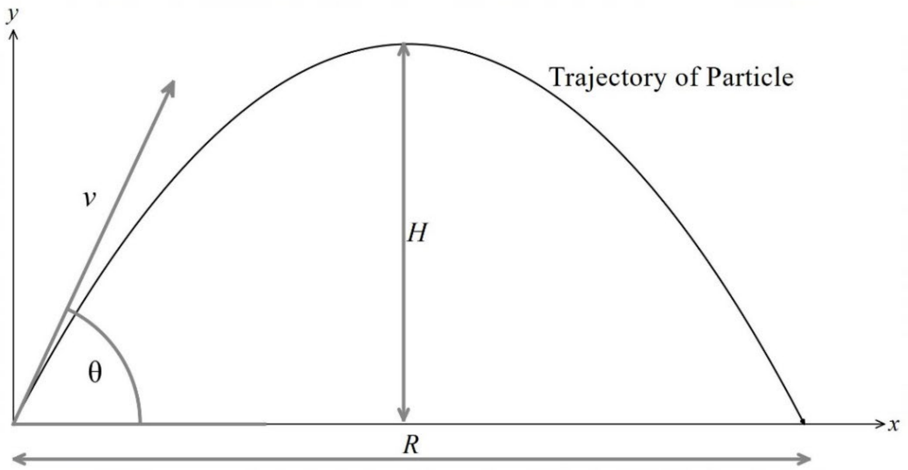
$$\binom{n}{1} + 3\binom{n}{2} + 9\binom{n}{3} + \ldots + 3^{n-1}\binom{n}{n} = \frac{1}{3}\left(2^{2n} - 1\right)$$

Question 14 continues on page 15

Question 14 continued…

(e) The position vector of a particle projected from the origin at an initial velocity of v m/s at an angle of θ° to the horizontal is given by: $x = vt\cos\theta$ and $y = vt\sin\theta - \frac{gt^2}{2}$.

This can also be represented in vector form: $r(t) = \begin{pmatrix} vt\cos\theta \\ vt\sin\theta - \frac{gt^2}{2} \end{pmatrix}$



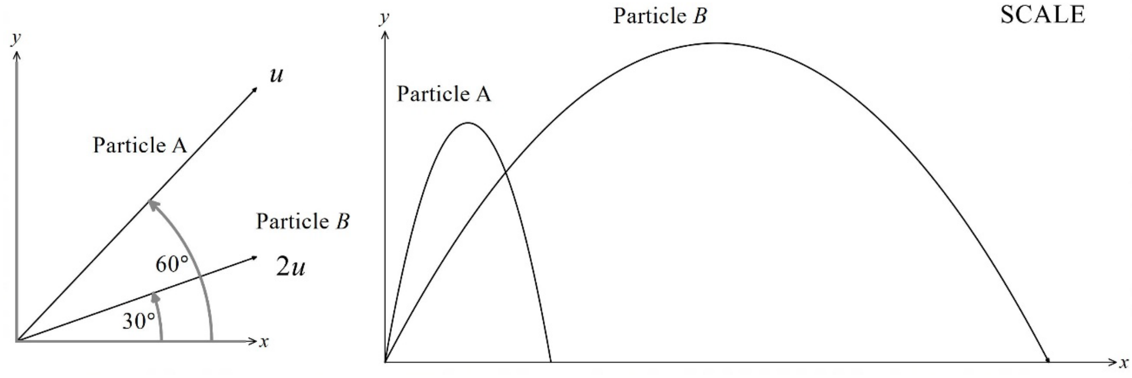
NOT TO SCALE

(i) Show that the time of flight of the particle is given by $t = \frac{2v\sin\theta}{g}$ and the horizontal range by $R = \frac{v^2\sin 2\theta}{g}$. **3**

(ii) Particle A is projected from the origin at a velocity of u m/s at an angle of 60° to the horizontal. **2**

For Particle B, the initial velocity is doubled to $2u$ m/s and the angle of elevation is halved to 30°.

NOT TO SCALE



Show that the range of Particle B is 4 times that of Particle A.

End of Question 14

End of paper

2024 Trial HSC Examination Mathematics Extension 1

Student Number:

Section I – Multiple Choice Answer Sheet

Allow about 15 (maximum) minutes for this section

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample: 2 + 4 = (A) 2 (B) 6 (C) 8 (D) 9

A ○ B ● C ○ D ○

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ● B ⊗ C ○ D ○

If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word **correct** and drawing an arrow as follows.

A ⊗ B ⊗ correct C ○ D ○

1. A ○ B ○ C ○ D ○
2. A ○ B ○ C ○ D ○
3. A ○ B ○ C ○ D ○
4. A ○ B ○ C ○ D ○
5. A ○ B ○ C ○ D ○
6. A ○ B ○ C ○ D ○
7. A ○ B ○ C ○ D ○
8. A ○ B ○ C ○ D ○
9. A ○ B ○ C ○ D ○
10. A ○ B ○ C ○ D ○

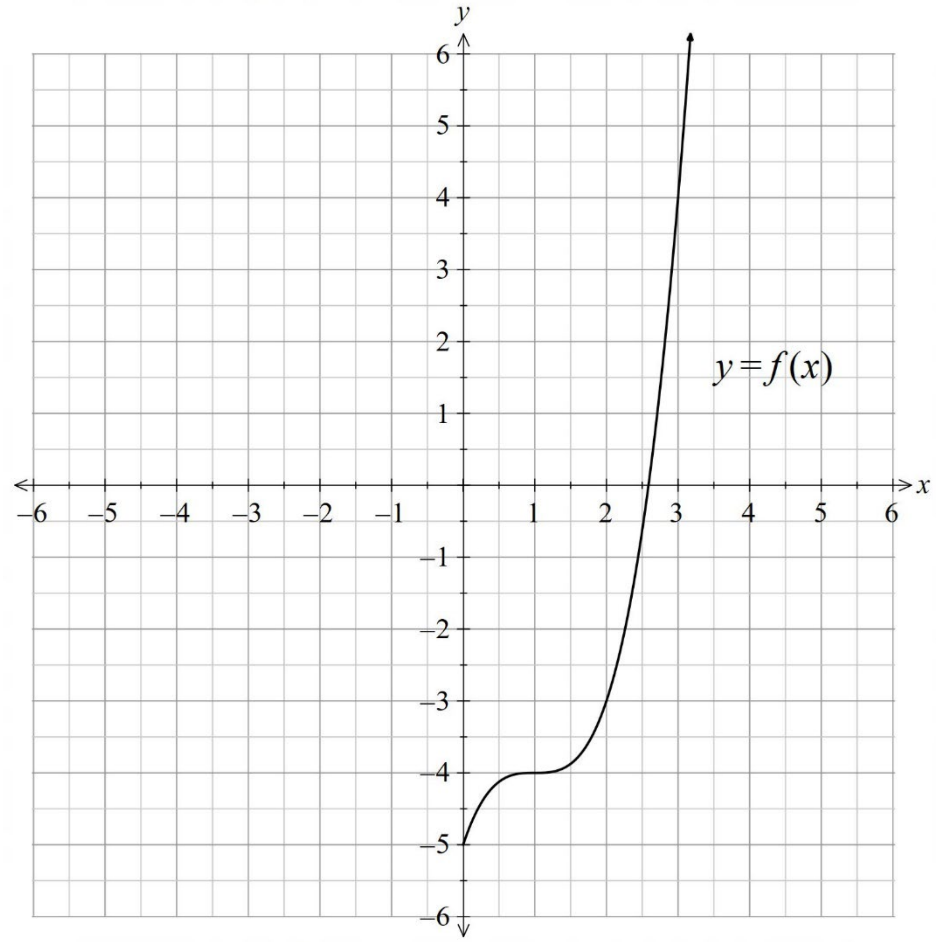
TOTAL:

/ 10

Student Number:

Question 12 (a)

(i) A function is defined as $f(x) = (x-1)^3 - 4; x \geq 0$.
On the diagram below, the graph of $y = f(x)$ has been drawn.
On the same diagram, draw the graph of $y = f^{-1}(x)$.



Cheltenham Girls High School

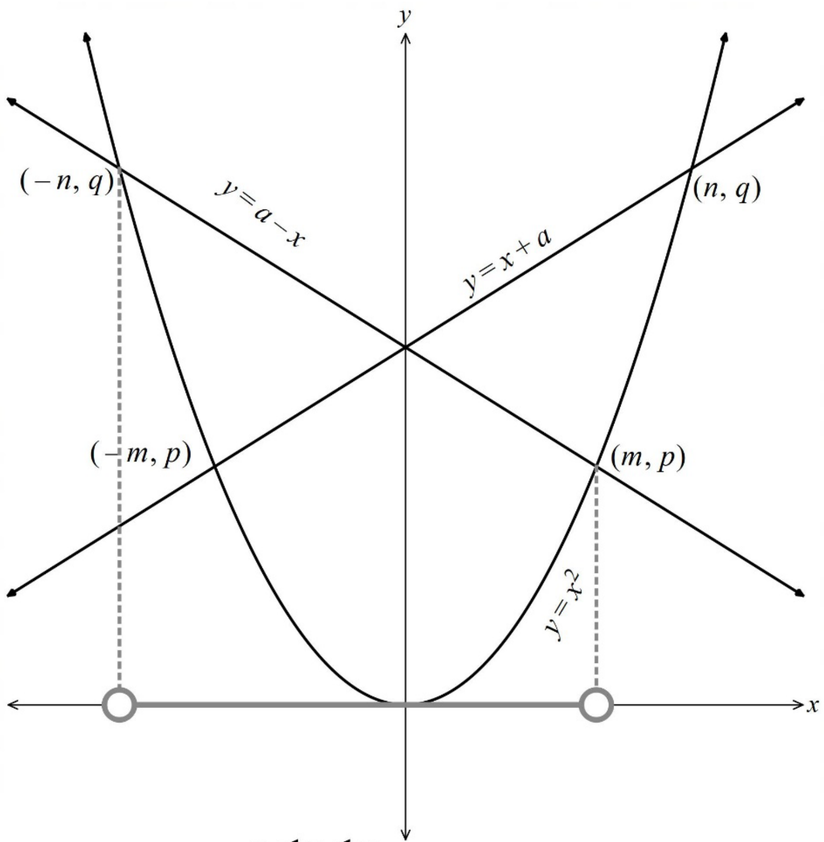
2024 | HSC TRIAL EXAMINATION

Mathematics - Extension 1
Marking Guidelines and Solutions

2024 Mathematics - Extension 1 Solutions

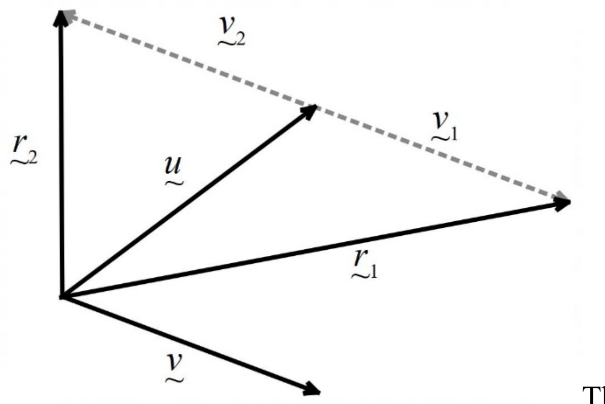
Section I – Multiple Choice Questions

No	Working	Answer
1.	$\int_0^{\frac{1}{2}} \frac{dx}{\sqrt{1-4x^2}} = \left[\frac{1}{2}\sin^{-1} 2x\right]_0^{\frac{1}{2}}$ $= \frac{1}{2}\left(\sin^{-1} 1 - \sin^{-1} 0\right)$ $= \frac{1}{2} \times \frac{\pi}{2}$ $= \frac{\pi}{4}$	**C**
2.	Consider the three statements in light of the pigeonhole principle which states : If there are n pigeonholes and $n+1$ pigeons to go into them, then at least one pigeonhole must hold 2 or more pigeons. Statement 1 There are 12 months and 13 people's birth-months to be matched with them, so at least one month must have at least two people. This is a correct application of the pigeonhole principle. Statement 2 There are 4 walls and 5 windows to be placed in them, so at least one wall must have at least two windows. This is a correct application of the pigeonhole principle. Statement 3 There are 7 colours and 10 socks to be placed in them, but only 6 are drawn, so all 6 could be different colours. To work correctly 8 socks would need to be drawn (7 +1). This is **not** a correct application of the pigeonhole principle. Statements 1 and 2 only are correct applications of the pigeonhole principle.	**A**

No	Working	Answer
3.	$x^2 + x - a < 0$ $x^2 < a - x$ The solution is where the graph of $y = x^2$ is below the graph of $y = a - x$  $-n < x < m$	**B**

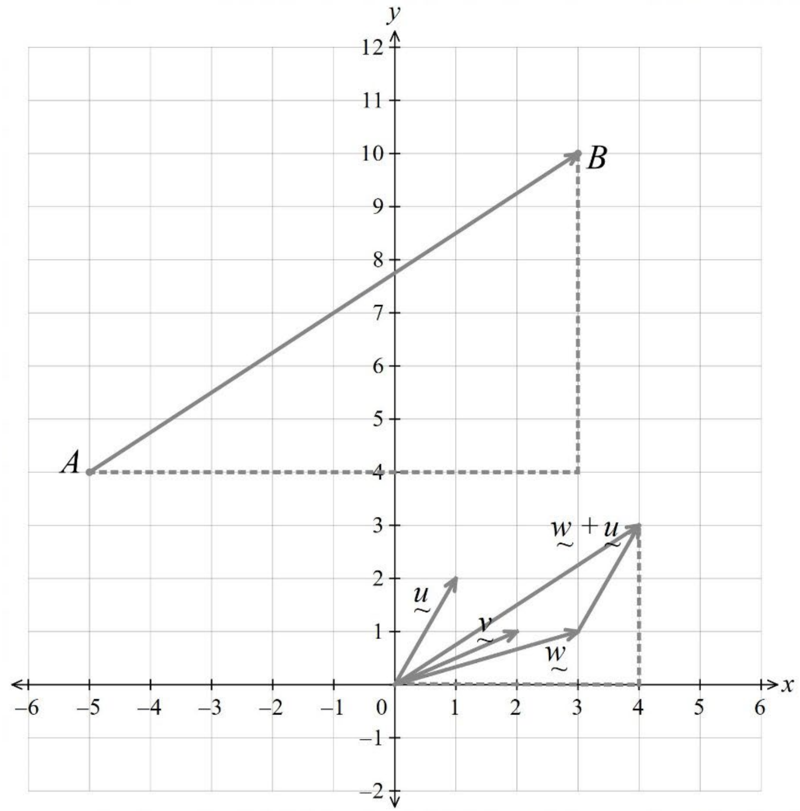
<table>
<tr><th>No</th><th>Working</th><th>Answer</th></tr>
<tr><td>4.</td><td>sin 2A = 2 sin A cos A so

sin 4A = 2 sin 2A cos 2A

$\therefore \sin 2A \cos 2A = \dfrac{1}{2} \sin 4A$</td><td>D</td></tr>
<tr><td>5.</td><td>

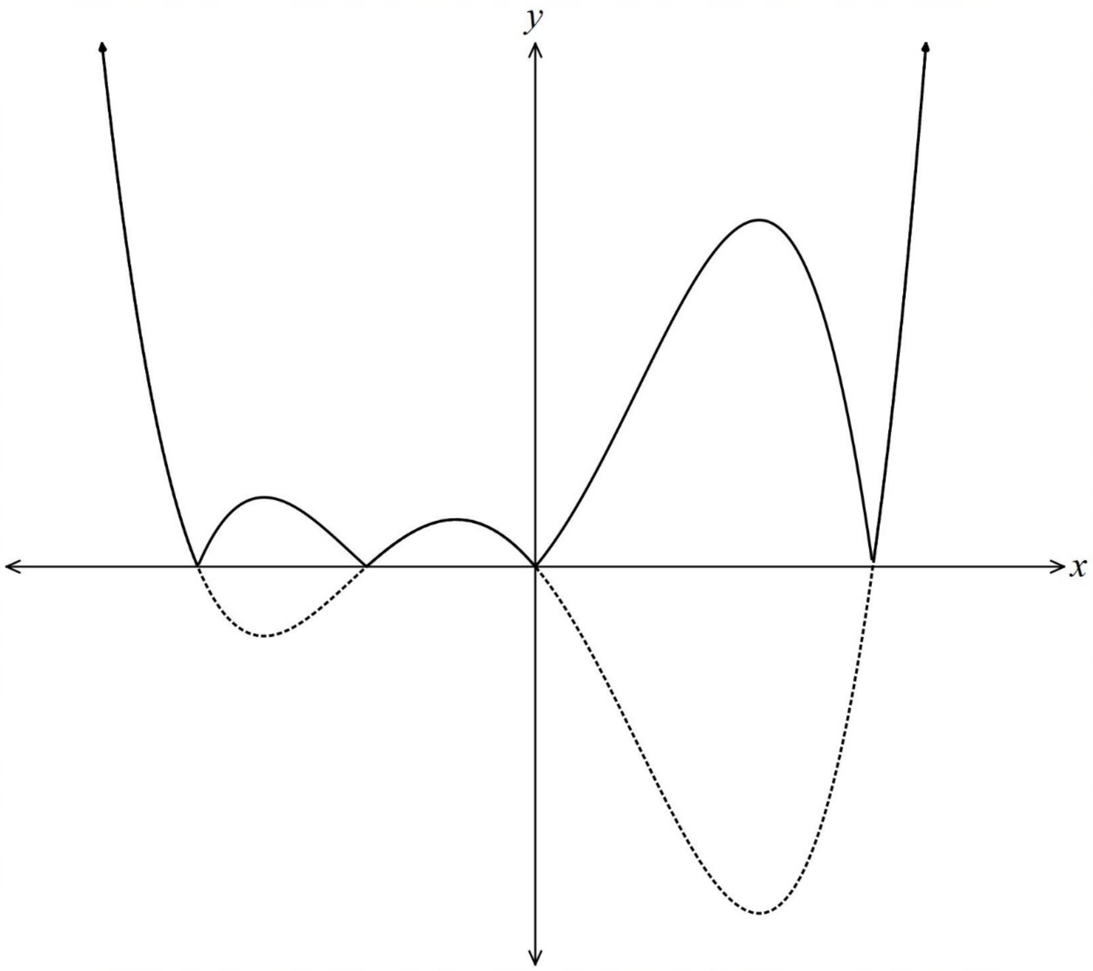
The addition of vectors is represented geometrically by translating the vectors top to tail, so $\underset{\sim}{v}$ is translated to $\underset{\sim}{v}_1$, giving the resultant $\underset{\sim}{r}_1$ and subtraction by reversing the direction of $\underset{\sim}{v}$ and translating it to $\underset{\sim}{v}_2$ giving the resultant $\underset{\sim}{r}_2$. So $\underset{\sim}{r}_1 = \underset{\sim}{u} + \underset{\sim}{v}$ and $\underset{\sim}{r}_2 = \underset{\sim}{u} - \underset{\sim}{v}$.</td><td>C</td></tr>
</table>

No	Working	Answer
6.	NECESSITIES has 11 letters with E and S both occurring 3 times and I occurring twice Distinct arrangements $= \dfrac{11!}{3!.3!.2!}$ $= \dfrac{39916800}{6 \times 6 \times 2}$ $= 554\ 400$	**B**

7.	 $\underset{\sim}{w} + \underset{\sim}{u}$ is inclined at $\theta = \tan^{-1}\left(\frac{3}{4}\right)$ $\overrightarrow{AB}$ is inclined at $\alpha = \tan^{-1}\left(\frac{6}{8}\right) = = \tan^{-1}\left(\frac{3}{4}\right) = \theta$ so the vectors are parallel **Simplest solution is to sketch the 4 resultant vectors and it is clear that $\underset{\sim}{w} + \underset{\sim}{u}$ is parallel to $\overrightarrow{AB}$.** **Other alternatives are outlined overleaf**	**D**

No	Working	Answer
	Q7 Alternative working **Alternative algebraically** $\overrightarrow{AB} = \begin{pmatrix} 8 \\ 6 \end{pmatrix}$ Gradient $= \frac{6}{8} = \frac{3}{4}$ $\underset{\sim}{u} - \underset{\sim}{v} = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$ $\underset{\sim}{u} + \underset{\sim}{v} = \begin{pmatrix} 3 \\ 3 \end{pmatrix}$ $\underset{\sim}{w} + \underset{\sim}{v} = \begin{pmatrix} 5 \\ 2 \end{pmatrix}$ $\underset{\sim}{w} + \underset{\sim}{u} = \begin{pmatrix} 4 \\ 3 \end{pmatrix}$ Gradient $= \frac{3}{4}$ Only $\underset{\sim}{w} + \underset{\sim}{u}$ has the same gradient as $\overrightarrow{AB}$ so the vectors are parallel **Further alternative** Can find scalar product $\overrightarrow{AB}.(\underset{\sim}{w} + \underset{\sim}{u}) = 8 \times 4 + 6 \times 3 = 50$ So $\lvert\overrightarrow{AB}\rvert . \lvert\underset{\sim}{w} + \underset{\sim}{u}\rvert . \cos\theta = 50$ $10 \times 5 \cos\theta = 50$ $\cos\theta = 1$ $\therefore \ \theta = 0$ Angle between the vectors is zero so they are parallel	

No	Working	Answer
8.	Take the point (1, 1) (which has a negative gradient in the direction field) and test with the four equations. $A. y' = (y - 2x) \quad y' = (1 - 2(1)) = -1$ ($-$ gradient) $B. y' = (y + 2x) \quad y' = (1 + 2(1)) = 3$ (+gradient) $C. y' = (y + x) \quad y' = (1 + 1) = 2$ (+ gradient) $D. y' = (x - y) \quad y' = 1 - 1 = 0$ (Horizontal gradient) Only A has a negative gradient at the point (1, 1).	**A**

9.	The sections where the curve dips below the x-axis are reflected in the x-axis (the negative y-values become positive when the absolute value is taken), while those above the x-axis remain unchanged.  	**C**

No	Working	Answer
10.	As the horizontal asymptote is at $N = 10,\ P = 10$ The vertical (N) intercept is 35, so $\mathrm{A} + P = 35$ $A + 10 = 35$ $A = 25$ So the equation is $N = 10 + 25\, e^{0.2t}$ The corresponding differential equation is of the form $\frac{dN}{dt} = k(N - P)$ Equation is $\frac{dN}{dt} = 0.2(N - 10)$	**A**

2024 Trial HSC Examination
Mathematics Extension 1 Course

Name ________________________________ Teacher ________________________

Section I – Multiple Choice Answer Sheet

Allow about 15 minutes for this section

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample:	2 + 4 =	(A) 2	(B) 6	(C) 8	(D) 9
		A ○	B ●	C ○	D ○

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ● B ~~●~~ C ○ D ○

If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word **correct** and drawing an arrow as follows.

correct

A ~~●~~ B ~~●~~ C ○ D ○

1.	A ○	B ○	C ●	D ○
2.	A ●	B ○	C ○	D ○
3.	A ○	B ●	C ○	D ○
4.	A ○	B ○	C ○	D ●
5.	A ○	B ○	C ●	D ○
6.	A ○	B ●	C ○	D ○
7.	A ○	B ○	C ○	D ●
8.	A ●	B ○	C ○	D ○
9.	A ○	B ○	C ●	D ○
10.	A ●	B ○	C ○	D ○

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Mathematics - Extension 1 Trial HSC

Solutions

Section II

Question 11.		WME Ext 1 HSC 2024 Worked Solution	Marks	Mark Allocation and Comments
11	a)	Show that the vectors $\underset{\sim}{s} = \begin{pmatrix} 5 \\ 10 \end{pmatrix}$ and $\underset{\sim}{t} = \begin{pmatrix} -4 \\ 2 \end{pmatrix}$ are perpendicular. $\underset{\sim}{s} \,.\, \underset{\sim}{t} = 5 \times -4 + 10 \times 2 = -20 + 20 = 0$ Now $\underset{\sim}{s} \,.\, \underset{\sim}{t} = \lvert\underset{\sim}{s}\rvert \times \lvert\underset{\sim}{t}\rvert \times \cos\theta = 0$ Clearly neither $\lvert\underset{\sim}{s}\rvert$ nor $\lvert\underset{\sim}{t}\rvert$ are equal to zero, (the length of the each vectors is greater than 1 $\therefore\ \cos\theta = 0$ $\theta = 90°$ so the vectors $\underset{\sim}{s}$ and $\underset{\sim}{t}$ are perpendicular	2	2 marks for correct use of dot product and explanation 1 mark for simply showing that the dot product is zero with no reasoning or equivalent merit

Question 11.		WME Ext 1 HSC 2024 Worked Solution	Marks	Mark Allocation and Comments
11	b)	$u = \tan\theta$ $\frac{du}{d\theta} = \sec^2\theta$ $\mathrm{d}\theta = \frac{du}{sec^2\theta}$ $\int_0^{\frac{\pi}{3}} \tan\theta\ \sec^2\theta\,\mathrm{d}\theta = \int u \sec^2\theta \frac{du}{\sec^2\theta}$ $= \int u\ du$ $= \left[\frac{u^2}{2}\right]$ $= \left[\frac{\tan^2\theta}{2}\right]_0^{\frac{\pi}{3}}$ $= \frac{1}{2}\left((\tan\frac{\pi}{3})^2 - (\tan 0)^2\right)(\#)$ $= \frac{1}{2}\left(\left(\sqrt{3}\right)^2 - 0\right)(*)$ $= \frac{3}{2}$	2	2 marks – shows all working out by finding $\frac{du}{d\theta}$ and substitutes correctly to integrate with complete working out including line (#) evident leading to line (*) correctly and finds the correct answer 1 mark – correctly finds $\frac{du}{d\theta}$ and substitutes correctly but makes an error integrating or substituting values when finding answer

Question 11.		WME Ext 1 HSC 2024 Worked Solution	Marks	Mark Allocation and Comments
11	c)	$\left(\frac{3x^3}{2}+\frac{1}{5}\right)^6$ $= {}^6C_0\left(\frac{3x^3}{2}\right)^6 + {}^6C_1\left(\frac{3x^3}{2}\right)^5\left(\frac{1}{5}\right)^1 + {}^6C_2\left(\frac{3x^3}{2}\right)^4\left(\frac{1}{5}\right)^2 + {}^6C_3\left(\frac{3x^3}{2}\right)^3\left(\frac{1}{5}\right)^3 + {}^6C_4\left(\frac{3x^3}{2}\right)^2\left(\frac{1}{5}\right)^4 + \ldots$ We require term in x^9 which will come from $(x^3)^3$ Required term will be $({}^6\mathbf{C}_3)\left(\frac{3x^3}{2}\right)^3 \left(\frac{1}{5}\right)^3 = 20\left(\frac{27x^9}{8}\right)\left(\frac{1}{125}\right)$ $= \frac{27x^9}{50}$ Coefficient $= \frac{27}{50}$	2	2 marks for correct answer 1 mark for stating the correct term with an error in calculation of coefficient or equivalent merit

11	d)	$\underset{\sim}{a} + \underset{\sim}{b} = \begin{pmatrix} 4-9 \\ 5-1 \end{pmatrix} = \begin{pmatrix} -5 \\ 4 \end{pmatrix}$ $\lvert\underset{\sim}{a}\rvert = \sqrt{4^2 + 5^2} = \sqrt{41}$ $\lvert\underset{\sim}{a} + \underset{\sim}{b}\rvert = \sqrt{(-5)^2 + 4^2} = \sqrt{41}$ $\therefore\ \lvert\underset{\sim}{a}\rvert = \lvert\underset{\sim}{a} + \underset{\sim}{b}\rvert$ $\therefore\ \angle OAC = \angle OCA$ (Base angles of isosceles triangle) $(\underset{\sim}{a}) . (\underset{\sim}{a} + \underset{\sim}{b}) = \lvert\underset{\sim}{a}\rvert \times \lvert\underset{\sim}{a} + \underset{\sim}{b}\rvert \times \cos(\angle AOC)$ $4 \times (-5) + 5 \times 4 = \sqrt{41} \times \sqrt{41} \times \cos(\angle AOC)$ $-20 + 20 = 41\cos(\angle AOC)$ $\cos(\angle AOC) = 0$ $\angle AOC = 90°$ $\therefore\ \angle OAC = \angle OCA = \frac{90}{2} = 45°$ ***Alternate Solution*** $\lvert\underset{\sim}{b}\rvert = \sqrt{(-9)^2 + (-1)^2} = \sqrt{82}$ $(\underset{\sim}{a}) . (\underset{\sim}{b}) = \lvert\underset{\sim}{a}\rvert \times \lvert\underset{\sim}{b}\rvert \times \cos(\angle AOB)$ $4 \times (-9) + 5 \times (-1) = \sqrt{82} \times \sqrt{41} \times \cos(\angle AOB)$ $-36 - 5 = 41\sqrt{2}\cos(\angle AOB)$ $-41 = 41\sqrt{2}\cos(\angle AOB)$ $\cos(\angle AOB) = -\frac{1}{\sqrt{2}}$ $\angle AOB = \arccos\left(-\frac{1}{\sqrt{2}}\right) = 135°$ $\angle AOB + \angle OAC = 180°$ (cointerior angles) $\angle OAC = 180° - 135° = 45° = \angle OCA$	3	3 marks for correct proof of triangle being isosceles and finding size of $\angle OAC$ 2 marks for showing triangle isosceles only or equivalent merit 1 mark for some relevant calculation using the vectors for lengths or angles

<table>
<tr><th colspan="2">Question 11.</th><th>WME Ext 1 HSC 2024

Worked Solution</th><th>Marks</th><th>Mark Allocation and Comments</th></tr>
<tr><td>11</td><td>e)</td><td>$$\int_0^1 \frac{2x^2+1}{25+(2x^3+3x)^2}\,dx = \frac{1}{3}\int_0^1 \frac{6x^2+3}{5^2+(2x^3+3x)^2}\,dx$$ $$= \frac{1}{3}\int_0^1 \frac{\frac{d}{dx}(2x^3+3x)}{5^2+(2x^3+3x)^2}\,dx$$ $$= \frac{1}{3}\left[\frac{1}{5}\tan^{-1}\left(\frac{2x^3+3x}{5}\right)\right]_0^1$$ $$= \frac{1}{15}[\tan^{-1}(1)-\tan^{-1}(0)] \qquad (*)$$ $$= \frac{1}{15}\left[\frac{\pi}{4}-0\right]$$ $$= \frac{\pi}{60}$$</td><td>4</td><td>4 marks for fully explicit and correct working out including line (*) n reaching the final answer as an exact value

3 marks for fully correct working out including line (*) but minor error in reaching the final answer with all lines of working clearly shown

2 marks for showing correct inverse tan integral, no substitution line (*) shown leading to solution with an error in the working or simplified working out

1 mark for correctly identifying use of inverse tan integral or equivalent merit but incomplete solution with many errors including incorrect or no substitutions used</td></tr>
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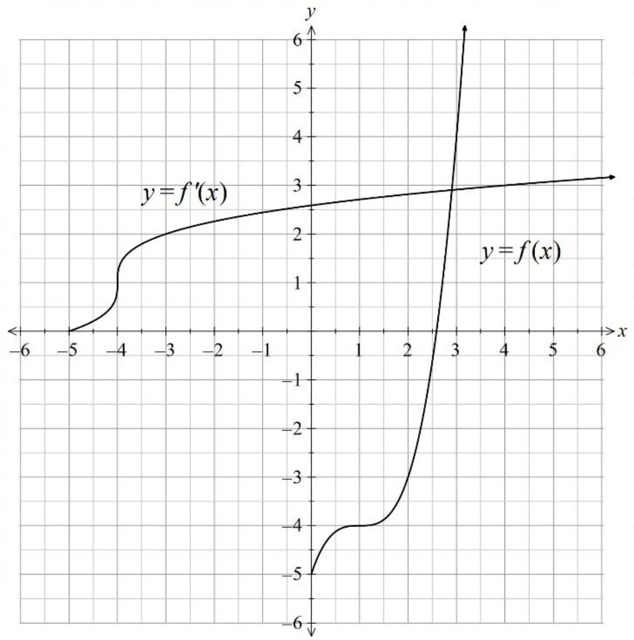
Question 11.		WME Ext 1 HSC 2024 Worked Solution	Marks	Mark Allocation and Comments
11	f)	$P(x) = x^4 - 9x^3 + 25x^2 - 27x + 10$ The roots must be factors of 10 i.e. $x = \pm 1, \pm 2, \pm 5$ $x = +1$ or -1 must be the double root, as 1^2 is a factor of 10, but 2^2 and 5^2 are not. $P(1) = (1)^4 - 9(1)^3 + 25(1)^2 - 27(1) + 10 = 0$ So $(x-1)^2 = x^2 - 2x + 1$ is a factor Could use division to get $P(x) = (x^2 - 2x + 1)(x^2 - 7x + 10)$ Then factorise to obtain $P(x) = (x-1)^2(x-2)(x-5)$ Or test the factors ± 2 and ± 5 $P(2) = (2)^4 - 9(2)^3 + 25(2)^2 - 27(2) + 10 = 0$ $P(5) = (5)^4 - 9(5)^3 + 25(5)^2 - 27(5) + 10 = 0$ So $(x-2)$ and $(x-5)$ are factors So factorisation is: $P(x) = (x-1)^2(x-2)(x-5)$	2	2 marks for correct factorisation 1 mark for minimum of two correct factors or correct method with an error that leads to incorrect factors

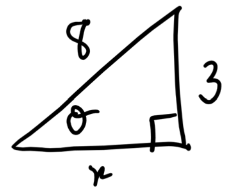
Question 12.	WME Ext 1 HSC 2024 Worked Solution		Marks	Mark Allocation and Comments
12	a)	(i)  	2	2 marks for correct diagram 1 mark for mainly correct graph with minor error, such as wrong domain, wrong shape (e.g. extending below the x-axis) or not intersecting at correct point(s)
12	(a)	(ii) $f(x) = (x-1)^3 - 4; x \geq 0$ Function $y = (x-1)^3 - 4\ ;\ x \geq 0$ Inverse $x = (y-1)^3 - 4$ $x + 4 = (y-1)^3;\ y \geq 0$ $\sqrt[3]{x+4} = y - 1;\ y \geq 0$ $f^{-1}(x) = \sqrt[3]{x+4} + 1;\ y \geq 0$ Domain $x \geq -5$	2	2 marks for correct equation and domain 1 mark for correct working toward equation and domain with a minor error

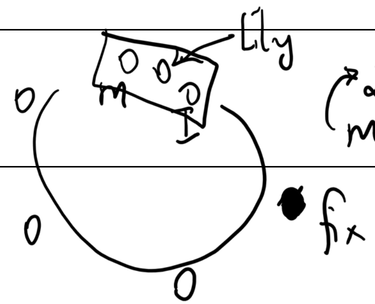
Question 12.		WME Ext 1 HSC 2024 Worked Solution	Marks	Mark Allocation and Comments
12	(b)	Let $\sqrt{6}\sin x + \sqrt{2}\cos x = R\sin(x + \alpha)$ Then $\tan\alpha = \dfrac{\sqrt{2}}{\sqrt{6}} = \dfrac{1}{\sqrt{3}}$ and $R^2 = \left(\sqrt{6}\right)^2 + \left(\sqrt{2}\right)^2 = 8$ $\alpha = \arctan\left(\dfrac{1}{\sqrt{3}}\right)$ and $R^2 = 2 + 6 = 8$ $\alpha = \dfrac{\pi}{6}$ and $R = \sqrt{8}$ $\sqrt{6}\sin x + \sqrt{2}\cos x = \sqrt{8}\sin\left(x + \dfrac{\pi}{6}\right)$ $\sqrt{6}\sin x + \sqrt{2}\cos x = 2$ $\therefore \quad \sqrt{8}\sin\left(x + \dfrac{\pi}{6}\right) = 2$ $\sin\left(x + \dfrac{\pi}{6}\right) = \dfrac{2}{\sqrt{8}} = \dfrac{2}{2\sqrt{2}} = \dfrac{1}{\sqrt{2}}$ $x + \dfrac{\pi}{6} = \dfrac{\pi}{4}, \dfrac{3\pi}{4}, \dfrac{9\pi}{4}, \dfrac{11\pi}{4} \ldots..$ $x = \dfrac{\pi}{4} - \dfrac{\pi}{6}, \dfrac{3\pi}{4} - \dfrac{\pi}{6}, \dfrac{9\pi}{4} - \dfrac{\pi}{6}, \dfrac{11\pi}{4} - \dfrac{\pi}{6} \ldots.$ $x = \dfrac{\pi}{12}, \dfrac{7\pi}{12}, \dfrac{25\pi}{12}, \dfrac{31\pi}{12} \ldots.$ For required domain of $0 \leq x \leq 2\pi$ $x = \dfrac{\pi}{12}, \dfrac{7\pi}{12}$	3	3 marks for showing all working out leading to correct exact solutions 2 marks for finding R or the phase angle correctly but not both 1 mark for one correct exact solutions only with limited or no working, or incorrect solutions with some correct and relevant working with significant errors

Question 12.		WME Ext 1 HSC 2024 Worked Solution	Marks	Mark Allocation and Comments
12	c)	${}^{n}\mathbf{C}_4 = p$ and ${}^{n}\mathbf{C}_5 = q$ ${}^{n}\mathbf{C}_{n-4} = p$ (Symmetry of Pascals Triangle) ${}^{n}\mathbf{C}_4 + {}^{n}\mathbf{C}_5 = {}^{n+1}\mathbf{C}_5 = p + q$ (Sums of terms below in P Triangle) ${}^{n+1}\mathbf{C}_5 = {}^{n+1}\mathbf{C}_{n+1-5} = {}^{n+1}\mathbf{C}_{n-4} = p + q$ (Symmetry of Pascals Triangle) ${}^{n+1}\mathbf{C}_{n-4} + {}^{n}\mathbf{C}_{n-4} = p + q + p$ $= 2p + q$	2	2 marks for correct solution developed with working out shown 1 mark for identifying the line (#) correctly only

Question 12.		WME Ext 1 HSC 2024 Worked Solution	Marks	Mark Allocation and Comments
12	d)	(i) $y = 4x^2$ $V = \pi \int_0^h x^2 \; dy$ $= \pi \int_0^h \frac{y}{4} \; dy$ $= \pi \left[\frac{y^2}{8}\right]_0^h$ $= \frac{\pi h^2}{8}$ $= \frac{\pi(4r^2)^2}{8}$ $= 2\pi r^4$	2	2 marks for correct derivation of both results clearly showing substitution of $h = 4r^2$ 1 mark for correct derivation of one result or equivalent merit
12	d)	(ii) $h = 4r^2$ $\frac{dh}{dr} = 8r$ $\frac{dh}{dt} = \frac{dh}{dr} . \frac{dr}{dt}$ $= 8r.(-2)$ $= -16r$	1	1 mark for correct derivation of required result

Question 12.		WME Ext 1 HSC 2024 Worked Solution	Marks	Mark Allocation and Comments
12	d)	(iii) $V = \frac{\pi h^2}{8} = 2\pi r^4$ $\frac{dV}{dh} = \frac{2\pi h}{8} = \frac{\pi h}{4}$ $\frac{dV}{dr} = 8\pi r^3$ $\frac{dV}{dt} = \frac{dV}{dh} . \frac{dh}{dt}$ $= \frac{\pi h}{4} . (-16r)$ $= \frac{\pi}{4} . (4r^2)(-16r)$ $= -16\pi r^3$ $OR \ \frac{dV}{dt} = \frac{dV}{dr} . \frac{dr}{dt}$ $= 8\pi r^3 . (-2)$ $= -16\pi r^3$ When $r = 5$ $\frac{dV}{dt} = -16\pi(5)^3$ $= -2000\pi \ m^3 / \text{h}$	2	2 marks for correct result 1 mark for correct use of related rates with a minor error

Question 13.		WME Ext 1 HSC 2024 Worked Solution	Marks	Mark Allocation and Comments
13	a)	$\cos 2\theta$ where $\sin^{-1}\frac{3}{8} = \theta$ so $\sin\theta = \frac{3}{8}$  $x = \sqrt{8^2 - 3^2}$ $= \sqrt{55}$ so $\cos 2\theta = 2\cos^2\theta - 1$ $= 2\left(\frac{\sqrt{55}}{8}\right)^2 - 1$ $= 2 \times \frac{55}{64} - 1$ $= \frac{55}{32} - 1$ $= \frac{23}{32}$ **Or** $\cos 2\theta = 1 - 2\sin^2\theta$ $= 1 - 2\left(\frac{3}{8}\right)^2$ $= 1 - 2 \times \frac{9}{64}$ $= 1 - \frac{9}{32}$ $= \frac{23}{32}$	2	2 marks for correct answer with full working out shown 1 mark for identifying right angled triangle and identifies correct equivalent rule for $\cos 2\theta$ but makes an error substituting into correct equivalent expression of $\cos 2\theta$
13	(b) (i)	$(7 - 1)! = 6!$ or 720	1	1 mark for correct answer
	(b) (ii)	$2 \times 4!$ or 48 M & I can swap	1	1 mark for correct answer



13.	c)	**Method 1** $\frac{dy}{dx} = \frac{3y}{x}$ $\frac{dy}{3y} = \frac{dx}{x}$ $\frac{1}{3}\int \frac{dy}{y} = \int \frac{dx}{x}$ $\frac{1}{3}\ln y = \ln x + c_1$ $\ln y = 3\ln x + c_2$ $\ln y = \ln x^3 + \ln c_3$ $\ln y = \ln (cx^3)$ $y = cx^3$ When $x = 2, \frac{dy}{dx} = 6$ $6 = \frac{3y}{2}$ $3y = 12$ $y = 4$ $y = cx^3$ $4 = c \times 2^3$ $c = \frac{4}{8} = \frac{1}{2}$ Solution $y = \frac{x^3}{2}$	**Method 2** $\frac{dy}{dx} = \frac{3y}{x}$ $\frac{dy}{3y} = \frac{dx}{x}$ $\frac{1}{3}\int \frac{dy}{y} = \int \frac{dx}{x}$ $\frac{1}{3}\ln y = \ln x + c_1$ at $(2,4)$ $\quad \frac{1}{3}\ln 4 = \ln 2 + c_1$ $c_1 = \frac{1}{3}\ln 4 - \ln 2$ $\frac{1}{3}\ln y = \ln x + \frac{1}{3}\ln 4 - \ln 2$ $\ln y = 3\ln x + \ln 4 - 3\ln 2$ $= \ln x^3 + \ln 4 - \ln 8$ $= \ln\left(\frac{4x^3}{8}\right)$ $\ln y = \ln\left(\frac{x^3}{2}\right)$ $y = \frac{x^3}{2}$	4	4 marks for correct answer 3 marks for basically correct solution with a minor error or equivalent merit 2 marks for separating the variables and integrating without simplifying to required result or equivalent merit. 1 mark for separating the variables and stating integration without proceeding or equivalent merit

<table>
<tr><th colspan="2">Question 13.</th><th>WME Ext 1 HSC 2024

Worked Solution</th><th>Marks</th><th>Mark Allocation and Comments</th></tr>
<tr><td>13</td><td>d)</td><td>

$\tan^{-1}\left(\frac{p}{q}\right) = \alpha$

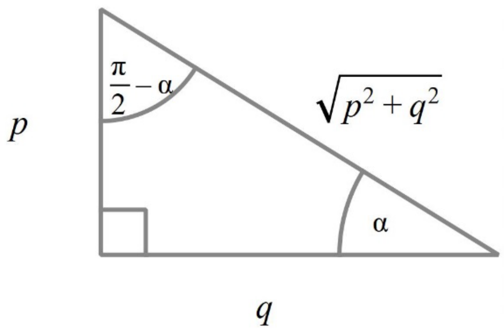
$\tan\alpha = \frac{p}{q}$

Based on this draw the right Δ and find hypotenuse (h)

$h^2 = p^2 + q^2$

$h = \sqrt{p^2 + q^2}$

$\cos\left(\frac{\pi}{2} - \alpha\right) = \frac{p}{\sqrt{p^2 + q^2}}$

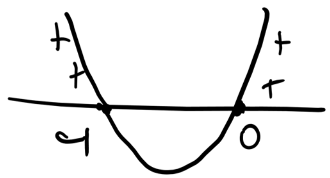


</td><td>2</td><td>2 marks for correct expression

1 mark for correct value of hypotenuse or equivalent working to assist with result</td></tr>
</table>

Question 13.		WME Ext 1 HSC 2024 Worked Solution	Marks	Mark Allocation and Comments
13	e)	(i) $x^3 - 5x^2 + 2x - 12 = 0$ $\alpha + \beta + \gamma = -\frac{b}{a} = \frac{5}{1} = 5$	1	1 mark for correct answer
13	e)	(ii) $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma} = \frac{\beta\gamma + \alpha\gamma + \alpha\beta}{\alpha\beta\gamma}$ $\alpha\beta + \beta\gamma + \alpha\gamma = \frac{c}{a} = 2$ (#) $\alpha\beta\gamma = \frac{-d}{a} = -(-12) = 12$ (^) So $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma} = \frac{\beta\gamma + \alpha\gamma + \alpha\beta}{\alpha\beta\gamma} = \frac{2}{12} = \frac{1}{6}$	2	2 marks for correct answer with full working out shown for each of lines (#) and (^) and substitution for each shown correctly leading to the right answer 1 mark for correct value of one expression but not both

Question 13.		WME Ext 1 HSC 2024 Worked Solution	Marks	Mark Allocation and Comments
13	f)	Show that $9^n - 4^n$ is divisible by 5 for $n \geq 1$ Show that it is true for $n = 1$ $9^1 - 4^1 = 9 - 4 = 5 \ = 5 \times 1$ $\therefore$ true for $n = 1$ Assume that it is true for $n = k$ i. e. that $9^k - 4^k = 5p$ where p is an integer Show that it is true for $n = k + 1$ i. e. that $9^{k+1} - 4^{k+1} = 5q$ where q is an integer LHS $= 9^{k+1} - 4^{k+1}$ $= 9 \times 9^k - 4 \times 4^k$ $= 9 \times 9^k - 4 \times 4^k - 5 \times 4^k + 5 \times 4^k$ $= 9 \times 9^k - 9 \times 4^k + 5 \times 4^k$ $= 9(9^k - 4^k) + 5 \times 4^k$ $= 9 \times 5p + 5 \times 4^k$ $= 5(9p + 4^k)$ $= 5q$ since $9p + 4^k$ is an integer, as p and k are integers $\therefore$ if true for $n = k$, it is also true for $n = k + 1$ But true for $n = 1$, so by induction is true for all integers $n \geq 1$	3	3 marks for correct proof 2 marks for minor mistake in algebraic manipulation or failing to state the conclusion or equivalent merit 1 mark for showing true for *n* = 1 and stating the assumption for *n* = *k* or equivalent merit

Question 14.		WME Ext 1 HSC 2024 Worked Solution	Marks	Mark Allocation and Comments
14	a)	$u = \sqrt{x} = x^{\frac{1}{2}}$ $\frac{du}{dx} = \frac{1}{2}x^{-\frac{1}{2}} = \frac{1}{2\sqrt{x}}$ $dx = 2\sqrt{x}\,du$ $\int \frac{dx}{\sqrt{x}\,e^{\sqrt{x}}} = \int \frac{2\sqrt{x}\,du}{\sqrt{x}\,e^{u}}$ $= \int 2e^{-u}du$ $= -2e^{-u} + \mathrm{C}$ $= -2e^{-\sqrt{x}} + \mathrm{C}$	2	2 marks for correct answer 1 mark for correct use of substitution with a minor error
14	b)	$\frac{x}{x+1} \geq 0$ $(x+1)^2 \times \frac{x}{x+1} \geq 0 \times (x+1)^2$ $x(x+1) \geq 0$ $x \leq -1$ and $x \geq 0$ but $x + 1 \neq 0$ so $x \neq -1$ so $x < -1$ and $x \geq 0$ 	2	2 marks – shows correct working out that leads to the correct solution, including showing the excluded value of $x = -1$ 1 mark – starts working out that leads to the correct values of x but does not find 2 inequalities or does not exclude $x = -1$ from solution

Question 14.		WME Ext 1 HSC 2024 Worked Solution	Marks	Mark Allocation and Comments
14	c)	Since $\cos 2x = 1 - 2\sin^2 x$ then $\cos 8x = 1 - 2\sin^2 4x$, so $2\sin^2 4x = 1 - \cos 8x$ $\displaystyle\int_{\frac{\pi}{12}}^{\frac{\pi}{4}} 2\sin^2 4x\ dx = \int_{\frac{\pi}{12}}^{\frac{\pi}{4}} 1 - \cos 8x\ dx$ $\displaystyle = \left[x - \frac{1}{8}\sin 8x\right]_{\frac{\pi}{12}}^{\frac{\pi}{4}}$ $\displaystyle = \frac{\pi}{4} - \frac{1}{8}\times \sin 2\pi - \left(\frac{\pi}{12} - \frac{1}{8}\times \sin\frac{2\pi}{3}\right)$ (#) $\displaystyle = \frac{\pi}{4} - \frac{\pi}{12} + \frac{1}{8}\times\frac{\sqrt{3}}{2}$ $\displaystyle = \frac{2\pi}{12} + \frac{\sqrt{3}}{16}$ $\displaystyle = \frac{\pi}{6} + \frac{\sqrt{3}}{16}$	3	3 marks - rearranges with double angle formulae correctly, integrates correctly and substitution explicitly shown which correctly leads to the correct answer in simplest exact form 2 marks - rearranges with double angle formulae correctly, integrates correctly and clearly shows substitution line (#) and makes only a minor error substituting correct values or when writing in simplest exact form 1 mark – rearranges with double angle formulae correctly but makes significant errors integrating and substituting (significantly simplifies the working needed to reach the answer)

14	d)	$(1+x)^n = \binom{n}{0} + \binom{n}{1}x + \binom{n}{2}x^2 + \binom{n}{3}x^3 + \ldots + \binom{n}{n}x^n$ Let $x = 3$ LHS $= (1+3)^n = 4^n = 2^{2n}$ RHS $= \binom{n}{0} + 3\binom{n}{1} + 3^2\binom{n}{2} + 3^3\binom{n}{3} + \ldots + \binom{n}{n}3^n$ $\therefore\ 2^{2n} = \binom{n}{0} + 3\binom{n}{1} + 9\binom{n}{2} + 27\binom{n}{3} + \ldots + \binom{n}{n}3^n$, but $\binom{n}{0} = 1$ so $2^{2n} = 1 + 3\binom{n}{1} + 9\binom{n}{2} + 27\binom{n}{3} + \ldots + \binom{n}{n}3^n$ $2^{2n} - 1 = 3\binom{n}{1} + 9\binom{n}{2} + 27\binom{n}{3} + \ldots + 3^n\binom{n}{n}$ $2^{2n} - 1 = 3\left(\binom{n}{1} + 3\binom{n}{2} + 9\binom{n}{3} + \ldots + 3^{n-1}\binom{n}{n}\right)$ $\frac{1}{3}(2^{2n} - 1) = \binom{n}{1} + 3\binom{n}{2} + 9\binom{n}{3} + \ldots + 3^{n-1}\binom{n}{n}$ *as required*		3 marks - expands $(1+x)^n$ correctly, substitutes $x = 3$ and works correctly with the subsequent equivalent lines, with all working out clearly shown, including explicitly stating $\binom{n}{0} = 1$ and then continues to clearly link each line of working, to achieve final line of $\binom{n}{1} + 3\binom{n}{2} + 9\binom{n}{3} + \ldots + 3^{n-1}\binom{n}{n} = \frac{1}{3}\left(2^{2n} - 1\right)$ 2 marks - expands $(1+x)^n$ correctly, substitutes $x = 3$ and works correctly with the subsequent equivalent lines, with all working out, including explicitly stating $\binom{n}{0} = 1$, clearly linking each line of working, but makes an error working towards $\frac{1}{3}\left(2^{2n} - 1\right)$ 1 mark – expands $(1+x)^n$ correctly and substitutes $x = 3$ but makes significant errors trying to prove as required or does not show sufficient working out to prove as required, including not explicitly stating $\binom{n}{0} = 1$

Question 14.		WME Ext 1 HSC 2024 Worked Solution	Marks	Mark Allocation and Comments
14	e)	(i) $r(t) = \begin{pmatrix} vt\cos\theta \\ vt\sin\theta - \frac{gt^2}{2} \end{pmatrix}$ or $x = vt\cos\theta$ and $y = vt\sin\theta - \frac{gt^2}{2}$ For this general projectile Range is reached when $y = 0$ $vt\sin\theta - \dfrac{gt^2}{2} = 0$ $t\left(v\sin\theta - \dfrac{gt}{2}\right) = 0$ so $t = 0$ [initial time at ground level] or $v\sin\theta = \frac{gt}{2}$ [When it returns to ground level] $t = \dfrac{2v\sin\theta}{g}$ Range $= v\left(\dfrac{2v\sin\theta}{g}\right)\cos\theta$ $= \dfrac{2v^2\sin\theta\cos\theta}{g}$ Range $= \dfrac{v^2\sin 2\theta}{g}$	3	3 marks for correct derivation **shown** with all lines of working out **shown** which link together explicitly to find both the time and range as required 2 marks for minor error in otherwise correct derivation with all lines of working out **shown** which link together explicitly to find both the time and range as required 1 mark for setting y component = 0 and attempting to solve or equivalent merit but makes significant errors in solution or incomplete solution

14.	(e) ii)	For Particle A For Particle B $v = u$ and $\theta = 60°$ $v = 2u$ and $\theta = 30°$ $R_A = \dfrac{u^2 \sin 120°}{g}$ $R_B = \dfrac{(2u)^2 \sin 60°}{g}$ (^) $R_A = \dfrac{u^2 \sin 60°}{g}$ $R_B = \dfrac{4u^2 \sin 60°}{g}$ (*) $R_A = \dfrac{u^2 \sin 60°}{g}$ $R_B = 4\left(\dfrac{u^2 \sin 60°}{g}\right) = 4R_A$ **Or** you can evaluate sin 60° and sin 120° $\sin 30° = \dfrac{1}{2}$ $\cos 30° = \dfrac{\sqrt{3}}{2}$ $\sin 60° = \dfrac{\sqrt{3}}{2}$ $\cos 60° = \dfrac{1}{2}$ $\sin 120° = \dfrac{\sqrt{3}}{2}$ $\cos 120° = -\dfrac{1}{2}$ Please turn over… solution continued	2	2 marks for **showing** correct derivation with explicit details with all working out leading to correctly show a factor of 4, <u>including</u> explicitly showing that $(2u)^2 = 4u^2$ from line (^) to line (*) 1 mark for starting with some correct and relevant working but makes errors or doesn't show explicit working out linking lines of working out to **show** required result

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		Particle A: $v = u$ and $\theta = 60°$ $R_A = \dfrac{u^2 \sin 120°}{g}$ $= \dfrac{u^2}{g}\left(\dfrac{\sqrt{3}}{2}\right)$ $= \dfrac{\sqrt{3}\,u^2}{2g}$ Particle B: $v = 2u$ and $\theta = 30°$ $R_B = \dfrac{(2u)^2 \sin 60°}{g}$ $= \dfrac{4u^2}{g}\left(\dfrac{\sqrt{3}}{2}\right)$ $= \dfrac{4\sqrt{3}\,u^2}{2g} = 4R_A$		