Teacher: Mrs Karagiannis / Mr Selby (Circle)

Student Number:

Mathematics Extension 1
Year 12 Examination Block 2025
Task 4 - Trial HSC Examination

General Instructions	
Write using black or blue pen Graphs can be drawn in pencil No liquid paper or correction tape are to be used.	
Reading Time	10 minutes
Working time	2 hours

Structure of the Paper		
Section 1	/10 Marks	Allow about 15 minutes for this section
Section 2	/ 60 Marks	Allow about 1 hour and 45 minutes for this section
Total	**/70 Marks**	**Weighting: 30%**

Special Instructions
NESA approved Calculator may be used. A reference sheet is provided at the back of this paper. A separate multiple choice answer sheet is provided. Attempt all questions This examination must not be removed from the Examination Room. In Question 11-14 show relevant mathematical reasoning and/or calculations. Answer each question in a separate writing booklet.

Section I

10 marks
Attempt questions 1 – 10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Question 1-10

1. A drawer contains 12 red and 12 blue socks, all unmatched.

   In the dark, a person randomly selects socks.

   How many socks must they take out to guarantee having at least two blue socks.

   (A) 12 (B) 13

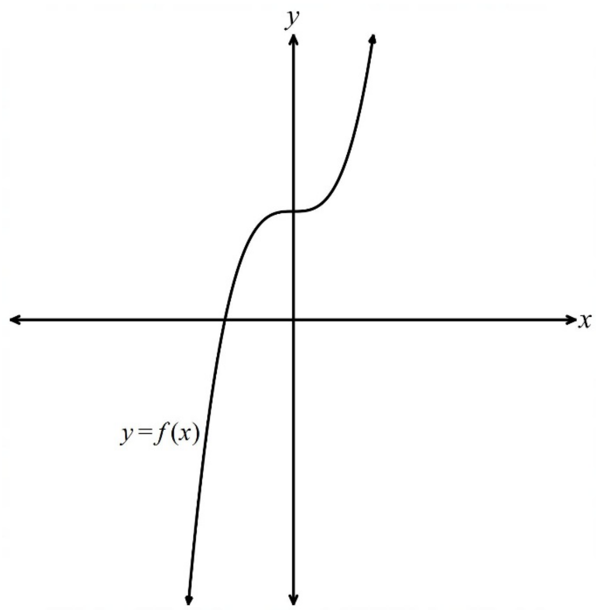
   (C) 14 (D) 15

2. If $\sqrt{8}\,\sin x + 2\sqrt{6}\,\cos x = R\sin(x+\alpha)$ find the values of R and α.

   (A) $\alpha = \frac{\pi}{6}$ and $R = \sqrt{24}$ (B) $\alpha = \frac{\pi}{6}$ and $R = \sqrt{32}$

   (C) $\alpha = \frac{\pi}{3}$ and $R = \sqrt{24}$ (D) $\alpha = \frac{\pi}{3}$ and $R = \sqrt{32}$

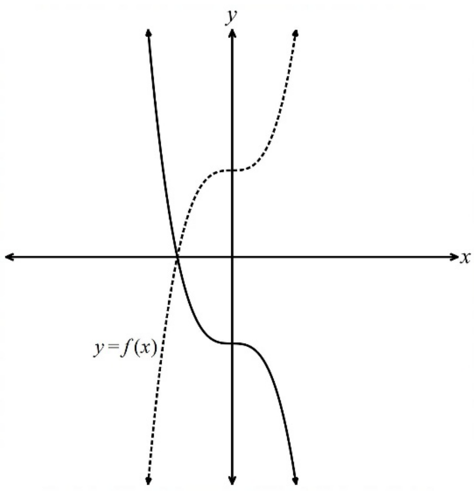
3. The graph below shows a function $y = f(x)$.



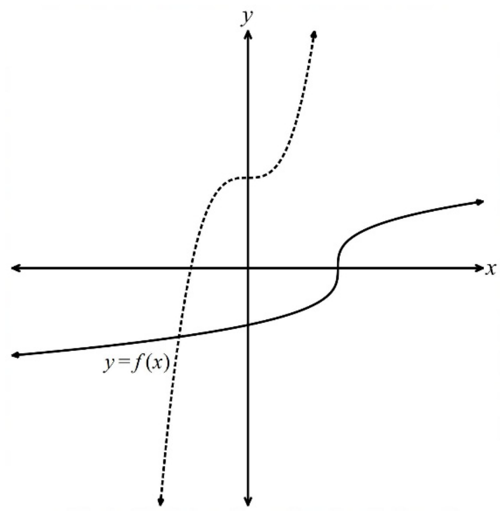
Which of these graphs could represent the inverse function $y = f^{-1}(x)$?

The graph of $y = f(x)$ is shown with a broken line in each graph.

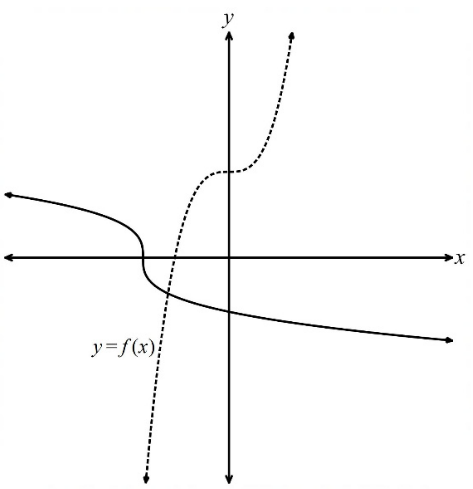
(A)



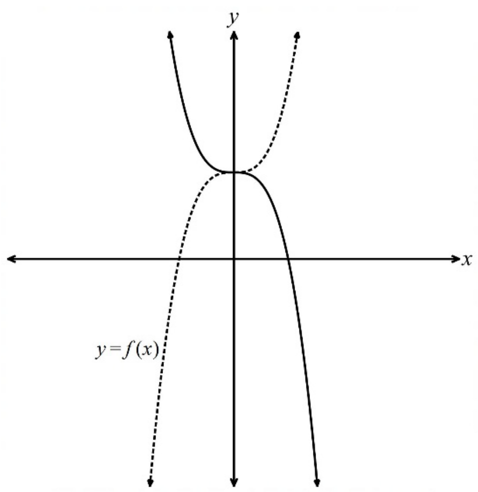
(B)



(C)



(D)



4. Find $\int \frac{6x^2 dx}{\sqrt{100-[4x^6]}}$

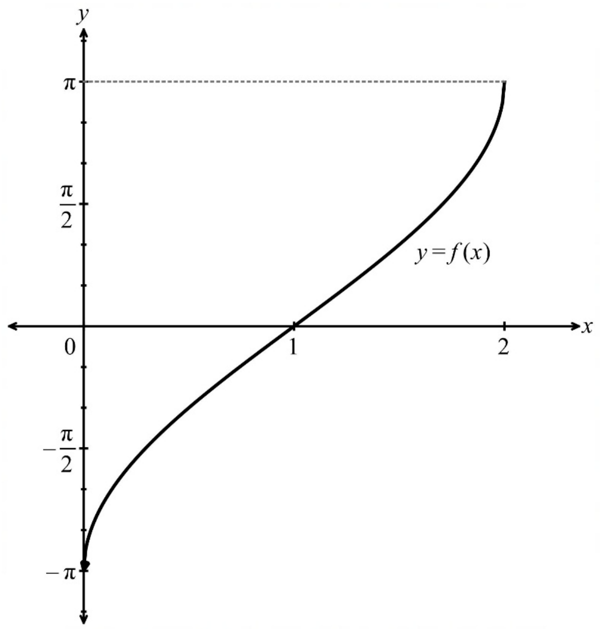
(A) $\cos^{-1}\left(\frac{2x^3}{10}\right) + C$

(B) $\cos^{-1}\left(\frac{4x^3}{100}\right) + C$

(C) $\sin^{-1}\left(\frac{2x^3}{10}\right) + C$

(D) $\sin^{-1}\left(\frac{4x^3}{100}\right) + C$

5. The graph of $y = f(x)$ is shown below.



Which equation describes $f(x)$?

(A) $f(x) = 2\cos^{-1}(x-1)$

(B) $f(x) = \frac{1}{2}\sin^{-1}(x-1)$

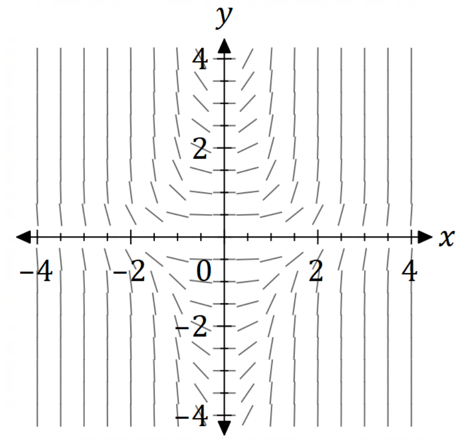
(C) $f(x) = 2\sin^{-1}(x-1)$

(D) $f(x) = 2\sin^{-1}(x+1)$

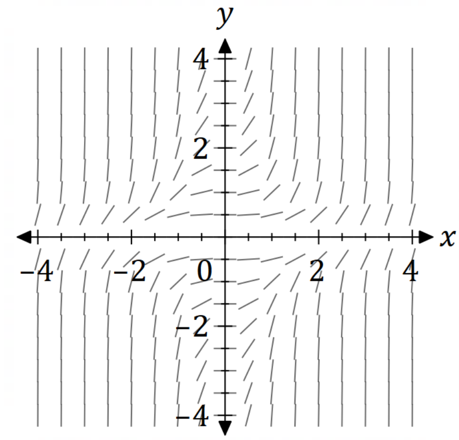
6. Consider the differential equation $\frac{dy}{dx} = x^2 y^2$.

Which of the following best represents its direction field?

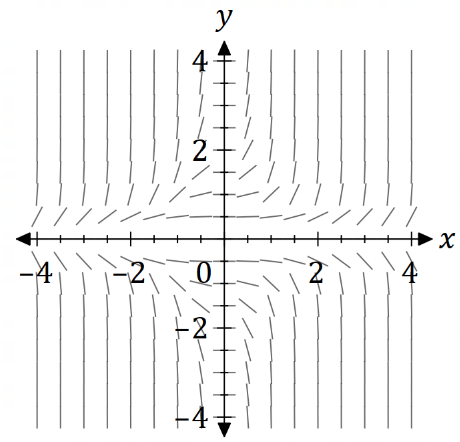
(A)



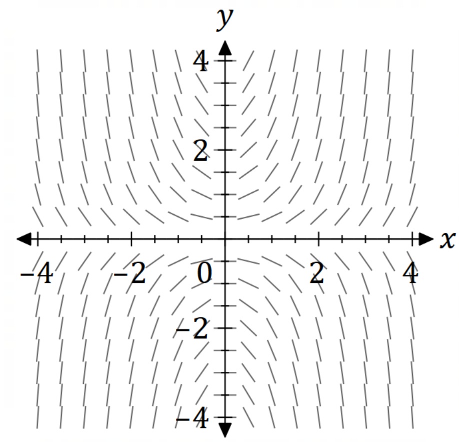
(B)



(C)



(D)



7. Which of these calculations will give the coefficient of $x^3 y^9$ in the expansion of $(3x - y)^{12}$?

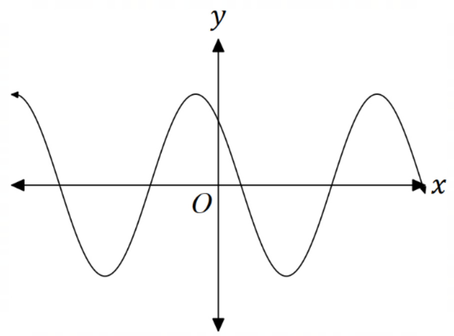
(A) $-27 \times {}^{12}C_8$

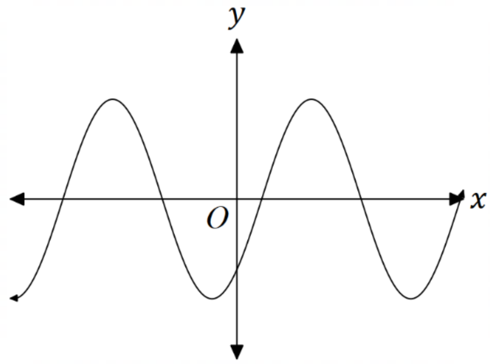
(B) $-27 \times {}^{12}C_9$

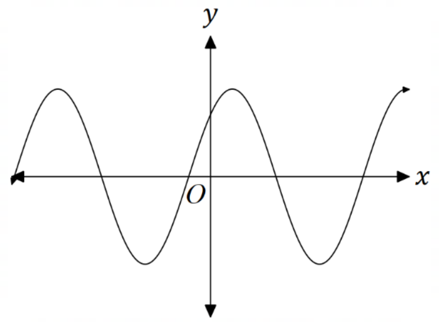
(C) $9 \times {}^{12}C_8$

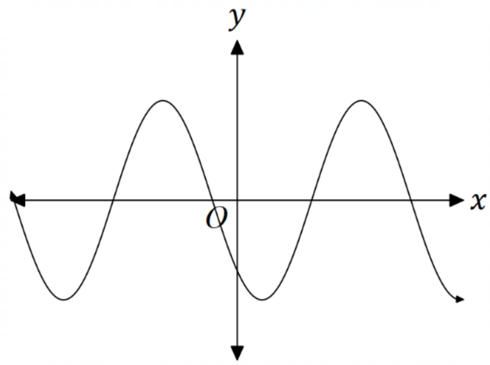
(D) $27 \times {}^{12}C_9$

8. Which graph best represents $y = a\sin x - b\cos x$, given a and b are positive constants?

(A) 

(B) 

(C) 

(D) 

9. The quantity of a substance in a container (in kg) at a time t seconds after the container is first filled is given by the equation $Q(t) = 20000e^{-0.025t}$.

Which expression gives the rate of change of the quantity (in kg/s) after two minutes?

(A) $-\dfrac{500}{e^3}$

(B) $-\dfrac{500}{e^{0.05}}$

(C) $\dfrac{500}{e^{0.05}}$

(D) $\dfrac{500}{e^3}$

10. Given $\cos(\theta + \phi) = \frac{4}{5}$ and $\cot\theta\cot\phi = \frac{1}{3}$, then $\cos(\theta - \phi)$ is equivalent to:

(A) $-\dfrac{8}{5}$

(B) $-\dfrac{4}{5}$

(C) $\dfrac{3}{5}$

(D) $\dfrac{6}{5}$

Section II

60 marks
Attempt all questions 11 – 14
Allow about 1 hour and 45 minutes for this section

Answer each question in the appropriate writing booklet. Extra writing booklets are available.
For questions in Section II, your responses should include relevant mathematical reasoning and/or calculations.

Marks

Question 11 (15 marks) **Use the Question 11 Writing booklet.**

(a) Eight teachers and seven students are arranged in a straight line on 15 chairs that are bolted to the ground and cannot be moved. **2**

What is the probability that none of the teachers will sit next to each other?

(b) Solve the inequality $\dfrac{2x}{x-3} \geq 5$. **3**

(c) Let $P(x) = 3x^3 - 12x^2 - x + 8$ have roots α, β and γ.

(i) What is the value of $\alpha^2 + \beta^2 + \gamma^2$? **2**

(ii) Find the value of $\frac{\alpha}{\beta\gamma} + \frac{\beta}{\alpha\gamma} + \frac{\gamma}{\alpha\beta}$. **2**

(d) Show that $y = \frac{1}{10}e^{3x}$ is a solution to the differential equation $y'' + y = e^{3x}$. **2**

(e) Use the substitution $u = 1 - x^3$ to find $\displaystyle\int \frac{x^2}{\sqrt{1-x^3}}\,dx$ **2**

(f) Find $\dfrac{d}{dx}(\log_e x)^2$ and hence, evaluate $\displaystyle\int_1^e \frac{\log_e x}{x}\,dx$ **2**

End of Question 11

Question 12 (15 marks) **Use the Question 12 Writing booklet** **Marks**

(a) The letters of the word A B R A C A D A B R A are arranged in a row.

(i) Find the number of different arrangements that can be made. **1**

(ii) Find the number of different arrangements in which the 2 B's are next to each other, the 2 R's are next to each other, exactly 4 of the A's are next to each other, and the C is next to the D. **3**

(iii) Given that the 11 letters are arranged randomly, find the probability that all 5 A's are together. **1**

(b) A snowball in the shape of a sphere is melting such that its radius is decreasing at a rate of 1 cm/min. **2**

At what rate is the volume of the snowball decreasing when its radius is 8 cm?

(c) Find $\int \sin^2 5x + \sin x \, dx$. **2**

(d) (i) Find the remainder, R, when $P(x) = x^3 + 2x^2 + 3x + 10$ is divided by $x + 2$. **1**

(ii) Find the polynomial $Q(x)$, such that $P(x) = (x + 2).Q(x) + R$ and express $P(x)$ in that form. **2**

(e) Use mathematical induction to prove $5^n + 31$ is divisible by 4, for all integers $n \geq 1$. **3**

End of Question 12

Question 13 (15 marks) **Use the Question 13 Writing booklet** **Marks**

(a) Let vector $\underset{\sim}{u} = \underset{\sim}{i} + 3\underset{\sim}{j}$ and vector $\underset{\sim}{v} = 4\underset{\sim}{j}$. Find $\text{proj}_{\underset{\sim}{u}}\underset{\sim}{v}$. **3**

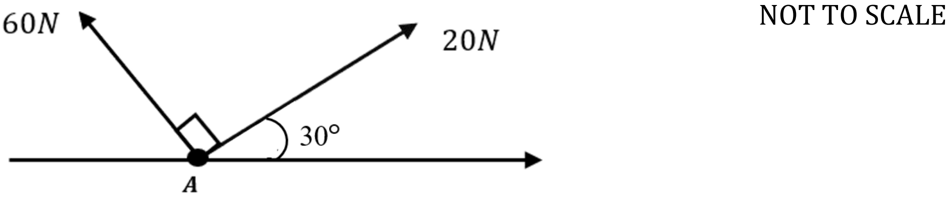
(b) Express $\cos\theta + \sqrt{3}\sin\theta$ in terms of t, where $t = \tan\frac{\theta}{2}$ and hence solve the equation $\cos\theta + \sqrt{3}\sin\theta = 1$ for $0 \le \theta \le 2\pi$. **3**

(c) A curve is described by the parametric equations given below: **3**

$$x = 12q$$
$$y = 6(q+1)^2$$

Find the gradient of the tangent to the curve at the point where $q = 1\frac{1}{2}$

(d) Forces of $60N$ and $20N$ act on an object, at point A, as shown in the diagram.



The vector sum of these forces acting on the object at point A is called the resultant force. Find the magnitude of the resultant force vector, correct to 3 decimal places, and its direction as a true bearing, correct to the nearest minute. **3**

(e) (i) Show that $\frac{\cos(a+b)}{\cos a \sin b} = \cot b - \tan a$. **1**

(ii) Hence, find the exact value of the following definite integral. **2**

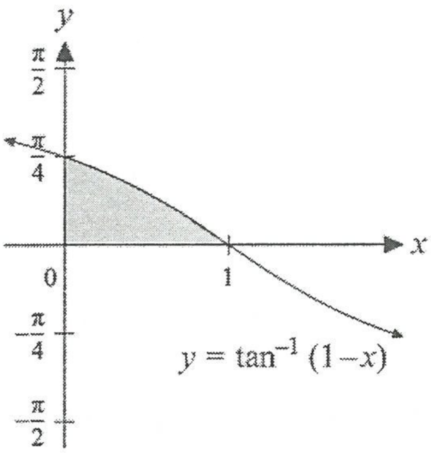
$$\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\cos\left(x + \frac{\pi}{6}\right)}{\cos\frac{\pi}{6}\sin x}\,dx$$

End of Question 13

Question 14 (15 marks) **Use the Question 14 Writing booklet** **Marks**

(a) Please refer to the separate page for this question. **3**

(b) The diagram shows the graph of the function $y = \tan^{-1}(1-x)$. The region in the first quadrant bounded by the curve and the coordinates axes is rotated through 2π radians about the y axis. Find, in simplest exact form, the volume of the solid formed. **3**



(c) A tank initially contains 1000 litres of water and 20 kg of dissolved salt.

Fresh water is pumped into the tank at a rate of 10 L/min. The solution is thoroughly mixed at all times and is drained from the tank at a rate of 5 L/min.

Let A be the amount of salt in the tank, in kg, at time t, in minutes.

(i) By considering the volume of liquid in the tank at time t, show that C, the concentration of salt in the tank, in kg/L, at time t can be given by **1**

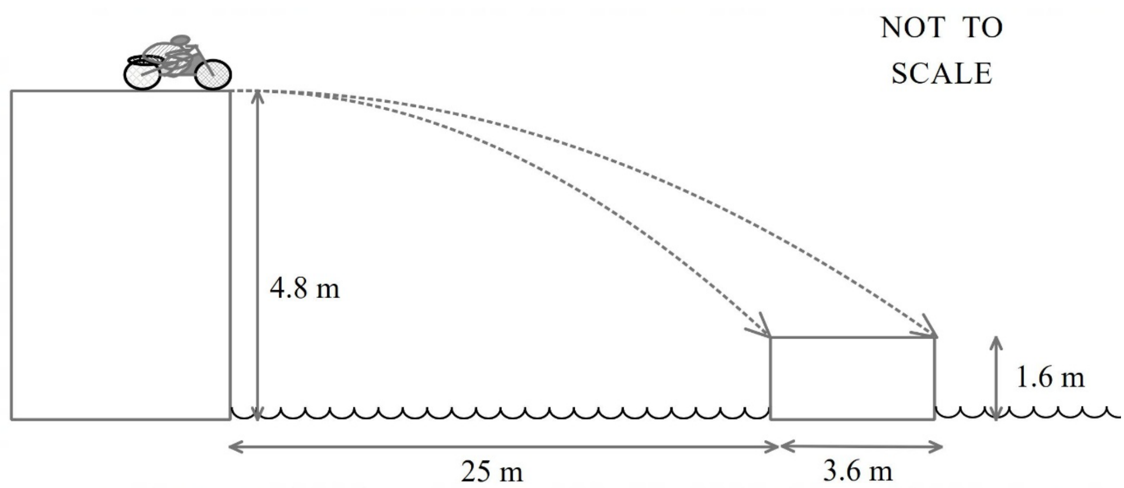
$$C = \frac{A}{1000 + 5t}$$

(ii) Explain why $\frac{dA}{dt} = -5C$. **1**

(iii) Hence find A as a function of t. **3**

Question 14 continues on page 10

(d) A motorcycle stunt rider is positioned on top of a 4.8 m high wharf and will ride off the end of the wharf at a constant speed with the intention of landing on a barge positioned in the water 25 m from the wharf. The barge is 3.6 m wide and sits 1.6 m above the water level. **4**



To stop safely, she needs to land in the centre of the barge.

For our calculations, we will use the point at which she leaves the wharf as the origin, gravity as $g = 10$ ms^{-2} and treat the bike and rider as a single particle.

Using t as the time in seconds after leaving the edge of the wharf, the position vector of the particle $r(t)$ is given by:

$$\underset{\sim}{r}(t) = \begin{pmatrix} Vt \\ -5t^2 \end{pmatrix} \qquad \text{[Do not prove this]}$$

where V is the velocity in ms^{-1} when she leaves the wharf.

Find the initial velocity required for her to land exactly in the centre of the barge and calculate the vertical component of her velocity when she lands.

End of paper

2025 Maths Ext 1 Task 4

1. 12red + 2 blue = 14

C

2. $a=\sqrt{8}$ $b=2\sqrt{6}$ $\sqrt{8}\sin x+2\sqrt{6}\cos x = R\sin(x+\alpha)$

$R\sin(x+\alpha) = R\sin x\cos\alpha + R\cos x\sin\alpha$

$R\cos\alpha = \sqrt{8}$

$R\sin\alpha = 2\sqrt{6}$

$\tan\alpha = \frac{2\sqrt{6}}{\sqrt{8}}$

$\alpha = \frac{\pi}{3}$

$R=\sqrt{(\sqrt{8})^2+(2\sqrt{6})^2}$

$R=\sqrt{32}$

D

3. B

4. $\int \frac{f'(x)}{\sqrt{a^2-[f(x)]^2}}\,dx$

$a=10$

$f(x)=2x^3$

$f'(x)=6x^2$

$= \sin^{-1}\left(\frac{2x^3}{10}\right)+C$

C

5. Original curve $f(x)=\sin^{-1}(x)$ translated 1 unit right and vertically dilated by a factor of 2

$\therefore f(x) = 2\sin^{-1}(x-1)$

C

6. At $(-2,1)$, $\frac{dy}{dx} > 0$

Eliminate A and D

At $(-2,-1)$, $\frac{dy}{dx} > 0$

Eliminate C

$\therefore$ B

7. $T_{r+1} = \binom{12}{r}(3x)^{12-r}(-y)^r$

$12 - r = 3$

$r = 9$

$T_{10} = \binom{12}{9} 3^3 x^3 (-1)^9 y^9$

coefficent is $-27 \times \binom{12}{9}$

B

8. When $x=0$

$y = -b\cos 0$

$y = -b$

Eliminate A and C

$y' = a\cos x + b\sin x$

When $x=0$

$y' = a\cos 0 + 0$

$y' = a$

$\therefore y' > 0$ since a is positive

B

9. $Q(t) = 20\,000 e^{-0.025t}$

$Q'(t) = 20\,000 \times -0.025 e^{-0.025t}$
$= -500 e^{-0.025t}$

$Q'(120) = -500 e^{-0.025 \times 120}$
$= \frac{-500}{e^3}$

A

10. $\cos(\theta + \phi) = \frac{4}{5}$ $\cot\theta \cot\phi = \frac{1}{3}$

$\cos\theta\cos\phi - \sin\theta\sin\phi = \frac{4}{5}$ (1) $\frac{\cos\theta\cos\phi}{\sin\theta\sin\phi} = \frac{1}{3}$

Sub (2) into (1)

$\frac{\sin\theta\sin\phi}{\cos\theta\cos\phi} = 3$

$\cos\theta\cos\phi - 3\cos\theta\cos\phi = \frac{4}{5}$

$\sin\theta\sin\phi = 3\cos\theta\cos\phi$ (2)

$-2\cos\theta\cos\phi = \frac{4}{5}$

$\cos\theta\cos\phi = \frac{-2}{5}$

$\cos(\theta - \phi) = \cos\theta\cos\phi + \sin\theta\sin\phi$
$= \frac{-2}{5} + 3 \times \frac{-2}{5}$

$= \frac{-8}{5}$

A

Question 11

(a)

no restrictions $= 15!$

$$P(\text{no teachers together}) = \frac{8! \times 7!}{15!}$$

$$= \frac{1}{6435}$$

(b) $\frac{2x}{x-3} \geq 5 \qquad x \neq 3$

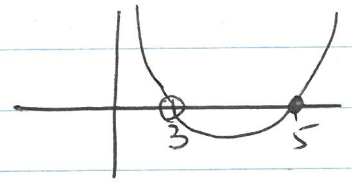
$$2x(x-3) \geq 5(x-3)^2$$

$$5(x-3)^2 - 2x(x-3) \leq 0$$

$$(x-3)[5(x-3) - 2x] \leq 0$$

$$(x-3)(3x-15) \leq 0$$

$$(x-3)(x-5) \leq 0$$



$\therefore\ 3 < x \leq 5$

(c) $P(x) = 3x^3 - 12x^2 - x + 8$

$\alpha + \beta + \gamma = -\frac{b}{a}$

$= \frac{12}{3}$

$\therefore \alpha + \beta + \gamma = 4$

$\alpha\beta\gamma = -\frac{d}{a}$

$\alpha\beta\gamma = \frac{-8}{3}$

$\alpha\beta + \alpha\gamma + \beta\gamma = \frac{c}{a}$

$= -\frac{1}{3}$

	α	β	γ
α	α^2	$\alpha\beta$	$\alpha\gamma$
β	$\alpha\beta$	β^2	$\beta\gamma$
γ	$\alpha\gamma$	$\beta\gamma$	γ^2

(i) $\alpha^2 + \beta^2 + \gamma^2 = (\alpha+\beta+\gamma)^2 - 2(\alpha\beta + \alpha\gamma + \beta\gamma)$

$= 4^2 - 2(-\frac{1}{3})$

$= 16\frac{2}{3}$

(ii) $\frac{\alpha}{\beta\gamma} + \frac{\beta}{\alpha\gamma} + \frac{\gamma}{\alpha\beta} = \frac{\alpha^2+\beta^2+\gamma^2}{\alpha\beta\gamma}$

$= \frac{16\frac{2}{3}}{-\frac{8}{3}}$

$= \frac{-25}{4}$

(d) $y = \frac{1}{10}e^{3x}$

$y' = \frac{3}{10}e^{3x}$

$y'' = \frac{9}{10}e^{3x}$

LHS $= y'' + y$

$= \frac{9}{10}e^{3x} + \frac{1}{10}e^{3x}$

$= e^{3x}$

RHS $= e^{3x}$

$=$ LHS

$\therefore y = \frac{1}{10}e^{3x}$ is a solution

(e) $u = 1 - x^3$

$\frac{du}{dx} = -3x^2$

$du = -3x^2\,dx$

$-\frac{1}{3}du = x^2\,dx$

$\int \frac{x^2}{\sqrt{1-x^3}}\,dx = \frac{-1}{3}\int u^{-\frac{1}{2}}\,du$

$= \frac{-1}{3}\left[2u^{\frac{1}{2}}\right] + C$

$= -\frac{2}{3}u^{\frac{1}{2}} + C$

$= -\frac{2}{3}\sqrt{1-x^3} + C$

(f) $\frac{d}{dx}(\log_e x)^2 = 2\log_e(x) \times \frac{1}{x}$

$= \frac{2}{x}\log_e(x)$

$$\int_1^e \frac{\log_e x}{x}\,dx = \frac{1}{2}\int_1^e \frac{2\log_e(x)}{x}\,dx$$

$$= \frac{1}{2}\Big[(\log_e x)^2\Big]_1^e$$

$$= \frac{1}{2}\Big[(\log_e e)^2 - (\log_e 1)^2\Big]$$

$$= \frac{1}{2}(1-0)$$

$$= \frac{1}{2}$$

Question 12

(a) (i) $\dfrac{11!}{5!2!2!} = 83\,160$

(ii) BB RR AAAA CD A

$5! \times 2! = 240$

All 5A's together

AAAAA BB RR CD

$4! \times 2! \times 2$

(A can be infront of the other 4 A's or at end of the other 4A's)

$\therefore$ 96 ways

$\therefore$ different arrangements where exactly 4 A's are next to each other are $240 - 96 = 144$ ways

(iii) AAAAA B B R R C D

$\dfrac{7!}{2!2!} = 1260$

$\therefore P(\text{all 5A's together}) = \dfrac{1260}{83160}$

$= \dfrac{1}{66}$

(b) $\frac{dr}{dt} = -1$

$V = \frac{4}{3}\pi r^3$

$\frac{dV}{dr} = 4\pi r^2$

$\frac{dV}{dt} = \frac{dV}{dr} \times \frac{dr}{dt}$

$= 4\pi r^2 \times -1$

$= -4\pi r^2$

When $r=8$

$\frac{dV}{dt} = -4\pi(8)^2$

$= -256\pi$

$\therefore$ it is decreasing at the rate of $256\pi\ cm^3/min$

(c) Find $\int \sin^2 5x + \sin x \, dx$

$= \frac{1}{2}\int (1-\cos 10x)\, dx + \int \sin x \, dx$

$= \frac{1}{2}\left(x - \frac{1}{10}\sin 10x\right) + -\cos x + C$

$= \frac{1}{2}x - \frac{1}{20}\sin 10x - \cos x + C$

(d) (i) $P(-2) = (-2)^3 + 2(-2)^2 + 3(-2) + 10$

$= -8 + 8 - 6 + 10$

$= 4$

$\therefore$ remainder is 4

(d) (ii) $(x+2)Q(x)+4 = x^3+2x^2+3x+10$

$$(x+2)Q(x) = x^3+2x^2+3x+6$$

$$\begin{array}{r}
x^2 \quad +3 \\
x+2 \overline{\smash{)}\, x^3+2x^2+3x+6} \\
x^3+2x^2 \quad - \\
\hline
0+3x+6 \\
3x+6 \quad - \\
\hline
0
\end{array}$$

$\therefore Q(x) = x^2+3$

$\therefore P(x) = (x+2)(x^2+3)+4$

(e) Prove 5^n+31 is divisible by 4 for all integers $n \geq 1$

Test for $n=1$

$5^n+31 = 5+31$

$= 36$, which is divisible by 4

Assume true for $n=k$

$5^k+31 = 4P$ where P is an integer

$5^k = 4P-31$

Prove true for $n=k+1$

$5^{k+1}+31 = 4Q$ where Q is an integer

LHS $= 5^{k+1}+31$

$= 5^k.5+31$

$= 5(4P-31)+31$, by assumption

$= 20P-155+31$

$= 20P-124$

$= 4(5P-31)$

$= 4Q$ which is divisible by 4 and $Q=5P-31$

$=$ RHS

$\therefore$ it is true for $n=k+1$

$\therefore$ Proven by PMI for all integers $n \geq 1$

Question 13

(a) $\underset{\sim}{u} = \underset{\sim}{i} + 3\underset{\sim}{j}$

$\underset{\sim}{v} = 4\underset{\sim}{j}$

$$\text{proj}_{\underset{\sim}{u}}\underset{\sim}{v} = (\underset{\sim}{v} \cdot \hat{\underset{\sim}{u}})\hat{\underset{\sim}{u}}$$

$$|\underset{\sim}{u}| = \sqrt{1^2+3^2} = \sqrt{10}$$

$$= \frac{1}{\sqrt{10}}(4\underset{\sim}{j})\cdot(\underset{\sim}{i}+3\underset{\sim}{j})\hat{\underset{\sim}{u}}$$

$$= \frac{1}{\sqrt{10}}(0+12)\left(\frac{\underset{\sim}{i}+3\underset{\sim}{j}}{\sqrt{10}}\right)$$

$$= \frac{12}{(\sqrt{10})^2}(\underset{\sim}{i}+3\underset{\sim}{j})$$

$$= \frac{12}{10}(\underset{\sim}{i}+3\underset{\sim}{j})$$

$$= \frac{6}{5}(\underset{\sim}{i}+3\underset{\sim}{j})$$

(b) $\cos\theta + \sqrt{3}\sin\theta$

$$\frac{1-t^2}{1+t^2} + \sqrt{3}\left(\frac{2t}{1+t^2}\right) = \frac{1-t^2+2\sqrt{3}t}{1+t^2}$$

Solve $\cos\theta + \sqrt{3}\sin\theta = 1$ for $0 \le \theta \le 2\pi$

$$\frac{1-t^2+2\sqrt{3}t}{1+t^2} = 1$$

$$1-t^2+2\sqrt{3}t = 1+t^2$$

$$-2t^2+2\sqrt{3}t = 0$$

$$2t(\sqrt{3}-t) = 0$$

$$\therefore t = 0, \quad t = \sqrt{3}$$

$$\tan\frac{\theta}{2} = 0, \quad \tan\frac{\theta}{2} = \sqrt{3}$$

(b) $\frac{\theta}{2} = 0, \pi, \frac{\pi}{3}$ $\qquad 0 \le \frac{\theta}{2} \le \pi$

$\theta = 0, 2\pi, \frac{2\pi}{3}$

Test $\theta = \pi$

$\cos\pi + \sqrt{3}\sin\pi = -1$

$\neq 1$

$\therefore \theta = \pi$ is not a solution

$\therefore \theta = 0, \frac{2\pi}{3}, 2\pi$

(c) $x = 12q$ ①

$y = 6(q+1)^2$ ②

Rearrange ①

$q = \frac{x}{12}$ ③

Sub ③ into ②

$y = 6\left(\frac{x}{12} + 1\right)^2$

$= 6\left(\frac{x^2}{144} + \frac{x}{6} + 1\right)$

$y = \frac{x^2}{24} + x + 6$

$y' = \frac{2x}{24} + 1$

$y' = \frac{x}{12} + 1$

When $q = 1\frac{1}{2}$, $x = 12q$

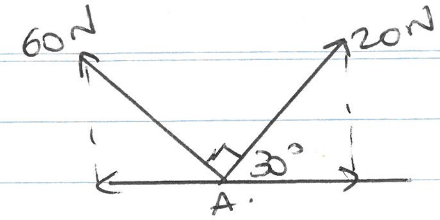
$x = 12(1\frac{1}{2})$

$x = 18$

$y' = \frac{18}{12} + 1$

$\therefore y' = 2\frac{1}{2}$

(d) Horizontal force



$= 20\cos 30^\circ \underset{\sim}{i} - 60\cos 60^\circ \underset{\sim}{i}$

$= 20 \times \frac{\sqrt{3}}{2} \underset{\sim}{i} - 60 \times \frac{1}{2} \underset{\sim}{i}$

$= (10\sqrt{3} - 30)\underset{\sim}{i}$ N

$\underset{\sim}{F} = (10\sqrt{3} - 30)\underset{\sim}{i} + 20\sin 30^\circ \underset{\sim}{j} + 60\sin 60^\circ \underset{\sim}{j}$

$= (10\sqrt{3} - 30)\underset{\sim}{i} + 10\underset{\sim}{j} + 60 \times \frac{\sqrt{3}}{2} \underset{\sim}{j}$

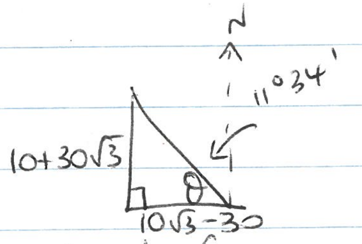
$= (10\sqrt{3} - 30)\underset{\sim}{i} + (10 + 30\sqrt{3})\underset{\sim}{j}$

$|\underset{\sim}{F}| = \sqrt{(10\sqrt{3}-30)^2 + (10 + 30\sqrt{3})^2}$

$= \sqrt{4000}$

$|\underset{\sim}{F}| \doteqdot 63.246$ N

$\tan\theta = \frac{10 + 30\sqrt{3}}{10\sqrt{3} - 30}$



$\theta \doteqdot -78.435^\circ$

We know it's in the 2nd quadrant (since x is negative, y positive)

True bearing is $360^\circ - 11^\circ 34' = 348^\circ 26'$ T

(e) (i.) $\text{LHS} = \dfrac{\cos(a+b)}{\cos a \sin b}$

$$= \frac{\cos a \cos b - \sin a \sin b}{\cos a \sin b}$$

$$= \frac{\cancel{\cos a} \cos b}{\cancel{\cos a} \sin b} - \frac{\sin a \cancel{\sin b}}{\cos a \cancel{\sin b}}$$

$$= \cot b - \tan a$$

$$= \text{RHS}$$

(ii) $\displaystyle\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\cos\left(x + \frac{\pi}{6}\right)}{\cos\frac{\pi}{6}\sin x}$

From part (i) $a = \frac{\pi}{6}$, $b = x$

$$= \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \left(\cot x - \tan\frac{\pi}{6}\right) dx$$

$$= \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \left(\frac{\cos x}{\sin x} - \frac{1}{\sqrt{3}}\right) dx$$

$$= \left[\log_e|\sin x| - \frac{1}{\sqrt{3}}x\right]_{\frac{\pi}{6}}^{\frac{\pi}{3}}$$

$$= \log_e\left|\sin\frac{\pi}{3}\right| - \frac{1}{\sqrt{3}}\left(\frac{\pi}{3}\right) - \left(\log_e\left|\sin\frac{\pi}{6}\right| - \frac{1}{\sqrt{3}}\left(\frac{\pi}{6}\right)\right)$$

$$= \log_e\left(\frac{\sqrt{3}}{2}\right) - \frac{\pi}{3\sqrt{3}} - \log_e\left(\frac{1}{2}\right) + \frac{\pi}{6\sqrt{3}}$$

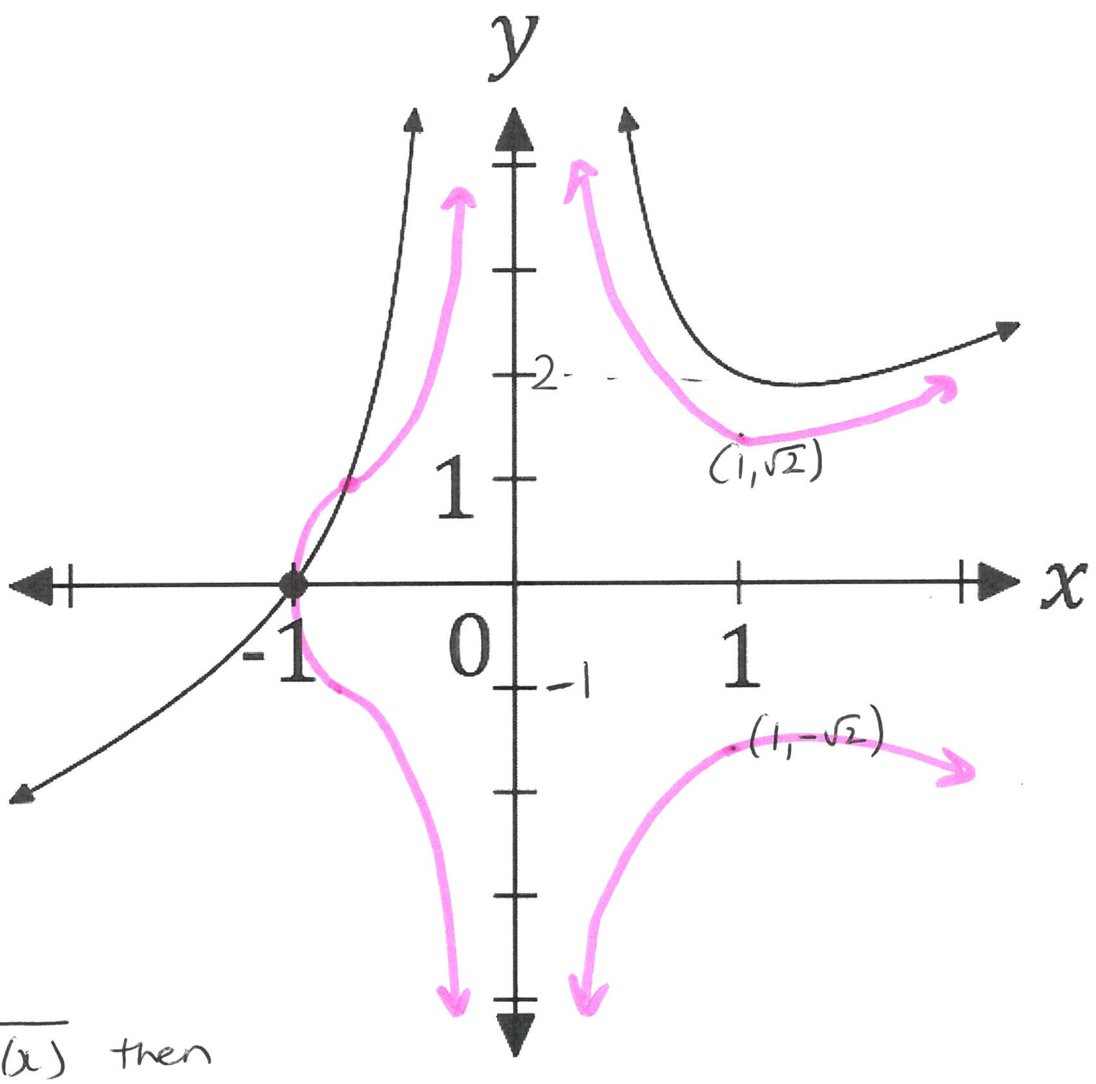
$$= \log_e\left(\frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}}\right) - \frac{2\pi}{6\sqrt{3}} + \frac{\pi}{6\sqrt{3}}$$

$$= \log_e(\sqrt{3}) - \frac{\pi}{6\sqrt{3}}$$

Use this page to answer Q14 (a), and place it inside your Question 14 answer booklet

(a) The graph of a function $f(x)$ is graphed below, which has a vertical asymptote at $x = 0$. **3**

Sketch the graph of $y^2 = f(x)$ on the same axes.



$*\ y = \sqrt{f(x)}$ then

a reflection in the x-axis

Question 14

(b) $y = \tan^{-1}(1-x)$

$\tan y = 1-x$

$x = 1 - \tan y$

$$V = \pi \int_0^{\frac{\pi}{4}} (1-\tan y)^2 \, dy$$

$$= \pi \int_0^{\frac{\pi}{4}} (1 - 2\tan y + \tan^2 y) \, dy$$

$$= \pi \int_0^{\frac{\pi}{4}} \left(\sec^2 y - 2\frac{\sin y}{\cos y}\right) dy \qquad \text{using } (1+\tan^2 y = \sec^2 y)$$

$$= \pi \left[\tan y + 2\log_e(\cos y)\right]_0^{\frac{\pi}{4}}$$

$$= \pi \left[\tan\frac{\pi}{4} + 2\log_e\left(\cos\frac{\pi}{4}\right) - \left(\tan 0 + 2\log_e(\cos 0)\right)\right]$$

$$= \pi\left(1 + 2\log_e\left(\tfrac{1}{\sqrt{2}}\right) - 0 - 2\log_e 1\right)$$

$$= \pi\left(1 + \log_e\left(\tfrac{1}{\sqrt{2}}\right)^2\right)$$

$$= \pi\left(1 + \log_e\left(\tfrac{1}{2}\right)\right) \quad \underline{\text{OR}}$$

$$= \pi(1 - \log_e 2)$$

(c) $R_{in} = q_{in} \times C_{in}$

(i) $= 10 \times 0$

$= 0$

$V = V_0 + (q_{in} - q_{out})t$

$V = 1000 + (10-5)t$

$V = 1000 + 5t$

$R_{out} = q_{out} \times C_{out}$

$= 5 \times \frac{A}{V}$

$= 5C$ where $C = \frac{A}{1000+5t}$

(ii) $\frac{dA}{dt} = R_{in} - R_{out}$

$= 0 - \frac{5A}{1000+5t}$

$= \frac{-5A}{1000+5t}$

$= -5C$

the amount of salt only changes via the water leaving the tank

$\therefore \frac{dA}{dt}$ is the rate of change of the amount of salt in the tank. It is the concentration of salt at time t multiplied by the rate of liquid leaving the tank.

(iii) $\frac{dA}{dt} = \frac{-5A}{1000+5t}$

$\int \frac{1}{A}\, dA = \int \frac{-5}{1000+5t}\, dt$

$\log_e|A| = -\log_e(1000+5t) + C$

$\log_e A = -\log_e(1000+5t) + C$, since $A>0$, $t>0$

When $t=0$, $A=20$

$\log_e 20 = -\log_e 1000 + C$

$C = \log_e 20 + \log_e 1000$

$= \log_e(20 \times 1000)$

$= \log_e(20\,000)$

(c) (iii)

$\therefore \log_e A = -\log_e(1000+5t) + \log_e 20\,000$

$= \log_e\left(\frac{1}{1000+5t}\right) + \log_e 20\,000$

$\log_e A = \log_e\left(\frac{20\,000}{1000+5t}\right)$

$A = \frac{20\,000}{1000+5t}$

$\therefore A = \frac{4000}{200+t}$

(d) $\underset{\sim}{r}(t) = \begin{pmatrix} vt \\ -5t^2 \end{pmatrix}$

$y = -3.2$

$-3.2 = -5t^2$

$t^2 = \frac{3.2}{5}$

$t = \pm\sqrt{\frac{3.2}{5}}$

since $t \geq 0$

$\therefore t = 0.8$

Middle of the barge $= 25 + \frac{3.6}{2}$

$= 26.8\text{m}$

$vt = 26.8$

$v = \frac{26.8}{0.8}$

$v = 33.5\text{ m/s}$

$\therefore$ initial velocity is 33.5 m/s

Vertical velocity is $\frac{d}{dt}(-5t^2) = -10t$

When $t = 0.8$

(d)

vertical velocity $= -10(0.8)$
$= -8$

$\therefore$ her vertical velocity is -8 m/s