NESA Number:

Fort Street
High School

Name:

2025 TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Extension 1

Teacher	Hussein	Kaur	Choi	Razzaghi	Trifunovic
	☐	☐	☐	☐	☐

General Instructions

- Reading time – 10 minutes
- Working time – 2 hours
- Write using black pen
- Calculators approved by NESA may be used
- A reference sheet is provided
- For questions in Section II, show relevant mathematical reasoning and/ or calculations
- Marks may be deducted for careless or badly arranged work.

Question	Mark
Multiple Choice	**/10**
Question 11	**/12**
Question 12	**/12**
Question 13	**/12**
Question 14	**/12**
Question 15	**/12**
Total	**/70**

Total marks: 70

Section I – 10 marks

- Attempt Questions 1 – 10
- Allow about 15 minutes for this section

Section II – 60 marks

- Attempt Questions 11 – 15
- Allow about 1 hours and 45 minutes for this section
- Write your student number on each answer booklet

Section I

10 marks
Attempt Questions 1-10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1-10.

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1. Which of the following vectors is perpendicular to $-2\underset{\sim}{i}+5\underset{\sim}{j}$?

A. $5\underset{\sim}{i} - 2\underset{\sim}{j}$

B. $5\underset{\sim}{i} + 2\underset{\sim}{j}$

C. $2\underset{\sim}{i} - 5\underset{\sim}{j}$

D. $2\underset{\sim}{i} + 5\underset{\sim}{j}$

2. Which expression is equivalent to $\sin\left(\frac{\pi}{4}-x\right)\cos\left(\frac{\pi}{4}-x\right)$?

A. $\frac{1}{2}\sin 2x$

B. $2\cos 2x$

C. $\frac{1}{2}\cos 2x$

D. $\frac{1}{2}\sin\left(\frac{\pi}{4}-2x\right)$

3. What are the equations of asymptotes of the function $y = \frac{e^x + 2}{e^x - 2}$?

A. $x = 2,\ y = 1$

B. $x = \ln 2,\ y = -1$

C. $x = \ln 2,\ y = 1$

D. $x = \ln 2,\ y = \pm 1$

4. 9 people are to be arranged into two groups to sit around two circular tables. One group will consist of 4 people and will sit at Table 1, while the remaining people will be sitting at Table 2.

In how many ways can they sit around the two tables?

A. 18 144

B. 90 720

C. 362 880

D. 8 709 120

5. Which one of the following functions, $f(x)$, and its inverse function, $f^{-1}(x)$, are equivalent, i.e. where $f(x) \equiv f^{-1}(x)$?

A. $f(x) = \frac{1}{x}$ for $(-\infty, 0) \cup (0, \infty)$

B. $f(x) = |e^x|$ for $(-\infty, \infty)$

C. $f(x) = \sin|x|$ for $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

D. $f(x) = x^2$ for $[0, \infty)$

6. Given $\cos(\theta + \phi) = \frac{4}{5}$ and $\cot\theta \cot\phi = -\frac{1}{3}$, then $\cos(\theta - \phi)$ is equivalent to:

A. $-\frac{3}{5}$

B. $-\frac{2}{5}$

C. $\frac{2}{5}$

D. $\frac{3}{5}$

7. $ABCD$ is a parallelogram. Vector $\overrightarrow{AB} = \begin{bmatrix} 4 \\ 3 \end{bmatrix}$ and Vector $\overrightarrow{BC} = \begin{bmatrix} 8 \\ -6 \end{bmatrix}$.

What is the area of the parallelogram $ABCD$?

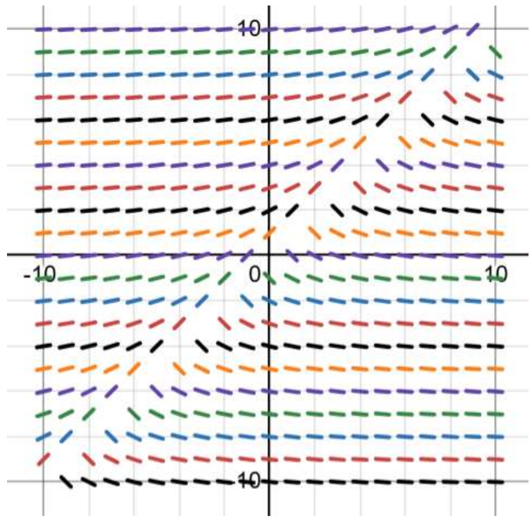
A. 14 square units

B. 24 square units

C. 28 square units

D. 48 square units

8. Which of the following differential equations matches the slope field shown?



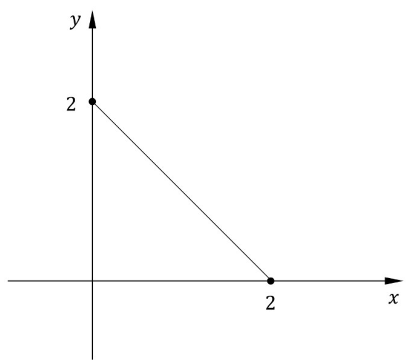
A. $\dfrac{dy}{dx} = y - x$

B. $\dfrac{dy}{dx} = x - y$

C. $\dfrac{dy}{dx} = \dfrac{1}{y - x}$

D. $\dfrac{dy}{dx} = \dfrac{1}{x - y}$

9. The diagram shows the interval from (0,2) to (2,0).



Which pair of parametric equations does NOT represent the interval shown?

A. $\begin{cases} x = e^t - 1 \\ y = 3 - e^t \end{cases}$ for $0 \le t \le \ln 2$

B. $\begin{cases} x = 2(1 - t^2) \\ y = 2t^2 \end{cases}$ for $0 \le t \le 1$

C. $\begin{cases} x = 2\sin t \\ y = 2 - 2\sqrt{1 - \cos^2 t} \end{cases}$ for $0 \le t \le \dfrac{\pi}{2}$

D. $\begin{cases} x = 2 - \dfrac{2}{\pi}\cos^{-1} t \\ y = \dfrac{2}{\pi}\cos^{-1} t \end{cases}$ for $-1 \le t \le 1$

10. The polynomial $P(x) = 6x^4 - x^3 - 17x^2 + 16x - 4$ has four real roots.
What is the sum of the roots of the polynomial $P\left(-\frac{1}{2}x - 1\right)$?

A. $\frac{5}{6}$

B. $\frac{1}{12}$

C. $-2\frac{1}{3}$

D. $-8\frac{1}{3}$

Section II

60 marks

Attempt Questions 11–15

Allow about 1 hour and 45 minutes for this section

Answer each question in the appropriate writing booklet. Extra writing booklets are available.

For questions in Section II, your responses should include relevant mathematical reasoning and/or calculations

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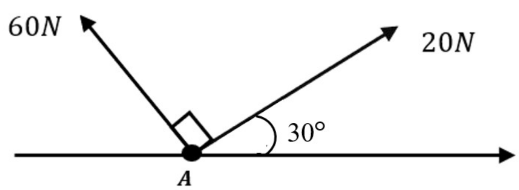
Question 11 (12 marks) Use the Question 11 Writing Booklet

(a) Find the exact value of $\cos\left(\sin^{-1}\frac{2}{3} + \sin^{-1}\frac{1}{4}\right)$. **3**

(b) Consider the $f(x) = 6\sin x + 2\cos x - 13$, for $0 \le x \le 2\pi$. **2**

Explain why the graph of $y = \frac{1}{f(x)}$ has no vertical asymptotes.

(c) Forces of $60N$ and $20N$ act on an object, at point A, as shown in the diagram. **2**



NOT TO SCALE

The vector sum of forces acting on an object is called the resultant force.

Find the magnitude of the resultant force vector acting on object A.

Question 11 continues on page 7

Question 11 (continued)

(d) Find the term independent of x in the expansion of $\left(2x - \frac{3}{x^2}\right)^6$. **3**

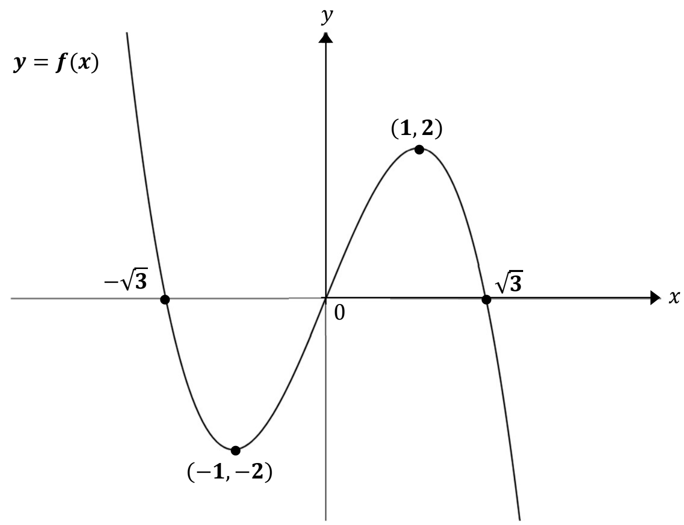
(e) Given $y = e^{\tan^{-1} x}$ show that: **2**

$$\frac{d^2y}{dx^2}\left(1+x^2\right)^2 = y - 2xy\,.$$

End of Question 11

Question 12 (12 marks) Use the Question 12 Writing Booklet

(a) The diagram shows the graph of $y = f(x)$. **3**



The diagram is reproduced in the Question 12 Answer Sheet provided.

Sketch the graph of $y = \frac{1}{f(x)}$ on the Question 12 Answer Sheet provided, labelling all turning points and asymptotes.

(b) The following letters are arranged in a row.

E	**F**	**T**	**E**	**G**	**E**	**H**	**E**	**F**	**T**	**E**

(i) Find the number of different arrangements that can be made. **1**

(ii) Given that the 11 letters are arranged randomly, find the probability that all 5 E's are together. **1**

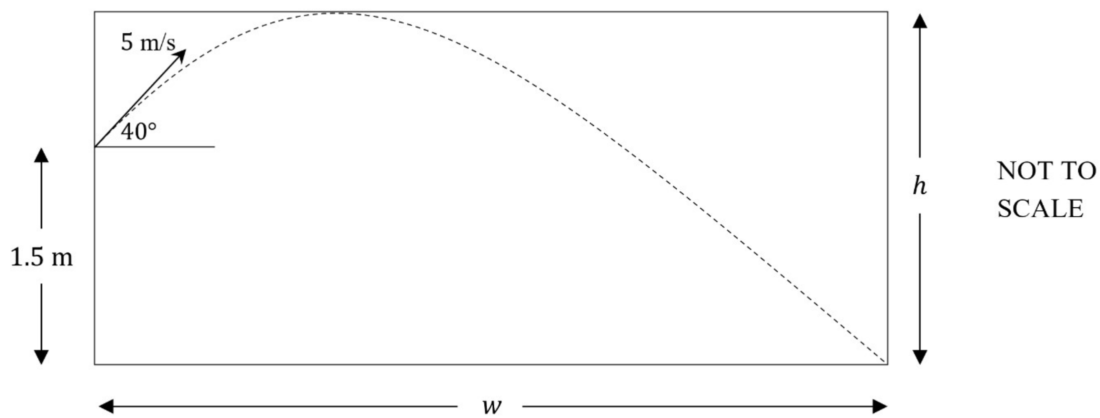
(iii) Find the number of different arrangements in which the 2 F's are next to each other, the 2 T's are next to each other, exactly 4 of the E's are next to each other, and the G is next to the H. **3**

Question 12 continues on page 9

(c) A student is standing against a wall in a room which is h metres high and w metres wide. They throw a ball from a height of 1.5 metres, at a speed of 5 m/s with an angle of projection of 40°. The ball just touches the ceiling before landing at the junction of the floor and the wall, as shown in the diagram. **4**

The horizontal and vertical displacement of the ball t seconds after projection are

$x = 5t\cos 40°$ and $y = 5t\sin 40° - 5t^2 + 1.5$. (Do NOT prove these.)



Find the width and height of the room, correct to the nearest centimetre.

End of Question 12

Question 13 (12 marks) Use the Question 13 Writing Booklet

(a) Using $t = \tan\frac{x}{2}$, prove that: **2**

$$\frac{2}{1+\cos x} = \sec^2\left(\frac{x}{2}\right).$$

(b) (i) Show that $\tan 3\theta = \dfrac{3\tan\theta - \tan^3\theta}{1 - 3\tan^2\theta}$. **2**

(ii) Hence, by letting $x = \tan\theta$, solve **3**

$$x^3 - 3x^2 - 3x + 1 = 0.$$

(c) Liz heats a mug of milk to 90 $^\circ$C in a microwave. She takes it out into a room where the temperature is a constant of 26 $^\circ$C. The milk cools to 70 $^\circ$C in 5 minutes. At time t minutes, its temperature T, in degrees Celsius, decreases according to the equation:

$$\frac{dT}{dt} = -k(T - 26), \text{ where } k \text{ is a positive constant.}$$

(i) Verify that $T = 26 + Ae^{-kt}$ is a solution of this equation, where A is a constant. **1**

(ii) Find the values of A and k. **2**

(iii) How long will it take for the temperature to cool to 30 $^\circ$C? Give your answer correct to the nearest minute. **2**

End of Question 13

Question 14 (12 marks) Use the Question 14 Writing Booklet

(a) A cone has a fixed height of 24 cm. It's curved surface area, given by $A = \pi r s$, is increasing at the rate of $36\pi\ \text{cm}^2 s^{-1}$. **3**

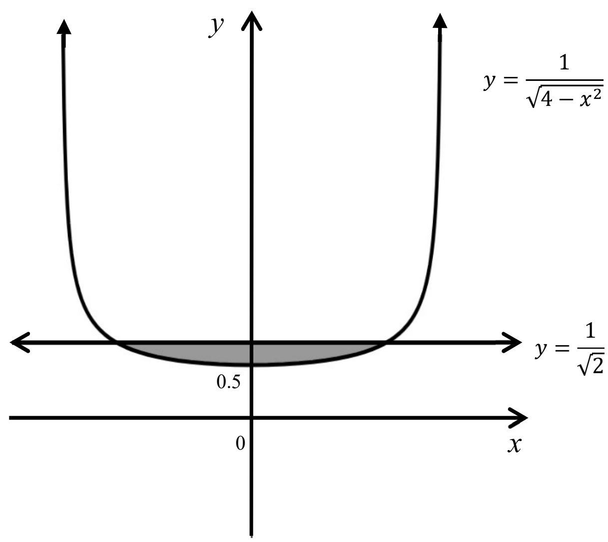
Find the rate at which its radius is increasing when its slant height is 30 cm?

(b) Find one possible set of values for a and b such that: **3**

$$\binom{102}{90} - \binom{100}{11} - \binom{100}{12} = \binom{a}{b}.$$

(d) (i) Show that $\dfrac{1}{4-x^2} = \dfrac{1}{4(2-x)} + \dfrac{1}{4(2+x)}$. **1**

(ii) Consider the graph of $y = \dfrac{1}{\sqrt{4-x^2}}$ and the line $y = \dfrac{1}{\sqrt{2}}$ shown below.



Show that the points of intersection are $\left(\sqrt{2}, \frac{1}{\sqrt{2}}\right)$ and $\left(-\sqrt{2}, \frac{1}{\sqrt{2}}\right)$. **2**

(iii) Hence, find the exact volume generated when the shaded region enclosed by the curve $y = \dfrac{1}{\sqrt{4-x^2}}$ and the line $y = \dfrac{1}{\sqrt{2}}$ is rotated about the x-axis. **3**

End of Question 14

Question 15 (12 marks) Use the Question 15 Writing Booklet

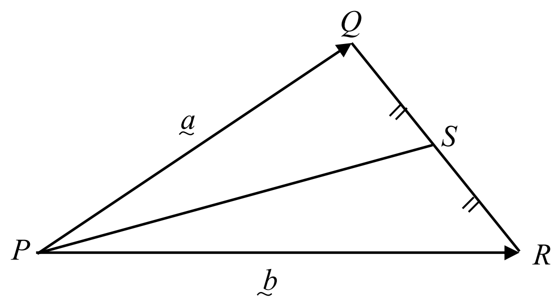
(a) A curve is defined by the parametric equations: **2**

$$x = \text{cosec}(\theta) + 1$$

$$y = 2\cot(\theta)$$

for $\theta \in [0,2\pi]$. Find the cartesian equation of the curve.

(b) PQR is a triangle, S is the midpoint of QR and $\overrightarrow{PQ} = \underset{\sim}{a}$, $\overrightarrow{PR} = \underset{\sim}{b}$.



(i) Show that $\overrightarrow{QS} = \frac{1}{2}\left(\underset{\sim}{b} - \underset{\sim}{a}\right)$ and $\overrightarrow{PS} = \frac{1}{2}\left(\underset{\sim}{a} + \underset{\sim}{b}\right)$. **2**

(ii) Hence, prove $\left|\overrightarrow{PQ}\right|^2 + \left|\overrightarrow{PR}\right|^2 = 2\left(\left|\overrightarrow{PS}\right|^2 + \left|\overrightarrow{QS}\right|^2\right)$. **2**

(c) Use the substitution $u = \dfrac{1+x}{1-x}$ to find $\displaystyle\int \frac{\sqrt{1-x^2}}{(1-x)^2(1+x)}\,dx$. **2**

(d) Use Mathematical Induction to prove that:

$\sin x + \sin 3x + \sin 5x + \ldots + \sin(2n-1)x = \dfrac{\sin^2(nx)}{\sin x}$ for all positive integers, $n \geq 1$. **4**

End of paper

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Mathematics Extension 1

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SOLUTIONS

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- Reading time – 10 minutes
- Working time – 2 hours
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Question	Mark
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Question 14	/12
Question 15	/12
Total	/70

Total marks: 70

Section I – 10 marks

- Attempt Questions 1 – 10
- Allow about 15 minutes for this section

Section II – 60 marks

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- Allow about 1 hours and 45 minutes for this section
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Section I

10 marks
Attempt Questions 1-10
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---

1. Which of the following vectors is perpendicular to $-2\underset{\sim}{i}+5\underset{\sim}{j}$?

A. $5\underset{\sim}{i}-2\underset{\sim}{j}$

B. $5\underset{\sim}{i}+2\underset{\sim}{j}$

C. $2\underset{\sim}{i}-5\underset{\sim}{j}$

D. $2\underset{\sim}{i}+5\underset{\sim}{j}$

$(-2\underset{\sim}{i}+5\underset{\sim}{j})(5\underset{\sim}{i}+2\underset{\sim}{j})$
$= -2\times5+5\times2$
$=0$

2. Which expression is equivalent to $\sin\left(\frac{\pi}{4}-x\right)\cos\left(\frac{\pi}{4}-x\right)$?

A. $\frac{1}{2}\sin 2x$

B. $2\cos 2x$

C. $\frac{1}{2}\cos 2x$

D. $\frac{1}{2}\sin\left(\frac{\pi}{4}-2x\right)$

$\left(\sin\frac{\pi}{4}\cos x-\sin x\cos\frac{\pi}{4}\right)\left(\cos\frac{\pi}{4}\cos x+\sin\frac{\pi}{4}\sin x\right)$
$=\left(\frac{\cos x}{\sqrt2}-\frac{\sin x}{\sqrt2}\right)\left(\frac{\cos x}{\sqrt2}+\frac{\sin x}{\sqrt2}\right)$
$=\frac{\cos^2 x}{2}-\frac{\sin^2 x}{2}$
$=\frac{1}{2}\cos 2x$

3. What are the equations of asymptotes of the function $y=\dfrac{e^x+2}{e^x-2}$?

A. $x=2,\ y=1$

B. $x=\ln 2,\ y=-1$

C. $x=\ln 2,\ y=1$

D. $x=\ln 2,\ y=\pm1$

- Vertical asymptote occurs when denominator $=0$
  ie, $e^x-2=0 \quad \therefore x=\ln 2$
- Horizontal asymptote:
  as $x\to-\infty \quad e^x\to0$ so $\frac{e^x+2}{e^x-2}\to-1$

  as $x\to\infty \quad e^x\to\infty$ so $\frac{e^x+2}{e^x-2}\to1$

  $\therefore y=\pm1$

4. 9 people are to be arranged into two groups to sit around two circular tables. One group will consist of 4 people and will sit at Table 1, while the remaining people will be sitting at Table 2.

In how many ways can they sit around the two tables?

A. 18 144

B. 90 720

C. 362 880

D. 8 709 120

$^{9}C_4 \times {}^{5}C_5 \times 3! \times 4! = 18\,144$

5. Which one of the following functions, $f(x)$, and its inverse function, $f^{-1}(x)$, are equivalent, i.e. where $f(x) \equiv f^{-1}(x)$?

A. $f(x) = \frac{1}{x}$ for $(-\infty, 0) \cup (0, \infty)$

B. $f(x) = |e^x|$ for $(-\infty, \infty)$

C. $f(x) = \sin|x|$ for $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

D. $f(x) = x^2$ for $[0, \infty)$

let $f(x) = y$

$y = \frac{1}{x}$

$x = \frac{1}{y}$

$\therefore y = \frac{1}{x}$

$\therefore f^{-1}(x) = f(x) = 1/x$

6. Given $\cos(\theta + \phi) = \frac{4}{5}$ and $\cot\theta \cot\phi = -\frac{1}{3}$, then $\cos(\theta - \phi)$ is equivalent to:

A. $-\frac{3}{5}$

B. $-\frac{2}{5}$

C. $\frac{2}{5}$

D. $\frac{3}{5}$

$\cos\theta\cos\phi - \sin\theta\sin\phi = \frac{4}{5}$ —— (1)

$\cot\theta\cot\phi = \frac{\cos\theta\cos\phi}{\sin\theta\sin\phi} = -1/3$

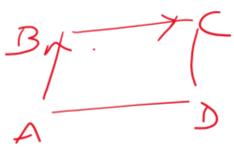
$\sin\theta\sin\phi = -3\cos\theta\cos\phi$ —(2)

sub (2) into (1)

$\cos\theta\cos\phi + 3\cos\theta\cos\phi = 4/5 \quad \therefore \cos\theta\cos\phi = \frac{1}{5}$

$\cos(\theta - \phi) = \cos\theta\cos\phi + \sin\theta\sin\phi = \frac{1}{5} - \frac{3}{5} = -2/5$

7. $ABCD$ is a parallelogram. Vector $\overrightarrow{AB} = \begin{bmatrix} 4 \\ 3 \end{bmatrix}$ and Vector $\overrightarrow{BC} = \begin{bmatrix} 8 \\ -6 \end{bmatrix}$.



What is the area of the parallelogram $ABCD$? let $\overrightarrow{AB} = \underset{\sim}{u}$ $\overrightarrow{BC} = \underset{\sim}{v}$

A. 14 square units

B. 24 square units

C. 28 square units

D. 48 square unitsm

$\underset{\sim}{u} \cdot \underset{\sim}{v} = |\underset{\sim}{u}||\underset{\sim}{v}| \cos\angle BAD$

$\begin{bmatrix} 4 \\ 3 \end{bmatrix} \cdot \begin{bmatrix} 8 \\ -6 \end{bmatrix} = (\sqrt{16+9})(\sqrt{64+36}) \cos\angle BAD$

$14 = 5 \times 10 \cos\angle BAD \rightarrow \angle BAD = \cos^{-1}\left(\frac{7}{25}\right)$

$\sin\angle BAD = \frac{24}{25}$

$\text{Area} = 2 \times \frac{1}{2} \times 5 \times 10 \sin\angle BAD = 50 \times \frac{24}{25}$

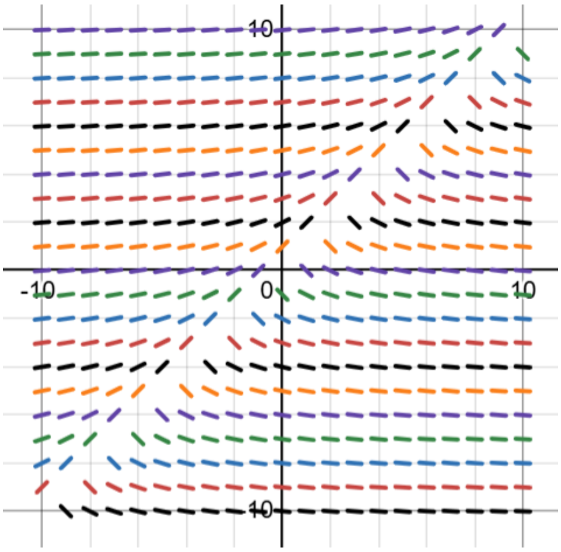
8. Which of the following differential equations matches the slope field shown?

A. $\dfrac{dy}{dx} = y - x$

B. $\dfrac{dy}{dx} = x - y$

C. $\dfrac{dy}{dx} = \dfrac{1}{y - x}$

D. $\dfrac{dy}{dx} = \dfrac{1}{x - y}$



slope field is not defined on $y=x$ so either C or D is the answer.

when x is a large number y is smaller than x and gradient is negative.

$\therefore \dfrac{dy}{dx} = \dfrac{1}{y-x}$

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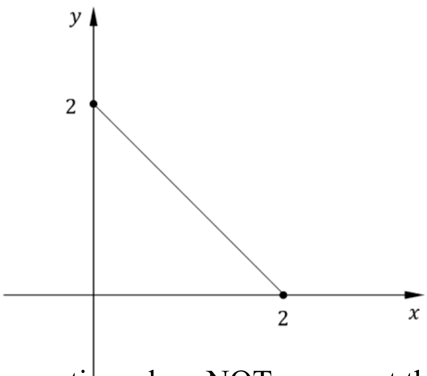
9. The diagram shows the interval from (0,2) to (2,0).



Which pair of parametric equations does NOT represent the interval shown?

A. $\begin{cases} x = e^t - 1 \\ y = 3 - e^t \end{cases}$ for $0 \le t \le \ln 2$

B. $\begin{cases} x = 2(1 - t^2) \\ y = 2t^2 \end{cases}$ for $0 \le t \le 1$

C. $\begin{cases} x = 2\sin t \\ y = 2 - 2\sqrt{1 - \cos^2 t} \end{cases}$ for $0 \le t \le \dfrac{\pi}{2}$

D. $\begin{cases} x = 2 - \dfrac{2}{\pi}\cos^{-1} t \\ y = \dfrac{2}{\pi}\cos^{-1} t \end{cases}$ for $-1 \le t \le 1$

When $t=0$ $x = e^0 - 1 = 1-1 = 0$, $y = 3 - e^0 = 3-1 = 2$ correct

When $x = 2$ $2 = e^t - 1$ $e^t = 3$

When $e^t = 3$ $y = 3-3 = 0$ correct

$\therefore$ A

10. The polynomial $P(x) = 6x^4 - x^3 - 17x^2 + 16x - 4$ has four real roots.

What is the sum of the roots of the polynomial $P\left(-\frac{1}{2}x - 1\right)$?

A. $\frac{5}{6}$

B. $\frac{1}{12}$

C. $-2\frac{1}{3}$

D. $-8\frac{1}{3}$

$P(-1/2(x+2)) \Rightarrow$ dilation factor 2, reflection in x axis and translation 2 units left.

let roots of $P(x)$ be $\alpha, \beta, \gamma, \delta$

Sum of roots: $\alpha + \beta + \gamma + \delta = 1/6$

Roots of $P(-1/2(x+2))$ are:

$-2\alpha - 2, -2\beta - 2, -2\gamma - 2, -2\delta - 2$

Sum of roots

$= -2\alpha - 2 - 2\beta - 2 - 2\gamma - 2 - 2\beta - 2$

$= -2(\alpha + \beta + \gamma + \delta) - 8$

$= -2 \times \frac{1}{6} - 8$

$= -8\,1/3$

Section II

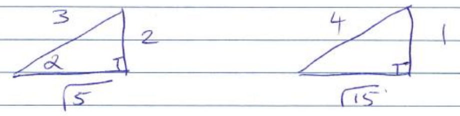
60 marks
Attempt Questions 11–15
Allow about 1 hour and 45 minutes for this section

Answer each question in the appropriate writing booklet. Extra writing booklets are available.
For questions in Section II, your responses should include relevant mathematical reasoning and/or calculations

Question 11 (12 marks) Use the Question 11 Writing Booklet

(a) Find the exact value of $\cos\left(\sin^{-1}\frac{2}{3}+\sin^{-1}\frac{1}{4}\right)$. **3**

let $\alpha = \sin^{-1}(2/3)$ $\quad \beta = \sin^{-1}(1/4)$



$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

$\cos\beta = \frac{\sqrt{15}}{4}$ $\quad \cos\alpha = \frac{\sqrt{5}}{3}$

hence,

$\cos(\alpha+\beta) = \frac{\sqrt{5}}{3}\times\frac{\sqrt{15}}{4} - \frac{2}{3}\times\frac{1}{4}$

$= \frac{5\sqrt{3}-2}{12}$

Marking Criteria:

3 – Correct Solution

2 – Applies an appropriate compound angle result

1 – Finds exact values of $\cos\left(\sin^{-1}\frac{2}{3}\right)$ and $\cos\left(\sin^{-1}\frac{1}{4}\right)$ or equivalent merit

(b) Consider the $f(x) = 6\sin x + 2\cos x - 13$, for $0 \le x \le 2\pi$. **2**

Explain why the graph of $y = \frac{1}{f(x)}$ has no vertical asymptotes.

$6\sin x + 2\cos x - 13 = R\sin(x+\alpha) - 13$

$R = 2\sqrt{10}$ $\quad \alpha$ is a constant

hence,

$6\sin x + 2\cos x - 13 = 2\sqrt{10}\sin(x+\alpha) - 13$

Vertical asymptotes occur when $f(x) = 0$

$2\sqrt{10}\sin(x+\alpha) = 13$

$\sin(x+\alpha) = \frac{13}{2\sqrt{10}}$

which has no solution since $-1 \le \sin\theta \le 1$

Since $f(x) \neq 0$ then $\frac{1}{f(x)}$ has no vertical asymptote.

Marking Criteria:

2 – Correct reason

1 – Attempts to write $2\sin x + 6\cos x$ as single function or equivalent merit

$f(x) \neq 0$

$f(x) = 0$

Alternative solution:

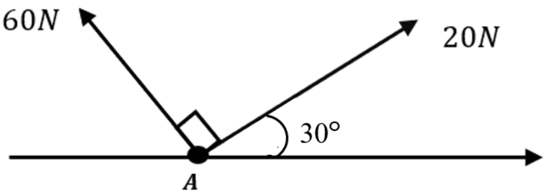
Since $-1 \le \sin x \le 1$; $-1 \le \cos x \le 1$

then $-6 \le 6\sin x \le 6$; $-2 \le 2\cos x \le 2$

$\therefore 6\sin x + 2\cos x - 13 < 0$

$\therefore f(x) \neq 0$

(c) Forces of $60N$ and $20N$ act on an object, at point A, as shown in the diagram. **2**



NOT TO SCALE

The vector sum of forces acting on an object is called the resultant force.

Find the magnitude of the resultant force vector acting on object A.

$\underset{\sim}{F_r} = (20\cos 30° - 60\cos 60°)\underset{\sim}{i} + (20\sin 30° + 60\sin 60°)\underset{\sim}{j}$

$= (10\sqrt{3} - 30)\underset{\sim}{i} + (10 + 30\sqrt{3})$

$|\underset{\sim}{F_r}| = \sqrt{(10\sqrt{3} - 30)^2 + (10 + 30\sqrt{3})^2}$

$= 63.25\,N$ or $20\sqrt{10}\,N$

Marking Criteria:

2 – Correct Solution

1 – Correctly finds forces in horizontal & vertical directions

Question 11 continues on page 7

Question 11 (continued)

(d) Find the term independent of x in the expansion of $\left(2x - \frac{3}{x^2}\right)^6$. **3**

$$(2x - 3x^{-2})^6 = \sum_{x=0}^{6} {}^6C_r (2x)^r (-3x^{-2})^{6-r}$$

$r - 12 + 2r = 0 \quad \therefore r = 4$

Term independent of x:

$${}^6C_4 \, (2x)^4 (-3x^{-2})^2$$

$$= 15 \times 2^4 \times (-3)^2$$

$$= 2160$$

Marking Criteria:
3 – Correct Answer
2 – Correct value of r
1 – Correctly sets up the general term

(e) Given $y = e^{\tan^{-1} x}$ show that: **2**

$$\frac{d^2y}{dx^2}(1+x^2)^2 = y - 2xy.$$

$$y = e^{\tan^{-1}x}$$

$$\frac{dy}{dx} = \frac{1}{1+x^2} e^{\tan^{-1}x} = (1+x^2)^{-1} e^{\tan^{-1}x}$$

$$\frac{d^2y}{dx^2} = \frac{-1 \times 2x}{(1+x^2)^2} e^{\tan^{-1}x} + \frac{1}{1+x^2} \times \frac{1}{1+x^2} e^{\tan^{-1}x}$$

$$= \frac{-2xe^{\tan^{-1}x}}{(1+x^2)^2} + \frac{1}{(1+x^2)^2} e^{\tan^{-1}x}$$

$$\frac{d^2y}{dx^2}(1+x^2)^2 = -2xe^{\tan^{-1}x} + e^{\tan^{-1}x}$$

but $y = e^{\tan^{-1}x}$

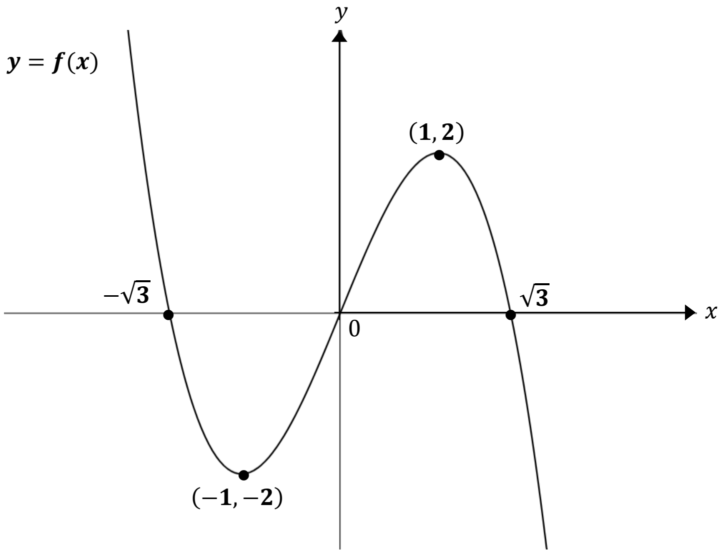
$\therefore \frac{d^2y}{dx^2}(1+x^2)^2 = y - 2xy$ as req'd.

Marking Criteria:
2 – Correct Solution
1 – obtains correct expression with one error

End of Question 11

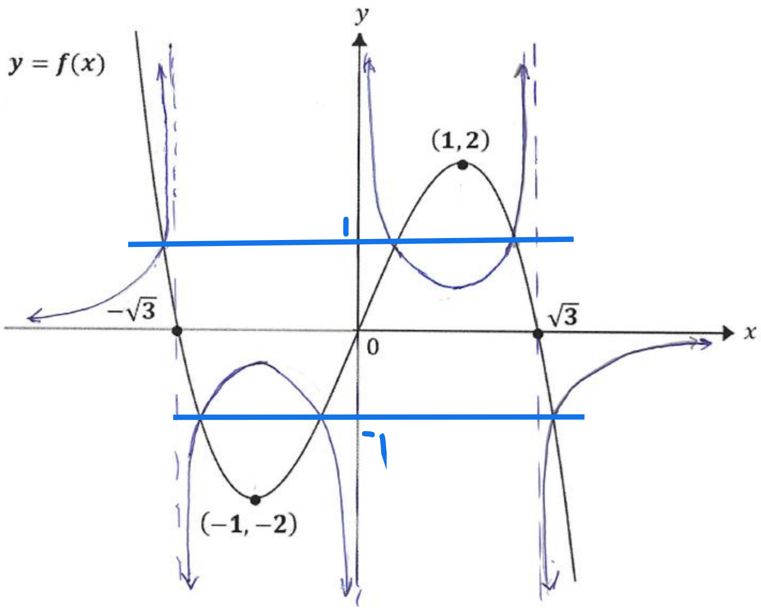
Question 12 (12 marks) Use the Question 12 Writing Booklet

(a) The diagram shows the graph of $y = f(x)$. **3**



The diagram is reproduced in the Question 12 Answer Sheet provided.

Sketch the graph of $y = \frac{1}{f(x)}$ on the Question 12 Answer Sheet provided, labelling all turning points and asymptotes.



Marking Criteria:

3 – Correct Solution

2 – Draws the correct curve **and** writes the coordinates of the turning points **or** shows the vertical and horizontal asymptotes, or equivalent merit

1 – Attempts to sketch a reciprocal curve or equivalent merit

(b) The following letters are arranged in a row.

E	F	T	E	G	E	H	E	F	T	E

(i) Find the number of different arrangements that can be made. **1**

$$\frac{11!}{2!2!5!} = 83160$$

Marking Criteria:
1 – Correct Solution

(ii) Given that the 11 letters are arranged randomly, find the probability that all 5 E's are together. **1**

$\boxed{E|E|E|E|E}$ _ _ _ _ _ _

Number of arrangements of 5 E's together:

$$\frac{7!}{2!2!} = 1260$$

$$\text{Prob.} = \frac{1260}{83160}$$

$$= \frac{1}{66}$$

Marking Criteria:
1 – Correct Solution

(iii) Find the number of different arrangements in which the 2 F's are next to each other, the 2 T's are next to each other, exactly 4 of the E's are next to each other, and the G is next to the H. **3**

_ $\boxed{F|F}$ _ $\boxed{T|T}$ _ $\boxed{G|H}$ _

4 spaces for $\boxed{EEEE}$

3 spaces for E

3! spaces for $\boxed{FF}$, $\boxed{TT}$, $\boxed{GH}$

2! for $\boxed{GH}$

No. of ways $= 4 \times 3 \times 3! \times 2!$
$= 144$

Marking Criteria:
3 – Correct Solution
2 – Partially correct solution
1 – Makes progress towards correct solution

Question 12 continues on page 9

Alternative Solution 1

No of arrangements $= \dfrac{11!}{2!\,2!\,5!}$

$= 83160$

No of arrangements of E's together

$= \dfrac{7!}{2!\,2!} = 1260$

$\therefore$ P(E's together) $= \dfrac{1260}{83160}$

$= \dfrac{1}{66}$

$\boxed{EEEE}$ $\boxed{T\ T}$ $\boxed{G\ H}$ $\boxed{F\ F}$ $\boxed{E}$

G & H

All possible combinations $= 5! \times 2!$

$= 240$

G & H

Cases where all E's are together $= 4! \times 2! \times 2!$

$= 96$

arrangement of $\boxed{E's}$ $\boxed{E}$

$\therefore$ No of ways $= 240 - 96$

$= 144$

Alternative Solution 2

Cases :

1. $\boxed{\qquad}$ $3 \times 3 \times 2 \times 1$

2. 3 $\boxed{\qquad}$ $\times 2 \times 2 \times 1$

3. 2×3 $\boxed{\qquad}$ $\times 2 \times 1$

4. $1 \times 2 \times 3$ $\boxed{\qquad}$ $\times 2$

5. $1 \times 2 \times 3 \times 3$ $\boxed{\qquad}$

G & H

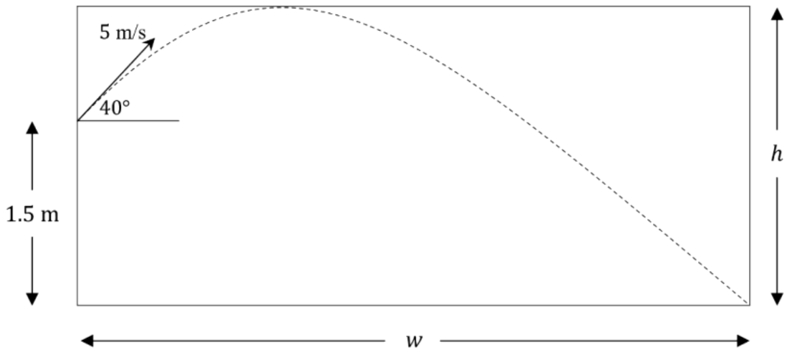
No. of ways $= [3! \times 3 \times 2 + 3! \times 2 \times 3] \times 2!$

$= 144$

(c) A student is standing against a wall in a room which is h metres high and w metres wide. They throw a ball from a height of 1.5 metres, at a speed of 5 m/s with an angle of projection of 40°. The ball just touches the ceiling before landing at the junction of the floor and the wall, as shown in the diagram. **4**

The horizontal and vertical displacement of the ball t seconds after projection are

$x = 5t\cos 40°$ and $y = 5t\sin 40° - 5t^2 + 1.5$. (Do NOT prove these.)



NOT TO SCALE

Find the width and height of the room, correct to the nearest centimetre.

$x = 5t\cos 40°$ $\quad y = 5t\sin 40 - 5t^2 + 1.5$

$\dot{y} = 5\sin 40 - 10t$ $\quad \dot{y} = 0$ when touches ceiling

$5\sin 40 = 10t$

$\therefore t = \frac{\sin 40}{2}$

When $t = \frac{\sin 40}{2}$

$H = 5 \times \frac{\sin 40}{2} \times \sin 40 - 5\left(\frac{\sin 40}{2}\right)^2 + 1.5$

$= 2.02\text{ m}$

When $y = 0$

$5t^2 - 5t\sin 40 - 1.5 = 0$

$t = \frac{5\sin 40 \pm \sqrt{(5\sin 40)^2 - 4(5)(-1.5)}}{10}$

$= 0.9564...$

$W = 5 \times 0.9564...\cos 40°$

$= 3.6634...$

$\therefore$ width $= 3.66\text{m}$.

Marking Criteria:
4 – Correct Solution
3 – Finds the height of the room, or finds the width of the room, or equivalent merit
2 – Finds the time to max height of the ball, or the width of the room, or equivalent merit
1 – Finds an expression for the vertical velocity or the time until $y = 0$, or equivalent merit

End of Question 12

Question 13 (12 marks) Use the Question 13 Writing Booklet

(a) Using $t = \tan\frac{x}{2}$, prove that: **2**

$$\frac{2}{1+\cos x} = \sec^2\left(\frac{x}{2}\right).$$

$t = \tan x/2.$

$LHS = \dfrac{2}{1+\cos x}$ $\quad \cos x = \dfrac{1-t^2}{1+t^2}$

$= \dfrac{2}{1 + \dfrac{1-t^2}{1+t^2}}$

$= \dfrac{2(1+t^2)}{1+t^2+1-t^2}$

$= \dfrac{2(1+t^2)}{2}$

$= 1+t^2$

$= 1+\tan^2\left(\frac{x}{2}\right)$

$= \sec^2\left(\frac{x}{2}\right)$ pythagorean identity

Marking Criteria:

2 – Correct Solution

1 – Prove the LHS or RHS = 1 + t^2, using the t substitution, or $\sec^2\left(\frac{x}{2}\right) = 1 + \tan^2\left(\frac{x}{2}\right) = 1 + t^2$ or equivalent merit

(b) (i) Show that $\tan 3\theta = \dfrac{3\tan\theta - \tan^3\theta}{1-3\tan^2\theta}$. **2**

$LHS = \tan 3\theta$

$= \tan(2\theta + \theta)$

$= \dfrac{\tan 2\theta + \tan\theta}{1 - \tan\theta\tan 2\theta}$

$= \dfrac{\dfrac{2\tan\theta}{1-\tan^2\theta} + \tan\theta}{1 - \tan\theta \times \dfrac{2\tan\theta}{1-\tan^2\theta}}$

$= \dfrac{2\tan\theta + \tan\theta - \tan^3\theta}{1-\tan^2\theta - 2\tan^2\theta}$

$= \dfrac{3\tan\theta - \tan^3\theta}{1-3\tan^2\theta}$ as req'd

Marking Criteria:

2 – Correct Solution

1 – Use correct compound angle or equivalent merit

(ii) Hence, by letting $x = \tan\theta$, solve **3**

$$x^3 - 3x^2 - 3x + 1 = 0.$$

$\tan^3\theta - 3\tan^2\theta - 3\tan\theta + 1 = 0$

$1 - 3\tan^2\theta = 3\tan\theta - \tan^3\theta$

$\dfrac{3\tan\theta - \tan^3\theta}{1 - 3\tan^2\theta} = 1.$

from part (i) $\tan 3\theta = 1$

$3\theta = k\pi + \dfrac{\pi}{4}$ where $k \in \mathbb{Z}$

$\theta = \dfrac{k\pi}{3} + \dfrac{\pi}{12}$

when $k=0$ $\theta = \pi/12$

$k=1$ $\theta = \dfrac{5\pi}{12}$

$k=2$ $\theta = \dfrac{3\pi}{4}$

$\therefore x = \tan\pi/12,\ \tan\dfrac{5\pi}{12},\ \tan 3\pi/4$

$= -1,\ \tan\dfrac{\pi}{12},\ \tan\dfrac{5\pi}{12}$

(accept: $-1,\ 0.2679,\ 3.73205$

Marking Criteria:

3 – Correct Solution

2 – Find three different values for 3θ, or one correct x value or equivalent merit

1 – Correctly moves two terms of $x^3 - 3x^2 - 3x + 1 = 0$ to the other side of the equation (can still be in terms of x or in terms of tan θ), or equivalent merit

(c) Liz heats a mug of milk to 90 °C in a microwave. She takes it out into a room where the temperature is a constant of 26 °C. The milk cools to 70 °C in 5 minutes. At time *t* minutes, its temperature *T*, in degrees Celsius, decreases according to the equation:

$$\frac{dT}{dt} = -k(T - 26),\ \text{where } k \text{ is a positive constant.}$$

(i) Verify that $T = 26 + Ae^{-kt}$ is a solution of this equation, where A is a constant. **1**

$T = 26 + Ae^{-kt} \Rightarrow T - 26 = Ae^{-kt}$

$\dfrac{dT}{dt} = -kAe^{-kt}$

but $Ae^{-kt} = T - 26$, hence.

$\dfrac{dT}{dt} = -k(T - 26)$ as req'd.

Marking Criteria:

1 – Correct Solution

(ii) Find the values of A and k. **2**

when $t=0$ $T=90°$

$90 = 26 + Ae^0$

$\therefore A = 64$

hence,

$T = 26 + 64e^{-kt}$

when $t=5$ $T=70°$

$70 = 26 + 64e^{-5k}$

$e^{-5k} = \frac{44}{64} = \frac{11}{16}$

$-5k = \ln\left(\frac{11}{16}\right)$

$\therefore k = -\frac{1}{5}\ln\left(\frac{11}{16}\right)$.

Marking Criteria:

2 – Correct Solution

1 – Finds correct value for A or k

Marker's comments:

Mostly well done, those got 1 mark usually only found A. Those who got the correct value of k usually find A correctly.

(iii) How long will it take for the temperature to cool to 30 °C? Give your answer correct to the nearest minute. **2**

$T = 30°$

$30 = 26 + 64e^{-\left(-\frac{1}{5}\ln\left(\frac{11}{16}\right)\right)t}$

$e^{\frac{1}{5}\ln\left(\frac{11}{16}\right)t} = \frac{1}{16}$

$\frac{1}{5}\ln\left(\frac{11}{16}\right)t = \ln\left(\frac{1}{16}\right)$

$\therefore t = 36.998\ldots$

$= 37$ mins

Marking Criteria:

2 – Correct Solution

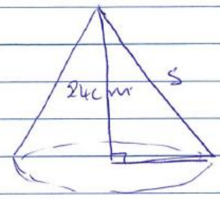
1 – Makes progress towards finding time.

End of Question 13

Question 14 (12 marks) Use the Question 14 Writing Booklet

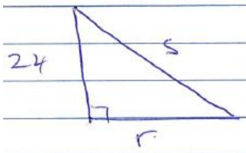
(a) A cone has a fixed height of 24 cm. It's curved surface area, given by $A = \pi r s$, is increasing at the rate of 36π cm$^2 s^{-1}$. **3**

Find the rate at which its radius is increasing when its slant height is 30 cm?



$S = \pi r s \qquad \frac{dS}{dt} = 36\pi$

$\frac{dr}{dt} = \frac{dS}{dt} \times \frac{dr}{dS}$



$s^2 = 24^2 + r^2$

hence

$S = \pi r (576 + r^2)^{1/2}$

$\frac{dS}{dr} = \pi\sqrt{576+r^2} + \frac{\frac{1}{2} \times \pi r \times 2r}{\sqrt{576+r^2}}$

$\frac{dS}{dr} = \frac{\pi(576+r^2) + \pi r^2}{\sqrt{576+r^2}}$

when $s = 30 \qquad r = \sqrt{30^2 - 24^2}$

$= 18$

$\frac{dr}{dt} = 36\cancel{\pi} \times \frac{\sqrt{576+18^2}}{\cancel{\pi}(576 + 2 \times 18^2)}$

$= \frac{15}{17}$ cm s^{-1}

Marking Criteria:

3 – Correct Solution

2 – Attempts to find $\frac{dr}{dt}$ using $\frac{dA}{dr}$ and $\frac{dS}{dt}$

1 – Indicates $\frac{dA}{dr} = \pi\sqrt{r^2+144} + \frac{\pi r^2}{\sqrt{r^2+144}}$ or equivalent merit

(b) Find one possible set of values for a and b such that: **3**

$$\binom{102}{90} - \binom{100}{11} - \binom{100}{12} = \binom{a}{b}.$$

$\binom{102}{90} - \left\{\binom{100}{11} + \binom{100}{12}\right\} = \binom{a}{b}$

$\binom{102}{90} - \binom{101}{12} = \binom{a}{b}$

$\binom{a}{b} + \binom{101}{12} = \binom{102}{90}$

$\therefore a = 101$

$b = 11$ or 90

Marking Criteria:

3 – Correct Solution

2 – Finds correct value of a or b

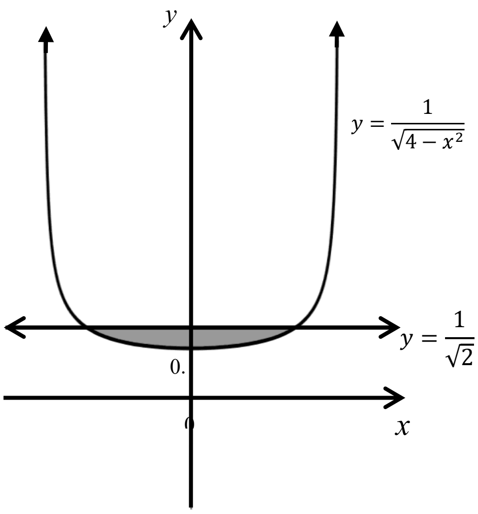
1 – Correctly applies Pascals' identity or equivalent merit

(d) (i) Show that $\frac{1}{4-x^2} = \frac{1}{4(2-x)} + \frac{1}{4(2+x)}$. **1**

$$RHS = \frac{2+x+2-x}{4(2-x)(2+x)}$$
$$= \frac{4}{4(4-x^2)}$$
$$= \frac{1}{4-x^2}$$
$$= RHS$$

Marking Criteria:
1 – Correct Solution

(ii) Consider the graph of $y = \frac{1}{\sqrt{4-x^2}}$ and the line $y = \frac{1}{\sqrt{2}}$ shown below.



Show that the points of intersection are $\left(\sqrt{2}, \frac{1}{\sqrt{2}}\right)$ and $\left(-\sqrt{2}, \frac{1}{\sqrt{2}}\right)$. **2**

$$\frac{1}{\sqrt{4-x^2}} = \frac{1}{\sqrt{2}}$$
$$\sqrt{4-x^2} = \sqrt{2}$$
$$4-x^2 = 2$$
$$x^2 = 2$$
$$\therefore x = \pm\sqrt{2}$$
$$y = \frac{1}{\sqrt{2}}$$

$\therefore$ Points of intersection are $(\sqrt{2}, 1/\sqrt{2})$ $(-\sqrt{2}, 1/\sqrt{2})$

Marking Criteria:
2 – Correct Solution
1 – Finds one correct value of x or y

(iii) Hence, find the exact volume generated when the shaded region enclosed by the curve $y = \frac{1}{\sqrt{4-x^2}}$ and the line $y = \frac{1}{\sqrt{2}}$ is rotated about the x-axis. **3**

$$V = 2\pi \int_0^{\sqrt{2}} \left(\frac{1}{\sqrt{2}}\right)^2 - \left(\frac{1}{\sqrt{4-x^2}}\right)^2 dx$$

$$= 2\pi \int_0^{\sqrt{2}} \frac{1}{2} - \frac{1}{4}\left(\frac{1}{2-x} + \frac{1}{2+x}\right) dx$$

$$= 2\pi \left[\frac{x}{2} - \frac{1}{4}\left(-\ln|2-x| + \ln|2+x|\right)\right]_0^{\sqrt{2}}$$

$$= 2\pi \left[\frac{x}{2} - \frac{1}{4}\ln\left(\frac{2+x}{2-x}\right)\right]_0^{\sqrt{2}}$$

$$= 2\pi \left\{\left(\frac{\sqrt{2}}{2} - \frac{1}{4}\ln\left(\frac{2+\sqrt{2}}{2-\sqrt{2}}\right)\right) - \left(0 - \frac{1}{4}\ln\left(\frac{2}{2}\right)\right)\right\}$$

$$= 2\pi \left[\frac{\sqrt{2}}{2} - \frac{1}{4}\ln\left(\frac{(2+\sqrt{2})^2}{2}\right)\right]$$

$$= \pi\sqrt{2} - \frac{\pi}{2}\ln\left(\frac{4+2\sqrt{2}+2}{2}\right)$$

$$= \pi\sqrt{2} - \frac{\pi}{2}\ln\left(3+2\sqrt{2}\right) \text{ units}^3$$

Marking Criteria:
- 3 – Finds exact volume
- 2 – Makes substantial progress
- 1 – Correctly sets up integral for volume

End of Question 14

Question 15 (12 marks) Use the Question 15 Writing Booklet

(a) A curve is defined by the parametric equations: **2**

$$x = \operatorname{cosec}(\theta) + 1$$
$$y = 2\cot(\theta)$$

for $\theta \in [0, 2\pi]$. Find the cartesian equation of the curve.

$x = \operatorname{cosec}\theta + 1 \Rightarrow \operatorname{cosec}\theta = x - 1$

$y = 2\cot\theta$

$y^2 = 4\cot^2\theta$

$\cot^2\theta = \frac{y^2}{4}$

$\operatorname{cosec}^2\theta - 1 = \frac{y^2}{4}$

$(x-1)^2 - 1 = \frac{y^2}{4}$

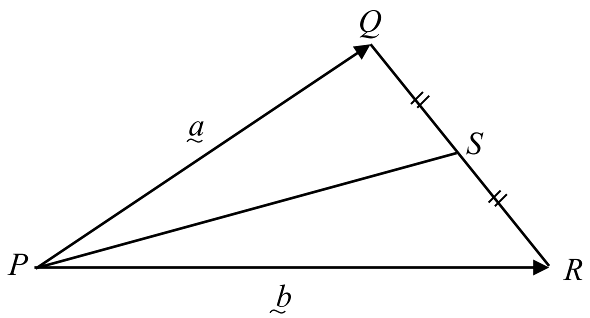
$\therefore y^2 = 4(x-1)^2 - 4$

Marking Criteria:

2 – Correct Solution

1 – Makes progress using Pythagorean identity or equivalent merit

(b) PQR is a triangle, S is the midpoint of QR and $\overrightarrow{PQ} = \underset{\sim}{a}$, $\overrightarrow{PR} = \underset{\sim}{b}$.



(i) Show that $\overrightarrow{QS} = \frac{1}{2}\left(\underset{\sim}{b} - \underset{\sim}{a}\right)$ and $\overrightarrow{PS} = \frac{1}{2}\left(\underset{\sim}{a} + \underset{\sim}{b}\right)$. **2**

i) $\overrightarrow{QS} = \frac{1}{2}\overrightarrow{QR}$

$\overrightarrow{QR} = \underset{\sim}{b} - \underset{\sim}{a}$

$\therefore \overrightarrow{QS} = \frac{1}{2}(\underset{\sim}{b} - \underset{\sim}{a})$ as req'd

$\overrightarrow{PS} = \underset{\sim}{a} + \cancel{\tfrac{1}{2}}\ \overrightarrow{QS}$

$= \underset{\sim}{a} + \frac{1}{2}(\underset{\sim}{b} - \underset{\sim}{a})$

$= \underset{\sim}{a} + \frac{1}{2}\underset{\sim}{b} - \frac{1}{2}\underset{\sim}{a}$

$= \frac{1}{2}(\underset{\sim}{a} + \underset{\sim}{b})$ as req'd.

Marking Criteria:

2 – Correct Solution

1 – Finds $\overline{QS}$ or $\overline{PS}$

(ii) Hence, prove $\left|\overrightarrow{PQ}\right|^2 + \left|\overrightarrow{PR}\right|^2 = 2\left(\left|\overrightarrow{PS}\right|^2 + \left|\overrightarrow{QS}\right|^2\right)$. **2**

$\overrightarrow{QS} = \frac{1}{2}(\underset{\sim}{b} - \underset{\sim}{a})$

$2\overrightarrow{QS} = \underset{\sim}{b} - \underset{\sim}{a}$ —— (1)

$\overrightarrow{PS} = \frac{1}{2}(\underset{\sim}{a} + \underset{\sim}{b})$

$2\overrightarrow{PS} = \underset{\sim}{a} + \underset{\sim}{b}$ —— (2)

$LHS = |\overrightarrow{PQ}|^2 + |\overrightarrow{PR}|^2$

$= |\underset{\sim}{a}|^2 + |\underset{\sim}{b}|^2$

$= \frac{1}{2}(2|a|^2 + 2|\underset{\sim}{b}|^2)$

$= \frac{1}{2}(|\underset{\sim}{a}|^2 + |\underset{\sim}{b}|^2 + |\underset{\sim}{a}|^2 + |\underset{\sim}{b}|^2)$

$= \frac{1}{2}(|\underset{\sim}{a}|^2 + |\underset{\sim}{b}|^2 + 2\underset{\sim}{a}\underset{\sim}{b} + |a|^2 + |b|^2 - 2\underset{\sim}{a}\underset{\sim}{b})$

$= \frac{1}{2}[(\underset{\sim}{a} + \underset{\sim}{b})^2 + (\underset{\sim}{a} - \underset{\sim}{b})^2]$

$= \frac{1}{2}[(\underset{\sim}{a} + \underset{\sim}{b})^2 + (\underset{\sim}{b} - \underset{\sim}{a})^2]$

sub (1) and (2)

$= \frac{1}{2}[(2|\overrightarrow{PS}|)^2 + (2\overrightarrow{QS})^2]$

$= 2|\overrightarrow{PS}|^2 + 2|\overrightarrow{QS}|$

Marking Criteria:
2 – Correct Solution
1 – Makes progress using algebraic manipulation progress or equivalent merit

(c) Use the substitution $u = \dfrac{1+x}{1-x}$ to find $\displaystyle\int \frac{\sqrt{1-x^2}}{(1-x)^2(1+x)}\,dx$. **2**

$u = \dfrac{1+x}{1-x}$

$\dfrac{du}{dx} = \dfrac{1(1-x) - (-1)(1+x)}{(1-x)^2}$

$= \dfrac{1-x+1+x}{(1-x)^2}$

$= \dfrac{2}{(1-x)^2}$

$\therefore \dfrac{du}{2} = \dfrac{dx}{(1-x)^2}$

$I = \displaystyle\int \frac{\sqrt{1-x^2}}{(1+x)(1-x)^2}\,dx$

$= \dfrac{1}{2}\displaystyle\int u^{-1/2}\,du$

$= \dfrac{1}{2}\left[2u^{1/2}\right] + C$

$= u^{1/2} + C$

$= \sqrt{\dfrac{1+x}{1-x}} + C$

$u = \dfrac{1+x}{1-x} \times \dfrac{(1+x)}{(1+x)}$

$= \dfrac{(1+x)^2}{1-x^2}$

$\sqrt{u} = \dfrac{1+x}{\sqrt{1-x^2}}$

Marking Criteria:
2 – Correct Solution
1 – Indicates $\dfrac{\sqrt{1-x^2}}{1+x} = \left(\dfrac{1+x}{1-x}\right)^{-\frac{1}{2}} = u^{-\frac{1}{2}}$ or
$I = \displaystyle\int \frac{\sqrt{1-x^2}}{(1-x)^2(1+x)}\,dx = \int \frac{\sqrt{1-x^2}}{1+x} \times \frac{1}{2} \times \frac{2}{(1-x)^2}\,dx$

(d) Use Mathematical Induction to prove that:

$$\sin x + \sin 3x + \sin 5x + \ldots + \sin(2n-1)x = \frac{\sin^2(nx)}{\sin x} \text{ for all positive integers, } n \geq 1.$$ **4**

d) $\sin x + \sin 3x + \sin 5x + \ldots + \sin(2n-1)x = \frac{\sin^2(nx)}{\sin x}$

Prove true for $n=1$

ie $\sin(2(1)-1)x = \frac{\sin^2(1\times x)}{\sin x}$

$LHS = \sin(2(1)-1)x$
$= \sin x$

$RHS = \frac{\sin^2(1\times x)}{\sin x}$
$= \frac{\sin^2 x}{\sin x}$
$= \sin x$

$\therefore LHS = RHS$
$\therefore$ true for $n=1$.

Assume true for $n=k$, $k \in \mathbb{Z}^+$

ie $\sin x + \sin 3x + \sin 5x + \ldots + \sin(2k-1)x = \frac{\sin^2(kx)}{\sin x}$

Prove true for $n=k+1$

ie $\sin x + \sin 3x + \sin 5x + \ldots + \sin(2k-1)x + \sin(2(k+1)-1)x = \frac{\sin^2(k+1)x}{\sin x}$

$= \frac{\sin^2(kx)}{\sin x} + \frac{\sin(2k+1)x \cdot \sin x}{}$ LHS

$\sin^2(kx) = \frac{1}{2}(1-\cos 2kx)$

Product to sum

$\sin x \sin(2k+1)x = \frac{1}{2}[\cos(x-2kx-x) - \cos(x+2kx+x)]$
$= \frac{1}{2}[\cos(2kx) - \cos(2(k+1)x)]$

Sub:

$= \frac{\frac{1}{2}(1-\cos 2kx) + \frac{1}{2}[\cos(2kx) - \cos(2k+2)x]}{\sin x}$

$= \frac{1 - \cos 2kx + \cos 2kx - \cos(2k+2)x}{2\sin x}$

$= \frac{1-\cos(2k+2)x}{2\sin x}$

$= \frac{1 - [\cos^2(k+1)x - \sin^2(k+1)x]}{2\sin x}$

$= \frac{1 - [1 - \sin^2(k+1)x - \sin^2(k+1)x]}{2\sin x}$

$= \frac{1 - 1 + 2\sin^2(k+1)x}{2\sin x}$

$= \sin^2(k+1)x$

Marking Criteria:

4 – Defines appropriate sequence of statements and proves them true by Mathematical Induction

3 – Makes substantially progress in the inductive step

2 – Correctly sets up the assumption stating k is a positive integer ≥ 1 or equivalent merit

1 – Correctly tests $n = 1$

End of paper