



GOSFORD HIGH SCHOOL

**2017 HIGHER SCHOOL CERTIFICATE
TRIAL EXAMINATION**

Mathematics Extension 1

General Instructions

- Reading Time – 5 minutes
- Working Time – 2 hours
- Write using black pen
- Board-approved calculators may be used
- A reference sheet is provided at the back of this paper
- In Questions 11 – 14, show relevant mathematical reasoning and/or calculations

Total Marks – 70

Section I

Pages 2 – 6

10 Marks

- Attempt Questions 1 – 10
- Allow about 15 minutes for this section

Section II

Pages 7 – 14

60 Marks

- Attempt Questions 11 – 14
- Allow about 1 hour and 45 minutes for this section

Section I

10 Marks

Attempt Questions 1 – 10.

Allow about 15 minutes for this section.

Use the multiple-choice answer sheet for Questions 1 – 10.

- 1) If $\cos x = \frac{2}{5}$ and $0 < x < \frac{\pi}{2}$, what is the value of $\cos 2x$?

(A) $\frac{4}{5}$

(B) $\frac{17}{25}$

(C) $-\frac{17}{25}$

(D) $-\frac{4}{5}$

- 2) The equation $ax^3 + bx^2 + cx + d = 0$, where $a \neq 0, b \neq 0, c \neq 0$ and $d \neq 0$, has roots α, β and γ . Which of the following is an expression for $\frac{1}{\alpha\beta} + \frac{1}{\alpha\gamma} + \frac{1}{\beta\gamma}$?

(A) $-\frac{b}{d}$

(B) $\frac{b}{d}$

(C) $-\frac{c}{d}$

(D) $\frac{c}{d}$

3) In how many ways can the letters of the word MATTYJAY be arranged in a line?

(A) 40 320

(B) 20 160

(C) 10 080

(D) 5 040

4) Which of the following is an expression for the inverse of the function $f(x) = \log_5(x + 3) - 2$?

(A) $f^{-1}(x) = \log_5(x + 2) - 2$

(B) $f^{-1}(x) = 5^{x+2} - 3$

(C) $f^{-1}(x) = 5^x - 1$

(D) $f^{-1}(x) = 10^x - 3$

5) Which expression is equal to

$$\int \sin^2 2x \, dx$$

(A) $\frac{1}{2} \left(x - \frac{1}{4} \sin 4x \right) + c$

(B) $\frac{1}{2} \left(x + \frac{1}{4} \sin 4x \right) + c$

(C) $\frac{\sin^3 2x}{6} + c$

(D) $-\frac{\cos^3 2x}{6} + c$

6) Which of the following is an expression for $\frac{d}{dx} (\tan^{-1}(2x + 1))$?

(A) $\frac{1}{4x^2 + 4x + 2}$

(B) $\frac{1}{2x^2 + 2x + 1}$

(C) $\frac{1}{4x^2 + 2}$

(D) $\frac{1}{2x^2 + 1}$

7) What is the general solution of the equation
 $2 \sin^2 x - 5 \sin x - 3 = 0$

(A) $n\pi - (-1)^n \frac{\pi}{3}$

(B) $n\pi + (-1)^n \frac{\pi}{3}$

(C) $n\pi - (-1)^n \frac{\pi}{6}$

(D) $n\pi + (-1)^n \frac{\pi}{6}$

8) A spherical hailstone is forming in a cloud with its radius increasing by 2 *mm* per second. At what rate is the volume of the stone increasing when the radius is 5 *mm*

(A) $\frac{500\pi}{3}$

(B) 200π

(C) 100π

(D) 50π

- 9) The original temperature of a body is 100°C and the temperature of the surroundings is 20°C . 10 minutes later the body is 70°C . Which of the following is the temperature $T^{\circ}\text{C}$ at time t minutes later?

(A) $T = 20 + 100e^{-0.069t}$

(B) $T = 20 + 100e^{0.069t}$

(C) $T = 20 + 80e^{-0.047t}$

(D) $T = 20 + 80e^{0.047t}$

- 10) Assume that the tides rise and fall in simple harmonic motion. At low tide a channel is 6 m deep and at high tide 18 m deep. Low tide is at 6 am with the next high tide at 4 pm. Which equation models the depth of the water d m at time t hours after 6 am?

(A) $d = -6 \cos \frac{\pi}{5}t$

(B) $d = -6 \cos \frac{\pi}{10}t$

(C) $d = -12 \cos \frac{\pi}{5}t$

(D) $d = -12 \cos \frac{\pi}{10}t$

Section II

60 Marks

Attempt Questions 11 – 14.

Allow about 1 hour and 45 minutes for this section.

Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.

In Questions 11 – 14, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Use a SEPARATE writing booklet.

- a) $A(-2, 8)$ and $B(4, -7)$ are two points. Find the coordinates of the point P that divides the interval AB internally in the ratio 5:2. 2

- b) Find in simplest exact form the value of 3

$$\int_{-1}^{\sqrt{2}} \frac{1}{\sqrt{2-x^2}} dx$$

- c) Use the substitution $t = \tan \frac{x}{2}$ to show that 3

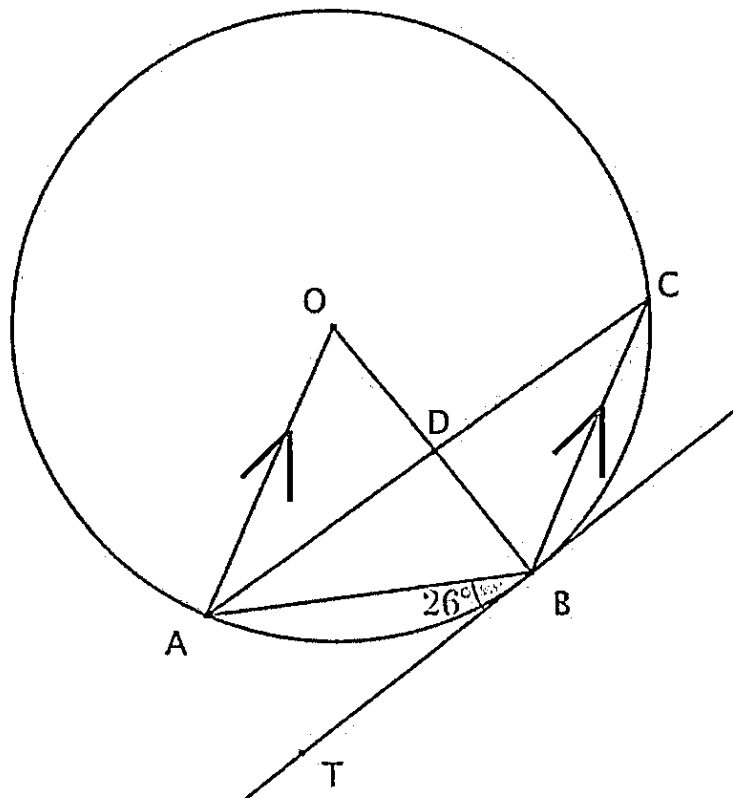
$$\sec x + \tan x = \tan \left(\frac{\pi}{4} + \frac{x}{2} \right)$$

- d) Find the term independent of x in the expansion of 3

$$\left(x - \frac{3}{x^2} \right)^{12}$$

**Question 11 Continues
On Next Page**

e)



The points A , B and C lie on a circle with centre O . The lines AO and BC are parallel, and OB and AC intersect at D . Also, $\angle TBA = 26^\circ$, as shown in the diagram.

Copy or trace the diagram into your writing booklet.

- i) Find the size of $\angle ACB$, giving reasons 1
- ii) Find the size of $\angle AOB$, giving reasons 1
- iii) Find $\angle BDC$. Justify your answer. 2

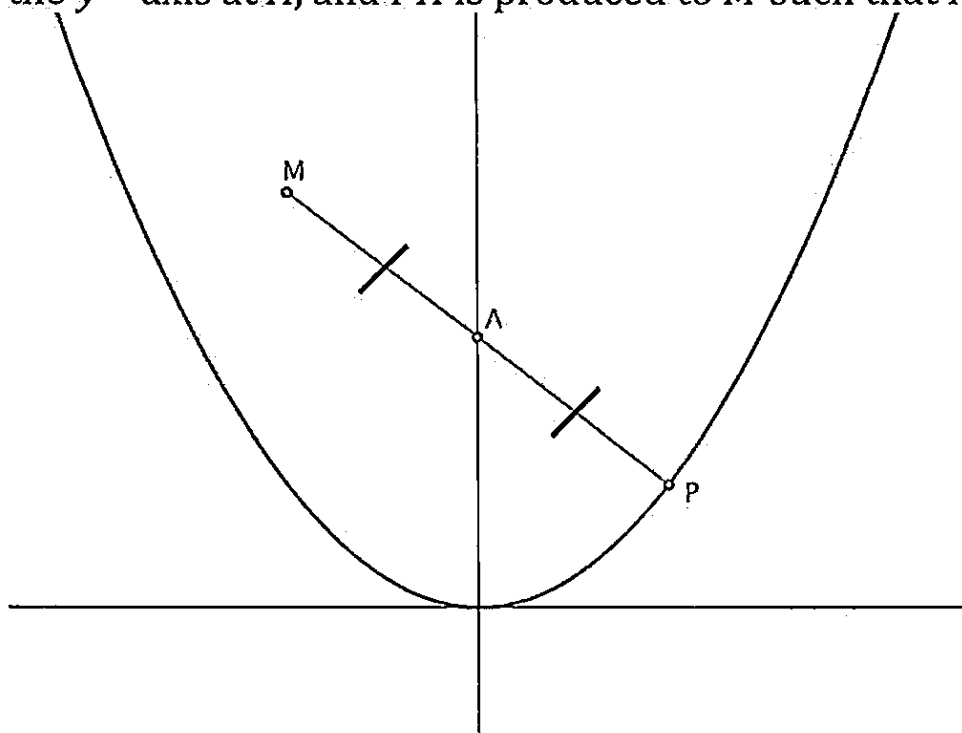
End of Question 11

Question 12 (15 marks) Use a SEPARATE writing booklet.

- a) Find correct to the nearest degree the acute angle between the lines $2x - y = 0$ and $x + 3y + 5 = 0$ 2
- b) i) Find the domain and range of the function $f(x) = -\sin^{-1}(x + 2)$ 2
- ii) Sketch the graph of the function $f(x) = -\sin^{-1}(x + 2)$ showing the coordinates of the end points. 2
- c) Show that $(x - 2)$ is a factor of $P(x) = x^3 + 2x^2 - 5x - 6$ 1
- d) Sketch the polynomial $P(x) = (x + 2)^2(1 - x)$ 2
- e) Use the substitution $u = x + 2$ to find $\int x\sqrt{x + 2} dx$ 2

**Question 12 Continues
On Next Page**

- f) The normal at $P(2ap, ap^2)$ on the parabola $x^2 = 4ay$ meets the y -axis at A , and PA is produced to M such that $PA = AM$



- i) Prove that M has coordinates $(-2ap, 4a + ap^2)$ 2
- ii) Hence find the locus of M 2

End of Question 12

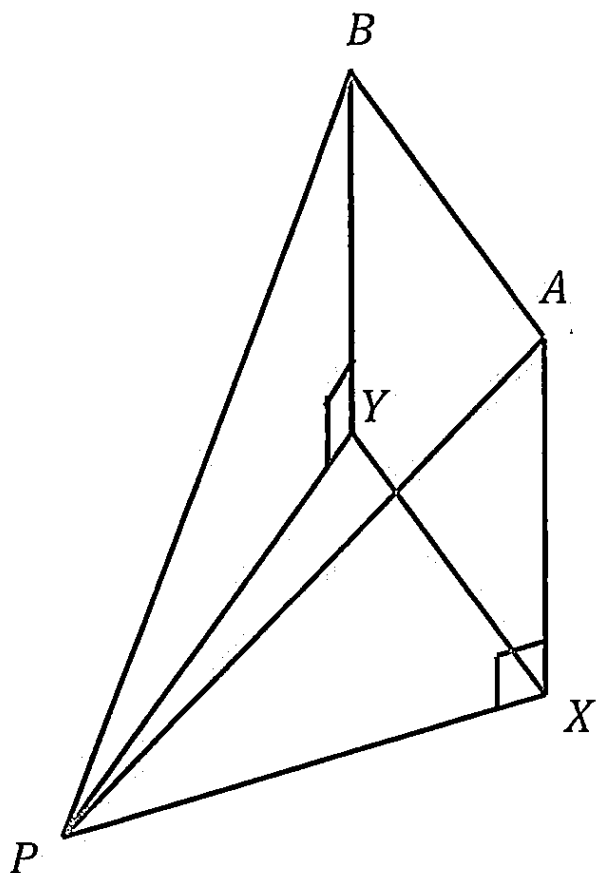
Question 13 (15 marks) Use a SEPARATE writing booklet.

- a) Prove by mathematical induction that, for all $n \geq 2$ 3
$$3^n > n^2$$
- b) i) Given that the function $f(x) = x^2 - \sin x$ is continuous for all real x , show that the equation $f(x) = 0$ has a root between $x = \frac{1}{2}$ and $x = 1$ 1
- ii) Use one application of Newton's method with an initial approximation 0.8 to find the next approximation to this root, giving your answer correct to 2 decimal places. 2
- c) A particle is moving in a straight line. At time t seconds it has displacement x metres to the right of a fixed point O on the line and velocity $v \text{ ms}^{-1}$ given by
$$v = \frac{8 - x^2}{x}$$

Initially the particle is 1 metre to the right of O .
- i) Show that 3
$$x = \sqrt{8 - 7e^{-2t}}$$
- ii) Find the limiting position of the particle 1
- iii) Describe the motion of the particle as $t \rightarrow \infty$ 2

**Question 13 Continues
on Next Page**

- d) A plane flies horizontally at a height h m over a distance of 8 000 m from a point A , vertically above X to a point B , vertically above Y .
An observer standing at P notices the angle of elevation to the plane at A was 5° and when the plane was at B the angle of elevation was 13° .



The observer notes that the initial bearing of the plane was 037°T and the final bearing was 290°T .

- i) Show that $PX = h \cot 5$ 1
- ii) Hence, find the value of h (to the nearest metre). 2

End of Question 13

Question 14 (15 marks) Use a SEPARATE writing booklet.

a) i) Write down the expansion of $(1+x)^n$ in ascending powers of x 1

ii) Hence show that 3

$$\binom{n}{0} + \frac{1}{2}\binom{n}{1} + \frac{1}{3}\binom{n}{2} + \dots + \frac{1}{n+1}\binom{n}{n} = \frac{2^{n+1} - 1}{n+1}$$

b) A particle moves according to the equation

$$x = 6 \sin^2 \left(4t + \frac{\pi}{3} \right)$$

i) Show that the acceleration can be expressed in the form 3
 $\ddot{x} = 64(3 - x)$

ii) Find the amplitude of the motion 1

iii) Find the period of the motion 1

**Question 14 Continues
on Next Page**

c) i) Simplify 1

$$\sin(2 \sin^{-1} x)$$

ii) Show that 2

$$\sin 3\theta = 3 \sin \theta - 4 \sin^3 \theta$$

iii) By using the substitution $\theta = \sin^{-1} x$ and the answers to the previous parts show that 3

$$\int \sin(2\sin^{-1} x) dx = \frac{1}{2} \int (\sin 3\theta + \sin \theta) d\theta$$

End of Exam

$$\begin{aligned}
 1) \quad \cos 2x &= 2\cos^2 x - 1 \\
 &= 2 \times \left(\frac{2}{5}\right)^2 - 1 \\
 &= \frac{8}{25} - 1 \\
 &= -\frac{17}{25}
 \end{aligned}$$

(C)

$$\begin{aligned}
 2) \quad \frac{1}{\alpha\beta} + \frac{1}{\alpha\gamma} + \frac{1}{\beta\gamma} &= \frac{\gamma + \beta + \alpha}{\alpha\beta\gamma} \\
 &= \frac{-b/a}{-d/a} \\
 &= b/d
 \end{aligned}$$

(B)

$$3) \quad \frac{8!}{2!2!2!} = 5040$$

(D)

$$\begin{aligned}
 4) \quad x &= \log_5(y+3) - 2 \\
 y+3 &= 5^{x+2} \\
 y &= 5^{x+2} - 3
 \end{aligned}$$

(B)

$$\begin{aligned}
 5) \quad \cos 2A &= 1 - 2\sin^2 A \\
 \sin^2 A &= \frac{1}{2}(1 - \cos 2A)
 \end{aligned}$$

$$\begin{aligned}
 \int \sin^2 2x \, dx &= \frac{1}{2} \int (1 - \cos 4x) \, dx \\
 &= \frac{1}{2} \left[x - \frac{1}{4} \sin 4x \right]
 \end{aligned}$$

(A)

$$\begin{aligned}
 6) \quad \frac{d}{dx} \tan^{-1}(2x+1) &= \frac{1}{1 + (2x+1)^2} \times 2 \\
 &= \frac{2}{1 + 4x^2 + 4x + 1} \\
 &= \frac{1}{2x^2 + 2x + 1}
 \end{aligned}$$

(B)

C B D B A | B C B C B

$$7) \quad 2\sin^2 x - 5\sin x - 3 = 0$$

$$(2\sin x + 1)(\sin x - 3) = 0$$

$$\sin x = -\frac{1}{2} \quad \text{or} \quad \sin x = 3$$

$$x = n\pi + (-1)^n \left(-\frac{\pi}{6}\right) \quad \text{no soln}$$

(C)

$$8) \quad V = \frac{4}{3}\pi r^3$$

$$\frac{dV}{dr} = 4\pi r^2$$

$$\frac{dV}{dt} = \frac{dV}{dr} \times \frac{dr}{dt}$$

$$= 4\pi r^2 \times 2$$

$$@ r = 5 \quad \frac{dV}{dt} = 200\pi$$

(B)

$$9) \quad T = 20 + 80e^{kt}$$

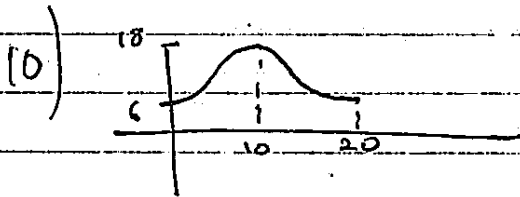
$$@ t = 10 \quad T = 70$$

$$70 = 20 + 80e^{10k}$$

$$k = \frac{1}{10} \ln \frac{5}{8}$$

$$\approx -0.047$$

(C)



$$2a = 12$$

$$a = 6$$

$$20 = \frac{2\pi}{\omega}$$

$$\omega = \frac{\pi}{10}$$

$$d = -6 \cos \frac{\pi}{10} t$$

(B)

11) a) $(-2, 8) \times (4, -7)$

$$S = 2$$

$$P = \left(\frac{-2 \times 2 + 4 \times 5}{5 + 2}, \frac{8 \times 2 + -7 \times 5}{5 + 2} \right)$$

$$= \left(\frac{16}{7}, \frac{-19}{7} \right)$$

b) $\int_1^{\sqrt{2}} \frac{dx}{\sqrt{2-x^2}} = \left[\sin^{-1} \frac{x}{\sqrt{2}} \right]_1^{\sqrt{2}}$

$$= \sin^{-1} 1 - \sin^{-1} \left(\frac{1}{\sqrt{2}} \right)$$

$$= \frac{\pi}{2} - \frac{\pi}{4}$$

$$= \frac{3\pi}{4}$$

c) LHS = $\sec x + \tan x$

$$= \frac{1+t^2}{1-t^2} + \frac{2t}{1-t^2}$$

$$= \frac{1+2t+t^2}{1-t^2}$$

$$= \frac{(1+t)^2}{(1+t)(1-t)}$$

$$= \frac{1+t}{1-t}$$

$$= \frac{\tan \frac{\pi}{4} + \tan \frac{x}{2}}{1 - \tan \frac{\pi}{4} \tan \frac{x}{2}}$$

$$= \tan \left(\frac{\pi}{4} + \frac{x}{2} \right)$$

$$= \text{RHS.}$$

$$\begin{aligned}
 d) T_{k+1} &= {}^{12}C_k x^{12-k} (-3x^{-2})^k \\
 &= {}^{12}C_k x^{12-k} (-3)^k x^{-2k} \\
 &= {}^{12}C_k (-3)^k x^{12-3k}
 \end{aligned}$$

for constant $12-3k=0$

$$k=4$$

$$\therefore \text{term is } {}^{12}C_4 (-3)^4 = 40\,095$$

e) i) $\angle ACB = 26^\circ$ ($\angle$ between tangent & chord equals $\angle$ in alternate segment)

ii) $\angle AOB = 52^\circ$ ($\angle$ @ the centre is twice $\angle$ standing on same arc @ the circumference)

iii) $\angle OAD = 26^\circ$ (alternate $\angle$'s on parallel lines)
 $\angle ODA = 102^\circ$ ($\angle$ sum of Δ is 180°)
 $\angle BDC = 102^\circ$ (vertically opposite)

12 a) $2x - y = 0$
 $y = 2x$
 $m_1 = 2$

$x + 3y + 5 = 0$
 $y = -\frac{1}{3}x - \frac{5}{3}$
 $m_2 = -\frac{1}{3}$

$\tan \theta = \left| \frac{2 - (-\frac{1}{3})}{1 + 2(-\frac{1}{3})} \right|$
 $= \left| \frac{\frac{7}{3}}{\frac{1}{3}} \right|$

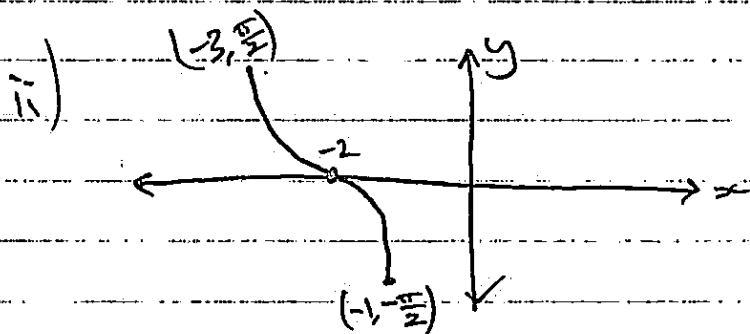
$= 7$

$\theta = \tan^{-1} 7$
 $= 81.86\dots$
 $= 82^\circ$

b) i) $-y = \sin^{-1}(x+2)$

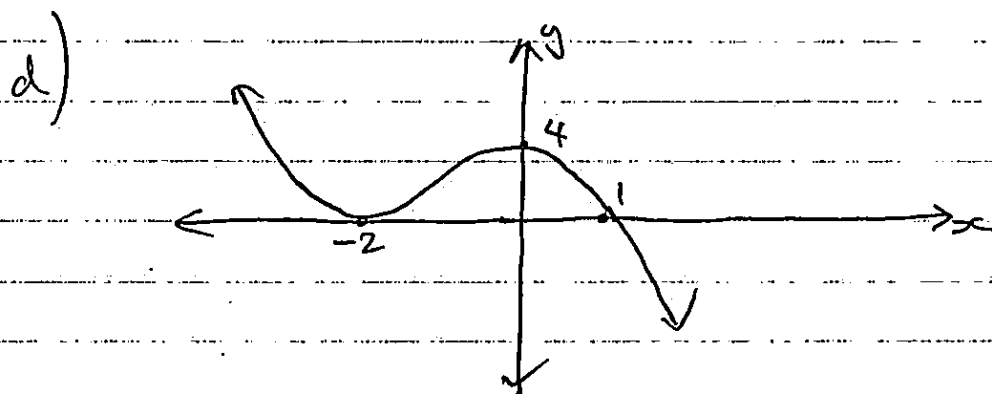
Domain: $-\frac{\pi}{2} \leq x+2 \leq \frac{\pi}{2}$
 $-3 \leq x \leq -1$

Range: $-\frac{\pi}{2} \leq -y \leq \frac{\pi}{2}$
 $\frac{\pi}{2} \geq y \geq -\frac{\pi}{2}$



c) $P(2) = 2^3 + 2 \times 2^2 - 5 \times 2 - 6$
 $= 8 + 8 - 10 - 6$
 $= 0$

$\therefore x-2$ is a factor



$$e) \quad u = x+2$$

$$du = dx$$

$$x = u-2$$

$$\int x \sqrt{x+2} \, dx = \int (u-2) u^{\frac{1}{2}} \, du$$

$$= \int u^{\frac{3}{2}} - 2u^{\frac{1}{2}} \, du$$

$$= \frac{2}{5} u^{\frac{5}{2}} - \frac{4}{3} u^{\frac{3}{2}} + C$$

$$= \frac{2}{5} (x+2)^{\frac{5}{2}} - \frac{4}{3} (x+2)^{\frac{3}{2}} + C$$

f) i) eqn of normal from ref. sheet

$$x + py = ap^3 + 2ap$$

A is @ $x=0$

$$py = ap^3 + 2ap$$

$$y = ap^2 + 2a$$

$$A = (0, 2a + ap^2)$$

$$A = \left(\frac{x_m + 2ap}{2}, \frac{y_m + ap^2}{2} \right)$$

$$\frac{x_m + 2ap}{2} = 0$$

$$\frac{y_m + ap^2}{2} = 2a + ap^2$$

$$x_m = -2ap$$

$$y_m + ap^2 = 4a + 2ap^2$$

$$y_m = 4a + ap^2$$

$$ii) \quad p = \frac{x}{-2a}$$

$$y = 4a + ap^2$$

$$= 4a + a \left(\frac{x}{-2a} \right)^2$$

$$= 4a + \frac{x^2}{4a}$$

$$4ay = 16a^2 + x^2$$

$$x^2 = 4ay - 16a^2$$

$$= 4a(y - 4a)$$

13) a) Prove true for $n=1$

$$\begin{aligned} \text{LHS} &= 3^1 \\ &= 3 \\ \text{RHS} &= 1^2 \\ &= 1 \end{aligned}$$

$\therefore$ true for $n=1$

Assume true for $n=k$

$$3^k \geq k^2$$

Prove true for $n=k+1$

$$\text{i.e. } 3^{k+1} > (k+1)^2$$

$$\begin{aligned} \text{LHS} &= 3^{k+1} \\ &= 3 \times 3^k \\ &> 3k^2 \quad \text{by assumption} \\ &= k^2 + k^2 + k^2 \\ &\geq k^2 + 2k + k^2 \quad \text{since } k > 1 \\ &\geq k^2 + 2k + 1 \quad \text{since } k \geq 1 \\ &= (k+1)^2 \\ &= \text{RHS} \end{aligned}$$

$\therefore$ the result is proven by the principle of mathematical induction.

b) i) $f\left(\frac{1}{2}\right) = \left(\frac{1}{2}\right)^2 - \sin \frac{1}{2}$ $f(1) = 1^2 - \sin(1)$

$$= -0.229\dots$$
 $= 0.158\dots$

$\therefore$ since $f\left(\frac{1}{2}\right)$ & $f(1)$ are opposite in sign & $f(x)$ is continuous there must be a root on $\frac{1}{2} < x < 1$

$$\text{ii)} \quad f'(x) = 2x - \cos x$$

$$f'(0.8) = 2 \times 0.8 - \cos 0.8$$

$$= 0.903 \dots$$

$$f(0.8) = (0.8)^2 - \sin 0.8$$

$$= -0.077 \dots$$

$$x_2 = 0.8 - \frac{f(0.8)}{f'(0.8)}$$

$$= 0.8856 \dots$$

$$= 0.89$$

$$\text{c) i)} \quad \frac{dx}{dt} = \frac{8-x^2}{x}$$

$$\frac{dt}{dx} = \frac{-1}{\frac{8-x^2}{x}} = -\frac{x}{8-x^2}$$

$$t = -\frac{1}{2} \ln(8-x^2) + C$$

$$\text{@ } t=0 \quad x=1$$

$$0 = -\frac{1}{2} \ln 7 + C$$

$$C = \frac{1}{2} \ln 7$$

$$t = -\frac{1}{2} \ln(8-x^2) + \frac{1}{2} \ln 7$$

$$-2t = \ln \left(\frac{8-x^2}{7} \right)$$

$$8-x^2 = 7e^{-2t}$$

$$x^2 = 8 - 7e^{-2t}$$

$$x = \pm \sqrt{8 - 7e^{-2t}}$$

since @ $t=0 \quad x=1$ it must be the +

$$x = \sqrt{8 - 7e^{-2t}}$$

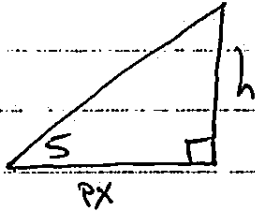
$$\text{ii)} \quad \text{as } t \rightarrow \infty \quad e^{-2t} \rightarrow 0$$

$$x \rightarrow \sqrt{8 - 7 \times 0}$$

$$= \sqrt{8} = 2\sqrt{2}$$

iii) the particle is travelling right towards $x=2\sqrt{2}$ with the acceleration left (-ve).

d) i)



$$\begin{aligned}\tan 5 &= \frac{h}{PX} \\ PX &= \frac{h}{\tan 5} \\ &= h \cot 5\end{aligned}$$

$$\begin{aligned}\text{ii)} \quad \tan 13 &= \frac{h}{PY} \\ PY &= \frac{h}{\tan 13} \\ &= h \cot 13\end{aligned}$$

$$\angle YPX = 107^\circ \text{ \& } XY = 8000\text{m}$$

$$\begin{aligned}8000^2 &= h^2 \cot^2 5 + h^2 \cot^2 13 - 2 \times h \cot 5 \times h \cot 13 \times \cos 107 \\ &= h^2 (\tan^2 85 + \tan^2 77 - 2 \tan 85 \tan 77 \cos 107)\end{aligned}$$

$$\begin{aligned}h &= \frac{8000}{\sqrt{\tan^2 85 + \tan^2 77 - 2 \tan 85 \tan 77 \cos 107}} \\ &= 278.20 \dots \\ &= 278\text{m}\end{aligned}$$

$$14) a) i) (1+x)^n = \binom{n}{0} + \binom{n}{1}x + \binom{n}{2}x^2 + \dots + \binom{n}{n}x^n$$

ii) integrating both sides

$$\int + \frac{(1+x)^{n+1}}{(n+1)x^1} = \binom{n}{0}x + \frac{1}{2}\binom{n}{1}x^2 + \frac{1}{3}\binom{n}{2}x^3 + \dots + \frac{1}{n+1}\binom{n}{n}x^{n+1}$$

$$\text{sub } x=0$$

$$\int + \frac{1}{n+1} = 0$$

$$\int = -\frac{1}{n+1}$$

$$\frac{(1+x)^{n+1}}{n+1} - \frac{1}{n+1} = \binom{n}{0}x + \frac{1}{2}\binom{n}{1}x^2 + \frac{1}{3}\binom{n}{2}x^3 + \dots + \frac{1}{n+1}\binom{n}{n}x^{n+1}$$

$$\text{sub } x=1$$

$$\frac{2^{n+1}-1}{n+1} = \binom{n}{0} + \frac{1}{2}\binom{n}{1} + \frac{1}{3}\binom{n}{2} + \dots + \frac{1}{n+1}\binom{n}{n}$$

$$b) i) x = 6 \sin^2\left(4t + \frac{\pi}{3}\right)$$

$$\dot{x} = 12 \sin\left(4t + \frac{\pi}{3}\right) \times \cos\left(4t + \frac{\pi}{3}\right) \times 4$$

$$= 48 \sin\left(4t + \frac{\pi}{3}\right) \cos\left(4t + \frac{\pi}{3}\right)$$

$$\ddot{x} = 48 \sin\left(4t + \frac{\pi}{3}\right) \times -\sin\left(4t + \frac{\pi}{3}\right) \times 4 + 48 \cos\left(4t + \frac{\pi}{3}\right) \times \cos\left(4t + \frac{\pi}{3}\right)$$

$$= 192 \cos^2\left(4t + \frac{\pi}{3}\right) - 192 \sin^2\left(4t + \frac{\pi}{3}\right)$$

$$= 192 (1 - 2\sin^2\left(4t + \frac{\pi}{3}\right))$$

$$= 64 (3 - 6\sin^2\left(4t + \frac{\pi}{3}\right))$$

$$= 64 (3 - x)$$

$$ii) \ddot{x} = -64(x-3)$$

$\therefore$ the particle is in SHM about $x=3$
since acceleration is proportional to distance from 3
and in the opposite direction.

$$\begin{aligned}
 \text{iii)} \quad x &= 6 \sin^2 \left(4 + \frac{\pi}{3} \right) \\
 &= 3 \times 2 \sin^2 \left(4 + \frac{\pi}{3} \right) \\
 &= 3 \left(1 - \cos \left(8 + \frac{2\pi}{3} \right) \right) \\
 &= 3 - 3 \cos \left(8 + \frac{2\pi}{3} \right)
 \end{aligned}$$

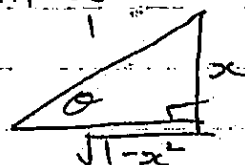
$\therefore$ amplitude is 3

$$\begin{aligned}
 \cos 2A &= 1 - 2 \sin^2 A \\
 2 \sin^2 A &= 1 - \cos 2A
 \end{aligned}$$

$$\begin{aligned}
 \text{iv)} \quad T &= \frac{2\pi}{8} \\
 &= \frac{\pi}{4}
 \end{aligned}$$

$$\begin{aligned}
 \text{c) i)} \quad &\sin(2 \sin^{-1} x) \\
 &= \sin 2\theta \\
 &= 2 \sin \theta \cos \theta \\
 &= 2x \sqrt{1-x^2}
 \end{aligned}$$

$$\text{let } \sin^{-1} x = \theta$$



$$\begin{aligned}
 \text{ii)} \quad \sin 3\theta &= \sin(2\theta + \theta) \\
 &= \sin 2\theta \cos \theta + \sin \theta \cos 2\theta \\
 &= 2 \sin \theta \cos^2 \theta + \sin \theta (1 - 2 \sin^2 \theta) \\
 &= 2 \sin \theta (1 - \sin^2 \theta) + \sin \theta - 2 \sin^3 \theta \\
 &= 2 \sin \theta - 2 \sin^3 \theta + \sin \theta - 2 \sin^3 \theta \\
 &= 3 \sin \theta - 4 \sin^3 \theta
 \end{aligned}$$

$$\begin{aligned}
 \text{iii)} \quad &\int \sin(2 \sin^{-1} x) dx \\
 &= \int 2x \sqrt{1-x^2} dx \\
 &= \int \frac{2x(1-x^2)}{\sqrt{1-x^2}} dx \\
 &= \int 2 \sin \theta (1 - \sin^2 \theta) \times d\theta \\
 &= \int 2 \sin \theta - 2 \sin^3 \theta d\theta \\
 &= \frac{1}{2} \int 4 \sin \theta - 4 \sin^3 \theta d\theta
 \end{aligned}$$

$$\begin{aligned}
 \theta &= \sin^{-1} x \\
 \frac{d\theta}{dx} &= \frac{1}{\sqrt{1-x^2}} \\
 x &= \sin \theta
 \end{aligned}$$

$$= \frac{1}{2} \int 3 \sin \theta - 4 \sin^3 \theta + \sin \theta \, d\theta$$

$$= \frac{1}{2} \int \sin 3\theta + \sin \theta \, d\theta$$

iii) Alternate solution

$$x = \sin \theta$$

$$dx = \cos \theta d\theta$$

$$\int \sin 2\theta \times \cos \theta d\theta = \int 2 \sin \theta \cos^2 \theta d\theta$$
$$= \frac{1}{2} \int 2 \sin \theta (1 - \sin^2 \theta) d\theta$$

$$= \frac{1}{2} \int 4 \sin \theta - 4 \sin^3 \theta d\theta$$

$$= \frac{1}{2} \int 3 \sin \theta - 4 \sin^3 \theta + \sin \theta d\theta$$

$$= \frac{1}{2} \int \sin 3\theta + \sin \theta d\theta$$

Alt soln. $\int \sin(2 \sin^{-1} x) dx = - \int 2x \sqrt{1-x^2} dx$

$$= -\frac{2}{3} (1-x^2)^{3/2} + C$$

$$= -\frac{2}{3} (1-\sin^2 \theta)^{3/2} + C$$

$$= -\frac{2}{3} (\cos^2 \theta)^{3/2}$$

$$= -\frac{2}{3} \cos^3 \theta$$

diff

$$= -2 \cos^2 \theta \times -\sin \theta$$

$$= 2 \sin \theta \cos^2 \theta$$