STUDENT'S NAME: ______________________________

TEACHER'S NAME: ______________________________

2025

HURLSTONE AGRICULTURAL HIGH SCHOOL

HIGHER SCHOOL CERTIFICATE TRIAL EXAMINATION

Mathematics Extension 1

General Instructions

- Reading time – 10 minutes
- Working time – 2 hours
- Write using a blue or black pen
- NESA approved calculators may be used
- A reference sheet is provided at the end of this booklet
- For questions in Section II, use the relevant booklet for writing your solutions.
- Show all relevant mathematical reasoning and /or calculations.

Total marks: 70

Section I – 10 marks (pages 2 – 6)

- Attempt Questions 1 – 10. The multiple-choice answer sheet has been provided
- Allow about 15 minutes for this section

Section II – 60 marks (pages 7 – 12)

- Attempt Questions 11 – 16, write your solutions in the spaces provided.
- You have been provided with 6 separate answer booklets, one for each question.
- Extra working pages are available if required.
- Allow about 1 hours and 45 minutes for this section.

SECTION I (10 marks)

Attempt Questions 1 - 10

Allow about 15 minutes on this section.

Use the multiple-choice answer sheet provided for Questions 1 – 10.

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1. What is the term independent of x in the expansion of $\left(x+\frac{2}{x}\right)^6$?

(A) 160 (B) 40

(C) 80 (D) 20

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2. Which of the following is the solution set of $\frac{2}{(1-3^x)} > -1$?

(A) $x<0$ (B) $x<1$

(C) $0<x<1$ (D) $x<0$ or $x>1$

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3. The parametric equations of a curve C are given by $x = \operatorname{cosec}^2 t$ and $y = \cos^2 t$.

What is the cartesian equation of C?

(A) $y=1-\frac{1}{x}$ (B) $y=1+\frac{1}{x}$

(C) $xy=1$ (D) $y^2=1-\frac{1}{x^2}$

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4. Given $t = \tan\dfrac{\theta}{2}$, which of the following is $\cot\left(\dfrac{\theta}{2}\right) - 2\cot\theta$ equal to?

(A) $\dfrac{1}{t}$

(B) $\dfrac{1-t^2}{t}$

(C) t

(D) $-t$

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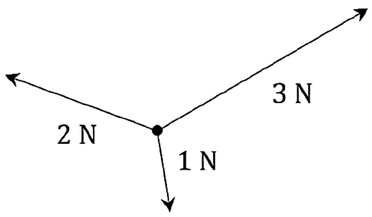
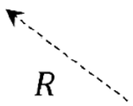
5. A particle is subject to forces of 1 N, 2 N, and 3 N in the directions shown.

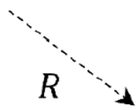


DRAWN TO SCALE

The sum of the three forces acting on the particle is called the resultant force.

Which diagram best shows the resultant force, R?

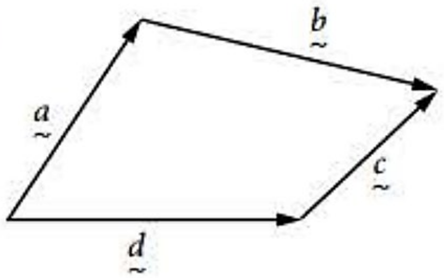
(A) 

(B) 

(C) 

(D) 

6. Four vectors, $\underset{\sim}{a}$, $\underset{\sim}{b}$, $\underset{\sim}{c}$ and $\underset{\sim}{d}$ are shown in the diagram.



Which one of the following statements is true?

(A) $\underset{\sim}{a}+\underset{\sim}{c}=\underset{\sim}{b}+\underset{\sim}{d}$

(B) $\underset{\sim}{a}+\underset{\sim}{b}=\underset{\sim}{c}+\underset{\sim}{d}$

(C) $\underset{\sim}{a}+\underset{\sim}{b}+\underset{\sim}{c}+\underset{\sim}{d}=0$

(D) $\underset{\sim}{b}+\underset{\sim}{c}=\underset{\sim}{a}+\underset{\sim}{d}$

---

7. Given that $N=80e^{-kt}$, which expression is equal to $\dfrac{dN}{dt}$?

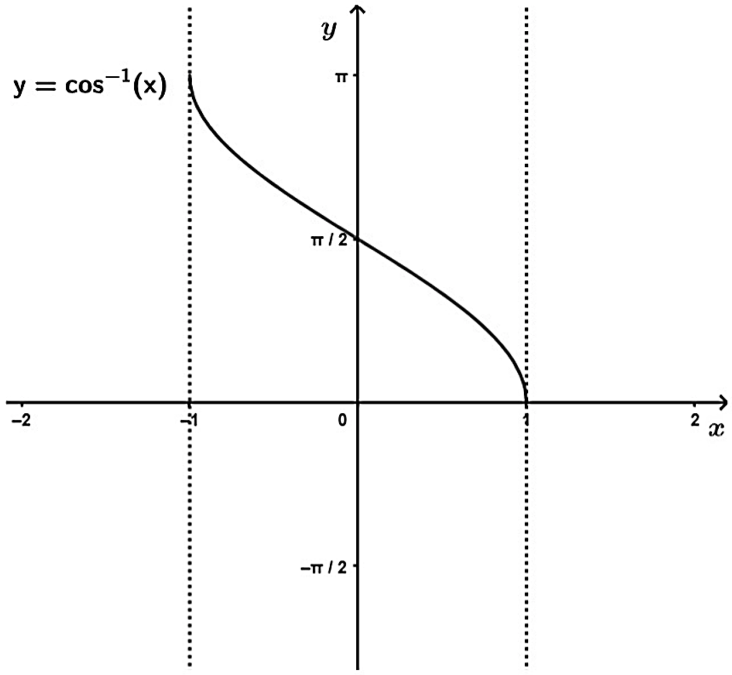
(A) $80kN$

(B) $-80kN$

(C) kN

(D) $-kN$

8. Which of the following gives the value of the area bounded by the graph $y = \cos^{-1}(x)$,
the $x-$axis and the line $x = -1$?



(A) 2π units^2

(B) π units^2

(C) $\frac{\pi}{2}$ units^2

(D) $\frac{\pi}{4}$ units^2

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9. Which of the following is the solution to $\int \cos^2 2x\,dx$?

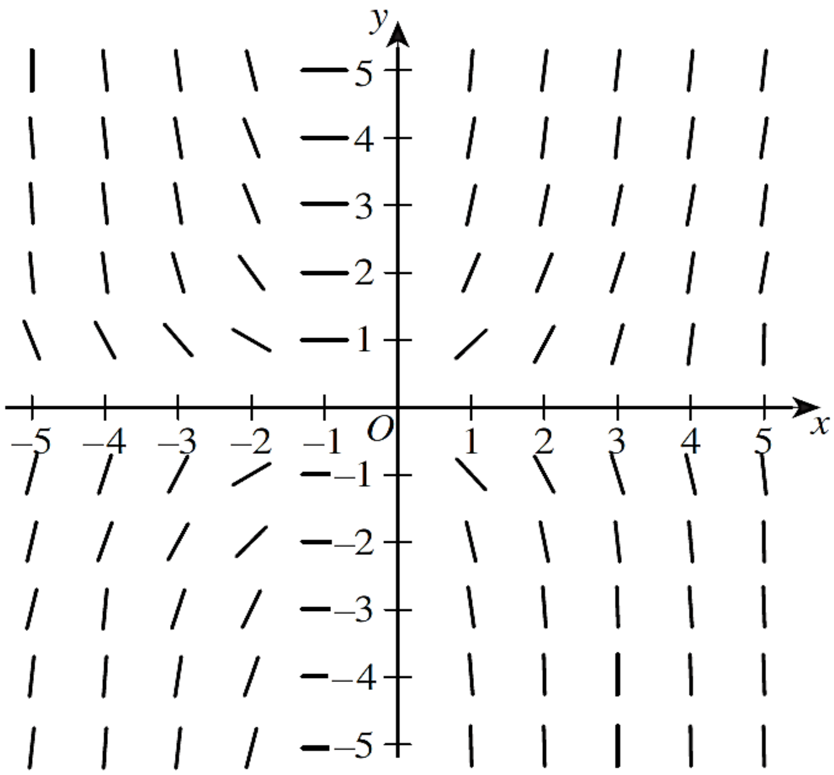
(A) $x + \frac{1}{4}\sin 4x + c$

(B) $\frac{x}{2} + \frac{1}{8}\sin 4x + c$

(C) $\frac{x}{2} + \frac{1}{8}\sin 2x + c$

(D) $x + \frac{1}{4}\sin 2x + c$

10. The direction (slope) field for a first order differential equation is shown.



Which of the following could be the differential equation represented?

(A) $\dfrac{dy}{dx} = (x+1)^3$

(B) $\dfrac{dy}{dx} = x(y+1)$

(C) $\dfrac{dy}{dx} = (x+1)y$

(D) $\dfrac{dy}{dx} = (x-1)y$

~ End of Section 1 ~

SECTION II

60 marks

Attempt Questions 11 - 16

Allow about 1 hours and 45 minutes on this section.
For questions in Section II, your responses should include relevant mathematical reasoning and/or calculations.
Answer each question in a separate answer booklet.

Question 11 (7 marks) Use the Question 11 BOOKLET **Marks**

(a) The letters of the word *CALCULUS* are arranged in a row.

(i) How many different arrangements are there? **1**

(ii) If one of these arrangements is selected at random, find the probability that it begins with *"U"* and ends with *"U"* **2**

(b)

(i) Use the sum of the terms of an arithmetic series to show that **1**

$$(1+2+3+.....+n)^2 = \frac{1}{4}n^2(n+1)^2$$

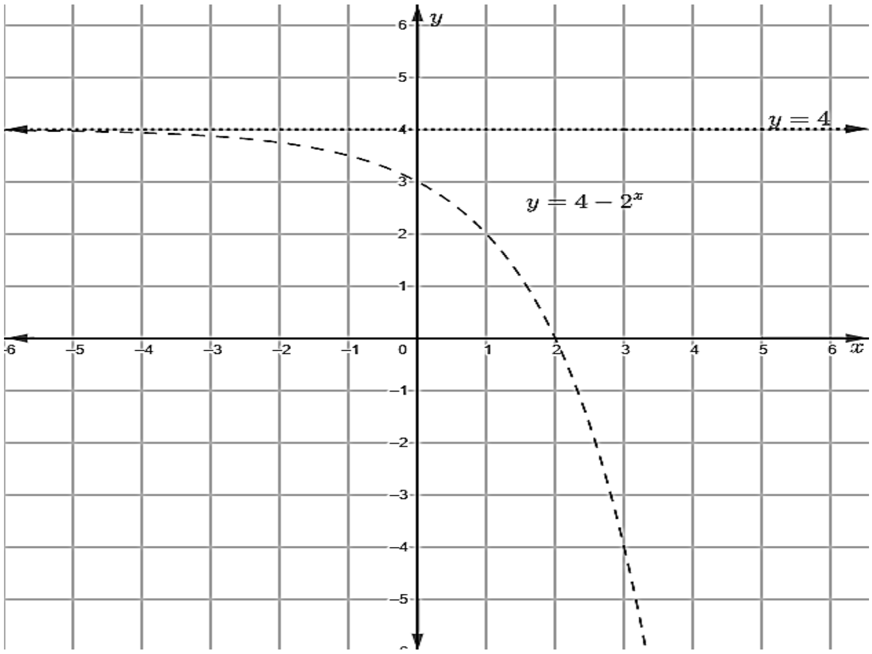
(ii) Hence, or otherwise, prove the following expression by mathematical induction: **3**

$1^3 + 2^3 + ... + n^3 = \left(1+2+...+n\right)^2$ for all integers $n \geq 1$

~ End of Question 11 ~

Question 12 (10 marks) Use the Question 12 BOOKLET **Marks**

(a) The graph $y = 4 - 2^x$ is shown below. **3**



In the Question 12 Answer Booklet, sketch the graph of $y^2 = 4 - 2^x$.

(b) Consider the function $f(x) = x^2 - 4x + 5$.

(i) Find the largest possible positive domain for which $f(x)$ has an inverse function. **1**

(ii) Find the equation of the inverse function $y = f^{-1}(x)$ for the domain determined in part (i). **2**

(c) Consider the function $y = 2\sin^{-1}(2x-1)$

(i) Find the domain and range. **2**

(ii) Sketch the curve, clearly showing the coordinates of the endpoints. **2**

~ End of Question 12 ~

Question 13 (9 marks) Use the Question 13 BOOKLET **Marks**

(a) Use the product to sum identity to write $\cos\theta\sin\left(2\theta\right)$ as a sum in simplest form. **2**

(b) It is given that $\sin A = \dfrac{1}{3}$ and $\cos B = \dfrac{1}{2}$. **3**

If $\angle A$ is obtuse and $\angle B$ is reflex, find the exact value of $\cos(A-B)$.

(c)

(i) Write $2\cos x + \sin x$ in the form $B\cos\left(x-\theta\right)$. **2**

(ii) Hence, or otherwise find, to the nearest minute, solutions to the equation **2**

$2\cos x + \sin x = 1$ for $0° \le x \le 360°$.

~ End of Question 13 ~

Question 14 (12 marks) Use the Question 14 BOOKLET **Marks**

(a) Let $\underset{\sim}{a} = -2\underset{\sim}{i} + 11\underset{\sim}{j}$ and $\underset{\sim}{b} = 8\underset{\sim}{i} + 6\underset{\sim}{j}$.

(i) Find the angle θ between $\underset{\sim}{a}$ and $\underset{\sim}{b}$. **2**

(ii) Find the projection of $\underset{\sim}{a}$ onto $\underset{\sim}{b}$. **2**

(b) 4 points A, B, C, and D on a plane have the following properties: **2**

$$\overrightarrow{AB} = \begin{pmatrix} -1 \\ 3 \end{pmatrix} \qquad \overrightarrow{AD} = \begin{pmatrix} 3 \\ -1 \end{pmatrix} \qquad \overrightarrow{DC} = \begin{pmatrix} 5 \\ 1 \end{pmatrix}$$

Prove that $ABCD$ is a trapezium

(c)

(i) Show that $\dfrac{d}{d\theta}\sec\theta = \sec\theta\tan\theta$ **1**

(ii) Hence, use the substitution $x = \sec\theta$ to find $\displaystyle\int_{\sqrt{2}}^{2} \frac{dx}{x^2\sqrt{x^2-1}}$ **3**

(d) Let $f(x) = 5x^2 + x$ for $x \geq \dfrac{-1}{10}$. Find the gradient of $f^{-1}(x)$ when $y = 1$ **2**

~ End of Question 14~

Question 15 (11 marks) Use the Question 15 BOOKLET **Marks**

(a) The volume, V, of a sphere of radius r mm is increasing at a constant rate of 200 mm^3 per second.

(i) Find $\dfrac{dr}{dt}$ in terms of r. **2**

(ii) Determine the rate of increase of the surface area, S, of the sphere when the radius is 50 mm. **2**

(b) A bottle of medicine which is initially at a temperature of $10°C$ is placed into a room which has a constant temperature of $20°C$. The medicine warms at a rate proportional to the difference between the temperature of the room and the temperature (T) of the medicine.

That is, T satisfies the equation $\dfrac{dT}{dt} = -k(T-20)$.

(i) Show that $T = 20 + Ae^{-kt}$ satisfies $\dfrac{dT}{dt}$. **1**

(ii) If the temperature of the medicine after 10 minutes is $16\,°C$, find its temperature after 40 minutes. Leave your answer correct to two decimal places. **3**

(c) Let $P(x) = (x+1)(x-3)Q(x) + ax + b$, where $Q(x)$ is a polynomial and a and b are real numbers.
The polynomial $P(x)$ has a factor of $x-3$.
When $P(x)$ is divided by $x+1$ the remainder is 8.

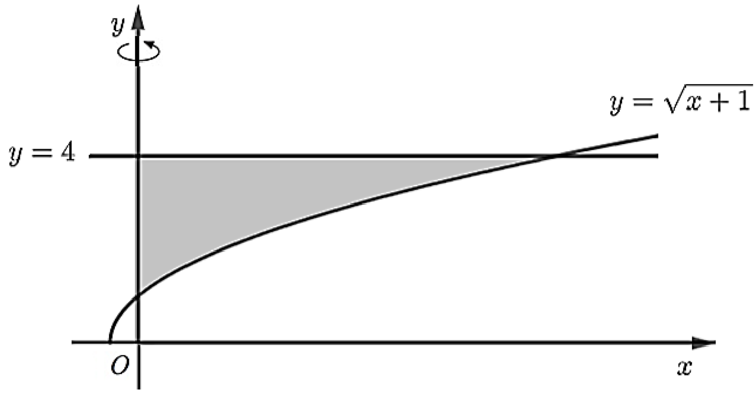
(i) Find the values of a and b. **2**

(ii) Find the remainder when $P(x)$ is divided by $(x+1)(x-3)$ **1**

~ End of Question 15~

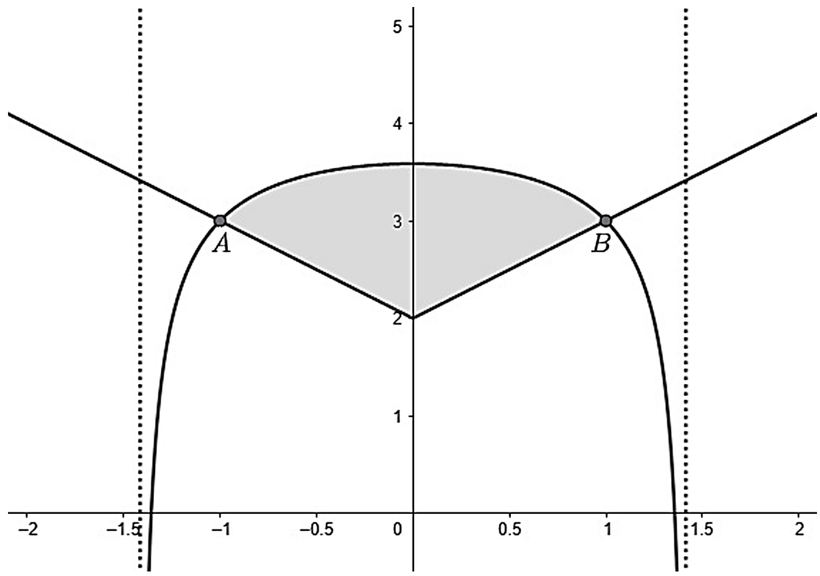
Question 16 (11 marks) Use the Question 16 BOOKLET **Marks**

(a) Find the exact volume of the solid formed by rotating the region between the curve $y=\sqrt{x+1}$, the y-axis and the line $y=4$ about the y-axis. **3**



(b) Find the domain and range of the function that is the solution to the differential equation $\dfrac{dy}{dx}=e^{x+2y}$ and whose graph passes through the origin. **4**

(c) The region enclosed by $f(x)=|x|+2$ and $g(x)=5-\dfrac{2}{\sqrt{2-x^2}}$ is shaded in the diagram. **4**



It is given that $A(-1,3)$ and $B(1,3)$ are the points of intersection of $f(x)$ and $g(x)$.

Find the exact value of the area of the shaded region.

Leave your answer in its simplest form.

~ End of Question 16~

~ End of Trial Examination ~

Year 12 Mathematics Extension 1 Assessment Task 4 Marking guidelines

MC Question 1:

The term independent of x in the expansion of $\left(x+\frac{2}{x}\right)^6$ is $6C_3(2)^3 = 160$

Answer: A

MC Question 2:

$$\frac{2}{(1-3^x)} > -1$$

$$\frac{2}{(1-3^x)} + 1 > 0$$

$$\frac{2+1-3^x}{1-3^x} > 0$$

$$\frac{3-3^x}{1-3^x} > 0$$

Critical points come from:

- Denominator $1-3^x = 0 \Rightarrow x = 0$ (undefined)
- Numerator $3-3^x = 0 \Rightarrow x = 1$ (equals 0)

Sign testing without detailed expansion

- Pick a test value in each region:
  - For $x < 0$: Denominator $+$, Numerator $+$ → fraction $+$ **True**
  - For $0 < x < 1$: Denominator $-$, Numerator $+$ → fraction - **False**
  - For $x > 1$: Denominator $-$, Numerator $-$ → fraction $+$ **True**

Answer: D

MC Question 3:

$x = \operatorname{cosec}^2 t \Rightarrow \sin^2 t = \frac{1}{x}$ and $y = \cos^2 t$

$$\sin^2 t + \cos^2 t = 1$$

$$\frac{1}{x} + y = 1$$

$$y = 1 - \frac{1}{x}$$

Answer: A

MC Question 4:

$$\cot\left(\frac{\theta}{2}\right)-2\cot\theta = \frac{1}{\tan\left(\frac{\theta}{2}\right)}-\frac{2}{\tan\theta}$$

$$= \frac{1}{t}-\frac{2}{\frac{2t}{1-t^2}}$$

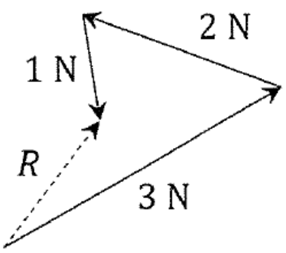
$$= \frac{1}{t}-\frac{1-t^2}{t}$$

$$= \frac{1-1+t^2}{t}=\frac{t^2}{t}=t$$

Answer: C

MC Question 5:

You can attempt to sketch these vectors end to end and achieve a resultant force in the "North-East" direction;



or it also suffices to see that of the 4 options, only one is between the 2 strongest forces, and most influenced by the direction of the strongest one.

Answer: C

MC Question 6:

Going around the figure gives:

$$\underset{\sim}{a}+\underset{\sim}{b}-\underset{\sim}{c}-\underset{\sim}{d}=\underset{\sim}{0}$$
$$\therefore \underset{\sim}{a}+\underset{\sim}{b}=\underset{\sim}{c}+\underset{\sim}{d}$$

Answer: B

MC Question 7:

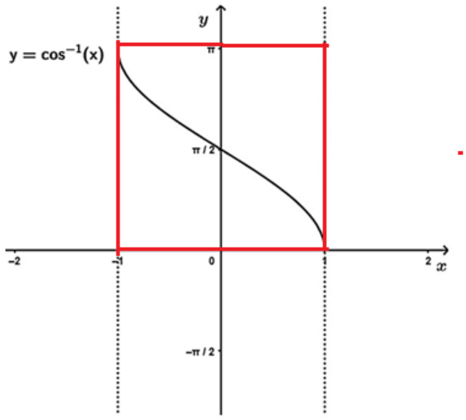
$$\frac{dN}{dt}=-80ke^{-kt}$$
$$=-k(80ke^{-kt})$$
$$=-kN$$

Answer: D

MC Question 8

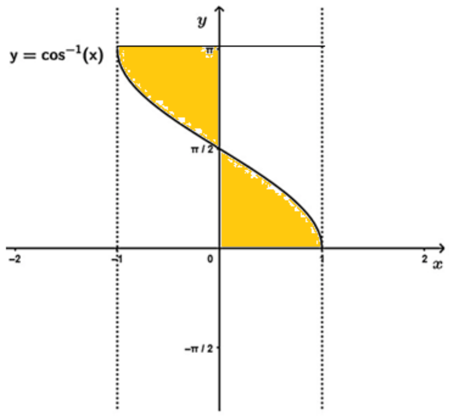
Area bounded by the graph $y = \cos^{-1}(x)$, the $x-$axis and the line $x = -1$ is

$$A = \frac{1}{2} \times \text{Area of the rectangle} = \frac{1}{2} \times 2 \times \pi = \pi \text{ units}^2$$



or

$$A = 1 \times \pi = \pi \text{ units}^2$$



Answer: B

MC Question 9:

Reference sheet: $\cos^2 nx = \frac{1}{2}\left(1 + \cos 2nx\right)$. We have n=2, so A and B are our only options, and they differ by a factor of $\frac{1}{2}$. The correct primitive is B **Answer: B**

MC Question 10

At $(0,0)$, $\frac{dy}{dx} = 0$ and so A is incorrect

At $(-1,1)$, $\frac{dy}{dx} = 0$ and so B and D are incorrect

Answer: C

<table>
<tr><td>Year 12</td><td colspan="2">Mathematics Extension 1</td><td>Assessment Task 4 2025</td></tr>
<tr><td colspan="4">Solutions and Marking Guidelines</td></tr>
<tr><td colspan="4">Outcomes Addressed in this Question</td></tr>
<tr><td colspan="4">ME11-5 uses concepts of permutations and combinations to solve problems involving counting or ordering
ME12-1 applies techniques involving proof or calculus to model and solve problems</td></tr>
<tr><td>Question 11</td><td colspan="2">Solutions</td><td>Marking Guidelines</td></tr>
<tr><td>(a)
(i)</td><td colspan="2">Total arrangements:
$\frac{8!}{2!\,2!\,2!} = 5040$</td><td>1 mark
Correct solution</td></tr>
<tr><td>(ii)</td><td colspan="2">Total arrangements that begins with “U” and ends with “U”:
$\frac{6!}{2!\,2!} = 180$
Total probability:
$= \frac{180}{5040} = \frac{1}{28}$
Students were awarded carry forward error if there was evidence of some understanding. The most common error was miscalculating the following $\frac{6!}{2!2!}$ as $\frac{6!}{2!2!} \times 2! = 360$. If you then determined the probability from that, one mark was awarded for showing some progress.</td><td>2 marks
Correctly determines the arrangements AND calculates the correct probability.

1 mark
Finds the correct arrangements OR makes significant progress towards the correct solution.</td></tr>
<tr><td>(b)
(i)</td><td colspan="2">Using the sum of an arithmetic sequence for
$1 + 2 + 3 + \cdots + n$
$a = 1, \quad d = 1, \quad n = n, \quad l = n$
$S_n = \frac{n}{2}(a + l)$
$= \frac{n}{2}(1 + n)$
$= \frac{n}{2}(n + 1)$
Since $1 + 2 + 3 ... + n = \frac{n}{2}(n+1)$,
$(1 + 2 + 3 + ...n)^2 = (S_n)^2$</td><td>1 mark
Correctly uses the sum of an arithmetic sequence to show the desired result.</td></tr>
</table>

	$$(1+2+3+...n)^2 = \left[\frac{n}{2}(n+1)\right]^2$$ ***see note*** $$= \frac{n^2}{4}(n+1)^2$$ $$= \frac{1}{4}n^2(n+1)^2$$ Evidence of using the formula for the sum of the arithmetic sequence must be shown, either by referencing the formula to be used and identifying the first term, last term OR number of terms. Students who went straight to this step (***) were not awarded full marks as this provides no evidence of utilising the formula, as this can be derived from the solution given in the question by backtracking.	
(ii)	<u>Step 1:</u> Show the result is true when $n = 1$ When $n = 1$, $$LHS = 1^3 = 1,\ RHS = 1^2 = 1$$ $$LHS = RHS$$ $$\therefore \text{ The result is true when } n = 1$$ <u>Step 2:</u> Assume the result is true for some integer k, where $k \geq 1$ i.e. assume $$1^3 + 2^3 + 3^3 + ... + k^3 = (1+2+3+...+k)^2$$ <u>Step 3:</u> Prove the result is true for $n = k + 1$ assuming it is true for $n = k$ i.e. prove $$1^3 + 2^3 + 3^3 + ... + (k+1)^3 = [1+2+3+...+(k+1)]^2$$ $$LHS = 1^3 + 2^3 + 3^3 + ... + (k+1)^3$$ $$= 1^3 + 2^3 + 3^3 + ... + k^3 + (k+1)^3$$	**3 marks** Utilises mathematical induction to correctly prove the result. **2 marks** Utilises mathematical induction, however, missing part of the solution OR makes ONE singular error in the proof. **1 mark** Correctly shows step 1 AND step 2 OR shows some progress towards the correct solution.

From assumption in (i)

$$= \frac{1}{4}k^2(k+1)^2 + (k+1)^3$$

$$= \frac{(k+1)^2}{4}\left[k^2 + 4(k+1)\right]$$

$$= \frac{(k+1)^2}{4}\left[k^2 + 4k + 4\right]$$

$$= \frac{1}{4}(k+1)^2(k+2)^2$$

$$= RHS$$

$\therefore$ The statement is true when $n = k$ if it is true when $n = k + 1$.

Step 4:

If the statement is true for $n = k$, then it is true for $n = k + 1$. Since the result is true for $n = 1 + 1 = 2$ and for $n = 2 + 1 = 3$ and so on, for all positive integers n.

The conclusion doesn't need to be as detailed as above, however, a conclusion is necessary for a complete solution. If all the algebra was correct, however, there was no conclusion, then a maximum of 2 marks was awarded.

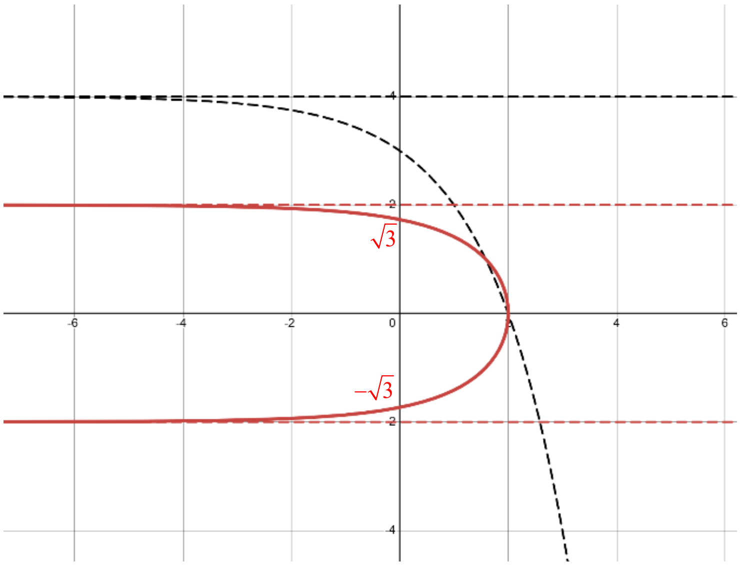
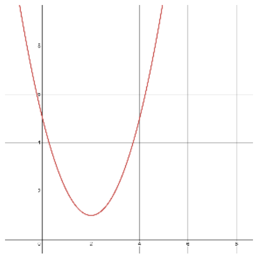
Year 12	Mathematics Extension 1	Ass Task 4 2025 HSC
Question 12	Solutions and Marking Guidelines	THE TRIAL

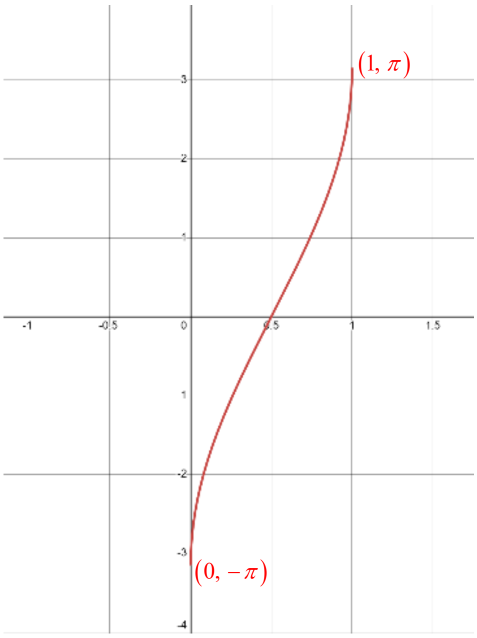
Outcomes Addressed in this Question

ME11-1 uses algebraic and graphical concepts in the modelling and solving of problems involving functions and their inverses

ME11-2 manipulates algebraic expressions and graphical functions to solve problems

ME11-3 applies concepts and techniques of inverse trigonometric functions and simplifying expressions involving compound angles in the solution of problems

Question	Solutions	Marking Guidelines
(a) **ME11-2**		3 marks: correct solution Provides a correct sketch with all intercepts and asymptotes noted, vertical gradient at x-intercept, and correct 'shape' in relation to given dotted function 2 marks: substantially correct solution Provides a correct sketch, but either intercepts, asymptotes or vertical gradient are missing or incorrect… or Provides a **correct** $y=\sqrt{4-2^x}$ graph without reflecting, including some of the above-mentioned features 1 mark: progress towards correct solution Recognises the asymptotes of $y=\pm 2$ or the intercepts of $\pm\sqrt{3}$, or equivalent merit
(b)(i) **ME11-1**	Vertex at (2, 1) So largest *positive* domain to have inverse functions is $x \geq 2$ 	1 mark: correct answer ie $x \geq 2$
(ii) **ME11-1**	$x = y^2 - 4y + 5 \qquad (x \geq 2)$ $y^2 - 4y + 4 = x - 1 \qquad (y \geq 2)$ $(y-2)^2 = x - 1$ $y - 2 = \pm\sqrt{x-1}$ $y = \pm\sqrt{x-1} + 2$ $y = \sqrt{x-1} + 2$ take $+ve$ as $y \geq 2$	2 marks: correct solution 1 mark: ***substantially*** correct solution Some attempt to make y the subject (such as completing the square) was required. Swapping x and y, while necessary, was not enough

	Question 12 continued…	
(c)(i) **ME11-3**	Domain $-1 \le 2x-1 \le 1$ $0 \le 2x \le 2$ $0 \le x \le \frac{1}{2}$ Range $-\frac{\pi}{2} \le \sin^{-1}(2x-1) \le \frac{\pi}{2}$ $-\pi \le 2\sin^{-1}(2x-1) \le \pi$ $-\pi \le y \le \pi$	<u>2 marks</u>: correct solution <u>1 mark</u>: substantially correct solution
(c)(ii)		<u>2 marks</u>: correct solution Including endpoints found in (i) *and* must show a sideways translation. <u>1 mark</u>: substantially correct solution

Year 12	**Mathematics Extension 1**	**Task 4 2025**
Section II	Solutions and Marking Guidelines	
	Outcomes Addressed in this Question	
ME12-3 applies advanced concepts and techniques in simplifying expressions involving compound angles and solving trigonometric equations		

Question13	**Solutions**	**Marking Guidelines**
a)	$\cos\theta\sin 2\theta = \frac{1}{2}\left[\sin(\theta+2\theta)-\sin(\theta-2\theta)\right]$ $= \frac{1}{2}\left[\sin(3\theta)-\sin(-\theta)\right] \quad (\sin(-\theta) = -\sin\theta)$ $= \frac{1}{2}\left[\sin(3\theta)+\sin(\theta)\right]$	1 mark for showing substitution of 2θ and θ into the correct formula and simplifying 1 mark for showing/ recognising that $\sin(-\theta) = -\sin\theta$ and further simplifying expression
b)	Since A is obtuse, cos A < 0: $\cos A = -\frac{\sqrt{8}}{3}$ Since B is reflex, sin B < 0 $\sin B = -\frac{\sqrt{3}}{2}$ $\cos(A-B) = \cos A\cos B + \sin A\sin B$ $= \left(-\frac{\sqrt{8}}{3}\right)\left(\frac{1}{2}\right)+\left(\frac{1}{3}\right)\left(-\frac{\sqrt{3}}{2}\right)$ $= -\frac{\sqrt{8}}{6} - \frac{\sqrt{3}}{6}$ $= \frac{-\sqrt{8}-\sqrt{3}}{6}$ $= -\frac{\sqrt{8}+\sqrt{3}}{6}$	1 mark for correct value of cosA 1 mark for correct value of sinB 1 mark for correct formula substitution and simplification to get correct answer.

c)	i) $2\cos x+\sin x = B[\cos x\cos\theta+\sin x\sin\theta]$ $B\cos\theta = 2$ .... (1) $B\sin\theta = 1$ ....(2) $\frac{(2)}{(1)} = B\tan\theta = \frac{1}{2}$ $\theta = \tan^{-1}\left(\frac{1}{2}\right) = 26°34'$ $(1)^2+(2)^2 = B^2\cos^2\theta+B^2\sin^2\theta$ $2^2+1^2 = B^2\left(\sin^2\theta+\cos^2\theta\right)$ $5 = B^2(1)$ $B = \sqrt{5}$ $2\cos x+\sin x = \sqrt{5}\cos\left(x-26°34'\right)$ Majority of students wrote θ as $\tan^{-1}\left(\frac{1}{2}\right)$. Mark was not awarded if its numerical value was not listed in part i) or ii).	1 mark for correct value of B. 1 mark for correct numerical value of θ
c)	ii) $2\cos x+\sin x = 1$ $\sqrt{5}\cos\left(x-26°34'\right) = 1$ $\cos\left(x-26°34'\right) = \frac{1}{\sqrt{5}}$ $x-26°34' = 63°26',\quad 296°34'$ $x = 90°,\ 323°8'$	1 mark for writing out line 3. 1 mark for both correct final answers to the nearest minute.

<table>
<tr><td colspan="3">Year 12 Mathematics Extension 1 Task 4 2025

Question 14 Solutions and Marking Guidelines</td></tr>
<tr><td colspan="3">Outcomes Addressed in this Question</td></tr>
<tr><td colspan="3">ME12-2: Applies concepts and techniques involving vectors to solve problems.

ME12-4 uses calculus in the solution of applied problems.</td></tr>
<tr><td>Outcome</td><td>Solutions</td><td>Marking Guidelines</td></tr>
<tr><td>(a)

ME12-2</td><td>(a)(i)

$\cos\theta = \dfrac{\underset{\sim}{a}\cdot\underset{\sim}{b}}{|\underset{\sim}{a}||\underset{\sim}{b}|} = \dfrac{-16+66}{\sqrt{(-2)^2+11^2}\sqrt{8^2+6^2}} = \dfrac{50}{\sqrt{125}\sqrt{100}}^{**} = \dfrac{50}{5\sqrt{5}\times 10}$

$= \dfrac{1}{\sqrt{5}}$

$\therefore \theta = 63^\circ 26'$

Degrees is the most practical form of answer in this context.

(ii)

$proj_{\underset{\sim}{b}}\underset{\sim}{a} = \dfrac{\underset{\sim}{a}\cdot\underset{\sim}{b}}{\underset{\sim}{b}\cdot\underset{\sim}{b}}\times\underset{\sim}{b} = \dfrac{50}{100}\times\left(8\underset{\sim}{i}+6\underset{\sim}{j}\right)$

$= 4\underset{\sim}{i}+3\underset{\sim}{j}$

OR

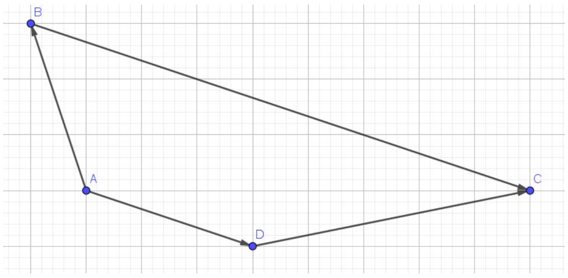
$proj_{\underset{\sim}{b}}\underset{\sim}{a} = \left(|\underset{\sim}{a}|\cos\theta\right)\hat{\underset{\sim}{b}} = \left(5\sqrt{5}\left(\dfrac{1}{\sqrt{5}}\right)\right)\times\dfrac{1}{10}\left(8\underset{\sim}{i}+6\underset{\sim}{j}\right)$

$= 4\underset{\sim}{i}+3\underset{\sim}{j}$</td><td>(a)(i) 2 marks: Correct solution. (Nearest degree and answer in radians also accepted.)

1 mark: Reaches ** as an expression for $\cos\theta$.

(ii) 2 marks: Correct solution giving the projection as a single vector. (Not factorised)

1 mark: A correct numerical expression for the calculation of the vector.</td></tr>
</table>

(b) ME 12-2	(b) Draw a sketch:  The sketch doesn't need to contain the grid, but it does give you a view of how the vectors join. Find the fourth side: $\overrightarrow{BC}=\overrightarrow{BA}+\overrightarrow{AD}+\overrightarrow{DC}$ $=\underset{\sim}{i}-3\underset{\sim}{j}+3\underset{\sim}{i}-\underset{\sim}{j}+5\underset{\sim}{i}+\underset{\sim}{j}$ $=9\underset{\sim}{i}-3\underset{\sim}{j}$ $\overrightarrow{BC}=3\left(3\underset{\sim}{i}-\underset{\sim}{j}\right)=3\overrightarrow{AD}$ Being a scalar multiple means that $\overrightarrow{BC}\parallel\overrightarrow{AD}$, and so the quadrilateral must be a trapezium. The above working without the sketch is the minimum required for 2 marks. This is all that was required for the question because it provided the "trapezium" information. If you were asked to classify the shape, it would have been necessary to show that it was not a parallelogram, and hence that "trapezium" would be the best classification. The most efficient way to show it's not a parallelogram is just to realise that the two parallel vectors are not equal in length. No need to calculate the length or slope of the other 2 sides.	**(b) 2 marks:** Proving that the two sidelengths are parallel, and hence form part of a trapezium. **1 mark:** Correct calculation of $\overrightarrow{BC}$, followed by either incorrect reasoning or incorrect calculation of scalar.

(c) ME 12-4	c) (i) $\frac{d}{d\theta}\sec\theta = \frac{d}{d\theta}(\cos\theta)^{-1} = -(\cos\theta)^{-2} \times -\sin\theta$ $= \frac{\sin\theta}{\cos^2\theta} = \frac{1}{\cos\theta} \times \frac{\sin\theta}{\cos\theta}$ $= \sec\theta\tan\theta$ as required. **NOTE: Use of dash notation after a function is NOT ACCEPTED NOTATION. The only time you can use it is for *y'* or *f'(x)*. There is no relevance and only confusion if you put a dash where it looks like a 1 in the index of a function.** (ii) After the substitution, the integral becomes: $\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{\sec\theta\tan\theta}{\sec^2\theta\sqrt{\sec^2\theta - 1}} d\theta = \int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{\sec\theta\tan\theta d\theta}{\sec^2\theta\sqrt{\tan^2\theta}}$ $= \int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{d\theta}{\sec\theta}$ $= \int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \cos\theta d\theta$ $= \sin\theta\Big]_{\frac{\pi}{4}}^{\frac{\pi}{3}}$ $= \sin\frac{\pi}{3} - \sin\frac{\pi}{4}$ $= \frac{\sqrt{3} - \sqrt{2}}{2}$	**(c) (i) 1 mark: Completely** correct solution. **(ii) 3 marks:** 3 components correct: Substitution of correct boundaries (This does not include ignoring the boundaries and then reverting back at the end). Simplification of the integrand. Correct numerical expression for the final answer. **2 marks**: 2 correct components shown. **1 mark** : 1 correct component shown.

(d) ME 12-4	(d) The inverse function satisfies: $x = 5y^2 + y$ $\frac{dx}{dy} = 10y + 1 \quad \rightarrow \quad \frac{dy}{dx} = \frac{1}{10y+1}$ When $y = 1$, $\frac{dy}{dx} = \frac{1}{11}$ NOTE: 1. Most people chose to evaluate x in $f(x)$ and did a lot of unrewarded work, because the awareness of the geometry of the inverse function was required before earning the first mark available. The above solution shows that such a method was not necessary. 2. Lots of responses think that the inverse function's gradient is perpendicular to that of its reflection in the line $y=x$, but that is not the case. It is the reciprocal of the tangent at the point of reflection. The answer above tells us that the gradient of the original function is 11 when $x=1$, and if that was clearly communicated, a response could receive full marks.	**(d) 2 marks:** Correct differential equation **for the inverse function.** Correct substitution of $y=1$ into expression. **1 mark:** Correct differential equation **for the inverse function only.** You do not receive a mark for $f'(x) = 10x + 1$ because this does not apply to the inverse function.

2025 Yr12 HSC Assessment Task 4	
Question 15 Solutions and Marking Guidelines	
Outcomes Addressed in this Question	
ME11-4 applies understanding of the concept of a derivative in the solution of problems, including rates of change, exponential growth and decay and related rates of change **ME11-2** manipulates algebraic expressions and graphical functions to solve problems	
Solutions	**Marking Guidelines**
a) i) $V=\frac{4}{3}\pi r^3$ $\frac{dV}{dt}=4\pi r^2$ $\frac{dr}{dV}=\frac{1}{4\pi r^2}$ $\frac{dr}{dt}=\frac{dr}{dV}\times\frac{dV}{dt}$ $=\frac{1}{4\pi r^2}\times 200$ $=\frac{50}{\pi r^2}$	**2 marks** Correct solution for finding $\frac{dr}{dt}$ **1 mark** Correctly finds $\frac{dr}{dv}$
ii) $\frac{dS}{dt}=\frac{dS}{dr}\times\frac{dr}{dt}$ $=8\pi(50)\times\frac{50}{\pi(50)^2}$ $=8\ \text{mm}^2/\text{s}$ 1 mark related rate 1 mark correct solution	**2 marks** Correct solution **1 mark** States correct related rate using the chain rule
b) i) $T=20+Ae^{-kt}$ and $Ae^{-kt}=T-20$ $\frac{dT}{dt}=-kAe^{-kt}$ $=-k(T-20)$	**1 mark** States $Ae^{-kt}=T-20$ and substitutes into $\frac{dT}{dt}=-kAe^{-kt}$ **OR** Substitutes into $\frac{dT}{dt}=-k(T-20)$

ii) At $t=0$, $T=10$ $10=20+Ae^{-k\times 0}$ $A=10$ At $t=10$, $T=16$ $16=20-15e^{-10k}$ $e^{-10k}=\dfrac{9}{15}$ At $t=40$ $T=20-10e^{-40k}$ $=20-10\left(e^{-10k}\right)^4$ (applying index laws) $=20-10\left(\dfrac{2}{5}\right)^4$ $=19.74$ Note: I have shown an alternative method without having to find the value of *k* explicitly.	**3 marks** Correct Solution **2 marks** Finds a value for *k* or e^{-10k} correct from prior working **1 mark** Finds correct *A*
c) i) $P(3)=0$ $P(-1)=8$ $0+3a+b=0$ $0-a+b=8$ So $a=-2$ and $b=6$	**2 marks** Correct *a* and *b* **1 mark** Uses factor theorem correctly
ii) Remainder when divided by $(x+1)(x-3)$ is $ax+b$ (from question). So, remainder is $-2x+6$. Note: Students need to review $P(x)=A(x)Q(x)+R(x)$ form of a polynomial	**1 mark** Finds correct remainder

Year 12 Extension 1 Trial	HSC Mathematics	Task 4 2025

Section II Solutions and Marking Guidelines

Outcomes Addressed in this Question:

ME12-4 uses calculus in the solution of applied problems, including differential equations and volumes of solids of revolution
ME12-7 evaluates and justifies conclusions, communicating a position clearly in appropriate mathematical forms

Part	Solutions	Marking Guidelines
Q16 (a)	$y=\sqrt{x+1}$ $y^2=x+1$ $x=y^2-1$ $x^2=\left(y^2-1\right)^2$ $x^2=y^4-2y^2+1$ $y=\sqrt{x+1}$ has y-intercept at $(0,1)$ $V=\pi\int_1^4 (y^4-2y^2+1)dy$ $V=\pi\left[\frac{y^5}{5}-2\times\frac{y^3}{3}+y\right]_1^4$ $V=\pi\left[\left(\frac{4^5}{5}-2\times\frac{4^3}{3}+4\right)-\left(\frac{1^5}{5}-2\times\frac{1^3}{3}+1\right)\right]$ $V=\frac{828}{5}\pi$ units^3 **Marker Notes:** **Areas for students to improve include**: • Understanding the difference between area and volume. • understanding that volume of solid obtained on rotation about y-axis is not going to be the same as rotation about x-axis even when the limits are changed. • understanding how to find the volume of the solid of revolution using $V=\pi\int y^2dy$.	**3 marks awarded for** • Identifying that the volume is found by rotating the region about the y-axis and correctly determining the y-boundaries of integration. • Writing the correct integrand in terms of y for the volume with respect to rotation about the y-axis. • Integrating accurately and simplifying to obtain the exact final volume. **2 marks awarded for** • Correctly finding the y-limits and writing a mostly correct volume expression in terms of y or making a mistake during integration that changes the final answer. Or • Writing the correct volume expression in terms of y but making an error in the limits or a mistake during integration that changes the final answer. **1 mark awarded for** • Showing no understanding of rotation about the y-axis but integrating and evaluating correctly without any mistakes. Or • Showing some understanding of rotation about the y-axis, but with an incomplete setup or major errors.

Q16(b)	$\frac{dy}{dx}=e^{x+2y}$ $\frac{dy}{dx}=e^{x}\times e^{2y}$ $\frac{dy}{e^{2y}}=e^{x}dx$ $\int\frac{dy}{e^{2y}}=\int e^{x}dx$ $\int e^{-2y}dy=\int e^{x}dx$ $\frac{e^{-2y}}{-2}=e^{x}+c$ since the graph passes through $(0,0)$ $\frac{e^{-2(0)}}{-2}=e^{(0)}+c$ $c=-\frac{1}{2}-1=-\frac{3}{2}$ $\therefore\frac{e^{-2y}}{-2}=e^{x}-\frac{3}{2}$ $e^{-2y}=-2\left(e^{x}-\frac{3}{2}\right)$ $e^{-2y}=-2e^{x}+3$ $-2y=\ln(-2e^{x}+3)$ $y=-\frac{1}{2}\ln(-2e^{x}+3)$ For domain consider $-2e^{x}+3>0$ $-2e^{x}>-3$ $e^{x}<\frac{3}{2}$ $x<\ln\left(\frac{3}{2}\right)$ $\quad\therefore$ Domain is $\left(-\infty,\ln\left(\frac{3}{2}\right)\right)$ For range as $x\to-\infty,\ e^{x}\to0,\ y\to-\frac{1}{2}\ln(3)$ As $x\to\ln\left(\frac{3}{2}\right),\ e^{x}\to\frac{3}{2},\ -2e^{x}+3\to0$ $\ln\left(-2e^{x}+3\right)\to-\infty$ $y=-\frac{1}{2}\ln\left(-2e^{x}+3\right)\to\infty$ $\therefore$ Range is $\left(-\frac{1}{2}\ln3,\infty\right)$	**4 marks awarded for** • Correctly separating the variables in the differential equation and integrating both sides accurately. • Using the given initial condition (graph passes through the origin) to correctly find the constant of integration. • Correctly determining the domain of the function found. • Correctly determining the range of the function found. **3 marks awarded for** • Correctly performing separation of variables and integration, and using the initial condition to find the constant, but making a minor mistake in either the domain or the range. Or • Correctly separating variables, integrating, and determining domain and range, but making an error when applying the initial condition. **2 marks awarded for** • Correctly separating and integrating but making mistakes in both applying the initial condition and finding either domain or range. Or • Separating and integrating with some errors but correctly determining both domain and range from their (possibly incorrect) expression. **1 mark awarded for** Showing understanding of separation of variables but with an incomplete or incorrect integration constant calculation and not finding domain or range correctly.
	Q16 (b) Marker Notes: **Areas for students to improve include:**	

	• understanding how to integrate terms such as e^{-y} • ensuring that they include a constant when completing an indefinite integral • understanding when an absolute value sign may or may not be needed when using natural logarithms • understanding how to find the domain and range of a function involving logarithms • understanding the difference between the symbols '[' and '(' when using interval notation • leaving answers in exact form when they are simple terms such as $\ln\left(\frac{3}{2}\right)$.	
Q16 (c)	$A=\int_{-1}^{1}\left(g(x)-f(x)\right)dx$ $A=\int_{-1}^{1}\left(\left(5-\frac{2}{\sqrt{2-x^2}}\right)-\left(\lvert x\rvert+2\right)\right)dx$ Since the shaded region is symmetrical about the $A=2\int_{0}^{1}\left(\left(5-\frac{2}{\sqrt{2-x^2}}\right)-\left(x+2\right)\right)dx$ $A=2\int_{0}^{1}\left(5-\frac{2}{\sqrt{2-x^2}}-x-2\right)dx$ $A=2\int_{0}^{1}\left(3-\frac{2}{\sqrt{2-x^2}}-x\right)dx$ $A=2\left[3x-2\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)-\frac{x^2}{2}\right]_0^1$ $A=2\left[\left(3-2\sin^{-1}\left(\frac{1}{\sqrt{2}}\right)-\frac{1^2}{2}\right)-0\right]$ $A=2\left(3-2\times\frac{\pi}{4}-\frac{1}{2}\right)$ $A=6-\pi-1=5-\pi\ units^2$ **Marker Notes:** **Areas for students to improve include**: • understanding how to approach integrals involving absolute values • recognising that their workload can be simplified through use of symmetry • identifying the axis along which the limits of integration should be chosen	**4 marks awarded for** • Correctly identifying the symmetry of the region and using it to simplify the calculation. • Correctly writing the integrand for the area, including the correct removal of the absolute value notation in f(x) for the given limits. • Performing the integration accurately without algebraic or arithmetic errors. • Evaluating the definite integral correctly and simplifying the result to its exact form. **3 marks awarded for** • Correctly identifying the symmetry and writing the correct integrand, but making an error during integration or in evaluation. Or • correctly identifying the symmetry and integrating accurately, but making an error in writing the integrand (e.g., mishandling the absolute value) that still leads to a partially correct result. **2 marks awarded for** • Correctly identifying the symmetry but making errors in both the integrand setup and either integration or evaluation. Or • correctly writing the integrand (with correct absolute value handling) and integrating correctly, but making an error in recognising the symmetry. **1 mark awarded for** • Identifying the symmetry or setting up part of the correct integrand, but

	• using the Reference Sheet for integrations which result in inverse trigonometric functions • understanding that inverse trigonometric functions should only be evaluated in radians	not completing the rest of the solution correctly. Or • integrating a partly correct expression but with major errors in the setup or evaluation