Killara High School

Student Number: ____________________

2025 Higher School Certificate Trial

Mathematics Extension 1

General Instructions

- Reading time - 10 minutes
- Working time - 120 minutes
- Write using black pen
- Calculators approved by NESA may be used
- A reference sheet is provided
- For questions in Section II, show mathematical reasoning and/or calculations

Total Marks: 70

Section I – 10 marks (Pages 1–6)

- **Attempt Questions 1–10**
- **Allow about 15 minutes for this section**

Section II – 60 marks (Pages 7–13)

- **Attempt Questions 11–14**
- **Allow about 105 minutes for this section**

Question:	1	2	3	4	5	6	7	8
Marks:	1	1	1	1	1	1	1	1
Awarded:								
Question:	9	10	11	12	13	14		Total
Marks:	1	1	14	16	15	15		70
Awarded:								

Do not write on this page

Section I

10 marks

Attempt Questions 1–10

Allow about 15 minutes for this section.

Use the multiple-choice answer sheet for Questions 1–10.

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1. What is the range of the function $f(x) = |\arcsin x| + \arccos x$? **1**

   A. $f(x) \in [0, 2\pi]$

   B. $f(x) \in [0, \pi]$

   C. $f(x) \in \left[\frac{\pi}{2}, \frac{3\pi}{2}\right]$

   D. $f(x) \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

2. Find the remainder when $x^4 + 2x^2 - 13x - 60$ is divided by $2x + 3$. **1**

   A. $-\frac{495}{16}$

   B. $-\frac{4070}{81}$

   C. 0

   D. 78

3. Let O be the point $(0, 0)$, R the point $(-2, 1)$ and S the point $(-c, 2c)$, where $c \in \mathbb{R}$. **1**

   The scalar product $OS \cdot RS$ is equal to:

   A. $4c - 5$

   B. $5 - 4c$

   C. $4c - 5c^2$

   D. $5c^2 - 4c$

4. The random variable X represents the number of successes in 10 Bernoulli trials. **1**

The probability of success is $p = 0.4$ in each trial.

Which of the following statements is true?

A. $P(X < 4) > P(X > 4)$

B. $P(X < 4) = P(X > 6)$

C. $P(X \leqslant 4) = 0.5$

D. $P(X = 4) < P(X > 5)$

5. The coefficient of a^5b^4 in the expansion of $\left(3a - \dfrac{b}{3}\right)^9$ is: **1**

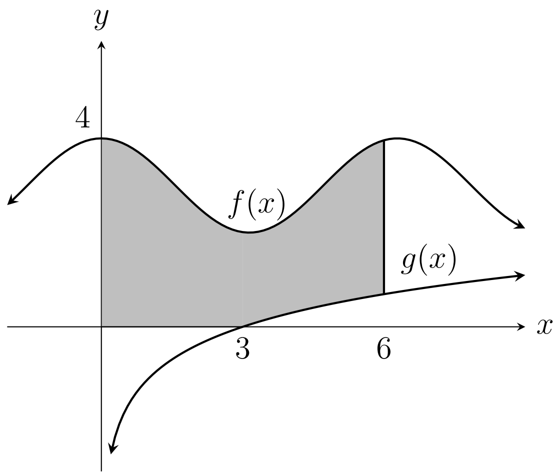
A. -42

B. 3

C. 42

D. 378

6. The shaded region shown is rotated round the x-axis. **1**



Which expression calculates the volume of revolution formed?

A. $\pi \displaystyle\int_0^6 \big(f(x) - g(x)\big)^2\,dx$

B. $\pi \displaystyle\int_0^3 \big(f(x)\big)^2\,dx + \pi \int_3^6 \big(f(x)\big)^2 - \big(g(x)\big)^2\,dx$

C. $\pi \displaystyle\int_0^6 \big(f(x)\big)^2 - \big(g(x)\big)^2\,dx$

D. $\pi \displaystyle\int_0^3 \big(f(x)\big)^2\,dx + \pi \int_3^6 \big(g(x)\big)^2\,dx$

7. The parametric equations **1**

$$x = 3\cos t$$

$$y = 4\sin t$$

are equivalent to which cartesian equation?

A. $x^2 + y^2 = 25$

B. $\dfrac{x^2}{3} + \dfrac{y^2}{4} = 1$

C. $\dfrac{x^2}{9} + \dfrac{y^2}{16} = 1$

D. $x^2 + y^2 = 16$

8. Given $g(x) = e^{2x} + 2$, $g^{-1}(x)$ is given by: **1**

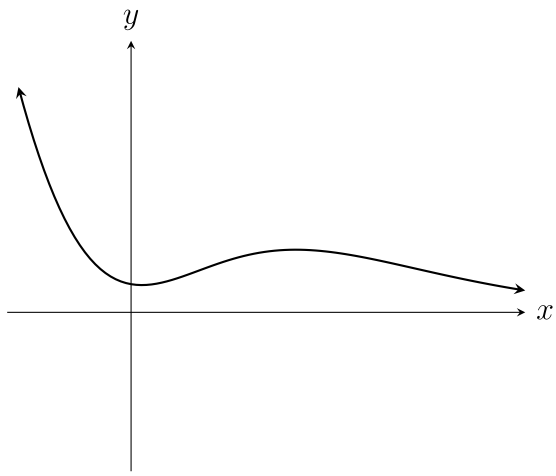
A. $y = \dfrac{\ln x - 2}{2}$

B. $y = \dfrac{\ln x}{2\ln 2}$

C. $y = \ln\left(x^2 - 2\right)$

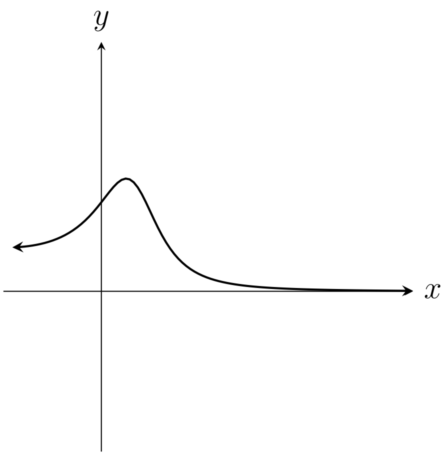
D. $y = \dfrac{\ln\left(x-2\right)}{2}$

9. The following graph shows $y = \dfrac{1}{f(x)}$ for a function $f(x)$. **1**

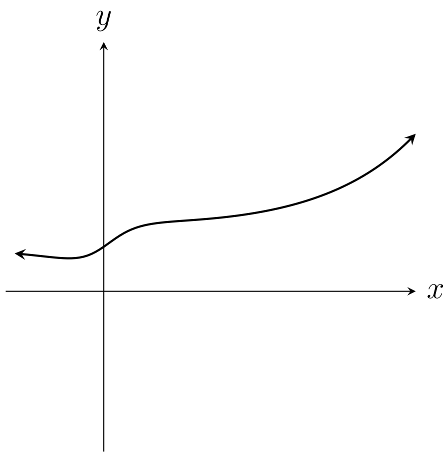


Which graph could represent $y = f(x)$?

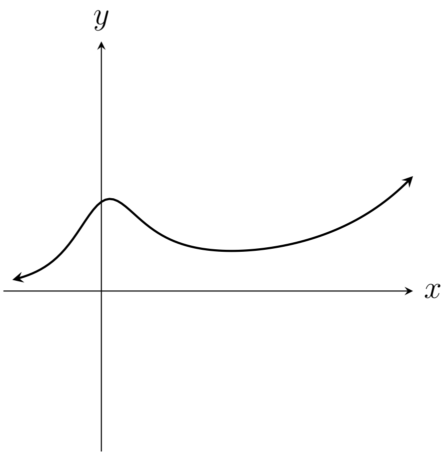
A.



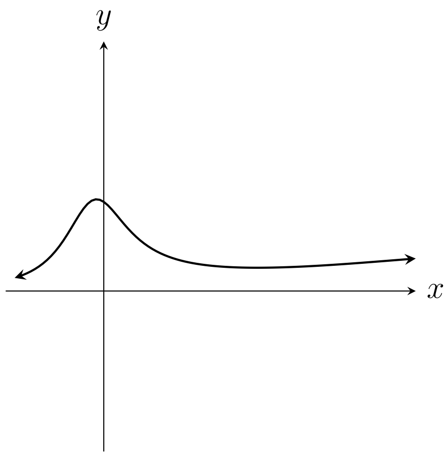
C.



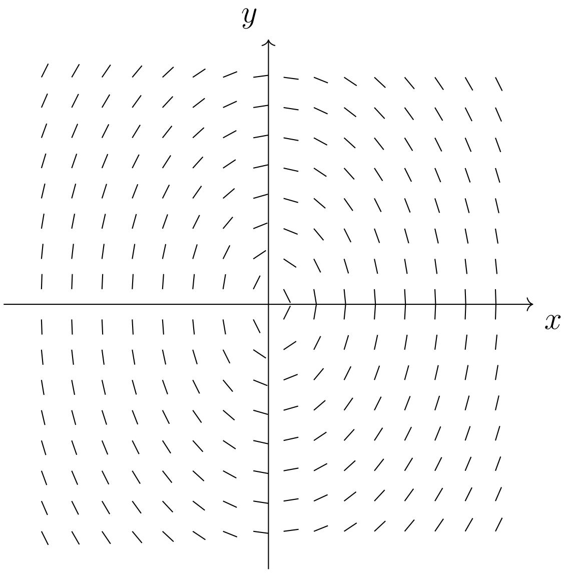
B.



D.



10. Which differential equation best represents the slope field below? **1**



A. $\dfrac{dy}{dx} = -\dfrac{2x}{y}$

B. $\dfrac{dy}{dx} = \dfrac{2x}{y}$

C. $\dfrac{dy}{dx} = \dfrac{y}{2x}$

D. $\dfrac{dy}{dx} = x^2 + y^2$

Section II

60 marks
Attempt Questions 11–14
Allow about 105 minutes for this section.
Answer each question in the appropriate writing booklet.
All necessary working should be shown in every question.

Question 11 (14 Marks) Use a SEPARATE writing booklet.

(a) Solve the inequality: **2**

$$|3x - 5| < 4$$

(b) Use the substitution $u = \sin x$ to find $\int -\cos x \sin^{\frac{3}{2}} x dx$. **2**

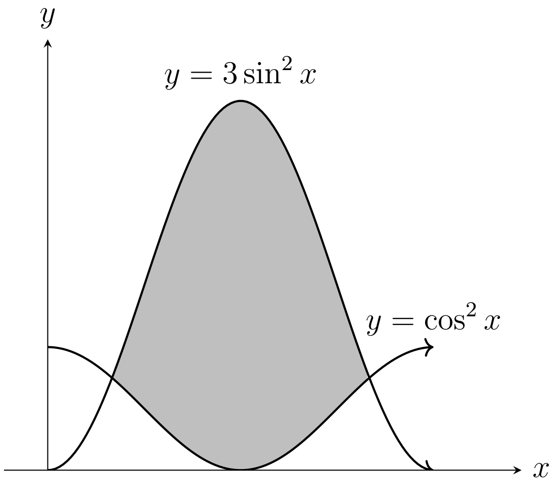
(c) The letters of the name NARUTO are arranged in a row.

(i) In how many ways can this be done using all the letters? **1**

(ii) What is the probability that the vowels are all together? **2**

(d) Determine the measure of angle A in triangle ABC, where $A(1, 1), B(4, -3)$, and $C(-3, 2)$. Express your answer in degrees to the nearest degree. **3**

(e) The graphs of $y = \cos^2 x$ and $y = 3\sin^2 x$ are shown below.



(i) Show that the points of intersection are at $x = \dfrac{\pi}{6}$ and $x = \dfrac{5\pi}{6}$. **2**

(ii) Find the exact shaded area. **2**

End of Question 11

Question 12 (16 Marks) Use a SEPARATE writing booklet.

(a) Using mathematical induction, verify that for all $n \geqslant 1$, the sum of the squares of the first $2n$ positive integers is given by the formula **3**

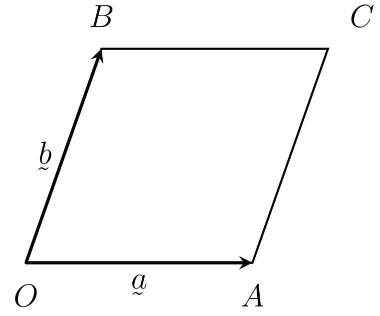
$$1^2 + 2^2 + 3^2 + \cdots + (2n)^2 = \frac{n(2n+1)(4n+1)}{3}$$

(b) Solve $\cos x - \sqrt{3}\sin x = 2$ for $0 \leqslant x \leqslant 2\pi$. **3**

(c) The temperature T, in degrees celcius, of tea, t minutes after it is brewed can be modelled using the differential equation $\dfrac{dT}{dt} = k\,(T - T_1)$, where k is the constant of proportionality and T_1 is a constant. **3**

In a particular teahouse, the temperature is kept at a comfortable 22°C and tea is poured out at 81°C. If the tea cools to 64°C in 5 minutes, use seperation of variables to solve the differential equation to determine a function $T(t)$ to model the temperature of the tea.

(d) Given a rhombus $OACB$ as seen below, and using vector methods, prove that the diagonals of a rhombus are perpendicular. **2**



(e) The tangent to the curve $y = \arcsin\left(\dfrac{x^2}{2}\right)$ is drawn at the point $\left(1, \dfrac{\pi}{6}\right)$. **3**

Find the y-intercept of the tangent.

(f) A biased coin has a 60% chance of landing on tails. If this coin is tossed 5 times, what is the probability that at least four of the tosses are tails? **2**

End of Question 12

Question 13 (15 Marks) Use a SEPARATE writing booklet.

(a) (i) Show that $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$. **2**

(ii) Consider the cubic equation $x^3 - 3x - \sqrt{3} = 0$. **2**

By letting $x = 2\cos\theta$, show that $\cos 3\theta = \dfrac{\sqrt{3}}{2}$.

(iii) Hence, prove that **2**

$$\sec\frac{\pi}{18} + \sec\frac{11\pi}{18} + \sec\frac{13\pi}{18} = -2\sqrt{3}$$

(b) The masses of mice are thought to be normally distributed. If 30% of mice have a mass of more than 65g and 20% have a mass of less than 40g, estimate the mean and the variance of the mass of the mice. **3**

(c) A ship is set to sail to an island which is on a bearing of 127°T. There is a current coming from the west which is flowing at a rate of 4 km/h. **3**

The ship is able to sail at a constant speed of 37 km/h.

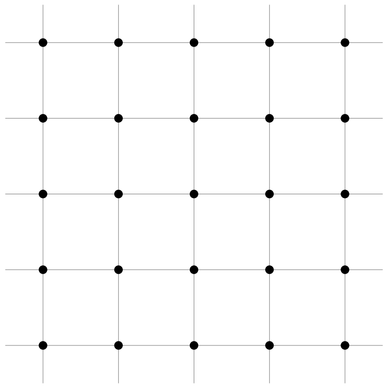
Determine the bearing the ship should sail on to reach the island, to the nearest degree.

(d) Find the general solution of $\dfrac{dy}{dx} = \dfrac{y^3+1}{\sqrt{xy}}$ using the substitution $y = u^{\frac{3}{2}}$. **3**

End of Question 13

Question 14 (15 Marks) Use a SEPARATE writing booklet.

(a) Five points are chosen at the nodes of an infinite square lattice (a grid). Using pigeonhole principle, determine why is it certain that at least one mid-point of a line joining a pair of chosen points is also a node of the square lattice? **2**



(b) Consider the function $f(x) = x - k\cos^2 x$ for a constant k.

(i) Show that $f(x)$ has an inverse for $-1 \leqslant k \leqslant 1$. **2**

(ii) The maximum value of the derivative of $f^{-1}(x)$ is 3. Find the value of k. **1**

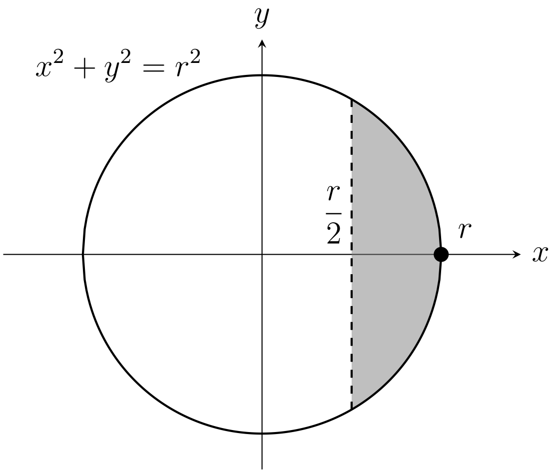
(c) A factory produces silicon wafers. For each silicon wafer produced, there is a 93% chance that it is considered acceptable. **4**

The factory produces at least n silicon wafers each day, so that they are 95% certain that at least 5000 silicon wafers are acceptable.

What is the least number of silicon wafers that need to be produced each day?

(d) (i) The region between the curve $x^2 + y^2 = r^2$ and the line $x = \frac{r}{2}$ is rotated around the x-axis. **2**

Show that the volume of revolution is given by $V = \dfrac{5\pi r^3}{24}$.



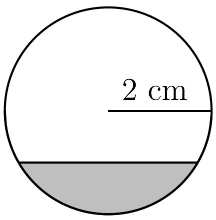
(ii) A spherical ball of radius 2 cm has water in it, so that the height of the water in the ball is 1 cm. **4**

The radius of the ball is increasing at a rate inversely proportional to its radius.

At the same time, water is flowing into the ball so that the height of water in the ball is exactly half the radius.

It is known that after 5 seconds, the radius of the ball is 4 cm.

Find the rate of change of water in the ball when the radius is 8 cm.



Cross-section view

END OF PAPER

KILLARA HIGH SCHOOL

Extension 1 Mathematics

2025 Higher School Certificate Trial

Student Number: ____________

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.
Sample: $2 + 4 =$

(A) 2 (B) 6 (C) 8 (D) 9

A ● C D

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

● ● C D

If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word correct and drawing an arrow as follows.

Correct

● ● C D

1. A B C D
2. A B C D
3. A B C D
4. A B C D
5. A B C D
6. A B C D
7. A B C D
8. A B C D
9. A B C D
10. A B C D

Question 11	Sample solutions	Marking guide	Feedback
(a)	$$\lvert 3x-5\rvert<4$$ $$-4<3x-5<5$$ $$1<3x<9$$ $$\frac{1}{3}<x<3$$	1 mark – getting part of the solution ($x<3$) 2 marks – getting both components of the solutions	Generally done well
(b)	$$u=\sin x \;\rightarrow\; du=\cos x\,dx$$ $$I=-\int u^{\frac{3}{2}}du$$ $$=-\left[\frac{2}{5}u^{\frac{5}{2}}\right]$$ $$=-\frac{2}{5}(\sin x)^{\frac{5}{2}}+c$$	1 mark – a correct process with a mistake, or leaving the answer in terms of u 2 marks – a correct solution with the answer in terms of x	Mostly done well. Some students forgot to put their answer back in terms of x
(c) I	$$6!$$	1 mark – a correct answer	Done well
(c) ii	$$P=\frac{4!\times 3!}{6!}=\frac{1}{5}$$	1 mark – the correct numerator 2 marks – the correct probability	Many students received 1 mark because they didn't give the answer as a probability
(d)	$$\overrightarrow{AB}=\begin{pmatrix}3\\-4\end{pmatrix},\quad \overrightarrow{AC}=\begin{pmatrix}-4\\1\end{pmatrix}$$ $$\lvert\overrightarrow{AB}\rvert=\sqrt{3^2+(-4)^2}=5$$ $$\lvert\overrightarrow{AC}\rvert=\sqrt{(-4)^2+1^2}=\sqrt{17}$$ $$\overrightarrow{AB}\cdot\overrightarrow{AC}=\lvert\overrightarrow{AB}\rvert\lvert\overrightarrow{AC}\rvert\cos\theta$$	1 mark for setting up the vectors at point A and trying to work with them. 2 marks – attempting to use the dot product to find the angle and making a minor error	This was generally done well across the cohort.

	$-12-4=5\sqrt{17}\cos\theta$ $\cos\theta=-\frac{16}{5\sqrt{17}}$ $\theta\approx 141°$	3 marks – finding the correct angle	
(e) i	$3\sin^2 x=\cos^2 x$ $\tan^2 x=\frac{1}{3}$ $\tan x=\pm\frac{1}{\sqrt{3}}$ $\tan x=\frac{1}{\sqrt{3}} \rightarrow x=\frac{\pi}{6}$ $\tan x=-\frac{1}{\sqrt{3}} \rightarrow x=\frac{5\pi}{6}$	1 mark – showing why $\frac{\pi}{6}$ is a solution 2 marks – showing both solutions	Many students received 1 mark. There were various ways to show this, and the most common mistake came when solving a quadratic trig equation. A number of students forgot the negative root which meant they couldn't show why $\frac{5\pi}{6}$ was a solution.
(e) ii	$\text{Area}=\int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} 3\sin^2 x-\cos^2 x\,dx$ $=\int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} \frac{3(1-\cos 2x)}{2}-\frac{(1+\cos 2x)}{2}\,dx$ $=\int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} 1-2\cos 2x\,dx$ $=[x-\sin 2x]_{\frac{\pi}{6}}^{\frac{5\pi}{6}}$	1 mark – setting up the integral and using the trig identities for $\sin^2 x$ / $\cos^2 x$ 2 marks – a correct solution to find the area	This was started well be students, but was fiddly with the different terms in the integral, which caused some errors. The most common error was realising that $\sin\frac{5\pi}{3}$ was negative.

	$= \left[\left(\frac{5\pi}{6} - \sin\frac{5\pi}{3}\right) - \left(\frac{\pi}{6} - \sin\frac{\pi}{6}\right)\right]$ $= \frac{2\pi}{3} + \sqrt{3}$ units^2		

12a)	Prove base case, $n = 1$ $LHS = 1^2 + 2^2$ $= 5$ $RHS = \frac{(1)(2+1)(4+1)}{3}$ $= 5$ $= LHS$ $\therefore true\ for\ n = 1$ Assume true for $n = k$ $1^2 + 2^2 + 3^2 + \cdots + (2k)^2 = \frac{k(2k+1)(4k+1)}{3}$ Prove true for $n = k + 1$ $LHS = 1^2 + 2^2 + 3^2 + \cdots + (2k)^2 + (2k+1)^2 + (2k+2)^2$ $= \frac{k(2k+1)(4k+1)}{3} + (2k+1)^2 + (2k+2)^2$ $(by\ assumption)$ $= \frac{k(2k+1)(4k+1) + 3(2k+1)^2 + 3(2k+2)^2}{3}$ $= \frac{(2k+1)\big(3(2k+1) + k(4k+1)\big) + 12(k+1)^2}{3}$ $= \frac{(2k+1)(4k^2+7k+3) + 12(k+1)^2}{3}$ $= \frac{(2k+1)(4k+3)(k+1) + 12(k+1)^2}{3}$ $= \frac{(k+1)\big((2k+1)(4k+3) + 12(k+1)\big)}{3}$ $= \frac{(k+1)(8k^2+18k+15)}{3}$ $= \frac{(k+1)(4k+5)(2k+3)}{3}$ $= RHS$ $\therefore n = k\ implies\ n = k + 1$ $\therefore$The statement is true for all $n \geq 1$byt the principal of mathematical induction	*base case * Assume n=k *set up n=k+1 *by assumption *One factor found *Complete inductive step *n=k implies n=k+1 *conclusion 1 mark 2 stars 2marks – Awarded if a factorisation of k+1 was made from an incorrect set up for k+1, and 3 other stars 3marks 8 stars Almost no students set up their $k + 1$ step correctly.
12b)	$\cos x - \sqrt{3}\sin x = 2$ $let\ R\cos(x+\alpha) = \cos x - \sqrt{3}\sin x$ $1 = R\cos\alpha, \sqrt{3} = R\sin\alpha$ $R^2 = 1 + 3$ $= 4$ $R = 2, (R > 0)$ $\tan\alpha = \sqrt{3}$	1 mark Found R and alpha correctly 2 marks Found either R or α

	$\alpha = \frac{\pi}{3}$ $2\cos\left(x + \frac{\pi}{3}\right) = 2$ $\cos\left(x + \frac{\pi}{3}\right) = 1$ $x + \frac{\pi}{3} = 0, 2\pi$ $x = \frac{5\pi}{3}\ (discard\ -\frac{\pi}{3}\ as\ out\ of\ the\ domain)$ N.B. students who chose to use sin had to work slightly harder to get the result they wanted. (Making sure the sign was correct for the $\sin\alpha\ \ and \cos\alpha$	correctly and solved for x correctly given error 3 marks Found a correct expression for $Rcos(x+\alpha)$, solved for x, discarding solutions outside of the domain
c)	$\frac{dT}{dt} = k(T - T_1)$ As temperature will not change if it arrives at room temperature $T_1 = 22$. $\int \frac{1}{T-22}\,dT = \int k\,dt$ $\ln\lvert T - 22\rvert = kt + c$ $\lvert T - 22\rvert = e^{kt}e^{c}$ $T = Ae^{kt} + 22 \qquad A \in \mathbb{R}$ $(0,81)$ $81 = A + 22$ $A = 59$ $T = 59e^{kt} + 22$ $(5,64)$ $64 = 59e^{5k} + 22$ $42 = 59e^{5k}$ $e^{k5} = \frac{59}{42}$ $5k = \ln\frac{59}{42}$ $k = \frac{1}{5}\ln\frac{59}{42}$ $64 = 59e^{\frac{1}{5}\ln\left(\frac{59}{42}\right)} + 22$	1 mark Correct integration 2 marks Correct integration and values 2 of $k,\ A$ and T_1 3 marks correct final answer from correct working. Generally well done. 22 can be inferred as the asymptote. No working needed.

d)	$\overrightarrow{OC} = \underset{\sim}{a} + \underset{\sim}{b}$ $\overrightarrow{AB} = \underset{\sim}{b} - \underset{\sim}{a}$ $\overrightarrow{OC} \cdot \overrightarrow{AB} = \left(\underset{\sim}{a} + \underset{\sim}{b}\right) \cdot \left(\underset{\sim}{b} - \underset{\sim}{a}\right)$ $= \lvert \underset{\sim}{a} \rvert^2 - \left\lvert \underset{\sim}{b} \right\rvert^2$ $= 0 \qquad (\lvert a \rvert = \lvert b \rvert in\ a\ rhombus)$ $\therefore The\ diagonals\ of\ a\ rhombus\ are\ perpendicular$	1 mark set up $\overrightarrow{AB}$ and $\overrightarrow{OC}$ and attempt to dot product. 2 marks – complete working with complete reasoning This question was generally well done.
e)	$y = \arcsin\left(\frac{x^2}{2}\right)$ $y' = \frac{\left(\frac{2x}{2}\right)}{\sqrt{1 - \left(\frac{x^2}{4}\right)^2}}$ $y' = \frac{2x}{\sqrt{4 - x^2}}$ $y'(1) = \frac{2}{\sqrt{3}}$ Using $\left(1, \frac{\pi}{6}\right)$ $y - \frac{\pi}{6} = \frac{2}{\sqrt{3}}(x - 1)$ $y = \frac{2x}{\sqrt{3}} - \frac{2}{\sqrt{3}} + \frac{\pi}{6}$ Therefore the y-intercept is $\left(0, \frac{\pi}{6} - \frac{2}{\sqrt{3}}\right)$	1 mark - differentiated correctly 2 marks – differentiated correctly and substituted into the point gradient form of a linear function (or equivalent). 3 – Above and Expresses the correct y-intercept as a point, or otherwise indicates the appropriate x and y values.
f)	$(\ )$	

Question 13

a) i) $\cos 3\theta = \cos(2\theta + \theta)$

$= \cos 2\theta \cos\theta - \sin 2\theta \sin\theta$ 1

$= (\cos^2\theta\cos\theta - \sin^2\theta\cos\theta) - 2\sin\theta\cos\theta\sin\theta$

$= \cos^3\theta - \sin^2\theta\cos\theta - 2\sin^2\theta\cos\theta$

$= \cos^3\theta - 3(\sin^2\theta\cos\theta)$

$= \cos^3\theta - 3((1-\cos^2\theta)\cos\theta)$

$= \cos^3\theta - 3\cos\theta + 3\cos^3\theta$

$= 4\cos^3\theta - 3\cos\theta$ 1

$=$ RHS as required. ②

① for $\cos(2\theta+\theta)$ expansion

① for answer with working

ii) given $x^3 - 3x - \sqrt{3} = 0$

for $x = 2\cos\theta$:

$(2\cos\theta)^3 - 3(2\cos\theta) - \sqrt{3} = 0$ 1

$2\times 4\cos^3\theta - 2\times 3\cos\theta - \sqrt{3} = 0$

$4\cos^3\theta - 3\cos\theta = \frac{\sqrt{3}}{2}$ 1

$\cos 3\theta = \sqrt{3}/2$ by part (i) ②

① for substitution

① for answer with working

should reference part 1

iii) given $\cos 3\theta = \sqrt{3}/2$

$3\theta = \pi/6, \frac{11\pi}{6}, 13\pi/6 \ldots$

S A
T C

$\theta = \pi/18, 11\pi/18, 13\pi/18, \ldots$

$\therefore$ roots of $x^3 - 3x - \sqrt{3} = 0$ are

$2\cos \pi/18,\ 2\cos 11\pi/18,\ 2\cos 13\pi/18$

letting $x = \frac{1}{y}$ gives equation $\sqrt{3}y^2 + 3y^2 - 1 = 0$

with roots $y = \frac{1}{2}\sec \pi/18, \frac{1}{2}\sec 11\pi/18, \frac{1}{2}\sec 13\pi/18$

sum of roots $\frac{1}{2}\sec\pi/18 + \frac{1}{2}\sec 11\pi/18 + \frac{1}{2}\sec 13\pi/18 = -\frac{3}{\sqrt{3}}$

$\sec\pi/18 + \sec 11\pi/18 + \sec 13\pi/18 = -2\sqrt{3}$ as required. ②

① for identifying roots of $x = \frac{1}{y}$ equation

① for using sum of roots

poorly done, often skipped

b) $\frac{65-\bar{x}}{\sigma} = 0.53$ $(>30\%)$

$\frac{40-\bar{x}}{\sigma} = -0.85$ ①

① for 0.53 & −0.85

① for $\bar{x}$

① for ~~$\bar{x}$~~ or Var

very poorly done, students should review

hence $\frac{65-\bar{x}}{0.53} = \frac{40-\bar{x}}{-0.85}$

$1.38\bar{x} = 76.45$

$\bar{x} = 55.3985\ldots$

$\boxed{\bar{x} = 55.4 \text{ g}}$ ①

var(x) = 328.19

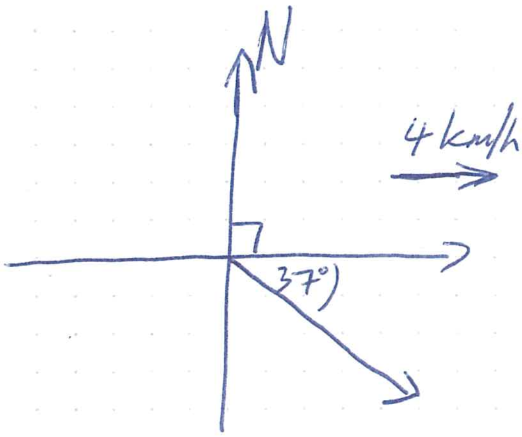
$\frac{65-55.4}{0.53} = \sigma$

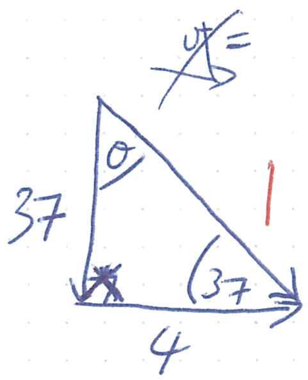
$\sigma = 18.1159\ldots$

$\boxed{\sigma = 18.12}$ ①

③

c)





① for diagram

① for angle

① for correct bearing

poorly done diagrams

$\frac{\sin\theta}{4} = \frac{\sin 37}{37}$ 1

$\sin\theta = \frac{4\sin 37}{37}$

$\theta = 3.73°$

∴ Ship sails on Bearing

$127 + 3.73$

$\boxed{\text{or } 131° \text{ T}}$ 1

③

d) $\frac{dy}{dx} = \frac{y^3+1}{\sqrt{xy}}$

$$\int \frac{\sqrt{y}}{y^3+1}\,dy = \int \frac{1}{\sqrt{x}}\,dx$$

1

let $y = u^{3/2}$

$\frac{dy}{du} = \frac{3}{2}u^{1/2}$

$dy = \frac{3}{2}u^{1/2}du$

① for substitution

① for integral to $\tan^{-1}$

① for correct rearrangement

$$\frac{3}{2}\int \frac{(u^{3/2})^{1/2}\,u^{1/2}\,du}{(u^{3/2})^3+1} = \int x^{-1/2}\,dx$$

Many substituted before seperating variables made question harder

$$\frac{4}{9}\times\frac{3}{2}\int \frac{\frac{9}{4}u^{5/4}}{1+u^{9/2}}\,du = 2\sqrt{x}+c$$

$$\frac{2}{3}\left[\tan^{-1}(u^{9/4})\right] = 2\sqrt{x}+c$$

1

$$\tan^{-1}(u^{9/4}) = 3\sqrt{x}+c_2$$

$$\tan^{-1}(y^{3/2}) = 3\sqrt{x}+c_2$$

$$y^{3/2} = \tan(3\sqrt{x}+c_2)$$

$$y = \tan^{2/3}(3\sqrt{x}+k) \quad k\in\mathbb{R}$$

1

— 3

Question 14

a) let each node be named as a point (x, y), $x, y \in \mathbb{Z}$.
This gives midpoint $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$, where the midpoint has integer values when x's and y's have same parity, so both x's odd or both x's even.

Consider then 4 cases: (odd, odd)
(even, odd)
(odd, even)
(even, even)

MOST students skipped this question

There must be, for a fifth point on the lattice, be a correspondence with one of these cases, thus having matching parity, so having a midpoint with integer values signifying it is on a node. 2

1 for partial reasoning or only 1 mistake in logic.

2 for correctly reasoned argument with pigeonhole principle.

b) i) $f'(x) = 1 - 2k\sin x\cos x$
$= 1 - k\sin 2x$

$1 - k\sin 2x \geq 0$ for $-1 \leq k \leq 1$
as $1 \leq \sin 2x \leq -1$

poorly done, few used deriv to prove monotonic

So $f(x)$ is monotonic increasing for $-1 \leq k \leq 1$ 2

ii) Maximum of $f^{-1\prime}(x) = 3$, $\therefore$ Maximum of $f'(x) = \frac{1}{3}$

$f'(x) = 1 - k\sin 2x \leq \frac{1}{3}$

Max $\sin 2x = 1$

so $k\sin 2x = \frac{2}{3}$

$k = \frac{2}{3}$. 1 for correct answer

c) 95% confidence at $z > -1.65$ / -1.64 1

$p = 93\%$

$q = 7\%$ 1

n = our unknown

① for -1.65.

① for 93% & 7% p & q.

① for expression for $\sqrt{n}$

① for final answer.

$$\frac{5000 - 0.93n}{\sqrt{n(0.93)(0.07)}} = -1.64$$

$$5000 - 0.93n = -0.41844107..\sqrt{n}$$

$$0.93n - 1.64\sqrt{0.93}\sqrt{0.07}\sqrt{n} - 5000 = 0.$$

$$\sqrt{n} = \frac{1.64\sqrt{0.93}\sqrt{0.07} \pm \sqrt{0.93 \times 0.07 \times 1.64^2 + 18600}}{2 \times 0.93}$$ 1

$\sqrt{n} = 73.5\ldots \quad (\sqrt{n} > 0)$

$n = 5409.4\ldots$

④

$n = 5410$ silicon wafers need to be produced each day. 1

poorly done, many students were satisfied with an answer of less than 5000 wafers.

d) i) $\pi \int_{r/2}^{r} y^2 \, dx$

$V = \pi \int_{r/2}^{r} r^2 - x^2 \, dx$ 1

$= \pi \left[r^2 x - \frac{x^3}{3} \right]_{r/2}^{r}$

$= \pi \left(r^3 - \frac{r^3}{3} - \frac{r^2}{2} + \frac{r^3}{24} \right)$ 1

$= \frac{5\pi r^3}{24}$

2

① for $\pi \int_{r/2}^{r} r^2 - x^2 \, dx$ expression

① for final answer with working.

well done

ii) $\frac{dr}{dt} = \frac{k}{r}$ ① 1

$\frac{dV}{dr} = \frac{5\pi r^2}{8}$

$\int_2^4 r \, dr = \int_0^5 k \, dt$. from ①

$\left[\frac{r^2}{2} \right]_2^4 = 5k$

$6 = 5k$

$k = \frac{6}{5}$ 1

$\frac{dr}{dV} = \frac{6}{5} r$

$\frac{dV}{dr} \frac{dr}{dt} = \frac{6r}{5} \times \frac{5\pi r^2}{8}$ 1

$= \frac{3\pi (8)^2}{4}$

$= 24\pi \times 2$

$= 48\pi \text{ cm}^3/\text{s}$ 1

4

① for $\frac{dr}{dt}$ expression with k

① for $k = 6/5$

① for $\frac{dV}{dr} \frac{dr}{dt}$ expression

① for final answer with working

Many students partially completed, and found k, but couldn't form the related rates