NAME: _____

Northern Beaches Secondary College

Manly Campus

2025 **HIGHER SCHOOL CERTIFICATE TRIAL EXAMINATION**

Mathematics Extension 1

General Instructions

- Reading time – 10 minutes
- Working time – 2 hours
- Write using black pen
- Calculators approved by NESA may be used
- A reference sheet is provided
- For questions in section II, show relevant mathematical reasoning and/or calculations

Total marks:

70

Section I – 10 marks

- Attempt Questions 1 – 10
- Allow about 15 minutes for this section

Section II – 60 marks

- Attempt Questions 11 – 14
- Allow about 1 hour and 45 minutes for this section

Section I

10 marks

Attempt Questions 1 – 10

Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1 – 10.

---

1. Let vector $\boldsymbol{v} = -3\boldsymbol{i} + 7\boldsymbol{j}$.

Which line is parallel to $\boldsymbol{v}$?

A. $y = 7x - 3$

B. $y = -3x + 7$

C. $y = -\frac{3}{7}x + 1$

D. $y = -\frac{7}{3}x + 1$

2. The polynomial $p(x) = ax^2 - 4x + 8a$ has roots α and β.

If $\alpha + \beta = \alpha\beta$, what is the value of a ?

A. $\frac{1}{2}$

B. 2

C. $-\frac{1}{2}$

D. –2

3. A discrete random variable X is binomially distributed with a mean of 6 and a variance of 1.5.

What is the number of independent trials?

A. 24

B. 8

C. 6

D. 5

4. Which of the following is equivalent to $\int \frac{1-\tan^2 x}{\sec^2 x}\,dx$?

A. $\int \sin 2x\,dx$

B. $\int \sin \frac{x}{2}\,dx$

C. $\int \cos \frac{x}{2}\,dx$

D. $\int \cos 2x\,dx$

5. How many distinct ways are there to arrange the letters of the word WOLLONGONG if the two Ls must remain adjacent?

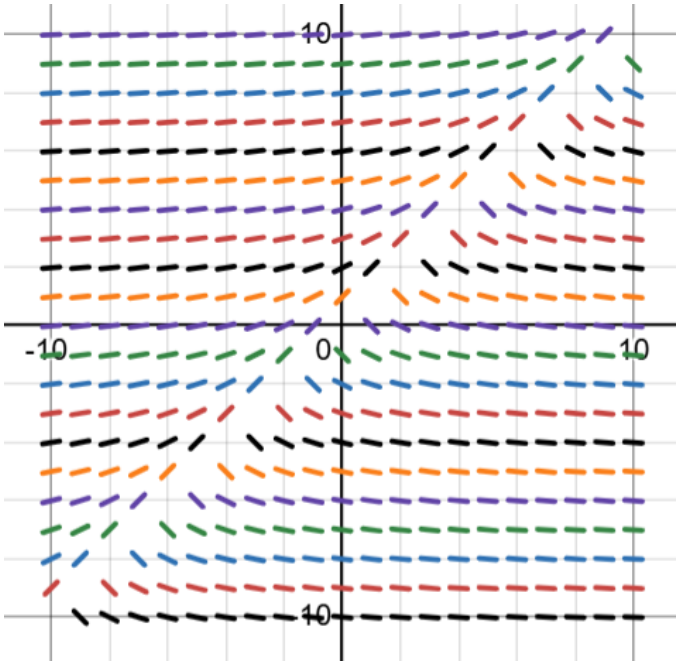
A. $\frac{9!}{3!\,2!\,2!\,2!}$

B. $\frac{10!}{3!\,2!\,2!\,2!}$

C. $\frac{9!}{3!\,2!\,2!}$

D. $\frac{10!}{3!\,2!\,2!}$

6. Which of the following differential equations matches the slope field shown?



A. $\frac{dy}{dx} = y - x$

B. $\frac{dy}{dx} = x - y$

C. $\frac{dy}{dx} = \frac{1}{y - x}$

D. $\frac{dy}{dx} = \frac{1}{x - y}$

7. If $\boldsymbol{a}$ and $\boldsymbol{b}$ are non-zero vectors, and $|\boldsymbol{a} + \boldsymbol{b}| = |\boldsymbol{a} - \boldsymbol{b}|$, which of the statements is true?

A. $\boldsymbol{a} \cdot (\boldsymbol{a} - \boldsymbol{b}) = 0$

B. $\boldsymbol{a} \cdot \boldsymbol{b} = 0$

C. $(\boldsymbol{a} - \boldsymbol{b}) \cdot \boldsymbol{b} = 0$

D. $\boldsymbol{a} = k\boldsymbol{b}$

8. The surface area of a sphere is decreasing at a rate of $8cm^2/s$ at the moment when its radius measures $R\ cm$.

What is the rate at which the volume of the sphere is decreasing at that moment?

A. $0.4R$

B. $2R$

C. $4R$

D. $8R$

9. The projection of the vector $\begin{pmatrix}0\\5\end{pmatrix}$ onto the line $y = \frac{x}{2}$ is the vector $\begin{pmatrix}2\\1\end{pmatrix}$.

The point (0, 5) is reflected over the line $y = \frac{x}{2}$. What are the coordinates of the new point?

A. (5, 0)

B. (4, –3)

C. (–5, 0)

D. (3, –4)

10. By considering the product of $(x + 1)^{100}$ and $(1 + x)^{100}$, what is the value of the expression

$$\binom{100}{0}\binom{100}{1} + \binom{100}{1}\binom{100}{2} + \binom{100}{2}\binom{100}{3} + \ldots + \binom{100}{99}\binom{100}{100} \; ?$$

A. $\binom{100}{50}$

B. $\binom{100}{50}^2$

C. $\binom{200}{100}$

D. $\binom{200}{101}$

Section II

60 marks

Attempt Questions 11 – 14

Allow about 1 hour and 45 minutes for this section.

Answer each question in a separate writing booklet. Extra writing booklets are available.

For questions is Section II, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Start a new Writing Booklet.

a) Solve $\frac{4}{2-x} \leq 3$ **2**

b) Find $\int \frac{3}{\sqrt{1-9x^2}}\,dx$ **2**

c) Use *t*-formulae to solve the equation $3\cos\theta - 4\sin\theta + 3 = 0$ where $0 \leq \theta \leq 360°$ giving your answer(s) correct to the nearest minute. **3**

d) The vectors $\begin{pmatrix} 3 \\ k-1 \end{pmatrix}$ and $\begin{pmatrix} -2 \\ 2k+1 \end{pmatrix}$ are parallel. **2**

What are the possible value(s) of k ?

e) Let $P(x) = 3x^3 - 12x^2 - x + 8$ have roots α, β and γ.

Find:

(i) $\alpha\beta\gamma$ **1**

(ii) $\alpha^2 + \beta^2 + \gamma^2$ **2**

f) Find the integral $\int x\sqrt{2x+1}\,dx$ using the substitution $u = 2x + 1$ **3**

Question 12 (15 marks) Start a new Writing Booklet.

a) A function is defined by:

$$f(x) = x^2 + 8x + 10 \ , \quad x \geq -4$$

(i) Find the equation of the inverse function $y = f^{-1}(x)$. **2**

(ii) Sketch the graph of $y = f^{-1}(x)$. Clearly show the coordinates of the endpoint and the x-intercept. **2**

(iii) Find the point(s) of intersection of the graphs of $y = f(x)$ and $y = f^{-1}(x)$. **2**

b) In Australia, voting is compulsory for anyone 18 years of age or older.

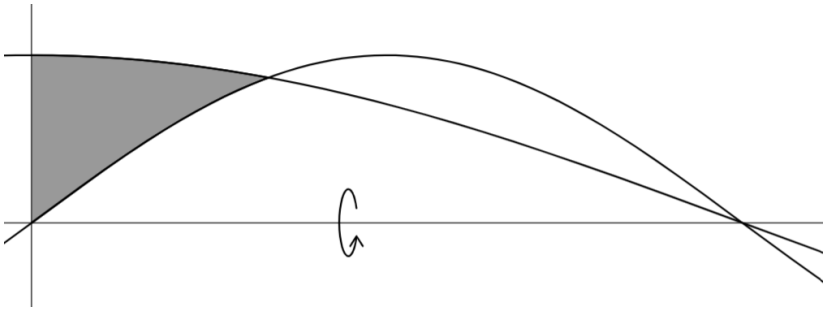
It is known that 91% of the Australian population is required to vote at the recent Federal election. 500 Australians are chosen at random.

By using a normal approximation to the sample proportion, find the probability that at least 450 Australians are eligible voters. **3**

You may use the extract shown from a table giving values of $P(Z < z)$, where Z has a standard normal distribution.

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830

c) The region bounded by the curves $y = \cos x$ and $y = \sin 2x$ and contained between the ordinates at $x = 0$ and $x = \frac{\pi}{6}$ is rotated about the x-axis.



3

Find the exact value of the volume of the solid of revolution.

d) A function $y = f(x)$ is defined by the parametric equations:

$$x = \frac{1}{t} + t^2 , \quad y = t^2 + t + 1$$

Find the gradient of the tangent to the graph of $y = f(x)$ at the point (2, 3). **3**

Question 13 (15 marks) Start a new Writing Booklet.

a) The graph of $y = \sin^{-1}(x-4)$ is dilated horizontally by a factor of 2 and then shifted right 1 unit.

Write the equation of the transformed graph. **2**

b) Prove by mathematical induction that $5^{2n+1} + 2^{2n+1}$ is divisible by 7 for integers $n \geq 0$. **3**

c) A company makes heating elements for electric ovens. A fixed number of randomly chosen heating elements are tested on each day and the number found to be faulty is denoted by X.

Assume that X has the distribution B(80, 0.02).

(i) Find the probability that more than 3 elements are found to be faulty on a randomly chosen day. **2**

(ii) Find the probability that, in a randomly chosen 5-day working week, more than 3 elements are found to be faulty on at least 2 days. **2**

d) The polynomial $P(x) = 3x^3 - 9x + 6$ has a zero of multiplicity 2.

Solve $P(x) = 0$. **3**

e) Evaluate $\int_0^{\frac{\pi}{4}} \sin 3x \sin x \, dx$ **3**

Question 14 (15 marks) Start a new Writing Booklet.

a) A standard pack of 52 cards consists of 13 cards from each of the four suits: Spades, Hearts, Clubs and Diamonds.

(i) In how many ways can six cards be selected without replacement so that exactly two are Spades and four are Clubs? **1**

(ii) In how many ways can six cards be selected without replacement if at least five cards must be of the same suit? **3**

b) A virus is spreading through a town with a population of 20 000 people. The disease is spreading at a rate proportional to the product of the number of people, N, who have the virus and the number, $20\,000 - N$, who don't have the virus. That is,

$$\frac{dN}{dt} = kN(20\,000 - N)$$

where k is a positive constant.

(i) Show that

$$\frac{1}{N(20\,000 - N)} = \frac{1}{20\,000}\left(\frac{1}{N} + \frac{1}{20\,000 - N}\right)$$ **1**

(ii) Deduce that $\frac{N}{20\,000 - N} = Ae^{\alpha t}$ where A and α are constants. **2**

Suppose that initially 100 people had the virus and that 400 people had the virus after 10 days.

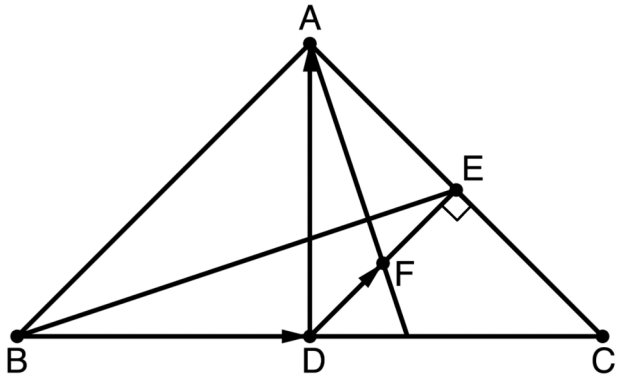
(iii) Find α correct to four decimal places. **2**

(iv) After how many days would exactly one half of the population have the virus?

State your answer correct to the nearest day. **1**

c) Consider the following isosceles triangle ABC where $AB = AC$.

D is the midpoint of BC and E is the foot of the perpendicular drawn from D to AC. F is the midpoint of DE.



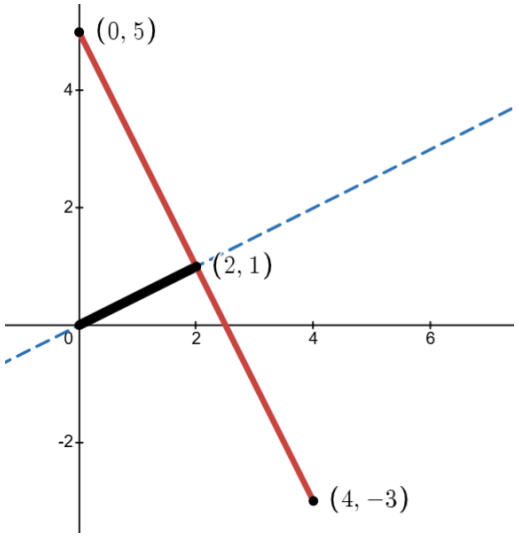
Let $\overrightarrow{BD} = \boldsymbol{a}$, $\overrightarrow{DA} = \boldsymbol{b}$ and $\overrightarrow{DF} = \boldsymbol{c}$.

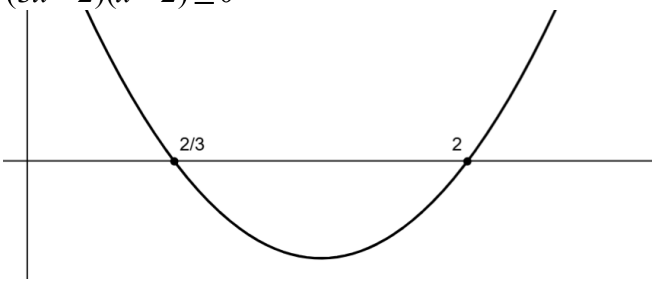
(i) Prove that $\boldsymbol{a} \cdot \boldsymbol{c} = \boldsymbol{b} \cdot \boldsymbol{c}$ **2**

(ii) Prove that $\overrightarrow{AF} \perp \overrightarrow{BE}$ **3**

End of paper

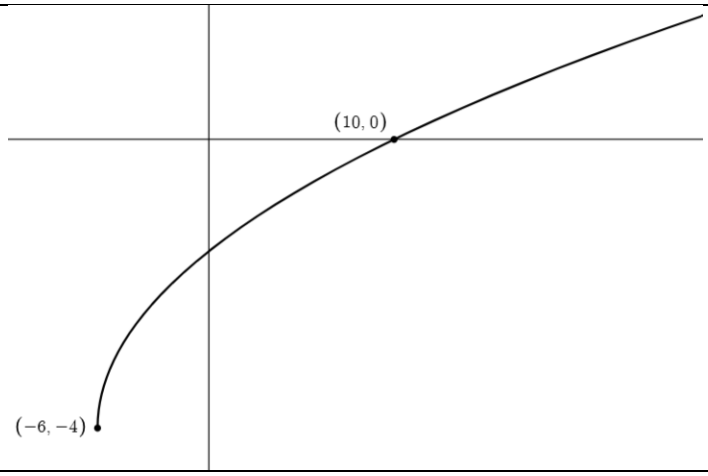
2025 Year 12 Mathematics Extension 1 marking guidelines		
Q	**Solution**	**Marks**
1	$\boldsymbol{v}$ connects (0,0) and (-3,7) $m = \frac{7-0}{-3-0} = -\frac{7}{3}$ $\therefore\ y = -\frac{7}{3}x + 1$ is parallel to $\boldsymbol{v}$	D
2	$p(x) = ax^2 - 4x + 8a$ $\alpha + \beta = -\frac{b}{a} = \frac{4}{a}$ $\alpha\beta = \frac{c}{a} = 8$ $\frac{4}{a} = 8 \Rightarrow a = \frac{1}{2}$	A
3	$\mu = np = 6$ $\sigma^2 = np(1-p) = 1.5$ $6(1-p) = 1.5$ $1-p = 0.25$ $p = 0.75$ $n \times 0.75 = 6$ $n = 8$	B
4	$\int \frac{1-\tan^2 x}{\sec^2 x}\,dx$ $= \int \cos^2 x(1-\tan^2 x)\,dx$ $= \int (\cos^2 x - \sin^2 x)\,dx$ $= \int \cos 2x\,dx$	D
5	Arrange W, O, O, O, LL, N, N, G, G in a line $\frac{9!}{3!2!2!}$	C
6	Only the DE in Option C gives a small positive gradient at the point (1, 10)	C
7	$\lvert\boldsymbol{a}+\boldsymbol{b}\rvert$ and $\lvert\boldsymbol{a}-\boldsymbol{b}\rvert$ are the two diagonals of a parallelogram whose sides are given by $\boldsymbol{a}$ and $\boldsymbol{b}$. A parallelogram with equal diagonals must be a rectangle, hence all angles are right angles, hence $\boldsymbol{a} \cdot \boldsymbol{b} = 0$ An algebraic method is:	B

	$$|\underset{\sim}{a}+\underset{\sim}{b}|^2=|\underset{\sim}{a}-\underset{\sim}{b}|^2$$ $$(\underset{\sim}{a}+\underset{\sim}{b})\cdot(\underset{\sim}{a}+\underset{\sim}{b})=(\underset{\sim}{a}-\underset{\sim}{b})\cdot(\underset{\sim}{a}-\underset{\sim}{b})$$ $$\underset{\sim}{a}\cdot\underset{\sim}{a}+2\,\underset{\sim}{a}\cdot\underset{\sim}{b}+\underset{\sim}{b}\cdot\underset{\sim}{b}=\underset{\sim}{a}\cdot\underset{\sim}{a}-2\,\underset{\sim}{a}\cdot\underset{\sim}{b}+\underset{\sim}{b}\cdot\underset{\sim}{b}$$ $$2\,\underset{\sim}{a}\cdot\underset{\sim}{b}+2\,\underset{\sim}{a}\cdot\underset{\sim}{b}=\underset{\sim}{a}\cdot\underset{\sim}{a}+\underset{\sim}{b}\cdot\underset{\sim}{b}-\underset{\sim}{a}\cdot\underset{\sim}{a}-\underset{\sim}{b}\cdot\underset{\sim}{b}$$ $$4\underset{\sim}{a}\cdot\underset{\sim}{b}=0$$ $$\underset{\sim}{a}\cdot\underset{\sim}{b}=0$$	
8	When $r=R$, $\frac{dA}{dt}=-8$. Find $\frac{dV}{dt}$ when $r=R$. $$\frac{dV}{dt}=\frac{dV}{dr}\times\frac{dr}{dA}\times\frac{dA}{dt}$$ $$V=\frac{4}{3}\pi r^3\Rightarrow\frac{dV}{dr}=4\pi r^2$$ $$A=4\pi r^2\Rightarrow\frac{dA}{dr}=8\pi r\Rightarrow\frac{dr}{dA}=\frac{1}{8\pi r}$$ $\therefore\quad\frac{dV}{dt}=4\pi r^2\times\frac{1}{8\pi r}\times-8=-4r$ $\therefore\ \frac{dV}{dt}(R)=-4R$ $\therefore$ decreasing at a rate of $4R$ cm^3/s	C
9	The situation is illustrated below:  The blue dotted line is $y=0.5x$. The black line segment represents the vector (2, 1), which is the projection of (0,5) onto the line.	B

	Start at (0, 5) and travel **right 2 and down 4** to get to (2, 1). Again travel **right 2 and down 4** to arrive at (4, -3).	
10	$(x+1)^{100} = {}^{100}\mathbf{C}_0\, x^{100} + {}^{100}\mathbf{C}_1\, x^{99} + {}^{100}\mathbf{C}_2\, x^{98} + \ldots + {}^{100}\mathbf{C}_{100}$ $(1+x)^{100} = {}^{100}\mathbf{C}_0 + {}^{100}\mathbf{C}_1\, x + {}^{100}\mathbf{C}_2\, x^2 + \ldots + {}^{100}\mathbf{C}_{100}\, x^{100}$ $\binom{100}{0}\binom{100}{1} + \binom{100}{1}\binom{100}{2} + \binom{100}{2}\binom{100}{3} + \ldots + \binom{100}{99}\binom{100}{100}$ = the coefficient of the x^{101} term in the expansion of $(x+1)^{100}(1+x)^{100}$ = the coefficient of the x^{101} term in the expansion of $(1+x)^{200}$ $= {}^{200}\mathbf{C}_{101}$	D
11a	$\frac{4}{2-x} \le 3$ $4(2-x) \le 3(2-x)^2$ $8-4x \le 3(4-4x+x^2)$ $8-4x \le 12-12x+3x^2$ $3x^2-8x+4 \ge 0$ $(3x-2)(x-2) \ge 0$  From the graph, and noting that $x \neq 2$, $x > 2$ or $x \le \frac{2}{3}$ ***Marker's feedback:*** *You have to draw the graph!* **Method 2: critical values** From the denominator, $x = 2$ is one critical value. Solving $\frac{4}{2-x} = 3$ gives the other critical value of 2/3	2 marks 1 mark for multiplying by the square of the denominator, or equivalent progress

	Test a value $x < 2/3$: $\frac{4}{2-0} = 2 \leq 3$ true Test a value $2/3 < x < 2$: $\frac{4}{2-1} = 4 \leq 3$ false Test a value $x > 2$: $\frac{4}{2-3} = -4 \leq 3$ true Therefore: $x \leq \frac{2}{3}$ or $x > 2$ ***Marker's feedback:*** *If you use this method, you have to check all three intervals, showing working for the substitutions.*	
11b	$\int \frac{3}{\sqrt{1-9x^2}}\,dx$ $= \int \frac{3}{\sqrt{1-(3x)^2}}\,dx$ $= \sin^{-1} 3x + C$ ***Marker's feedback:*** *Mostly done well. Carefully use the formula on the reference sheet.*	2 marks 1 mark for a primitive involving arcsin
11c	$3\cos\theta - 4\sin\theta + 3 = 0, \quad 0 \leq \theta \leq 360°$ Since $3\cos 180° - 4\sin 180° + 3 = -3 - 0 + 3 = 0$, $\theta = 180°$ is a solution Supposing $\theta \neq 180°$, let $t = \tan\frac{\theta}{2}$: $3 \times \frac{1-t^2}{1+t^2} - 4 \times \frac{2t}{1+t^2} + 3 = 0$ $3 - 3t^2 - 8t + 3 + 3t^2 = 0$ $6 - 8t = 0$ $8t = 6$ $t = \tan\frac{\theta}{2} = \frac{3}{4}, \quad 0 \leq \frac{\theta}{2} \leq 180°$ $\frac{\theta}{2} = \tan^{-1}\frac{3}{4}$ $\theta = 2\tan^{-1}\frac{3}{4} \approx 73°44'$ $\therefore \quad \theta = 73°44', \ 180°0'$	3 marks for both solutions 2 marks for finding 73°44' 1 mark for obtaining t=0.75 or equivalent

	Marker's feedback: *Of course the most common error was forgetting to check 180. Some students checked 90 but this is irrelevant. You have to check any value of θ between 0 and 360 for which tan(θ/2) is undefined. Some students also had difficulty solving tan(θ/2)=3/4 where θ/2 is between 0 and 180.*	
11d	$\begin{pmatrix} 3 \\ k-1 \end{pmatrix}$ parallel to $\begin{pmatrix} -2 \\ 2k+1 \end{pmatrix}$ $\therefore \quad \frac{k-1}{3} = \frac{2k+1}{-2}$ $-2k+2 = 6k+3$ $8k = -1$ $k = -\frac{1}{8}$ ***Marker's feedback:*** *If two vectors are parallel then $y_1 / x_1 = y_2 / x_2$. Draw the situation and you'll see this is the same as parallel lines having equal gradients. Note there's only one case – no need to apply ± . Another condition for parallel is that the dot product = ±the product of magnitudes, but this is harder to use for this question.*	2 marks 1 mark for forming correct equation in *k*
11ei	$\alpha\beta\gamma = -\frac{d}{a} = -\frac{8}{3}$	1 mark
11eii	$\alpha^2+\beta^2+\gamma^2 = (\alpha+\beta+\gamma)^2 - 2(\alpha\beta+\alpha\gamma+\beta\gamma)$ $= \left(-\frac{b}{a}\right)^2 - 2\left(\frac{c}{a}\right)$ $= \left(-\frac{-12}{3}\right)^2 - 2\left(-\frac{1}{3}\right)$ $= \frac{50}{3}$	2 marks 1 mark for some progress in terms of -b/a and/or c/a

11f	$\int x\sqrt{2x+1}\ dx$ $u = 2x+1$ $du = 2dx$ $dx = \frac{1}{2}\ du$ $2x = u-1$ $x = \frac{1}{2}(u-1)$ $\therefore \quad I = \int \frac{1}{2}(u-1)\sqrt{u}\ \times \frac{1}{2}\ \text{du}$ $I = \frac{1}{4}\int u^{\frac{3}{2}} - u^{\frac{1}{2}}\ \text{du}$ $I = \frac{1}{4}\left(\frac{2}{5}\ u^{\frac{5}{2}} - \frac{2}{3}\ u^{\frac{3}{2}}\right) + C$ $I = \frac{1}{10}(2x+1)^{\frac{5}{2}} - \frac{1}{6}(2x+1)^{\frac{3}{2}} + C$	3 marks for correct primitive in terms of x (must convert back to x) 2 marks for correct integral in terms of u 1 mark for integral in terms of u with one error
12ai	$f(x) = x^2 + 8x + 10,\ \ x \geq -4$ inverse: $x = y^2 + 8y + 10$ $x + 6 = (y+4)^2$ $y + 4 = \pm\sqrt{x+6}$ $y = -4 \pm \sqrt{x+6}$ but $y \geq -4$ (since f has domain $x \geq -4$) $\therefore \quad y = -4 + \sqrt{x+6}$	2 marks, for correct working and solution. 1 mark for interchanging x and y, however, makes no further progress or leaves the function in terms of $\pm$.
12aii		2 marks, for correct shape and correct coordinates. 1 mark for correct shape. 1 mark for correct coordinates.
12aiii	The graphs intersect on the line $y = x$.	2 marks for correct working and solution.

	$x^2+8x+10=x$ $x^2+7x+10=0$ $(x+2)(x+5)=0$ $x=-2 \quad (\text{as } x\geq -4)$ $POI=(-2, f(-2))=(-2, -2)$	1 mark for equating f(x) to x, however, makes no further progress.
12b	$P(X\geq 450)$ $=P\left(\hat{p}\geq\frac{450}{500}\right)$ $=P\left(\hat{p}\geq 0.9\right)$ $\approx P(Z\geq z_{0.9})$ $=P\left(Z\geq\frac{0.9-\mu}{\sigma}\right)$ $=P\left(Z\geq\frac{0.9-p}{\sqrt{\frac{pq}{n}}}\right)$ $=P\left(Z\geq\frac{0.9-0.91}{\sqrt{\frac{0.91\times 0.09}{500}}}\right)$ $=P(Z\geq -0.781345\ldots)$ $\approx P(Z\geq -0.78)$ $=P(Z\leq 0.78)$ ≈ 0.7823 OR using Normnal Approximation $P(x\geq 450)$ $np=500\times 0.91$ $=455$ $\sigma=\sqrt{500\times 0.91\times 0.09}$ $=6.40$ $z=\frac{450-455}{6.4}$ $z=-0.78$ $P(z\geq -0.78)$ $=P(z\leq 0.78)$ $=0.7823$	3 marks for correct working and solution. 2 marks for calculating the standard deviation and the z-score, however, makes no further progress. 1 mark for calculating the standard deviation.

12c	$V = \pi \int_0^{\frac{\pi}{6}} \cos^2 x \, dx - \pi \int_0^{\frac{\pi}{6}} \sin^2 2x \, dx$ $V = \pi \int_0^{\frac{\pi}{6}} (\cos^2 x - \sin^2 2x) \, dx$ $V = \pi \int_0^{\frac{\pi}{6}} \left[\frac{1}{2}(1 + \cos 2x) - \frac{1}{2}(1 - \cos 4x)\right] dx$ $V = \frac{\pi}{2} \int_0^{\frac{\pi}{6}} (1 + \cos 2x - 1 + \cos 4x) \, dx$ $V = \frac{\pi}{2} \int_0^{\frac{\pi}{6}} (\cos 2x + \cos 4x) \, dx$ $V = \frac{\pi}{2}\left[\frac{1}{2}\sin 2x + \frac{1}{4}\sin 4x\right]_0^{\frac{\pi}{6}}$ $V = \frac{\pi}{8}[2\sin 2x + \sin 4x]_0^{\frac{\pi}{6}}$ $V = \frac{\pi}{8}\left[2\sin\frac{\pi}{3} + \sin\frac{2\pi}{3}\right]_0^{\frac{\pi}{6}}$ $V = \frac{\pi}{8}\left[2 \times \frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2}\right]$ $V = \frac{\pi}{8} \times \frac{3\sqrt{3}}{2}$ $V = \frac{3\pi\sqrt{3}}{16}$	3 marks for correct working and solution. 2 marks for setting up the correct integral and integrating correctly, however, makes no further progress. 1 mark for setting up correct integral.

12d	$x = t^{-1} + t^2 \Rightarrow \frac{dx}{dt} = -t^{-2} + 2t$ $y = t^2 + t + 1 \Rightarrow \frac{dy}{dt} = 2t + 1$ $m = \frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx} = \frac{2t+1}{-t^{-2}+2t}$ When $y = 3$, $3 = t^2 + t + 1$ $t^2 + t - 2 = 0$ $(t+2)(t-1) = 0$ $t = 1$ or -2 $x(1) = \frac{1}{1} + 1^2 = 2$ and $x(-2) = -\frac{1}{2} + 4 = 3.5$ $\therefore$ at (2, 3), $t = 1$ $\therefore$ Gradient at (2, 3) $= \frac{dy}{dx} = \frac{2(1)+1}{-(1)^{-2}+2(1)} = 3$	3 marks for correct working and solution. 2 marks for differentiating x and y in terms of t, plus finding the correct t value, however, makes no further progress. 1 mark for differentiating x and y in terms of t, however, makes no further progress.
13a	$y = \arcsin(x - 4)$ Dilated horizontally by a factor of 2 $\rightarrow y = \sin^{-1}\left(\frac{x}{2} - 4\right)$ Shifted right 1 unit $\rightarrow y = \sin^{-1}\left(\frac{x-1}{2} - 4\right)$ $y = \sin^{-1}\left(\frac{x-9}{2}\right)$	2 marks for correct answer 1 mark for obtaining an expression of the form $\frac{x-\alpha}{2}$, where $\alpha \neq 9$

13b	For $n=0$, $5^{2n+1}+2^{2n+1}=5^{2(0)+1}+2^{2(0)+1}=7=7(1)$ $\therefore$ true for $n=0$ Assume true for $n=k$: i.e. $5^{2k+1}+2^{2k+1}=7A$, for some integer A For $n=k+1$, $5^{2n+1}+2^{2n+1}$ $=5^{2k+3}+2^{2k+3}$ $=25(5^{2k+1})+4(2^{2k+1})$ $=21(5^{2k+1})+4(5^{2k+1})+4(2^{2k+1})$ $=21(5^{2k+1})+4(5^{2k+1}+2^{2k+1})$ $=21(5^{2k+1})+4(7A)$ from assumption $=7[3\times 5^{2k+1}+4A]$ $3\times 5^{2k+1}+4A\in\mathbb{Z}$ since $2k+1\in\mathbb{Z}^+$, $A\in\mathbb{Z}$ $\therefore$ true for $n=k+1$ if true for $n=k$ $\therefore$ true by induction for all integers $n\geq 0$	3 marks for correct answer – must show initial case 2 marks for correct statement of $S(k)$ and $S(k+1)$ **and** correct substitution of a variant of $S(k)$ into $S(k+1)$ 1 mark for correct statement of $S(k)$ and $S(k+1)$ **or** correct substitution of a variant of $S(k)$ into $S(k+1)$
13ci	$P(X>3)$ $=1-P(X=0,1,2,3)$ $=1-\left[0.98^{80}+{}^{80}\mathbf{C}_1\,0.02^1\,0.98^{79}+{}^{80}\mathbf{C}_2\,0.02^2\,0.98^{78}+{}^{80}\mathbf{C}_3\,0.02^3\,0.98^{77}\right]$ ≈ 0.07685	2 marks for correct answer 1 mark for correct expression for $\sum_{k=0}^{3}{}^{80}\mathbf{C}_k(0.02)^k(0.98)^{80-k}$ **or** 1 mark for correct expression for three of the four possible terms **and** correct evaluation
13cii	Let Y be the number of days on which more than 3 faulty elements are found in a 5-day working week. Then $Y\sim$ B(5, 0.07685) $P(Y\geq 2)$ $=1-P(Y=0,1)$ $=1-\left[(0.92315)^5+{}^5\mathbf{C}_1(0.07685)(0.92315)^4\right]$ ≈ 0.05049	2 marks for correct evaluation using results from part (i) 1 mark for correct evaluation of an expression involving $P(X=2)$

13d	The double zero of $P(x)$ must be a zero of $P'(x)$ $P'(x) = 9x^2 - 9$ $P'(x) = 0 \Rightarrow 9x^2 - 9 = 0 \Rightarrow x = \pm 1$ $P(1) = 3 - 9 + 6 = 0, \therefore x = 1$ is the double zero $\therefore \quad 3x^3 - 9x + 6 \equiv (x-1)^2(ax+b)$ for some constants a,b $3x^3 - 9x + 6 \equiv (x^2 - 2x + 1)(ax+b)$ Considering the coefficients, $a = 3$ and $b = 6$ $P(x) = 0$ $\Rightarrow (x-1)^2(3x+6) = 0$ $x = 1$ or -2	3 marks for correct answer – must show $P(1)=0$ 2 marks for obtaining $x=\pm 1$, and showing $P(1)=0$, and attempting to factorise $P(x)$ 1 mark for obtaining one zero
13e	$\int_0^{\frac{\pi}{4}} \sin 3x \sin x \, dx$ $= \frac{1}{2}\int_0^{\frac{\pi}{4}} [\cos(3x-x) - \cos(3x+x)]dx$ $= \frac{1}{2}\int_0^{\frac{\pi}{4}} (\cos 2x - \cos 4x)\, dx$ $= \frac{1}{2}\left[\frac{1}{2}\sin 2x - \frac{1}{4}\sin 4x\right]_0^{\frac{\pi}{4}}$ $= \frac{1}{8}[2\sin 2x - \sin 4x]_0^{\frac{\pi}{4}}$ $= \frac{1}{8}\left(2\sin\frac{\pi}{2} - \sin\pi - (0-0)\right)$ $= \frac{1}{8}(2)$ $= \frac{1}{4}$	3 marks for correct answer 2 marks for correctly transforming $\sin 3x \sin x$ and finding the correct primitive function 1 mark for correctly transforming $\sin 3x \sin x$ **or** 1 mark for obtaining $\sin 3x \sin x = 3\sin^2 x - 4\sin^4 x$ and substituting $\frac{1}{2}(1-\cos 2x)$ into $\sin^2 x$ and $\sin^4 x$
14ai	${}^{13}\mathbf{C}_2 \times {}^{13}\mathbf{C}_4 = 55770$	1 mark
14aii	n(at least 5 cards the same suit) $= n$(exactly 5 the same suit) $+ n$(exactly 6 the same suit) $= {}^{13}\mathbf{C}_5 \times {}^{39}\mathbf{C}_1 \times 4 + {}^{13}\mathbf{C}_6 \times 4$ $= 207\,636$ **Note:** the answer is not $(4)({}^{13}\mathbf{C}_5)(47)$ because this repeats some combinations.	3 marks: +1 mark for answer including ${}^{13}\mathbf{C}_5$ or ${}^{13}\mathbf{C}_6$ +1 mark for adding together two cases (5 same, 1 different) + (6 same)

		+1 mark for multiplying by 4 (as there are 4 suits)
14bi	$\frac{1}{20\,000}\left(\frac{1}{N}+\frac{1}{20\,000-N}\right)$ $=\frac{1}{20\,000}\times\frac{20\,000-N+N}{N(20\,000-N)}$ $=\frac{1}{20\,000}\times\frac{20\,000}{N(20\,000-N)}$ $=\frac{1}{N(20\,000-N)}$	1 mark
14bii	$\frac{dN}{dt}=kN(20\,000-N)$ $\frac{dt}{dN}=\frac{1}{kN(20\,000-N)}$ $k\frac{dt}{dN}=\frac{1}{N(20\,000-N)}$ $kt+C=\int\frac{1}{N(20\,000-N)}\,dN$ $kt+C=\frac{1}{20\,000}\int\frac{1}{N}+\frac{1}{20\,000-N}\,dN$ $20\,000\,kt+20\,000\,C=\int\frac{1}{N}+\frac{1}{20\,000-N}\,dN$ $20\,000\,kt+20\,000\,C=\ln N-\ln(20\,000-N)$ $20\,000\,kt+20\,000\,C=\ln\frac{N}{20\,000-N}$ $e^{20\,000\,kt+20\,000\,C}=\frac{N}{20\,000-N}$ $\frac{N}{20\,000-N}=Ae^{\alpha t}$ where $A=e^{20\,000\,C}$ and $\alpha=20\,000\,k$	2 marks 1 mark for correctly integrating

14biii	When $t = 0$, $N = 100$: $\frac{100}{20\,000 - 100} = Ae^0$ $A = \frac{1}{199}$ When $t = 10$, $N = 400$: $\frac{400}{20\,000 - 400} = \frac{1}{199}e^{10\alpha}$ $\frac{1}{49} = \frac{1}{199}e^{10\alpha}$ $\frac{199}{49} = e^{10\alpha}$ $10\alpha = \ln\left(\frac{199}{49}\right)$ $a = \frac{1}{10}\ln\left(\frac{199}{49}\right) \approx 0.1401$	2 marks 1 mark for A, 1 mark for α
14biv	$N = 10000$, $t = ?$ $\frac{10\,000}{10\,000} = \frac{1}{199}e^{0.1401\,t}$ $199 = e^{0.1401\,t}$ $t = \frac{1}{0.1401}\ln 199 = 37.769\ldots$ $\therefore$ It takes 38 days to infect half the population	1 mark
14ci	Since $\overrightarrow{DE} \perp \overrightarrow{AC}$, $\overrightarrow{DE} \cdot \overrightarrow{AC} = 0$ $2\underline{c} \cdot (\underline{a} - \underline{b}) = 0$ $\underline{c} \cdot \underline{a} - \underline{c} \cdot \underline{b} = 0$ $\underline{a} \cdot \underline{c} = \underline{b} \cdot \underline{c}$	2 marks 1 mark for $2\underset{\sim}{\mathbf{c}} \cdot (\underset{\sim}{\mathbf{a}} - \underset{\sim}{\mathbf{b}})$ or equivalent
14cii	Required to prove that $\overrightarrow{AF} \cdot \overrightarrow{BE} = 0$. $\overrightarrow{AF} = \underline{c} - \underline{b}$ $\overrightarrow{BE} = \underline{a} + 2\underline{c}$ Therefore, $\overrightarrow{AF} \cdot \overrightarrow{BE} = (\underline{c} - \underline{b}) \cdot (\underline{a} + 2\underline{c}) = \underline{a} \cdot \underline{c} + 2c \cdot \underline{c} - \underline{a} \cdot \underline{b} - 2\underline{b} \cdot \underline{c}$ Since $\overrightarrow{AD} \perp \overrightarrow{BC}$, $\underline{a} \cdot \underline{b} = 0$, and since $\underline{a} \cdot \underline{c} = \underline{b} \cdot \underline{c}$ (from (i)), $\overrightarrow{AF} \cdot \overrightarrow{BE} = \underline{a} \cdot \underline{c} + 2c \cdot \underline{c} - 2\underline{a} \cdot \underline{c} = 2c \cdot \underline{c} - \underline{a} \cdot \underline{c}$ Now, since $\overrightarrow{DE} \perp \overrightarrow{AC}$, $\overrightarrow{DE} \cdot \overrightarrow{EC} = 0$. Thus, $2\underline{c} \cdot (\underline{a} - 2\underline{c}) = 0$ $2\underline{c} \cdot \underline{c} = \underline{a} \cdot \underline{c}$	3 marks for correct proof 1st mark for expressing $\overrightarrow{AF}$ and $\overrightarrow{BE}$ in terms of $\underset{\sim}{\mathbf{a}}$, $\underset{\sim}{\mathbf{b}}$, $\underset{\sim}{\mathbf{c}}$ 2nd mark for obtaining $\overrightarrow{AF} \cdot \overrightarrow{BE} = 2c \cdot \underline{c} - \underline{a} \cdot \underline{c}$ or equivalent. Must have used part (i) and $\underline{a} \cdot \underline{b} = 0$

	Hence, $\overrightarrow{AF} \cdot \overrightarrow{BE} = 2c \cdot \underline{c} - \underline{a} \cdot \underline{c} = \underline{a} \cdot \underline{c} - \underline{a} \cdot \underline{c} = 0$ as required	