	Year 12/ Year 11 acc **Student Number** ……………………………..…………………………………

2021 HSC Trial Examination

Year 12 Mathematics Extension 1

Examiner: AC

Reading time – 10 minutes

Writing time – 2 hours

General Instructions

- Write using black or blue pen.
- Read the instructions carefully.
- Multiple choice answers should be written on the multiple choice answer sheet.
- You are to write solutions in writing booklets, starting each question in a separate writing booklet.
- Write your student name clearly on each booklet.
- A reference sheet is given with the examination.
- Board-approved calculators may be used.
- All diagrams must be drawn in pencil.
- Do not remove this question paper from the examination room.

Section	Guidance	Marks Available	Your Score
SECTION I	• *Type of Questions – Multiple Choice* • *Attempt Questions 1 - 10* • *Timing 15 minutes*	10	
SECTION II	• *Type of Questions – Extended Response* • *Attempt Question 11 - 14* • *Timing 1hour 45 minutes*	60	
	Totals	70	

FINAL MARK	/ 70	%

Your examination paper begins overleaf.

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<u>Section 1</u>: 10 marks

Attempt Questions 1 – 10

Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1 – 10.

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1 Which of the following expressions is equivalent to $3\cos x - \sqrt{3}\sin x$?

(A) $2\sqrt{3}\cos\left(x+\frac{\pi}{3}\right)$

(B) $2\sqrt{3}\cos\left(x-\frac{\pi}{3}\right)$

(C) $2\sqrt{3}\cos\left(x+\frac{\pi}{6}\right)$

(D) $2\sqrt{3}\cos\left(x-\frac{\pi}{6}\right)$

2 $3^n + 7^{n+1}$ for integers $n \geq 1$ is divisible by:

(A) 3

(B) 4

(C) 5

(D) 10

3 The number of different words you can make by rearranging the letters in the word **WOODWARD** is:

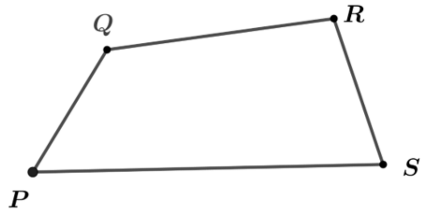
(A) 40320

(B) 20160

(C) 10080

(D) 5040

4 $PQRS$ is a quadrilateral where $\overrightarrow{PQ}=\vec{p}$, $\overrightarrow{QR}=\vec{q}$, $\overrightarrow{RS}=\vec{r}$ and $\overrightarrow{SP}=\vec{s}$



Which of the following is true?

(A) $\vec{p}+\vec{q}=\vec{s}+\vec{r}$

(B) $\vec{p}+\vec{q}=-\left(\vec{s}+\vec{r}\right)$

(C) $\vec{p}+\vec{s}=\vec{q}+\vec{r}$

(D) $\vec{p}+\vec{s}=-\left(\vec{q}-\vec{r}\right)$

5 The graph $y=f\left(x\right)$ is reflected about the x-axis and dilated horizontally with a scale factor of 3. The transformed graph has the equation:

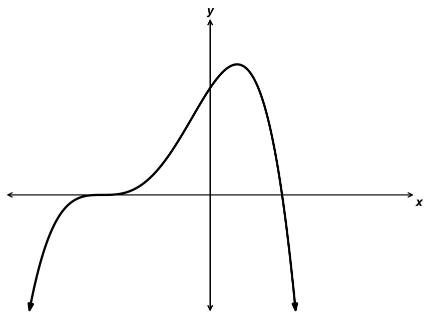
(A) $y=-f\left(\dfrac{x}{3}\right)$

(B) $y=f\left(\dfrac{-x}{3}\right)$

(C) $y=-f\left(x-3\right)$

(D) $y=-f\left(3x\right)$

6 The graph of a polynomial $y = P(x)$ is shown below.



What is the minimum possible degree of the polynomial $\left[P(x)\right]^2$?

(A) 2

(B) 4

(C) 8

(D) 16

7 Given $f(x) = \dfrac{3x-1}{2-x}$, which of the following is $f^{-1}(x)$?

(A) $f^{-1}(x) = \dfrac{2x+1}{3+x}$

(B) $f^{-1}(x) = \dfrac{2x+1}{3-x}$

(C) $f^{-1}(x) = \dfrac{3+x}{2x+1}$

(D) $f^{-1}(x) = \dfrac{3-x}{2x+1}$

8 The area enclosed between the curve $y = x^3 - 1$, the y-axis, and the lines $y = 1$ and $y = 2$ is given by:

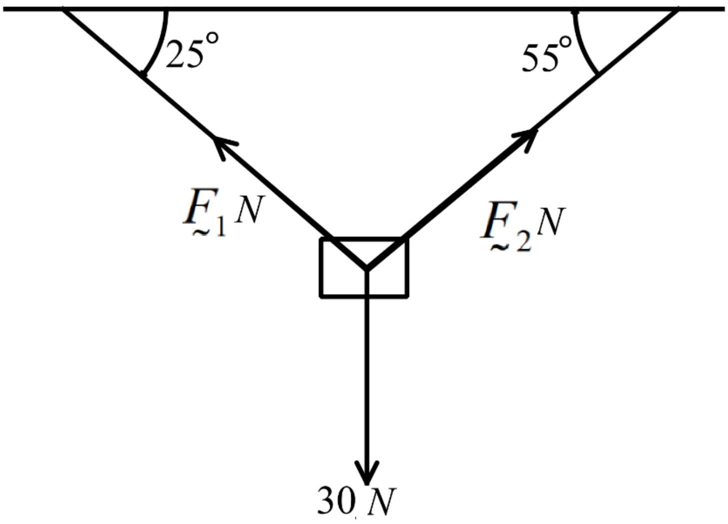
(A) $\int_1^2 (x^3 - 1)\,dy$

(B) $\int_1^2 (\sqrt[3]{y+1})\,dy$

(C) $\int_1^2 (\sqrt[3]{y} + 1)\,dy$

(D) $\int_1^2 (y + 1)\,dy$

9 A particle of mass with 30 N of force is suspended by two strings attached to two points in the same horizontal plane. The two strings make angles of 25° and 55° respectively to the horizontal. The magnitude of acceleration due to gravity is $10\,ms^{-2}$.



The magnitude of the tension force $\underset{\sim}{F}_1$, correct to the nearest newton, N, is given by:

(A) 12 N

(B) 15 N

(C) 17 N

(D) 21 N

10 The population, P, of animals in an environment in which there are scarce resources is increasing such that $\frac{dP}{dt} = P(100 - P)$, where t is time.
The initial population is 20 animals. Which of the following is true?

(A) $P = 100 - 80e^{100t}$

(B) The population is increasing most rapidly when $P = 50$.

(C) The population is increasing most rapidly when $t = 50$.

(D) The maximum population is $P = 50$.

End of Section I

<u>Section II:</u> 60 marks

Attempt Questions 11 – 14

Allow about 1 hour and 45 minutes for this section

Answer each question in a separate writing booklet.

Your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Use a SEPARATE writing booklet.

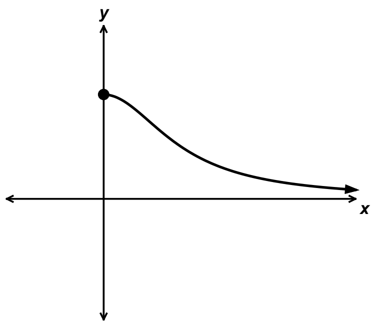
(a) Consider the polynomial $P(x) = x^3 + 4x^2 + x - 6$.

(i) Use the factor theorem to show that $(x + 2)$ is a factor of $P(x)$. **1**

(ii) Hence, or otherwise, factorise $P(x)$ completely. **2**

(b) Solve $\dfrac{2}{1-x} \geq -1$. **3**

(c)



The diagram above shows the function $f(x) = \dfrac{1}{x^2+1}$ for $x \geq 0$.

State the domain and range of the inverse function $f^{-1}(x)$. **2**

Question 11 continues on page 7

Question 11 (continued)

(d) Water is poured into a container at a rate of 8 cm^3 per second, and the volume of the water in the container is given by:

$V = \frac{3}{2}(h^2 + 8h)$, where h cm is the depth of the water.

Find the rate of change of the depth of the water when $V = 72$ cm^3. **3**

(e) (i) The vectors $\vec{p} = \begin{pmatrix} a \\ 3 \end{pmatrix}$ and $\vec{q} = \begin{pmatrix} 2 \\ 4 \end{pmatrix}$ are parallel. Find a. **1**

(ii) The vector $\vec{r} = \begin{pmatrix} x \\ y \end{pmatrix}$ is perpendicular to vector $\vec{q}$ and $\left|\vec{q} - \vec{r}\right| = \sqrt{65}$.

Find the possible values of x and y. **3**

End of Question 11

Question 12 (15 marks) Use a SEPARATE writing booklet.

(a) A roast chicken is cooked in an oven at $240^\circ C$ and then taken out of the oven and left to cool to room temperature. The rate at which the chicken cools is given by $\frac{dT}{dt} = -k\left(T-22\right)$, where T is the temperature of the chicken in $^\circ C$, and k is a constant.

(i) Show that $T = 22 + 218e^{-kt}$ is a solution to the differential equation, where t is measured in minutes. **1**

(ii) If it takes 10 minutes for the chicken to cool to $180^\circ C$, how long will it take for the chicken to cool to $60^\circ C$? Give your answer correct to the nearest second. **2**

(b) A curve has gradient given by $\frac{dy}{dx} = 2xy$. Find an expression for y in terms of x given that the curve passes through the point $\left(0,4\right)$. **3**

(c)

(i) Find $\frac{d}{dx}\left(x\tan^{-1}\left(x\right)\right)$. **1**

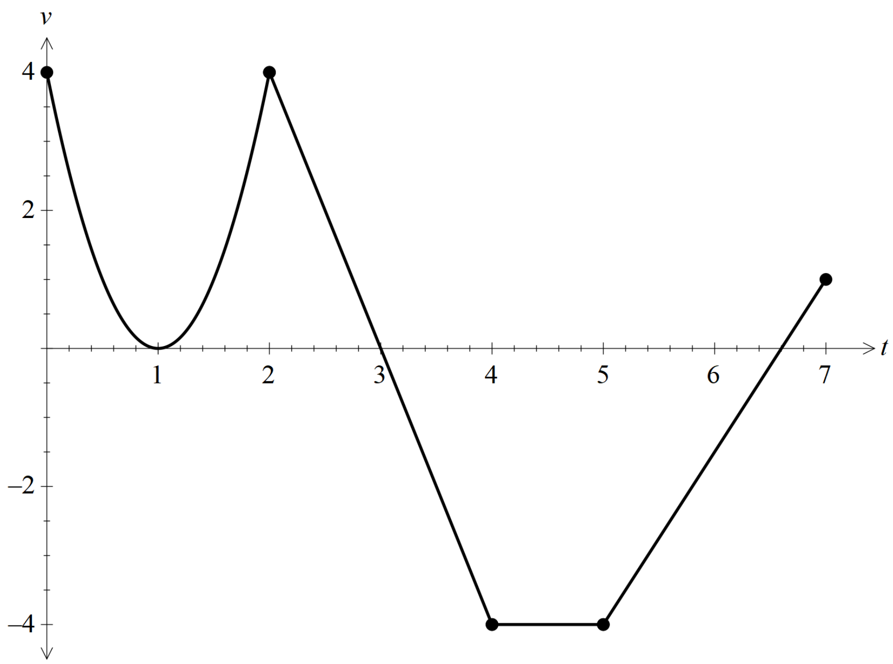
(ii) Hence, evaluate $\int \tan^{-1}\left(x\right)\, dx$. **2**

(d) Show that $\frac{1-\cos 2\theta}{1+\cos 2\theta} + 1 = \sec^2\theta$. **2**

Question 12 continues on page 9

Question 12 (continued)

(e) A particle P moves along the x-axis with velocity v ms^{-1}, such that the following sketch shows the graph of v for $0 \le t \le 7$, where t is in seconds.



(i) Using the graph provided, state the first time when P is at rest. **1**

(ii) Determine the number of times P changes directions. **1**

(iii) When is the particle P a maximum distance to the right of the origin? **1**

(iv) Determine the acceleration of the particle when $t = 3$. **1**

End of Question 12

Question 13 (15 marks) Use a SEPARATE writing booklet.

(a) What is the derivative of $f(x) = 4\cos^{-1}\frac{x}{2}$? Write your answer in the form $\dfrac{a}{\sqrt{b-x^2}}$. **2**

(b) A NSW census showed that 40% of NSW adults completed at least 30 minutes of exercise each day.

(i) A random sample of 26 NSW adults is to be conducted to find the proportion who completed at least 30 minutes of exercise each day. Show that the mean and standard deviation for the distribution of sample proportions are 0.4 and 0.0961 respectively. **2**

(ii) Part of a table giving values of $P(Z < z)$, where Z is a standard normal variable is shown below.

z	+0.00	+0.01	+0.02	+0.03	+0.04	+0.05	+0.06	+0.07	+0.08	+0.09
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9931	0.9934	0.9936
2.5	0.9938	0.9940	0.9943	0.9945	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964

Estimate the probability that a random sample of 21 NSW adults will find at most 2 who have completed at least 30 minutes of exercise each day. **2**

(c) The function $f(x) = \dfrac{2}{\sqrt{x-1}}$ is rotated around the x-axis between $x = 3\sqrt{2}$ and $x = \sqrt{2}$.

Calculate the volume of the solid of revolution formed. **3**

(d) Evaluate $\displaystyle\int \frac{\left(\sqrt{x}+1\right)^{\frac{1}{3}}}{4\sqrt{x}}\,dx$ using $u = \sqrt{x}+1$. **3**

Question 13 continues on page 11

Question 13 (continued)

(e) Find the constant term in $\left(x^2 - \frac{4}{3x^6}\right)^8$. **3**

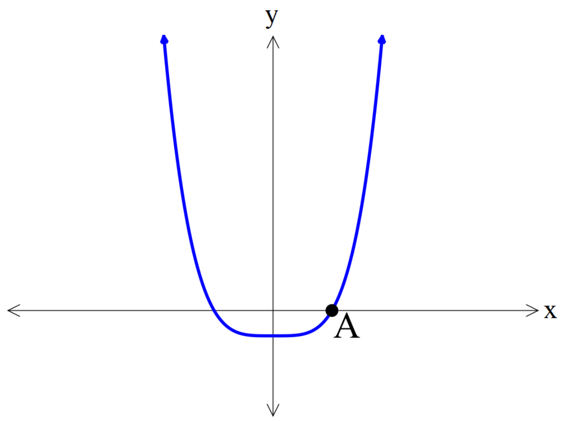
End of Question 13

Question 14 (15 marks) Use a SEPARATE writing booklet.

(a) An examination has 30 multiple-choice questions, each with 4 options. In each question, only one option is correct. For each question a student chooses one option at random. What is the probability that the student achieves a mark of at least 10% for the examination? Answer correct to three decimal places. **3**

(b) Prove that $2\times1! + 5\times2! + 10\times3! + \ldots + \left(n^2+1\right)n! = n\left(n+1\right)!$ for $n\in\mathbb{Z}^+$ by mathematical induction. **3**

(c) Consider the graph of $y=\dfrac{1}{\sqrt{1-4x^2}}-2$ below. Point $A\left(a,\,0\right)$ is an x-intercept of the curve.



(i) Find the coordinates of point A. **1**

(ii) Hence, find the area bound by the curve, the x-axis and the line $x=\dfrac{1}{2}$.

Leave your answer correct to two decimal places. **3**

(d) Using the t-formulae solve the equation $\sin 2x = \tan x$ for $0\le x\le \pi$. **3**

Question 14 continues on page 13

Question 14 (continued)

(e) Shares in company X are particularly volatile. On any given trading day, the probability that the share price will increase by $1 is $\frac{3}{5}$ and the probability that the share price will fall by $1 is $\frac{2}{5}$.

If the share price is now $6, calculate the probability that it will be less than $6 after 5 trading days. **2**

End of Question 14

End of Examination

2021 Ext1 Trial Sols

Monday, 5 July 2021 10:05 AM

MC

Q1.) let $x = \pi$ & test each answer.

test a.) $2\sqrt{3}\cos\left(\pi + \frac{\pi}{3}\right) = 3\cos(\pi) - \sqrt{3}\sin(\pi)$

$-\sqrt{3} = -3$ ✗

test c.) $2\sqrt{3}\cos\left(\pi - \frac{\pi}{6}\right) = 3\cos\pi - \sqrt{3}\sin\pi$

$-3 = -3$ ✓

$\therefore$ C

Q2.) ① Prove true for $n = 1$

test 3: $3^1 + 7^{1+1} = 52$ (not ÷ 3) ✗

test 4: $3^1 + 7^2 = 52$ (÷ by 4) ✓

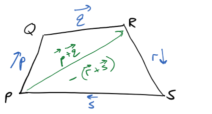
$\therefore$ B

Q3.)

$= \frac{8!}{2!2!2!} \leftarrow$ WW, OO, DD ($2!$ $2!$ $2!$)

$= 5040$ D

Q4.)



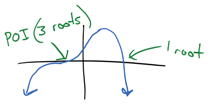
$\therefore \vec{p} + \vec{q} = -(\vec{r} + \vec{s})$ B

Q5.) $y = f(x)$

$y = -f(x)$ (reflected)

$y = -f\left(\frac{1}{3}x\right)$ (hor. dil.) A

Q6.)



$p(x) = (x-a)^3(x-b)$ (4 roots total)

$= (x^4 + \ldots\ldots)$

$[P(x)]^2 = (x^4 \ldots)(x^4 \ldots\ldots)$

$= x^8 \ldots$ when expanded.

C

Q7.) $y = \frac{3x-1}{2-x}$

$x = \frac{3y-1}{2-y}$

$2x - xy = 3y - 1$

$3y + xy = 2x + 1$

$y = \frac{2x+1}{3+x}$ A

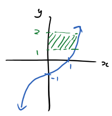
Q8.) $y = x^3 - 1$

$y + 1 = x^3$

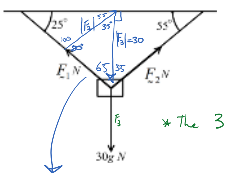
$x = \sqrt[3]{y+1}$

$\boxed{A = \int_a^b f(x)\,dy}$

$\therefore A = \int_1^2 \left(\sqrt[3]{y+1}\right) dy$ B

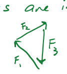


Q9.)



* The 3 vectors are in equilibrium.

i.e.)



$\therefore \frac{30}{\sin 80°} = \frac{|F_2|}{\sin 65°} = \frac{|F_1|}{\sin 35°}$

$\frac{|F_1|}{\sin 35°} = \frac{30}{\sin 80°}$

$|F_1| = \frac{30 \times \sin(35°)}{\sin(80°)}$

$\doteqdot 17$ C

Q10) $\frac{dP}{dt} = P(100 - P)$

This function, $(P(100-P))$, has a max value when $P = 50$. → i.e.

$$\frac{dP}{dt} = 100P - P^2$$

$$\frac{d^2P}{dt^2} = 100 - 2P$$

$$-2P + 100 = 0$$

$$\underline{P = 50}$$

$\therefore$ The pop. is increasing most rapidly at $P = 50$. (B)

11.) a.) i) $P(x) = x^3 + 4x^2 + x - 6$

$P(-2) = (-2)^3 + 4(-2)^2 + (-2) - 6$
$= 0$

$\therefore (x+2)$ is a factor of $P(x)$

ii.)

$$\begin{array}{r|l} & x^2 + 2x - 3 \\ x+2 & x^3 + 4x^2 + x - 6 \\ & x^3 + 2x^2 \\ \hline & 2x^2 + x \\ & 2x^2 + 4x \\ \hline & -3x - 6 \\ & -3x - 6 \\ \hline & 0 \end{array}$$

$\therefore P(x) = (x+2)(x^2+2x-3)$

$P(x) = (x+2)(x+3)(x-1)$

b.) $\frac{2}{1-x} \geq -1$ $\quad x \neq 1$

$\times (1-x)^2 \quad \times (1-x)^2$

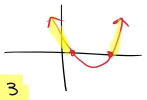
$2(1-x) \geq -(1-x)^2$

$2 - 2x \geq -(1 - 2x + x^2)$

$2 - 2x \geq -x^2 + 2x - 1$

$x^2 - 4x + 3 \geq 0$

$(x-3)(x-1) \geq 0$

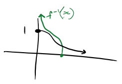


$\therefore$ $x \leq 1$ or $x \geq 3$

$\therefore$ $x \in (-\infty, 1) \cup [3, \infty)$

c.) $y = \frac{1}{x^2+1}$

for $x \geq 0$



D: $[0, \infty)$
R: $(0, 1]$

$\therefore f^{-1}(x) \longrightarrow$ D: $(0, 1]$
R: $[0, \infty)$

d.) $\frac{dV}{dt} = 8$

$V = \frac{3}{2}(h^2 + 8h)$

$\frac{dV}{dh} = \frac{3}{2}(2h + 8)$

$\frac{dh}{dt} = \frac{dh}{dV} \times \frac{dV}{dt}$

$\frac{dh}{dt} = \frac{2}{3(2h+8)} \times 8$

$\frac{dh}{dt} = \frac{16}{3(2h+8)}$

when $V = 72$, $72 = \frac{3}{2}(h^2 + 8h)$

$h^2 + 8h - 48 = 0$

$(h+12)(h-4) = 0$

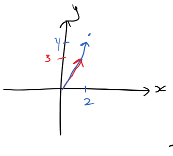
$h = 4$

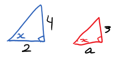
$\therefore$ when $h = 4$, $\frac{dh}{dt} = \frac{16}{3(2(4)+8)}$

$= \frac{1}{3}$

$\therefore$ height changing at $\frac{1}{3}$ cm/s.

e.) i) $\vec{P} = \begin{pmatrix} a \\ 3 \end{pmatrix}$ & $\vec{q} = \begin{pmatrix} 2 \\ 4 \end{pmatrix}$ parallel.





$\tan x = \frac{4}{2}$ $\quad \tan x = \frac{3}{a}$

$\therefore \frac{4}{2} = \frac{3}{a}$

$a = \frac{3}{2}$

ii.) $\rightarrow$ /x\

$\therefore \frac{4}{2} = \frac{3}{a}$

$a = \frac{3}{2}$

ii.) $\vec{r} = \begin{pmatrix} x \\ y \end{pmatrix}$ is perp to $\vec{q} = \begin{pmatrix} 2 \\ 4 \end{pmatrix}$ & $|\vec{q} - \vec{r}| = \sqrt{65}$

$\vec{r} \cdot \vec{q} = 0$

$2x + 4y = 0$

$x = -2y$

Also $\vec{q} - \vec{r} = \begin{pmatrix} 2 - x \\ 4 - y \end{pmatrix}$

sub in $x = -2y$

$\vec{q} - \vec{r} = \begin{pmatrix} 2 + 2y \\ 4 - y \end{pmatrix}$

$|\vec{q} - \vec{r}| = \sqrt{(2+2y)^2 + (4-y)^2} = \sqrt{65}$

square   square

$(2+2y)^2 + (4-y)^2 = 65$

$4 + 8y + 4y^2 + 16 - 8y + y^2 = 65$

$5y^2 + 20 - 65 = 0$

$5y^2 - 45 = 0$

$y^2 = 9$

$y = \pm 3$

$x = -2(\pm 3)$

$x = \pm 6$

$\therefore x = 6, y = -3$ or $x = -6, y = 3$

Q12. a.) i) $\frac{dT}{dt} = -k(T - 22)$

$\frac{1}{T-22} \, dT = -k \, dt$

$\int \quad \int$

$\ln|T - 22| = -kt + c$

$|T - 22| = e^{-kt + c}$

$T - 22 = Ae^{-kt}$ where $A = \pm e^c$

$T = 22 + Ae^{-kt}$

$t = 0, T = 240$

$240 = 22 + A$

$A = 218$

$\therefore T = 22 + 218e^{-kt}$

ii.) $t = 10, T = 180$

$180 = 22 + 218e^{-k \times 10}$

$158 = 218e^{-10k}$

$\frac{79}{109} = e^{-10k}$

$-10k = \ln\left(\frac{79}{109}\right)$

$k = -\frac{1}{10}\ln\left(\frac{79}{109}\right)$

$\therefore T = 22 + 218e^{\frac{1}{10}\ln\left(\frac{79}{109}\right)t}$

When $T = 60$,

$60 = 22 + 218e^{\frac{1}{10}\ln\left(\frac{79}{109}\right)t}$

$\frac{38}{218} = e^{\frac{1}{10}\ln\left(\frac{79}{109}\right)t}$

$\ln\left(\frac{19}{109}\right) = \frac{1}{10}\ln\left(\frac{79}{109}\right)t$

$t = \frac{10\ln\left(\frac{19}{109}\right)}{\ln\left(\frac{79}{109}\right)}$

$t \doteqdot 54.2686\ldots$

$\therefore$ Takes 54 mins 16 secs.

b.) $\frac{dy}{dx} = 2xy$

$\frac{1}{y}\,dy = 2x\,dx$

$\ln|y| = x^2 + c$

$x = 0 \quad y = 4$

$\ln(4) = c$

$\therefore \ln|y| = x^2 + \ln(4)$

$y = e^{x^2 + \ln(4)}$

$y = e^{\ln 4} \times e^{x^2}$

$y = 4e^{x^2}$

c.) i) $\frac{d}{dx}\left(x\tan^{-1}x\right)$

$u = x \quad v = \tan^{-1}x$

$u' = 1 \quad v' = \frac{1}{1+x^2}$

$= \tan^{-1}x + \frac{x}{1+x^2}$

ii.) As $\frac{d}{dx}\left(x\tan^{-1}x\right) = \tan^{-1}x + \frac{x}{1+x^2}$

$\int \qquad \int \qquad \int$

$x\tan^{-1}x = \int \tan^{-1}x\,dx + \frac{1}{2}\int\frac{2x}{1+x^2}\,dx$

$x\tan^{-1}x = \int \tan^{-1}x\,dx + \frac{1}{2}\ln|1+x^2| + c$

$\int \tan^{-1}x\,dx = x\tan^{-1}x - \frac{1}{2}\ln|1+x^2| + c$

d.) $LHS = \frac{1-(1-2\sin^2\theta)}{1+(2\cos^2\theta-1)} + 1$

$= \frac{2\sin^2\theta}{2\cos^2\theta} + 1$

$= \tan^2\theta + 1$

$= \sec^2\theta$

$= RHS$

e.) i.) P is at rest at 1 sec. $(v=0)$

ii.) At $t = 3$ & 6.6 sec ...

e.) i.) P is at rest at 1 sec. $(v=0)$

ii.) At $t=3$ & $6\cdot6$ seconds. (cuts x-axis)

iii.) Farthest right at $t=3$ seconds.

iv.) At $t=3$, $a=-4ms^{-2}$.

13.) a.) $\frac{d}{dx}\left(4\cos^{-1}\left(\frac{x}{2}\right)\right)$

$$= 4\times\frac{d}{dx}\left(\cos^{-1}\left(\frac{x}{2}\right)\right)$$

$$= 4\times -\frac{\frac{1}{2}}{\sqrt{1-\left(\frac{x}{2}\right)^2}}$$

$$= -\frac{2}{\sqrt{1-\frac{x^2}{4}}}$$

$$= -\frac{2}{\sqrt{\frac{1}{4}}\sqrt{4-x^2}}$$

$$= -\frac{2}{\frac{1}{2}\sqrt{4-x^2}}$$

$$= -\frac{4}{\sqrt{4-x^2}}$$

b.) i) $p=0.4$ $n=26$

$\hat{p}=p=0.4$

$$\sigma\hat{p}=\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

$$=\sqrt{\frac{0.4(0.6)}{26}}$$

$$=0.0960768...$$

$$=0.0961 \text{ (4d.p.)}$$

ii.) At most 2, $x=\frac{2}{26}=0.0769...$

$$z=\frac{x-\mu}{\sigma}$$

$$=\frac{0.0769.....-0.4}{0.0960768...}$$

$$=-2.5620....$$

$$\doteq -2.56 \text{ (2d.p.)}$$

$$P(Z<z)=P(Z<-2.56)$$

$$=1-P(Z<2.56)$$

$$=1-0.9948$$

$$=0.0052$$

$$=0.52\%$$

$\therefore$ 0.52% chance of at most 2 people

c.) $y=\frac{2}{\sqrt{x-1}}$ $\quad y^2=\frac{4}{x-1}$

$$\boxed{V=\pi\int_a^b y^2\,dx}$$

$$V=\pi\int_{\sqrt{2}}^{3\sqrt{2}}\frac{4}{x-1}\,dx$$

$$V=4\pi\int_{\sqrt{2}}^{3\sqrt{2}}\frac{1}{x-1}\,dx$$

$$V=4\pi\left[\ln|x-1|\right]_{\sqrt{2}}^{3\sqrt{2}}$$

$V=4\pi[\ln$ ... (cut off)

$$V = 4\pi\left[\ln|x-1|\right]_{\sqrt{2}}^{3\sqrt{2}}$$

$$V = 4\pi\left[\ln|3\sqrt{2}-1| - \ln|\sqrt{2}-1|\right]$$

$$V = 4\pi\left[\ln\left|\frac{3\sqrt{2}-1}{\sqrt{2}-1}\right|\right] \ u^3$$

d.)

$u = \sqrt{x}+1$

$\frac{du}{dx} = \frac{1}{2}x^{-\frac{1}{2}}$

$du = \frac{1}{2\sqrt{x}}dx$

$$\therefore \int \frac{(\sqrt{x}+1)^{\frac{1}{3}}}{4\sqrt{x}}dx$$

$$= \frac{1}{2}\int \frac{(\sqrt{x}+1)^{\frac{1}{3}}}{2\sqrt{x}}dx$$

$$= \frac{1}{2}\int u^{\frac{1}{3}}\,du$$

$$= \frac{1}{2}\left[\frac{u^{\frac{4}{3}}}{4/3}\right]+C \overset{\text{or}}{\Longrightarrow} \frac{1}{2}\left[\frac{u^1 \times u^{\frac{1}{3}}}{4/3}\right]+C$$

$$= \frac{1}{2}\times\frac{3\sqrt[3]{u^4}}{4}+C$$

$$= \frac{3\sqrt[3]{u^4}}{8}+C$$

$$= \frac{3\sqrt[3]{(\sqrt{x}+1)^4}}{8}+C$$

$$= \frac{1}{2}\times\frac{3u\sqrt[3]{u}}{4}+C$$

$$= \frac{3u\sqrt[3]{u}}{8}+C$$

$$= \frac{3(\sqrt{x}+1)\sqrt[3]{\sqrt{x}+1}}{8}+C$$

e.) $\left(x^2 - \frac{4}{3x^6}\right)^8$

$$\boxed{T_{r+1} = {}^nC_r (a)^{n-r}(b)^r}$$

$$T_{r+1} = {}^8C_r (x^2)^{8-r}\left(-\frac{4}{3x^6}\right)^r$$

$$T_{r+1} = {}^8C_r (x^2)^8 \times (x^2)^{-r} \times (-4)^r \times (3)^{-r} \times (x^6)^{-r}$$

$$= {}^8C_r\, x^{16}\, x^{-2r}\, x^{-6r}\, (-4)^r\, (3)^{-r}$$

$$= {}^8C_r\, x^{16-8r}\, (-4)^r\, (3)^{-r}$$

$$\boxed{16-8r=0}$$

$$\therefore T_{r+1} = {}^8C_2\, x^{16-8(2)}\, (-4)^2\, (3)^{-2}$$

$$T_{r+1} = {}^8C_2 \times x^0 \times 16 \times \frac{1}{9}$$

$$T_{r+1} = 28 \times \frac{16}{9}$$

$$T_{r+1} = \frac{448}{9}$$

Q14.

a.) $n=30$ $p=0.25$ $q=0.75$

* At least 10% correct = 3 or more correct.

$$P(\geq 3 \text{ correct}) = 1 - [P(0)+P(1)+P(2)]$$

$$= 1 - \left[{}^{30}C_0(0.25)^0(0.75)^{30} + {}^{30}C_1(0.25)^1(0.75)^{29} + {}^{30}C_2(0.25)^2(0.75)^{28}\right]$$

$$= 0.9894\ldots$$

$$= 0.989\ (3\text{ d.p.}) \quad \text{or} \quad 98.940\%\ (3\text{dp})$$

$= 0.9894\ldots$

$= 0.989$ (3 d.p.) or 98.940% (3 d.p.)

b.) ① Prove true for $n=1$

$LHS = 2 \times 1!$
$= 2$

$RHS = 1(1+1)!$
$= 2$

$\therefore$ True for $n=1$

② Assume true for $n=k$

ie.) $2\times 1! + 5\times 2! + \ldots + (k^2+1)k! = k(k+1)!$

③ Prove true for $n=k+1$

RTP: $2\times 1! + 5\times 2! + \ldots + (k^2+1)k! + ((k+1)^2+1)(k+1)! = (k+1)((k+1)+1)!$

$LHS = 2\times 1! + 5\times 2! + \ldots + (k^2+1)k! + ((k+1)^2+1)(k+1)!$

$= k(k+1)! + ((k+1)^2+1)(k+1)!$ from assumption

$= k(k+1)! + (k^2+2k+2)(k+1)!$

$= (k+1)!(k + k^2 + 2k + 2)$

$= (k+1)!\,(k+1)(k+2)$

$= (k+2)!\,(k+1)$

$= (k+1)((k+1)+1)!$

$= RHS$

④ Explain: By PMI, the statement is true for all integers $n \geq 1$.

c.) i.) $y=0$ at A.

$0 = \dfrac{1}{\sqrt{1-4x^2}} - 2$

$2\sqrt{1-4x^2} = 1$

$1-4x^2 = \frac{1}{4}$

$4x^2 = \frac{3}{4}$

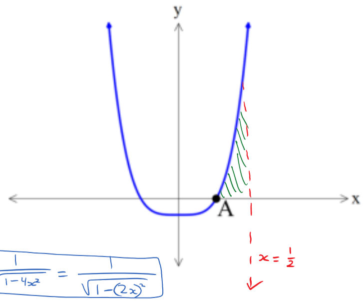
$x^2 = \frac{3}{16}$

$x = \pm\sqrt{\frac{3}{16}}$

$x = \frac{\sqrt{3}}{4}$ (in 1st quad)

$\therefore A\left(\frac{\sqrt{3}}{4}, 0\right)$

ii.)



$* \ \dfrac{1}{\sqrt{1-4x^2}} = \dfrac{1}{\sqrt{1-(2x)^2}}$

$A = \int_{\frac{\sqrt{3}}{4}}^{\frac{1}{2}} \left(\dfrac{1}{\sqrt{1-(2x)^2}} - 2\right) dx$

$A = \left[\frac{1}{2}\sin^{-1}(2x) - 2x\right]_{\frac{\sqrt{3}}{4}}^{\frac{1}{2}}$

$A = \left[\frac{1}{2}\sin^{-1}(1) - 1\right) - \left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) - \frac{\sqrt{3}}{2}\right)\right]$

$A = \left[\frac{\pi}{4} - 1 - \frac{\pi}{6} + \frac{\sqrt{3}}{2}\right]$

$A = \left[\frac{\pi}{12} - 1 + \frac{\sqrt{3}}{2}\right]$

$$A = \left(\frac{\pi - 12 + 6\sqrt{3}}{12}\right)$$

$$A = 0.13 \quad (2\ d.p)$$

d.) Let $\tan x = t$

$$\boxed{\text{i.e. } \sin 2x = \frac{2t}{1+t^2}}$$

$$\sin 2x = \tan x$$

$$\frac{2t}{1+t^2} = t$$

$$2t = t + t^3$$

$$t^3 - 2t + t = 0$$

$$t(t^2 - 1) = 0$$

$$t = 0 \text{ or } \pm 1$$

$\therefore \tan x = 0$ $\quad \tan x = 1$ $\quad \tan x = -1$

$x = 0, \pi$ $\quad x = \frac{\pi}{4}$ $\quad x = \frac{3\pi}{4}$

or

Let $\tan\left(\frac{x}{2}\right) = t$

$$\sin 2x = \tan x$$

$$2 \sin x \cos x = \tan x$$

$$\cancel{2}\left(\frac{2t}{1+t^2}\right)\left(\frac{1-t^2}{1+t^2}\right) = \frac{\cancel{2}t}{1-t^2}$$

$\times\ 1-t^2$ $\quad \times\ 1-t^2$

$\times\ (1+t^2)(1+t^2)$ $\quad \times\ (1+t^2)(1+t^2)$

$$2t(1-t^2)(1-t^2) = t(1+t^2)^2$$

$$2t(1-2t^2+t^4) = t(1+2t^2+t^4)$$

$$2t - 4t^3 + 2t^5 = t + 2t^3 + t^5$$

$$t^5 - 6t^3 + t = 0$$

$$t(\underline{t^4 - 6t^2 + 1}) = 0$$

$u = t^2$

$\therefore u^2 - 6u + 1 = 0$

$u = \frac{6 \pm \sqrt{36-4}}{2}$

$u = 3 \pm 2\sqrt{2}$

$t = \pm\sqrt{3 \pm 2\sqrt{2}}$

$\therefore t = 0,\ \pm\sqrt{3+2\sqrt{2}},\ \pm\sqrt{3-2\sqrt{2}}$

$\tan\left(\frac{x}{2}\right) = 0,\ \pm\sqrt{3+2\sqrt{2}},\ \pm\sqrt{3-2\sqrt{2}}$

$\frac{x}{2} = 0°, 180°, \pm 67.5°, \pm 22.5°,$

$x = 0°, 180°, \pm 135°, \pm 45°$

but $0 \le x \le \pi$

$\therefore x = 0, \frac{\pi}{4}, \frac{3\pi}{4}, \pi$

e.) $\text{Prob}(<\$6 \text{ after 5 days}) = \text{Prob}(\text{most movements are down})$

$= \text{Prob}(\text{up movements 2 or less})$

$= P(\leq 2)$

$= P(0) + P(1) + P(2)$

$= \left(\frac{2}{5}\right)^5 + {}^5C_1 \left(\frac{2}{5}\right)^4 \left(\frac{3}{5}\right)^1 + {}^5C_2 \left(\frac{2}{5}\right)^3 \left(\frac{3}{5}\right)^2$

$= \frac{992}{3125}$

$\therefore \frac{992}{3125}$ chance will be less than \$6 after 5 days.