NEWCASTLE GRAMMAR SCHOOL

Newcastle Grammar School

Student Number

2024 HSC TRIAL EXAMINATION

Year 12 **Extension 1 Mathematics**

Examiner: AF

READING TIME – 10 MINUTES

WORKING TIME – 2 HOURS

General Instructions

- Write using black pen.
- Section I answers are to be completed on the multiple choice answer sheet provided.
- Responses to Section II must be completed in the writing booklets provided, showing all relevant reasoning and calculations.
- Start each question in Section II in a new writing booklet.
- Write your student number clearly on each writing booklet and the multiple choice answer sheet.
- NESA-approved calculators may be used, unless stated otherwise.
- A NESA Reference Sheet is provided.
- All diagrams should be drawn in pencil.
- Do not remove this question paper from the examination room.

Section	Guidance	Marks Available	Your Score
SECTION I	• Type of Questions – Multiple Choice • Attempt Questions 1-10 • Allow about 15 mins for this section	10	
SECTION II	• Type of Questions – Short Answer and Extended Response • Attempt Questions 11-14 • Allow about 1 hr 45 mins for this section	60	
	TOTALS	70	

FINAL MARK	/ 70	%

Your examination paper begins overleaf.

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<u>SECTION I</u>

<u>Questions 1-10</u> (10 marks)

On the **multiple choice answer sheet provided**, shade in the best answer for each question.

1 Consider the function $f(x)=\cos\left(\frac{2x-1}{3}\right)$. Over which domain does an inverse function exist for f?

(A) $[0, \pi]$

(B) $\left[0, \frac{1+3\pi}{2}\right]$

(C) $\left[\frac{1-3\pi}{2}, \frac{1+3\pi}{2}\right]$

(D) $\left[\frac{1}{2}, \frac{1+3\pi}{2}\right]$

---

2 What is the minimum number of people required in a group so that four of them are guaranteed to share the same birth month?

(A) 13

(B) 36

(C) 37

(D) 366

---

3 Which of the following is a correct expression for $\int \cos^3 x\, dx$ using the substitution $u=\sin x$?

(A) $\sin x-\frac{\sin^3 x}{3}+C$

(B) $\sin x-\frac{\cos^3 x}{3}+C$

(C) $\cos x-\frac{\sin^3 x}{3}+C$

(D) $\frac{1}{2}\left[\sin x+\sin^2 x\right]+C$

4 Let X be a random variable that follows a Binomial Distribution with $E(X)=7$ and $Var(X)=6$. Which of the following is the probability of success, p?

(A) $\frac{36}{49}$

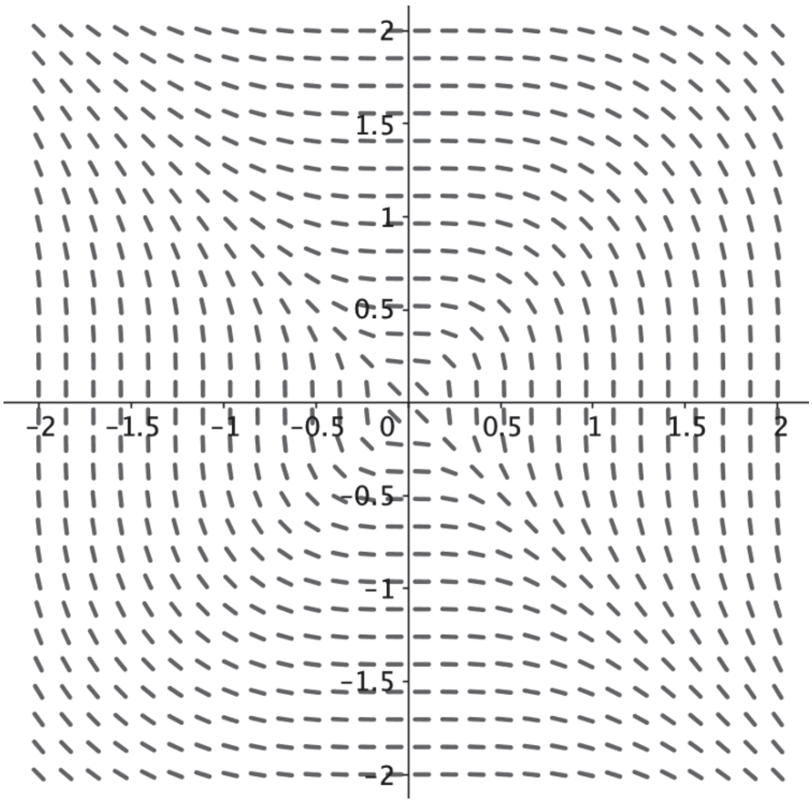
(B) $\frac{6}{7}$

(C) $\frac{1}{49}$

(D) $\frac{1}{7}$

---

5 Which of the following differential equations represents a correct solution to the slope field below?



(A) $\frac{dy}{dx}=\frac{-x}{y^2}$

(B) $\frac{dy}{dx}=\frac{y^2}{x^2}$

(C) $\frac{dy}{dx}=\frac{-x^2}{y^2}$

(D) $\frac{dy}{dx}=\frac{-y^2}{x}$

6 What is the vector projection of $\underset{\sim}{u}=\underset{\sim}{i}+4\underset{\sim}{j}$ onto $\underset{\sim}{v}=-4\underset{\sim}{i}+3\underset{\sim}{j}$?

(A) $\dfrac{-32}{17}\underset{\sim}{i}+\dfrac{24}{17}\underset{\sim}{j}$

(B) $\dfrac{8}{25}\underset{\sim}{i}+\dfrac{32}{25}\underset{\sim}{j}$

(C) $\dfrac{8}{17}\underset{\sim}{i}+\dfrac{32}{17}\underset{\sim}{j}$

(D) $\dfrac{-32}{25}\underset{\sim}{i}+\dfrac{24}{25}\underset{\sim}{j}$

---

7 If $\displaystyle\int_0^k \frac{dx}{\sqrt{4-9x^2}}=\frac{\pi}{6}$, what is the value of k ?

(A) $\dfrac{2}{3}$

(B) $\dfrac{\sqrt{3}}{2}$

(C) $\dfrac{1}{2}$

(D) 1

---

8 From a standard deck of cards, a card is drawn 30 times with replacement.
What is the probability that a black card is drawn 20 times?

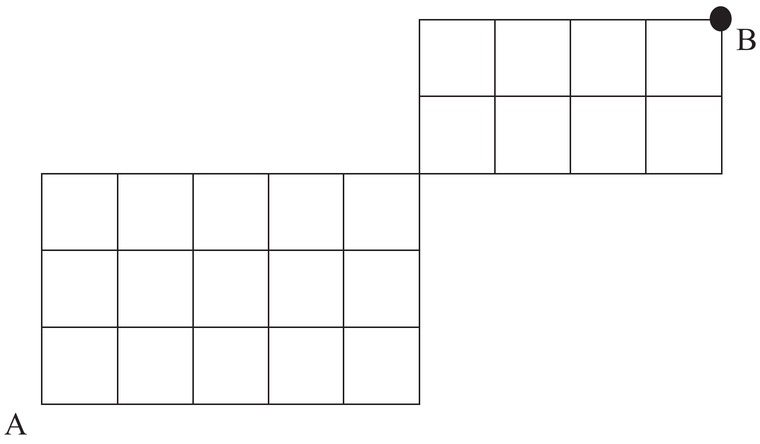
(A) $\dfrac{{}^{30}C_{20}}{(2!)^{20}}$

(B) $\dfrac{{}^{30}C_{20}}{2^{30}}$

(C) ${}^{30}C_{20}\left(\dfrac{1}{2}\right)^{-30}$

(D) $\dfrac{{}^{30}C_{20}}{2}$

9 Moving only right or up, how many different paths exist to get from Point A to Point B?



(A) 45

(B) 840

(C) 2002

(D) 14!

---

10 Brody and Ben are playing a badminton match where five games must be won in order to win the match. The probability that Brody wins (each game) is 0.9 and the probability that Ben wins is 0.1.

Which of the following is the approximate probability of Brody winning the match after exactly 7 games are played?

(A) 8.9%

(B) 9.8%

(C) 47.8%

(D) 12.4%

END OF SECTION I

SECTION II

Question 11 (15 marks) **START A NEW BOOKLET**

a) What is the range of the function $y = 2\sin x + \cos x - 1$? 2

[HINT: First consider $2\sin x + \cos x = R\sin(x+\alpha)$]

b) Show that $\cos\left(2\sin^{-1}\left(\frac{3}{8}\right)\right) = \frac{23}{32}$, including all working. 2

c) Consider the inequality:

$$\frac{x^2 - 6}{x} \le 1$$

The first line of Xanthe's solution is:

$$x^2 - 6 \le x$$

i) Explain why this is incorrect 1

ii) Solve $\frac{x^2-6}{x} \le 1$ 3

d) Find the Cartesian equation of the curve with the parametric equations: 2

$$\begin{cases} x = \dfrac{t}{1+t^2} \\ y = \dfrac{1}{1+t^2} \end{cases}$$

e) Evaluate $\int_3^{18} \frac{x}{\sqrt{x-2}}\,dx$ using the substitution $u = x - 2$ 3

f) Prove $\sin x + \sin 2x + \sin 3x = \sin 2x(2\cos x + 1)$. 2

END OF QUESTION 11

<u>Question 12</u> (15 marks) **START A NEW BOOKLET**

a) Consider the following vectors: $\underset{\sim}{u}=\begin{pmatrix}3\\7\end{pmatrix}$ and $\underset{\sim}{v}=\begin{pmatrix}5\\-6\end{pmatrix}$.

i) Find the size of the angle between vectors $\underset{\sim}{u}$ and $\underset{\sim}{v}$, correct to the nearest degree. 2

ii) **Use the answer sheet for Q12a)ii)** provided to graph the vectors $\underset{\sim}{u}$ and $\underset{\sim}{v}$, then sketch and label $proj_{\underset{\sim}{u}}\underset{\sim}{v}$ on the same diagram. 2

iii) Find $proj_{\underset{\sim}{u}}\underset{\sim}{v}$ 2

b) A polynomial $P(x)$ has a remainder of 4 when divided by $x+1$. Given that $P(2)=3$, find the remainder when $P(x)$ is divided by x^2-x-2. 2

c) Newton's law of cooling states that the rate of change of temperature of a body is proportional to the difference between the temperature of the body (T) and its surrounding air temperature (A). This is shown by the differential equation:

$$\frac{dT}{dt}=-k(T-A)$$

i) Show that $T=A+Be^{-kt}$, where B is a constant, satisfies Newton's law. 1

ii) A piece of hot metal has an initial temperature of 1350^oC. After 10 minutes in an environment at 25^oC its temperature has fallen to 720^oC. What time elapses for the metal to cool to 50^oC ? Answer correct to the nearest minute. 3

d) Evaluate $\displaystyle\int_0^{\frac{1}{4}} \frac{-3}{\sqrt{1-4x^2}}\,dx$ 3

END OF QUESTION 12

Question 13 (15 marks) **START A NEW BOOKLET**

a) The wind chill index, W, measures the apparent temperature $\left({}^{o}C\right)$ by taking into account the speed of the wind, v km/h in a location. A meteorologist suggests that the wind chill index in the region where his laboratory is located is given by the equation 3

$$W = 19.5 - 7.4v^{0.17}$$

Find the rate of change of W, given that the wind speed is 10 km/h and is increasing at a rate of 5 km/h per hour. Give your answer correct to two decimal places

b) i) Show that the function defined by 3

$$f(x) = \begin{cases} \dfrac{8}{\pi\left(1+4x^2\right)} & 0 \le x \le \dfrac{1}{2} \\ 0 & \text{otherwise} \end{cases}$$

is a probability density function.

ii) Determine the mode. 2

c) i) By considering $\sin(\sin^{-1}x + \cos^{-1}x)$, prove that 2

$$\sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}$$

ii) Sketch the curve $y = \sin^{-1}x + \cos^{-1}x + \tan^{-1}x - \dfrac{\pi}{2}$ 1

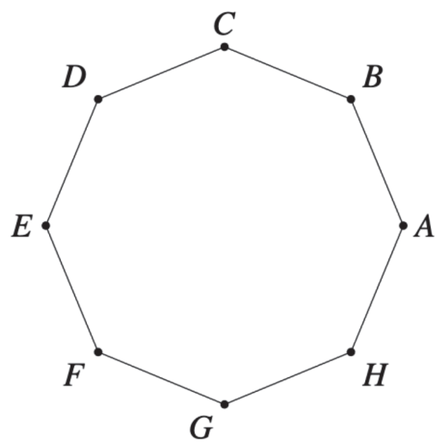
iii) Hence, or otherwise, evaluate $\displaystyle\int_{\frac{-\pi}{4}}^{\frac{\pi}{4}} \sin^{-1}x + \cos^{-1}x + \tan^{-1}x \, dx$ 2

QUESTION 13 CONTINUES ON NEXT PAGE

Question 13 (15 marks) **CONTINUED**

d) The diagram below shows how 8 buildings, A to G, are connected to form a network. Each edge represents a cable between two buildings. Each end of a cable is connected to an access point. The connection between each end of a cable and an access point has a 1.4% probability of being defective.

For the network to be operational, each building must be able to communicate with every other building in the network either directly or through another building(s).



Show that there is less than a 2% probability of the network not being operational. 2

END OF QUESTION 13

<u>Question 14</u> (15 marks) **START A NEW BOOKLET**

a) Solve the differential equation $\dfrac{dy}{dx}=\dfrac{\cos^2 y}{e^x}$, given that $y(0)=\dfrac{\pi}{4}$. 3

Express your answer in the form $y=f(x)$.

b) i) Use Mathematical Induction to prove that

$$\frac{1}{2^2-1}+\frac{1}{3^2-1}+\frac{1}{4^2-1}+\frac{1}{5^2-1}+\ .\ .\ .\ .\qquad +\frac{1}{n^2-1}=\frac{(n-1)(3n+2)}{4n(n+1)}$$ 3

for all integers $n\geq 2$.

ii) Hence, or otherwise, evaluate:

$$\frac{1}{4^2-1}+\frac{1}{5^2-1}+\frac{1}{6^2-1}+.\ .\ .\ .\qquad +\frac{1}{50^2-1}$$ 1

c) An online retailer claims that they ship 90% of orders within 24 hours of the order being lodged. An internal review is triggered if this rate falls below 87%.

A sample of 500 orders was taken in order to determine the sample proportion of orders that were shipped within 24 hours of lodgement. 3

Let $\hat{p}$ be the sample proportion of orders that were shipped within 24 hours.

Use the table of standard normal probabilities provided to approximate the probability that an internal review is triggered. Give your answer correct to four decimal places.

QUESTION 14 CONTINUES ON NEXT PAGE

<u>Question 14</u> (15 marks) **CONTINUED**

d) The points A, B and C have position vectors $\underset{\sim}{a}$, $\underset{\sim}{b}$ and $\underset{\sim}{c}$ respectively.
λ and μ are non-zero numbers such that $\lambda\underset{\sim}{a}+\mu\underset{\sim}{b}-\underset{\sim}{c}=0$ and $\lambda+\mu=1$.

i) Show that points A, B and C are collinear. 2

ii) It is known that $|\underset{\sim}{a}|=2$, the angle between $\underset{\sim}{a}$ and $\underset{\sim}{b}$ is acute, and the area of triangle OAB is k units^2.

Show that $\left(\underset{\sim}{a}\bullet\underset{\sim}{b}\right)^2=4\left(|\underset{\sim}{b}|^2-k^2\right)$ 3

END OF QUESTION 14

END OF EXAMINATION

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SOLUTIONS

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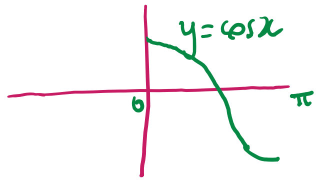


(A) $[0, \pi]$

(B) $\left[0, \frac{1+3\pi}{2}\right]$

(C) $\left[\frac{1-3\pi}{2}, \frac{1+3\pi}{2}\right]$

(D) $\left[\frac{1}{2}, \frac{1+3\pi}{2}\right]$

$0 \leq \frac{2x-1}{3} \leq \pi$

$0 \leq 2x-1 \leq 3\pi$

$\frac{1}{2} \leq x \leq \frac{3\pi+1}{2}$

2 What is the minimum number of people required in a group so that four of them are guaranteed to share the same birth month?

(A) 13

(B) 36

(C) 37

(D) 366

$3 \times 12 + 1$

3 Which of the following is a correct expression for $\int \cos^3 x\, dx$ using the substitution $u = \sin x$?

(A) $\sin x - \dfrac{\sin^3 x}{3} + C$

(B) $\sin x - \dfrac{\cos^3 x}{3} + C$

(C) $\cos x - \dfrac{\sin^3 x}{3} + C$

(D) $\dfrac{1}{2}\left[\sin x + \sin^2 x\right] + C$

$= \int \cos^2 x \times \cos x\, dx$

$= \int (1 - \sin^2 x) \cos x\, dx$

$= \int 1 - u^2\, du$

$= u - \dfrac{u^3}{3} + C$

$= \sin x - \dfrac{\sin^3 x}{3} + C$

$u = \sin x$

$du = \cos x\, dx$

4 Let X be a random variable that follows a Binomial Distribution with $E(X) = 7$ and $Var(X) = 6$. Which of the following is the probability of success, p?

(A) $\dfrac{36}{49}$

(B) $\dfrac{6}{7}$

(C) $\dfrac{1}{49}$

(D) $\dfrac{1}{7}$

$np = 7 \quad -(1)$

$npq = 6 \quad -(2)$

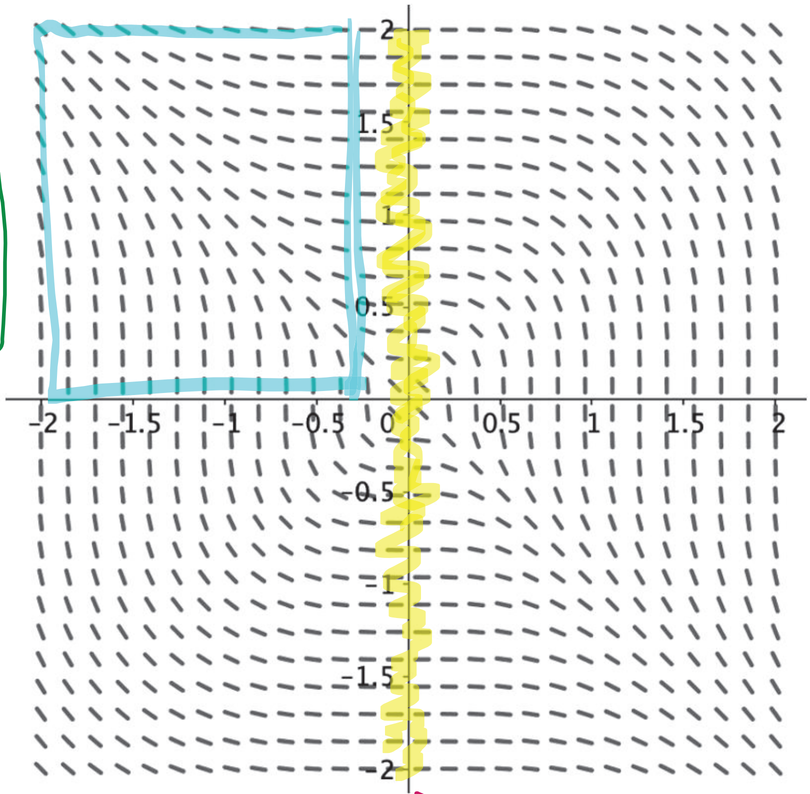
$\dfrac{(2)}{(1)} = \dfrac{\cancel{np}q}{\cancel{np}} = \dfrac{6}{7}$

$q = \dfrac{6}{7}$

$\therefore p = \dfrac{1}{7}$

5 Which of the following differential equations represents a correct solution to the slope field below?

$x < 0$
$y > 0$
$\frac{dy}{dx} < 0$



$\frac{dy}{dx} = 0$ @ $x = 0$

(A) $\frac{dy}{dx} = \frac{-x}{y^2}$

(B) $\frac{dy}{dx} = \frac{y^2}{x^2}$

(C) $\frac{dy}{dx} = \frac{-x^2}{y^2}$

(D) $\frac{dy}{dx} = \frac{-y^2}{x}$

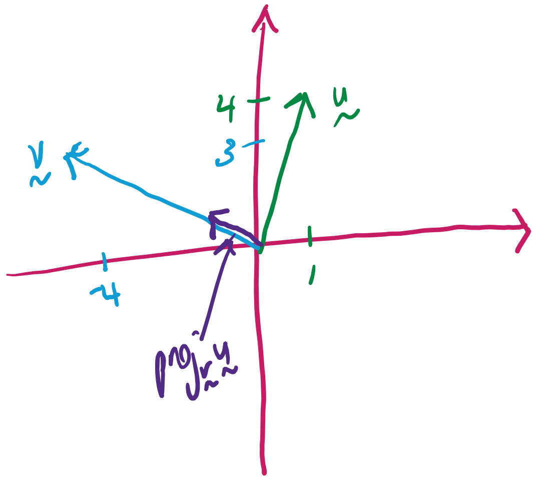
6 What is the vector projection of $\underset{\sim}{u} = \underset{\sim}{i} + 4\underset{\sim}{j}$ onto $\underset{\sim}{v} = -4\underset{\sim}{i} + 3\underset{\sim}{j}$?

(A) $\dfrac{-32}{17}\underset{\sim}{i} + \dfrac{24}{17}\underset{\sim}{j}$

(B) $\dfrac{8}{25}\underset{\sim}{i} + \dfrac{32}{25}\underset{\sim}{j}$ ✗

(C) $\dfrac{8}{17}\underset{\sim}{i} + \dfrac{32}{17}\underset{\sim}{j}$ ✗

(D) $\dfrac{-32}{25}\underset{\sim}{i} + \dfrac{24}{25}\underset{\sim}{j}$



$$\text{proj}_{\underset{\sim}{v}}\underset{\sim}{u} = (\underset{\sim}{u} \cdot \hat{\underset{\sim}{v}})\,\hat{v}$$
$$= \frac{8}{5} \times \frac{1}{5}\begin{pmatrix} -4 \\ 3 \end{pmatrix}$$
$$= \frac{8}{25}\begin{pmatrix} -4 \\ 3 \end{pmatrix}$$

$$\underset{\sim}{u} \cdot \hat{\underset{\sim}{v}}$$
$$= \frac{1 \times -4 + 4 \times 3}{\sqrt{4^2 + 3^2}}$$
$$= \frac{8}{5}$$

7 If $\displaystyle\int_0^k \frac{dx}{\sqrt{4 - 9x^2}} = \frac{\pi}{6}$, what is the value of k ?

(A) $\dfrac{2}{3}$

(B) $\dfrac{\sqrt{3}}{2}$

(C) $\dfrac{1}{2}$

(D) 1

$$\frac{1}{3}\int_0^k \frac{3}{\sqrt{2^2 - (3x)^2}}\,dx = \frac{\pi}{6}$$
$$\frac{1}{3}\left[\sin^{-1}\left(\frac{3x}{2}\right)\right]_0^k = \frac{\pi}{6}$$
$$\left[\sin^{-1}\left(\frac{3k}{2}\right) - \sin^{-1}(0)\right] = \frac{\pi}{2}$$
$$\sin^{-1}\left(\frac{3k}{2}\right) = \frac{\pi}{2}$$
$$\frac{3k}{2} = 1 \Rightarrow k = \frac{2}{3}$$

8 From a standard deck of cards, a card is drawn 30 times with replacement. What is the probability that a black card is drawn 20 times?

$X \sim Bin\left(30, \frac{1}{2}\right)$

(A) $\dfrac{{}^{30}C_{20}}{(2!)^{20}}$

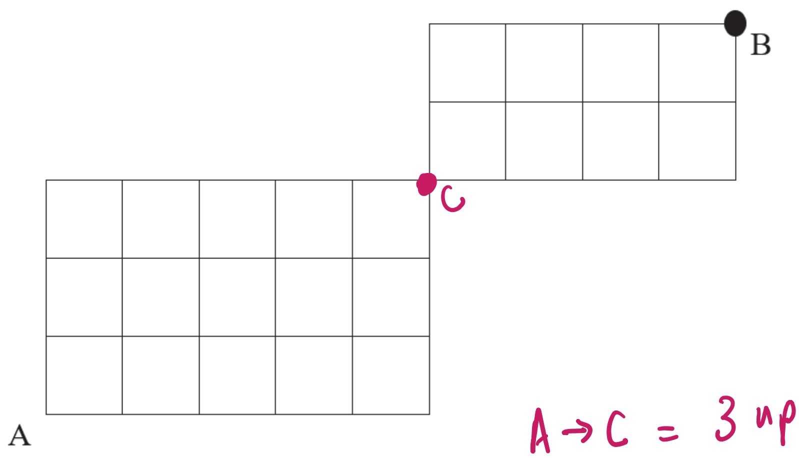
(B) $\dfrac{{}^{30}C_{20}}{2^{30}}$

(C) ${}^{30}C_{20}\left(\dfrac{1}{2}\right)^{-30}$

(D) $\dfrac{{}^{30}C_{20}}{2}$

$$P(x=20) = {}^{30}C_{20}\left(\frac{1}{2}\right)^{20}\left(\frac{1}{2}\right)^{10}$$
$$= {}^{30}C_{20}\left(\frac{1}{2}\right)^{30}$$
$$= \frac{{}^{30}C_{20}}{2^{30}}$$

9 Moving only right or up, how many different paths exist to get from Point A to Point B?



(A) 45

(B) 840

(C) 2002

(D) 14!

$A \to C = 3$ up, 5 right

\# ways $= {}^{8}C_{3}$

$C \to B = 2$ up, 4 right

\# ways $= {}^{6}C_{2}$

Total $= {}^{8}C_{3} \times {}^{6}C_{2} = 840$

10 Brody and Ben are playing a badminton match where five games must be won in order to win the match. The probability that Brody wins (each game) is 0.9 and the probability that Ben wins is 0.1.

Which of the following is the approximate probability of Brody winning the match after exactly 7 games are played?

(A) 8.9%

(B) 9.8%

(C) 47.8%

(D) 12.4%

Brody must win 4 of the first 6 games and then the 7th

$$p = \left[{}^{6}C_{4}\,(0.9)^{4}(0.1)^{2}\right] \times (0.9)$$

$$= 0.0885735$$

END OF SECTION I

SECTION II

Question 11 (15 marks) **START A NEW BOOKLET**

a) What is the range of the function $y = 2\sin x + \cos x - 1$? 2

[HINT: First consider $2\sin x + \cos x = R\sin(x+\alpha)$]

$$y = \sqrt{2^2+1^2}\,\sin(x+\alpha) - 1$$
$$= \sqrt{5}\sin(x+\alpha) - 1$$

1 mark - writing $\sqrt{5}\sin(x+\alpha)$

Range: $\boxed{y \in [-\sqrt{5}-1, \sqrt{5}-1]}$

1 mark - correct range

b) Show that $\cos\left(2\sin^{-1}\left(\frac{3}{8}\right)\right) = \frac{23}{32}$, including all working. 2

let $\alpha = \sin^{-1}\frac{3}{8}$

$\Rightarrow \sin\alpha = \frac{3}{8}$

$\frac{1}{2}$ mark

$\cos 2\alpha = 1 - 2\sin^2\alpha$

$\frac{1}{2}$ mark

$$= 1 - 2\left(\frac{3}{8}\right)^2$$
$$= 1 - 2\times\frac{9}{64}$$
$$= 1 - \frac{18}{64}$$
$$= \frac{46}{64}$$
$$= \boxed{\frac{23}{32}}$$

1 mark (shows substitution)

c) Consider the inequality:

$$\frac{x^2-6}{x} \leq 1$$

The first line of Xanthe's solution is:

$$x^2 - 6 \leq x$$

i) Explain why this is incorrect 1

ii) Solve $\dfrac{x^2-6}{x} \leq 1$ 2

i) x is not always positive. (consider $x<0$). multiplying both sides of an inequality by a negative number means the direction of the inequality has to change.. Xanthe should consider separate positive $(x>0)$ and negative $(x<0)$ cases if using this method.

1 mark

ii) $x^2 \times \dfrac{x^2-6}{x} \leq 1 \times x^2$ $\quad \boxed{x \neq 0}$ ← $\frac{1}{2}$ mark (either stated explicitly or incorporated in final answer)

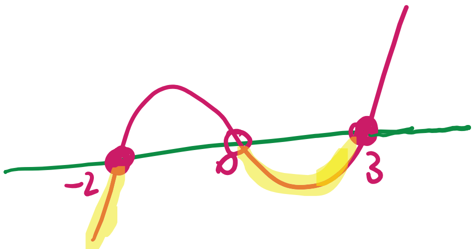
$x(x^2-6) \leq x^2$] $\frac{1}{2}$ mark (multiply by x^2 and simplifies)

$x^3 - x^2 - 6x \leq 0$

$x(x^2 - x - 6) \leq 0$

$x(x-3)(x+2) \leq 0$

1 mark (rearrange & factorise)



$$\boxed{x \in (-\infty, -2] \cup (0, 3]}$$

1 mark (interpret inequality from factorised polynomial)

d) Find the Cartesian equation of the curve with the parametric equations: 2

$$\begin{cases} x = \dfrac{t}{1+t^2} \\ y = \dfrac{1}{1+t^2} \end{cases}$$

can't isolate t in either equation

1 mark (squaring x & y)

$$x^2 = \frac{t^2}{(1+t^2)^2} \quad \text{— ①} \qquad y^2 = \frac{1}{(1+t^2)^2} \quad \text{— ②}$$

① + ② :

$$x^2 + y^2 = \frac{\cancel{t^2+1}}{(1+t^2)^{\cancel{2}}}$$

$$x^2 + y^2 = \frac{1}{1+t^2}$$

$$\boxed{x^2 + y^2 = y}$$

1 mark (fully eliminating t)*

*only $\frac{1}{2}$ mark if $\pm$ is just ignored

OR

$$x^2 + y^2 - y + \frac{1}{4} = \frac{1}{4}$$

$$\boxed{x^2 + \left(y - \tfrac{1}{2}\right)^2 = \left(\tfrac{1}{2}\right)^2}$$

Do not penalise if not re-arranged into $(x-h)^2 + (y-k)^2 = r^2$ form

e) Evaluate $\int_{3}^{18} \frac{x}{\sqrt{x-2}}\,dx$ using the substitution $u=x-2$

$u = x-2 \Rightarrow x = u+2$

$du = dx$

$x=3 \rightarrow u=1$

$x=18 \rightarrow u=16$

1 mark - find dx
- change bounds

$= \int_{1}^{16} \frac{u+2}{\sqrt{u}}\,du$

$= \int_{1}^{16} u^{\frac{1}{2}} + 2u^{-\frac{1}{2}}\,du$

1 mark (integral in terms of u, simplified index form)

$= \left[\frac{2}{3}u^{\frac{3}{2}} + 2\times 2u^{\frac{1}{2}}\right]_1^{16}$

$\frac{1}{2}$ mark (integration)

$\frac{1}{2}$ mark

$= \left[\frac{2}{3}\sqrt{u^3} + 4\sqrt{u}\right]_1^{16}$

$= \left(\frac{2}{3}\sqrt{16^3} + 4\sqrt{16}\right) - \left(\frac{2}{3}\sqrt{1^3} + 4\sqrt{1}\right)$

$= \boxed{54}$

f) Prove $\sin x + \sin 2x + \sin 3x = \sin 2x(2\cos x + 1)$. 2

LHS $= \sin x + \sin 2x + \sin 3x$

$= \sin 3x + \sin x + \sin 2x$

$= [\sin(2x+x) + \sin(2x-x)] + \sin 2x$ → using sums to products result [NESA ref. sheet]

$= 2\sin 2x \cos x + \sin 2x$

1 mark (shows sufficient detail)

$= \sin 2x(2\cos x + 1)$

$=$ RHS

1 mark (factorise)

END OF QUESTION 11

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a) Consider the following vectors: $\underset{\sim}{u} = \begin{pmatrix} 3 \\ 7 \end{pmatrix}$ and $\underset{\sim}{v} = \begin{pmatrix} 5 \\ -6 \end{pmatrix}$.

i) Find the size of the angle between vectors $\underset{\sim}{u}$ and $\underset{\sim}{v}$, correct to the nearest degree. 2

ii) **Use the answer sheet for Q12a)ii)** provided to graph the vectors $\underset{\sim}{u}$ and $\underset{\sim}{v}$, then sketch and label $proj_{\underset{\sim}{u}}\underset{\sim}{v}$ on the same diagram. 2

iii) Find $proj_{\underset{\sim}{u}}\underset{\sim}{v}$ 2

i)

$\frac{1}{2}$ mark for $\underset{\sim}{u} \cdot \underset{\sim}{v}$

$$\underset{\sim}{u} \cdot \underset{\sim}{v} = |\underset{\sim}{u}||\underset{\sim}{v}| \cos\theta$$

$\frac{1}{2}$ mark for $|\underset{\sim}{u}|, |\underset{\sim}{v}|$

$$\cos\theta = \frac{3 \times 5 + 7 \times -6}{\sqrt{3^2+7^2} \times \sqrt{5^2+6^2}} = \frac{-27}{\sqrt{58 \times 61}}$$

$\frac{1}{2}$ for subst. into correct formula

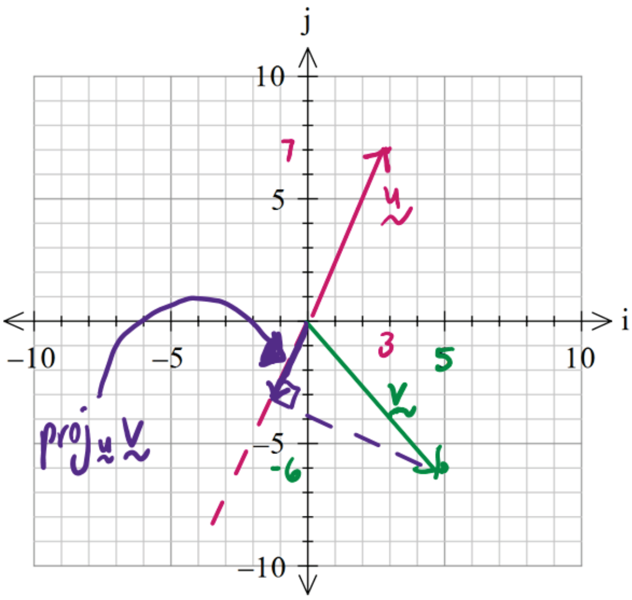
$$\theta = \cos^{-1}\left(\frac{-27}{\sqrt{3538}}\right)$$

$$\theta = 116^\circ 59' 45.02''$$

$$\boxed{\theta \approx 117^\circ} \quad (\text{nd})$$

$\frac{1}{2}$ mark

ii)



1 mark for $\underset{\sim}{u}$ and $\underset{\sim}{v}$

1 mark for projection (showing right L)

iii)

1 mark - Correct subst. into correct formula

$$\text{proj}_{\underset{\sim}{u}} \underset{\sim}{v} = \left(\underset{\sim}{v} \cdot \hat{\underset{\sim}{u}}\right) \hat{\underset{\sim}{u}}$$

$$= \frac{-27}{\sqrt{58}} \times \frac{1}{\sqrt{58}} \begin{pmatrix} 3 \\ 7 \end{pmatrix}$$

1 mark - correct simplification

$$= \frac{-27}{58} \begin{pmatrix} 3 \\ 7 \end{pmatrix}$$

OR $\boxed{-\frac{81}{58}\underset{\sim}{i} - \frac{189}{58}\underset{\sim}{j}}$

$\underset{\sim}{u} = \begin{pmatrix} 3 \\ 7 \end{pmatrix}$ $\underset{\sim}{v} = \begin{pmatrix} 5 \\ -6 \end{pmatrix}$

$$\underset{\sim}{v} \cdot \hat{\underset{\sim}{u}} = \frac{3 \times 5 + 7 \times -6}{\sqrt{3^2 + 7^2}} = \frac{-27}{\sqrt{58}}$$

b) A polynomial $P(x)$ has a remainder of 4 when divided by $x+1$. Given that $P(2)=3$, find the remainder when $P(x)$ is divided by $x^2 - x - 2$. 2

$\frac{1}{2}$ mark correct eqn for $P(x)$ using linear remainder

$$P(x) = (x^2 - x - 2)\,Q(x) + ax + b$$
$$= (x-2)(x+1)\,Q(x) + ax + b$$

$\frac{1}{2}$ mark correct use of $P(-1) = 4$

$P(-1) = 4$:
$$4 = \cancel{(-1-2)(-1+1)Q(-1)} + a(-1) + b$$
$$4 = -a + b$$
$$a = b - 4 \quad \text{— (1)}$$

$\frac{1}{2}$ mark correct use of $P(2) = 3$

$P(2) = 3$:
$$3 = \cancel{(2-2)(2+1)Q(2)} + a(2) + b$$
$$3 = 2a + b \quad \text{— (2)}$$

Sub (1) in (2): $3 = 2(b-4) + b$
$$3 = 3b - 8$$
$$b = 11/3$$

Sub in (1): $a = \frac{11}{3} - 4 = -\frac{1}{3}$

$\frac{1}{2}$ mark

$$\therefore \boxed{R(x) = -\frac{1}{3}x + \frac{11}{3}}$$

c) Newton's law of cooling states that the rate of change of temperature of a body is proportional to the difference between the temperature of the body (T) and its surrounding air temperature (A). This is shown by the differential equation:

$$\frac{dT}{dt} = -k(T - A)$$

i) Show that $T = A + Be^{-kt}$, where B is a constant, satisfies Newton's law. 1

ii) A piece of hot metal has an initial temperature of $1350^{o}C$. After 10 minutes in an environment at $25^{o}C$ its temperature has fallen to $720^{o}C$. What time elapses for the metal to cool to $50^{o}C$? Answer correct to the nearest minute. 3

i) $\frac{dT}{dt} = -kBe^{-kt}$

$= -k(A + Be^{-kt} - A)$

$= -k(T - A)$ #

1 mark [shows differentiation and manipulation]

ii) $1350 = 25 + Be^{0}$ $(A = 25)$

$B = 1325$ → ½ mark

$T = 25 + 1325e^{-kt}$

$720 = 25 + 1325e^{-10k}$ — $\frac{1}{2}$ mark

$\frac{695}{1325} = e^{-10k}$

$k = -\frac{1}{10}\ln\left(\frac{695}{1325}\right)$ → ½ mark

@ T=50 : $50 = 25 + 1325e^{-kt}$ → ½ mar

$\frac{25}{1325} = e^{-kt}$

$$t = -\frac{1}{k}\ln\left(\frac{25}{1325}\right)$$

1 mark

$$= \frac{-1}{-\frac{1}{10}\ln\left(\frac{695}{1325}\right)} \times \ln\left(\frac{25}{1325}\right)$$

$$= \frac{10}{\ln\left(\frac{695}{1325}\right)} \times \ln\left(\frac{25}{1325}\right)$$

$$= 61.5305\ldots$$

$$\approx \boxed{\text{62 mins (n. min) to cool to } 50^\circ C}$$

d) Evaluate $\displaystyle\int_0^{\frac{1}{4}} \frac{-3}{\sqrt{1-4x^2}}\,dx$ 3

$$= -\frac{3}{2}\int_0^{\frac{1}{4}} \frac{2}{\sqrt{1^2-(2x)^2}}\,dx$$

1 mark; correct use of chain rule

$$= -\frac{3}{2}\Big[\sin^{-1}(2x)\Big]_0^{1/4}$$

1 mark for correct use of inverse sine

$$= -\frac{3}{2}\left[\sin^{-1}\left(2\times\frac{1}{4}\right) - \sin^{-1}(2\times 0)\right]$$

½ mark

$$= -\frac{3}{2}\left[\sin^{-1}\frac{1}{2} - \sin^{-1}0\right] = \underbrace{-\frac{3}{2}\times\frac{\pi}{6} = \boxed{-\frac{\pi}{4}}}_{\text{½ mark}}$$

END OF QUESTION 12

Question 13 (15 marks) **START A NEW BOOKLET**

a) The wind chill index, W, measures the apparent temperature $\left({}^{o}C\right)$ by taking into account the speed of the wind, v km/h in a location. A meteorologist suggests that the wind chill index in the region where his laboratory is located is given by the equation

$$W = 19.5 - 7.4v^{0.17}$$ 3

Find the rate of change of W, given that the wind speed is 10 km/h and is increasing at a rate of 5 km/h per hour. Give your answer correct to two decimal places.

$\underbrace{\frac{dv}{dt} = 5 \; ; \; v = 10}_{\text{½ mark}}$

$W = 19.5 - 7.4v^{0.17}$

$\frac{dW}{dv} = \underbrace{-7.4 \times 0.17v^{-0.83}}_{\text{1 mark}}$

@ $v = 10$: ½ mark

$\frac{dW}{dv} = -7.4 \times 0.17(10)^{-0.83}$

$= -1.258 \times 10^{-0.83}$

$\frac{dW}{dt} = \frac{dW}{dv} \times \frac{dv}{dt}$ ½ mark

$= \left[-1.258 \times 10^{-0.83}\right] \times 5$

$= -0.9303591762$

≈ -0.93 (2dp)

½ mark

$\therefore$ The wind chill factor is decreasing by $\boxed{0.93°c/h}$ when the wind speed is 10 km/h. & increasing at a rate of 5 km/h

b) i) Show that the function defined by

$$f(x)=\begin{cases} \dfrac{8}{\pi\left(1+4x^2\right)} & 0\le x\le \dfrac{1}{2} \\ 0 & \text{otherwise} \end{cases}$$ 3

is a probability density function.

ii) Determine the mode. 2

1 mark

i) Given $0 \le x \le \frac{1}{2}$, $\frac{8}{\pi(1+4x^2)} \ge 0$

$\boxed{f(x) \geq 0}$

$\frac{1}{2}$ mark $\int_0^{1/2} \frac{8}{\pi(1+4x^2)}\,dx$

$\boxed{\int_0^{1/2} f(x)\,dx = 1}$

$\therefore f(x)$ is a PDF.

$= \frac{4}{\pi}\int_0^{1/2} \frac{2}{1^2+(2x)^2}\,dx$

1 mark (integration) $= \frac{4}{\pi} \times \left[\frac{1}{1}\tan^{-1}\frac{2x}{1}\right]_0^{1/2}$

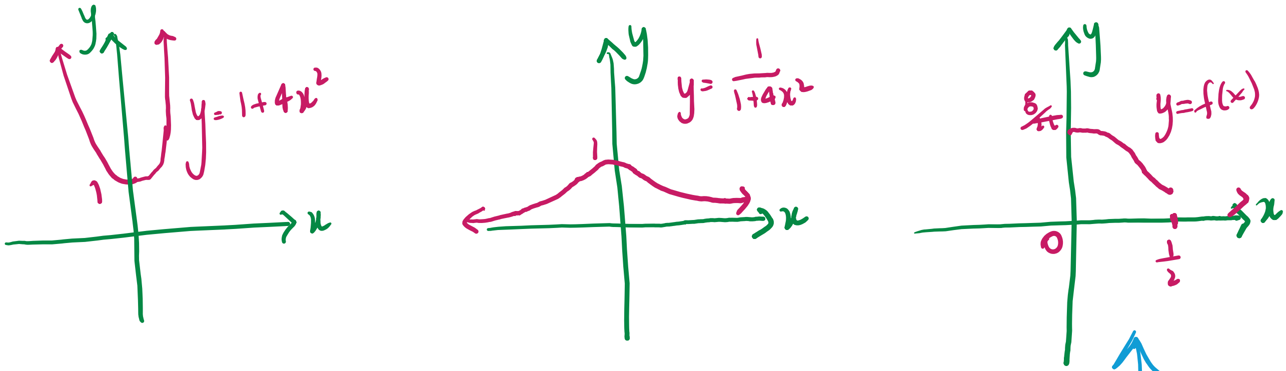
$\frac{1}{2}$ mark (substitution & simplification)

$= \frac{4}{\pi}\left[\tan^{-1} 2\left(\tfrac{1}{2}\right) - \tan^{-1} 2(0)\right]$

$= \frac{4}{\pi}\left[\tan^{-1} 1 - \tan^{-1} 0\right]$

$= \frac{4}{\pi} \times \left[\frac{\pi}{4} - 0\right] = 1$

ii) $f(x) = \dfrac{8}{\pi(1+4x^2)}$ $\quad 0 \le x \le \frac{1}{2}$



$\boxed{\text{Mode} = 0}$ (see graph of $y=f(x)$)

1 mark (must show some supporting evidence)

1 mark (or equivalent merit) to show max (not min)

c) i) By considering $\sin(\sin^{-1}x+\cos^{-1}x)$, prove that 2

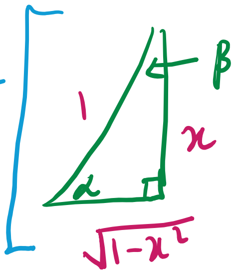
$$\sin^{-1}x+\cos^{-1}x=\frac{\pi}{2}$$

ii) Sketch the curve $y=\sin^{-1}x+\cos^{-1}x+\tan^{-1}x-\dfrac{\pi}{2}$ 1

iii) Hence, or otherwise, evaluate $\displaystyle\int_{\frac{-\pi}{4}}^{\frac{\pi}{4}} \sin^{-1}x+\cos^{-1}x+\tan^{-1}x\,dx$ 2

i) Let $\sin^{-1}x = \alpha$ $\quad \cos^{-1}x = \beta$

$\frac{1}{2}$ mark



$\sin(\sin^{-1}x + \cos^{-1}x)$

$= \sin(\alpha+\beta)$

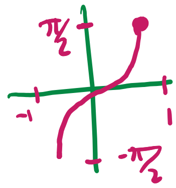
$= \sin\alpha\cos\beta + \cos\alpha\sin\beta$] $\frac{1}{2}$ mark

$\frac{1}{2}$ mark $\rightarrow$ $= \underbrace{\frac{x}{1}\times\frac{x}{1} + \frac{\sqrt{1-x^2}}{1}\times\frac{\sqrt{1-x^2}}{1}}$ $= x^2 + (1-x^2)$

$= 1$

$\therefore \sin(\sin^{-1}x + \cos^{-1}x) = 1$

$\therefore \sin^{-1}x + \cos^{-1}x = \underbrace{\sin^{-1}1}_{\frac{1}{2}\text{ mark}}$



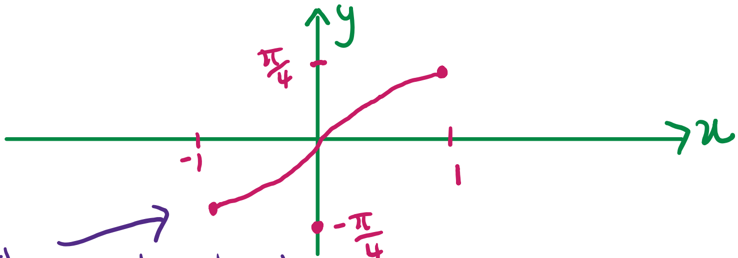
$$\boxed{\sin^{-1}x + \cos^{-1}x = \pi/2} \quad \#$$

ii) $y = \underbrace{\sin^{-1}x + \cos^{-1}x} + \tan^{-1}x - \pi/2$

$= \pi/2 + \tan^{-1}x - \pi/2$

$y = \tan^{-1}x$

NB: Domain is restricted to $x \in [-1, 1]$ due to restrictions on $\sin^{-1}x$, $\cos^{-1}x$



$y = \sin^{-1}x + \cos^{-1}x + \tan^{-1}x - \pi/2$

1 mark
(−½ if domain not restricted)

iii) $\displaystyle\int_{-\pi/4}^{\pi/4} \underbrace{\sin^{-1}x + \cos^{-1}x} + \tan^{-1}x \, dx$

½ mark (use i))

$= \displaystyle\int_{-\pi/4}^{\pi/4} \frac{\pi}{2}\,dx + \int_{-\pi/4}^{\pi/4} \tan^{-1}x\,dx$

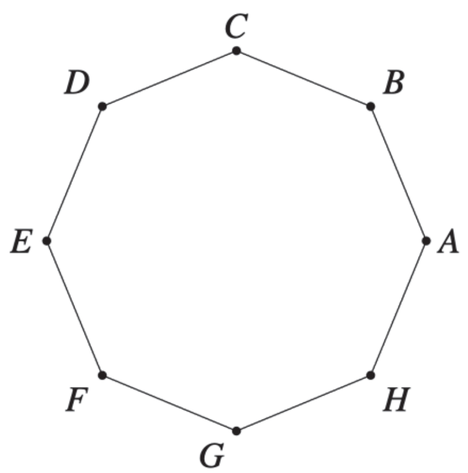
½ mark (subst/simpl)

½ mark ← ½ mark

$= \left[\frac{\pi}{2}x\right]_{-\pi/4}^{\pi/4} + \underset{\text{(odd function)}}{0} = \frac{\pi}{2}\left[\frac{\pi}{4} - \left(-\frac{\pi}{4}\right)\right] = \frac{\pi}{2} \times \frac{\pi}{2} = \boxed{\frac{\pi^2}{4}}$

d) The diagram below shows how 8 buildings, A to G, are connected to form a network. Each edge represents a cable between two buildings. Each end of a cable is connected to an access point. The connection between each end of a cable and an access point has a 1.4% probability of being defective.

For the network to be operational, each building must be able to communicate with every other building in the network either directly or through another building(s).



Show that there is less than a 2% probability of the network not being operational. 2

½ mark (realise that at least 2 must be defective)

For the network to not be operational, at least 2 access points must be defective (otherwise the signal could get through via the opposite direction from any building to any other). However, if the 2 access points are on the same cable, the signal can still get through. ½ mark (complement)

$$P(\text{defective}) = 1 - [P(X=0) + P(X=1) + P(2 \text{ on same cable})]$$

1 mark (correct exp. or equivalent ment)

$$= 1 - [{}^{16}C_0 (0.014)^0 (0.986)^{16} + {}^{16}C_1 (0.014)^1 (0.986)^{15} + {}^{8}C_1 (0.014)^2 (0.986)^{14}]$$

$$= 0.019360 \ldots$$

$$\leq 0.02 \ \#$$

END OF QUESTION 13

<u>Question 14</u> (15 marks) **START A NEW BOOKLET**

a) Solve the differential equation $\frac{dy}{dx} = \frac{\cos^2 y}{e^x}$, given that $y(0) = \frac{\pi}{4}$. **3**

Express your answer in the form $y = f(x)$.

$$\int \frac{dy}{\cos^2 y} = \int \frac{dx}{e^x}$$

1 mark (separation of variables)

$\frac{1}{2}$ mark

$$\int_{\frac{\pi}{4}}^{y} \sec^2 y \, dy = \int_0^x e^{-x} \, dx$$

$\frac{1}{2}$ mark

$$\left[\tan y\right]_{\frac{\pi}{4}}^{y} = \left[-e^{-x}\right]_0^x$$

$$\tan y - \tan\tfrac{\pi}{4} = -\left[e^{-x} - e^{0}\right]$$

$$\tan y - 1 = -e^{-x} + 1$$

$$\tan y = 2 - e^{-x}$$

$$\boxed{y = \tan^{-1}\left(2 - e^{-x}\right)}$$

1 mark

b) i) Use Mathematical Induction to prove that

$$\frac{1}{2^2-1}+\frac{1}{3^2-1}+\frac{1}{4^2-1}+\frac{1}{5^2-1}+\ .\ .\ .\ . \qquad +\frac{1}{n^2-1}=\frac{(n-1)(3n+2)}{4n(n+1)}$$ 3

for all integers $n \geq 2$.

ii) Hence, or otherwise, evaluate:

$$\frac{1}{4^2-1}+\frac{1}{5^2-1}+\frac{1}{6^2-1}+.\ .\ .\ . \qquad +\frac{1}{50^2-1}$$ 1

i) $P(n)$: $\frac{1}{2^2-1}+\frac{1}{3^2-1}+\frac{1}{4^2-1}+\cdots+\frac{1}{n^2-1}=\frac{(n-1)(3n+2)}{4n(n+1)}$

$\forall n \in \mathbb{Z} : n \geqslant 2$

1 mark

Test $P(2)$: LHS $= \frac{1}{2^2-1}$ $\qquad$ RHS $= \frac{(2-1)(3(2)+2)}{4(2)(2+1)}$

$= \frac{1}{4-1}$ $\qquad$ $= \frac{1\times \cancel{8}}{\cancel{4\times2}\times3} = \frac{1}{3}$

$= \frac{1}{3}$

$\therefore P(2)$ holds

Assume $P(k)$ is true:

i.e. $\frac{1}{2^2-1}+\frac{1}{3^2-1}+\frac{1}{4^2-1}+\cdots+\frac{1}{k^2-1}=\frac{(k-1)(3k+2)}{4k(k+1)}$

Prove $P(k+1)$ is true:

R.T.P:

$\frac{1}{2^2-1}+\frac{1}{3^2-1}+\frac{1}{4^2-1}+\cdots+\frac{1}{k^2-1}+\frac{1}{(k+1)^2-1}=\frac{(k+1-1)(3(k+1)+2)}{4(k+1)(k+1+1)}$

$= \frac{k(3k+5)}{4(k+1)(k+2)}$

$$LHS = \frac{(k-1)(3k+2)}{4k(k+1)} + \frac{1}{(k+1)^2 - 1}$$

1 mark from assumption

$$= \frac{(k-1)(3k+2)}{4k(k+1)} + \frac{1}{k^2 + 2k + \cancel{1} \cancel{-1}}$$

$$= \frac{(k-1)(3k+2)}{4k(k+1)} + \frac{1}{k(k+2)}$$

$$= \frac{(k-1)(3k+2)(k+2) + 4(k+1)}{4k(k+1)(k+2)}$$

$\frac{1}{2}$ mark (correct common denom.)

$$= \frac{(3k^2-k-2)(k+2) + 4k + 4}{4k(k+1)(k+2)}$$

$$= \frac{3k^3 + 5k^2 \cancel{-4k} \cancel{-4} \cancel{+4k} \cancel{+4}}{4k(k+1)(k+2)}$$

$\frac{1}{2}$ mark (shows = RHS)

$$= \frac{k^{\cancel{2}}(3k+5)}{4\cancel{k}(k+1)(k+2)} = \frac{k(3k+5)}{4(k+1)(k+2)} = RHS$$

Conclusion: Therefore $p(n)$ is true $\forall n \in \mathbb{Z} : n \geq 2$ by mathematical induction.

ii) $\frac{1}{4^2-1}+\frac{1}{5^2-1}+\frac{1}{6^2-1}+\cdots+\frac{1}{50^2-1}$

$$= \overbrace{\frac{(50-1)(3(50)+2)}{4(50)(50+1)}}^{\frac{1}{2}\text{ mark}} - \overbrace{\left[\frac{1}{2^2-1}+\frac{1}{3^2-1}\right]}^{\frac{1}{2}\text{ mark}}$$

$$= \frac{931}{1275} - \left(\frac{1}{3}+\frac{1}{8}\right) = \boxed{\frac{2773}{10200}}$$

c) An online retailer claims that they ship 90% of orders within 24 hours of the order being lodged. An internal review is triggered if this rate falls below 87%.

A sample of 500 orders was taken in order to determine the sample proportion of orders that were shipped within 24 hours of lodgement. 3

Let $\hat{p}$ be the sample proportion of orders that were shipped within 24 hours.

Use the table of standard normal probabilities provided to approximate the probability that an internal review is triggered. Give your answer correct to four decimal places.

$p = 0.9$ $\quad n = 500$ $\quad \underbrace{\hat{p} = p = 0.9}$ $\frac{1}{2}$ mark

$$\sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{0.9(0.1)}{500}} = \underbrace{\sqrt{0.00018}}_{\frac{1}{2}\text{ mark}}$$

For $x = 0.87$, $z = \overbrace{\frac{0.87-0.9}{\sqrt{0.00018}}}^{1\text{ mark}} = -2.24$ (2dp)

1 mark

$$P(z<-2.24) = 1 - P(z<2.24)$$
$$= 1 - 0.9875$$
$$= 0.0125$$

$\therefore$ Prob. of an internal review is 0.0125

d) The points A, B and C have position vectors $\underset{\sim}{a}$, $\underset{\sim}{b}$ and $\underset{\sim}{c}$ respectively.
λ and μ are non-zero numbers such that $\lambda\underset{\sim}{a}+\mu\underset{\sim}{b}-\underset{\sim}{c}=0$ and $\lambda+\mu=1$.

i) Show that points A, B and C are collinear. **2**

i) For A, B, C to be collinear, $\overrightarrow{AB} = k\overrightarrow{BC}$ (k is a constant).

½ mark

$\overrightarrow{AB} = \underset{\sim}{b} - \underset{\sim}{a}$ — (1) $\quad \overrightarrow{BC} = \underset{\sim}{c} - \underset{\sim}{b}$ — (2)

Given: $\lambda\underset{\sim}{a} + \mu\underset{\sim}{b} - \underset{\sim}{c} = 0$

$\therefore \underset{\sim}{c} = \lambda\underset{\sim}{a} + \mu\underset{\sim}{b}$ — (3)

$\lambda + \mu = 1$

$\lambda = 1 - \mu$ — (4)

Sub (3) in (2): $\overrightarrow{BC} = \lambda\underset{\sim}{a} + \mu\underset{\sim}{b} - \underset{\sim}{b}$

$= \lambda a + (\mu - 1)\underset{\sim}{b}$

$= \lambda a - (1-\mu)\underset{\sim}{b}$

$= \lambda\underset{\sim}{a} - \lambda\underset{\sim}{b}$ from (4)

$= \lambda(\underset{\sim}{a} - \underset{\sim}{b})$

$= -\lambda(\underset{\sim}{b} - \underset{\sim}{a})$

$= -\lambda\overrightarrow{AB}$ from (1)

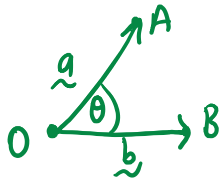
1 mark

½ mark (expression for $\overrightarrow{BC}$ in terms of $\underset{\sim}{a}$, $\underset{\sim}{b}$ OR $\overrightarrow{AB}$ in terms of $\underset{\sim}{b}$, $\underset{\sim}{c}$ using other given data)

$\therefore$ A, B, C are collinear as $\overrightarrow{AB} \parallel \overrightarrow{BC}$ and they share a common point B

ii) It is known that $|\underset{\sim}{a}|=2$, the angle between $\underset{\sim}{a}$ and $\underset{\sim}{b}$ is acute, and the area of triangle OAB is k units^2.

Show that $(\underset{\sim}{a}\cdot\underset{\sim}{b})^2 = 4\left(|\underset{\sim}{b}|^2 - k^2\right)$ 3



$\underset{\sim}{a}\cdot\underset{\sim}{b} = |\underset{\sim}{a}||\underset{\sim}{b}|\cos\theta$ where θ is $A\hat{O}B$

½ mark $\left[\ \underset{\sim}{a}\cdot\underset{\sim}{b} = 2|\underset{\sim}{b}|\cos\theta \right.$

Note: $0 < \cos\theta < 1$ (θ is acute)

½ mark $\left[\ (\underset{\sim}{a}\cdot\underset{\sim}{b})^2 = 4|\underset{\sim}{b}|^2\cos^2\theta \right.$

½ mark $\left[\ (\underset{\sim}{a}\cdot\underset{\sim}{b})^2 = 4|\underset{\sim}{b}|^2(1-\sin^2\theta) \right.$ — ①

Consider Area ΔAOB:

$A = \frac{1}{2}|\underset{\sim}{a}||\underset{\sim}{b}|\sin\theta$

½ mark $\left[\ k = \frac{1}{2}\times 2\times|\underset{\sim}{b}|\sin\theta \right.$ using given data

$k = |\underset{\sim}{b}|\sin\theta$

$\frac{k}{|\underset{\sim}{b}|} = \sin\theta$ — ② ½ mark

Sub ② in ①: $(\underset{\sim}{a}\cdot\underset{\sim}{b})^2 = \overbrace{4|\underset{\sim}{b}|^2\left(1 - \left(\frac{k}{|\underset{\sim}{b}|}\right)^2\right)}$

½ mark

$= 4|\underset{\sim}{b}|^2\left(1 - \frac{k^2}{|\underset{\sim}{b}|^2}\right)$

$= 4\left(|\underset{\sim}{b}|^2\times 1 - \cancel{|\underset{\sim}{b}|^2}\times\frac{k^2}{\cancel{|\underset{\sim}{b}|^2}}\right)$

$= 4\left(|\underset{\sim}{b}|^2 - k^2\right)$ #

END OF QUESTION 14

END OF EXAMINATION