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|---|---|---|
| **Mr Feng** | **Ms Davis** | **Name:** ………………………………………..……….. |
| **Mrs Greenberg** | **Mrs Kerr** | |
| **Ms Lau** | | **Teacher:**…………………………………………. |

# HIGHER SCHOOL CERTIFICATE TRIAL EXAMINATION

## 2019

# Mathematics Extension 1

**General Instructions**

- Reading time – 5 minutes.
- Working time – 2 hours.
- Write using pencil for Questions 1-10.
- Write using black or blue pen for Questions 11-14. Black pen is preferred.
- Erasable pens are **not** to be used.
- Board approved calculators may be used.
- A reference sheet is provided.
- In Questions 11-14, show relevant mathematical reasoning and/or calculations.

**Total Marks – 70**

**Section I** (Pages 1 – 4)

**10 marks**

- Attempt all Questions 1-10 on the multiple choice answer sheet provided.
- Allow about 15 minutes for this section

**Section II** (Pages 5 – 12)

**60 marks**

- Attempt Questions 11-14
- Allow about 1 hour 45 minutes for this section

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| **Mark** | **/70** |
| **Highest Mark** | **/70** |
| **Rank** | |

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**Section I**

**10 marks**
Attempt Questions 1-10
Allow about 15 minutes for this section.

Use the multiple choice answer sheet for Questions 1-10.

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**1** The point $A$ has coordinates $(2,-3)$ and the point $B$ has coordinates $(-4,5)$.
Find the coordinates of the point which divides $AB$ internally in the ratio 4:3.

(A) $\left(-\frac{10}{7},\frac{11}{7}\right)$

(B) $\left(-\frac{4}{7},\frac{3}{7}\right)$

(C) $(-22,9)$

(D) $\left(-\frac{25}{7},\frac{26}{7}\right)$

**2** What is the remainder when $x^3-10x$ is divided by $x+5$?

(A) $-75$

(B) $75$

(C) $x^2-2x$

(D) $x^2-5x+15$

**3** What is the cartesian equation of the tangent to the parabola $x=t-3, y=t^2+2$ at $t=-3$?

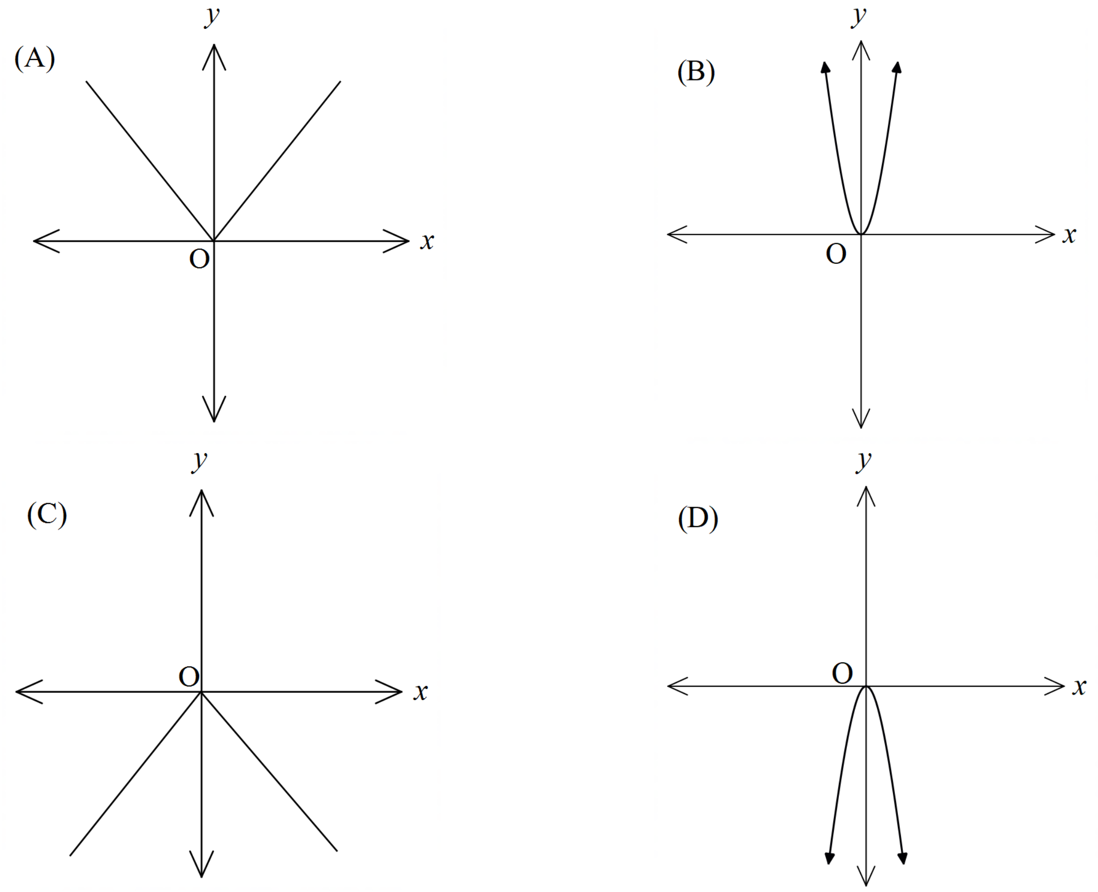
(A) $6x-y-25=0$

(B) $6x-y-36=0$

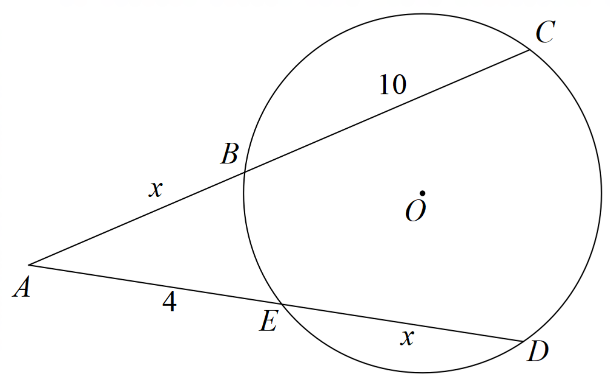
(C) $6x+y+25=0$

(D) $6x+y+36=0$

**4** What is the best representation of the graph $y = x\tan^{-1}x$?




**5**




Not to Scale

Two secants from the point $A$ cut the circle centre $O$ at $BC$ and $ED$.
Given $AB = DE = x$, $BC = 10$ and $EA = 4$, which of the following is the value of $x$?

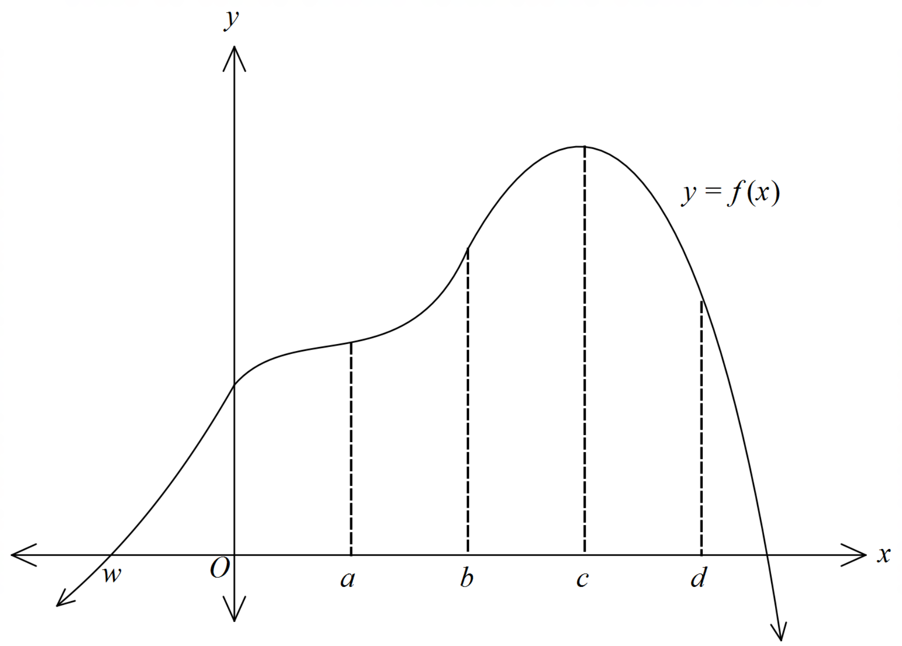
(A) $x = 6$

(B) $x = 8$

(C) $x = 2$

(D) $x = 5$

**6** The diagram shows the graph of $y = f(x)$. $f(x) = 0$ has a solution at $x = w$.




Newton's method can be used to give an approximation close to the solution $x = w$. Which initial approximation, $x_1$ will give the second approximation that is closest to the solution $x = w$?

(A) $x_1 = a$

(B) $x_1 = b$

(C) $x_1 = c$

(D) $x_1 = d$

**7** A particle moves in a straight line. Its position at any time $t$ is given by $x = 4\sin 2t + 3\cos 2t$.
What is the acceleration in terms of $x$?

(A) $\ddot{x} = -4x$

(B) $\ddot{x} = -8x$

(C) $\ddot{x} = -16x^2$

(D) $\ddot{x} = -16\sin 2x - 12\cos 2x$

**8** What is the general solution to the equation $2\sin^2\theta - 3\sin\theta - 2 = 0$?

(A) $\theta = n\pi + (-1)^n \dfrac{\pi}{6}$

(B) $\theta = n\pi + (-1)^n \dfrac{5\pi}{6}$

(C) $\theta = n\pi + (-1)^n \dfrac{7\pi}{6}$

(D) $\theta = n\pi + (-1)^n \dfrac{11\pi}{6}$

**9** What is the domain of $f(x) = \sin^{-1}(4x)$?

(A) $-\pi \le x \le \pi$

(B) $-4 \le x \le 4$

(C) $-\dfrac{\pi}{8} \le x \le \dfrac{\pi}{8}$

(D) $-\dfrac{1}{4} \le x \le \dfrac{1}{4}$

**10** What is the value of $k$ such that $\displaystyle\int_2^k \frac{dx}{\sqrt{16-x^2}} = \frac{\pi}{6}$?

(A) $\dfrac{2\pi}{3}$

(B) $2\sqrt{3}$

(C) $\dfrac{4\pi}{3}$

(D) $\dfrac{\sqrt{3}}{2}$

**Section II**

**60 marks**
Attempt Questions 11-14

Answer each question in a SEPARATE writing booklet. Extra booklets are available.
Your responses should include relevant mathematical reasoning and/or calculations.

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**Question 11** (15 marks) Use a Separate Booklet. **Marks**

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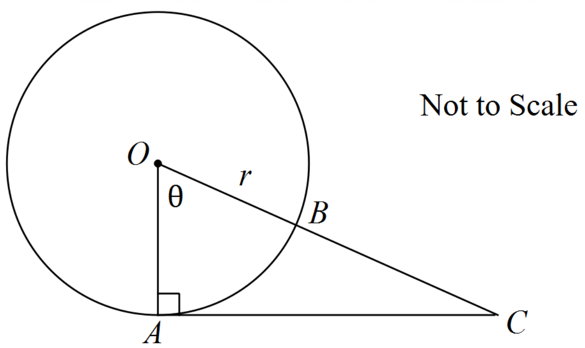
**(a)** Solve $x - \dfrac{2}{x} \leq 1$. **2**

**(b)** Evaluate $\displaystyle\int_{-1}^{0} x\sqrt{1+x}\; dx$ using the substitution $u = 1 + x$. **3**

**(c)** Find the exact value of $\tan\left(\dfrac{5\pi}{12}\right)$, leaving your answer in simplified exact form. **3**

**(d)** The diagram shows points $A$ and $B$ lying on a circle with centre $O$ and radius $r$. The arc $AB$ subtends an angle $\theta$ at $O$. The line $OB$ produced and the tangent at $A$ intersect at $C$.

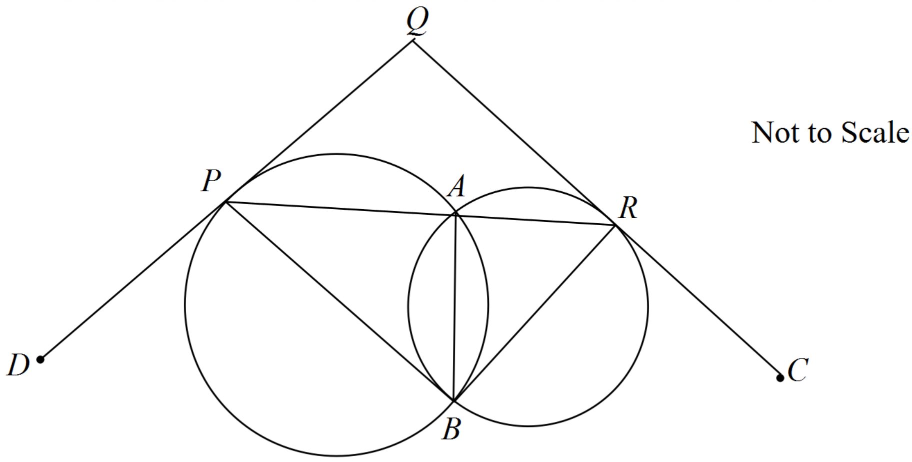



(i) It is known that $AC$ is twice the length of the arc $AB$. **2**
Show that $\tan\theta = 2\theta$

(ii) One solution to the equation $\tan\theta = 2\theta$ is approximately $\theta = 1.2$. **2**
Use one application of Newton's method to find a second approximation.
Give your answer correct to two decimal places.

**Question 11 continues on page 6**

**(e)** Two circles intersect at $A$ and $B$. $PAR$ is a straight line where $P$ is a point on the first circle and $R$ is a point on the second circle. The tangent at $P$ and the tangent at $R$ meet at $Q$. Prove that $BPQR$ is a cyclic quadrilateral. **3**



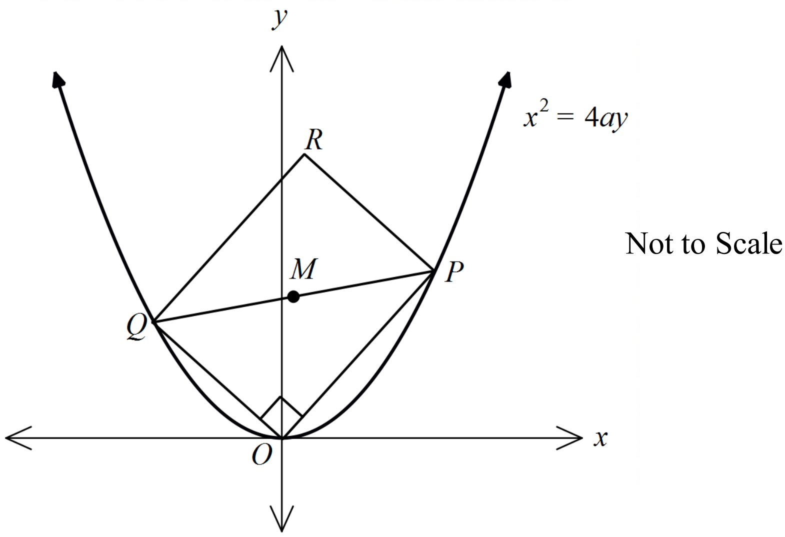

**End of Question 11**

**Question 12** (15 marks) Use a Separate Booklet. **Marks**

**(a)** Use mathematical induction to prove **3**

$$\left(1-\frac{2}{2\times3}\right)\left(1-\frac{2}{3\times4}\right)\left(1-\frac{2}{4\times5}\right)\ldots\left(1-\frac{2}{n(n+1)}\right)=\left(\frac{n+2}{3n}\right) \text{ for all integers } n\geq 2.$$

**(b)**




$P\left(2ap,ap^2\right)$ and $Q\left(2aq,aq^2\right)$ are two points which move on the parabola $x^2=4ay$ such that $\angle POQ=90°$, where $O(0,0)$ is the origin. $M\left(a(p+q),\frac{1}{2}a\left(p^2+q^2\right)\right)$ is the midpoint of $PQ$. $R$ is the point such that $OPRQ$ is a rectangle.

(i) Show that $pq=-4$. **1**

(ii) Show that $R$ has coordinates $\left(2a(p+q),\ a\left(p^2+q^2\right)\right)$. **1**

(iii) Find the equation of the locus of $R$. **2**

**Question 12 continues on page 8**

**(c)** A sphere is being heated so that its surface area is increasing at a rate of 1.5 $mm^2/s$.
Using the formulae for the surface area and volume of a sphere as $A = 4\pi r^2$ and $V = \frac{4}{3}\pi r^3$ respectively:

(i) Show that $\frac{dr}{dt} = \frac{3}{16\pi r}$. **2**

(ii) Hence, find the rate at which the volume is increasing when the radius is 60 mm. **2**

**(d)** A particle is moving in a straight line with $v^2 = 36 - 4x^2$.

(i) Prove that the motion is simple harmonic. **1**

(ii) Find the period and amplitude of the motion. **2**

(iii) If the particle is initially at the origin, write an expression for its displacement in terms of $t$. **1**

**End of Question 12**

**Question 13** (15 marks) Use a Separate Booklet. **Marks**

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**(a)** A particle moves in a straight line so that at time $t$ seconds, it has acceleration $\alpha \text{ms}^{-2}$, velocity $v$ ms$^{-1}$ and position $x$ metres relative to a fixed point on the line. The velocity and position of the particle at any time $t$ seconds are related by $v = -x^2$. Initially $x = 1$.

(i) Find the initial acceleration of the particle. **2**

(ii) Express $x$ in terms of $t$. **2**

**(b)** Suppose $(7+3x)^{25} = \sum_{k=0}^{25} t_k x^k$.

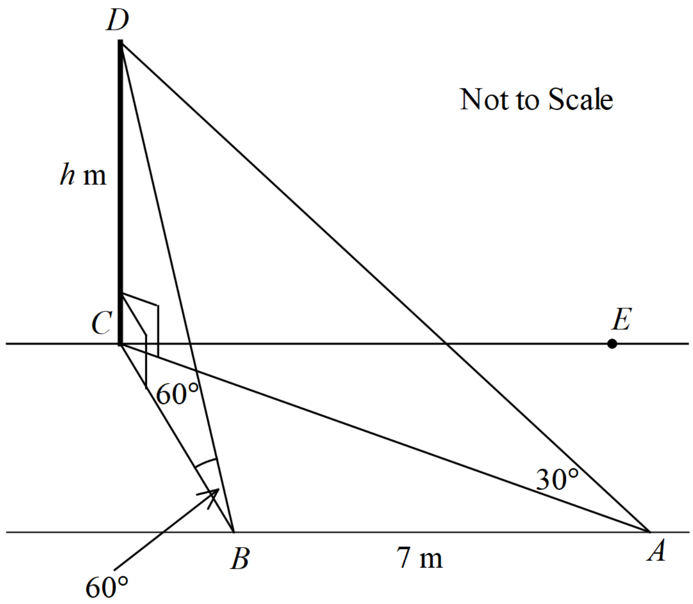
(i) Use the Binomial Theorem to write an expression for $t_k,\ 0 \le k \le 25$. **1**

(ii) Show that $\dfrac{t_{k+1}}{t_k} = \dfrac{3(25-k)}{7(k+1)}$. **2**

(iii) Hence, or otherwise, find the largest coefficient $t_k$. You may leave your answer in the form $\dbinom{25}{k} 7^c 3^d$. **2**

**Question 13 continues on page 10**

**(c)**




A footpath on horizontal ground has two parallel edges, $AB$ and $CE$.
$CD$ is a vertical flagpole of height $h$ metres which stands with its base $C$ on one edge of the footpath.
$A$ and $B$ are two points on the other edge of the footpath such that $AB = 7$ m and $\angle ACB = 60°$. From $A$ and $B$ the angles of elevation of the top ($D$) of the flagpole are $30°$ and $60°$ respectively.

(i) Find the exact height of the flagpole. **4**

(ii) Find the width of the footpath correct to one decimal place. **2**

**End of Question 13**

**Question 14** (15 marks) Use a Separate Booklet. **Marks**

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**(a)** Solve $\sqrt{3}\sin x - \cos x = 1,\ 0 \le x \le 2\pi$, using $t = \tan\dfrac{x}{2}$ as a substitution. **3**

**(b)** A cup of hot coffee at temperature $T$°C loses heat when placed in a cooler environment. It cools according to the law

$$T = T_0 + Ae^{-kt}$$

where $t$ is time elapsed in minutes, and $T_0$ is the temperature of the environment in degrees in Celsius.

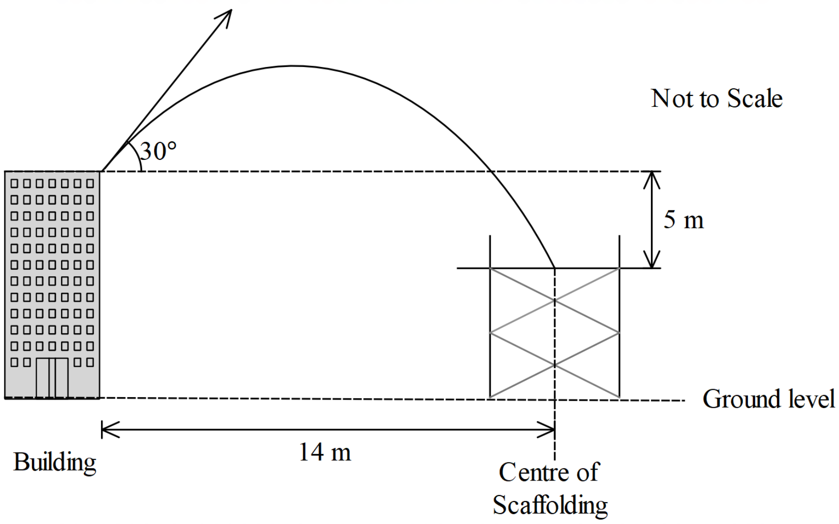
(i) A cup of coffee at $100°C$ is placed in an environment at $-20°C$ for 3 minutes, and cools to $70°C$. Find the exact value of $k$. **2**

(ii) The same cup of coffee, at $70°C$, is then placed in an environment at $20°C$. Assuming $k$ remains the same, find the temperature of the coffee after a further 15 minutes. Give your answer to the nearest degree. **2**

**(c)** Evaluate $\displaystyle\int_0^1 \frac{e^{\cos^{-1} x}}{\sqrt{1-x^2}}\,dx$ using the substitution $u = \cos^{-1} x$ **2**

**Question 14 continues on page 12**

**(d)** A film director must decide whether a stuntman is able to perform a dangerous stunt. The stuntman must leap from a building onto the centre of some erected scaffolding. The centre of the scaffolding is 5 m below his initial position and at a horizontal distance of 14 m. The stuntman jumps at an angle of $30°$ above the horizontal. Let the stuntman's initial velocity be $V$, and let $x$ and $y$ be his horizontal and vertical displacements respectively from his initial position.




You may assume that the velocity and displacement equations are:

$$\dot{x} = V\cos 30° \qquad \dot{y} = -10t + V\sin 30°$$

$$x = Vt\cos 30° \qquad y = -5t^2 + Vt\sin 30°$$

(i) Show that the Cartesian equation of the stuntman's path is **2**

$$y = \frac{-20x^2}{3V^2} + \frac{x}{\sqrt{3}}.$$

(ii) Hence, determine the required initial velocity $V$, so that he lands in the centre of the scaffolding. Write your answer to the nearest m/s. **2**

(iii) Safety requirements are such that if the impact velocity is greater than 15m/s, then padding must be placed on the scaffolding. Assuming that the stuntman leaps at the required speed, determine whether padding is needed or not. **2**

**End of Paper**

Extension 1 Trial 2019 Sol$^n$s and marking guidelines.

1. $A(2,-3)$ $B(-4,5)$ $4:3$

$$\left(\frac{4\times -4+3\times 2}{4+3}, \frac{4\times 5+3\times -3}{4+3}\right)$$

$$= \left(\frac{-10}{7}, \frac{11}{7}\right)$$ A

2. $P(x) = x^3 - 10x$

$P(-5) = (-5)^3 - 10\times -5$

$= -125 + 50$

$= -75$ A

3. $x = t - 3$

$y = t^2 + 2$

$$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$$

$= 2t \times 1$

$= 2t$

When $t = -3$ $x = -6$

$y = 11$

$\frac{dy}{dx} = -6$

Eq$^n$ of tangent

$y - 11 = -6(x+6)$

$6x + y + 25 = 0$ C

4. B

2019 Trial Exam Mathematics Ext 1. solutions.

Q11.

a). $x - \frac{2}{x} \le 1$. $(x \ne 0)$ } multiply by $x^2$.

1 mark. Simplify with $x^2$.

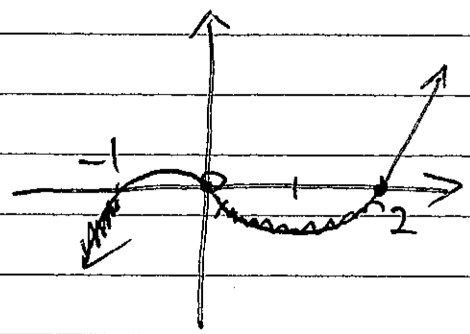
$x^3 - 2x \le x^2$

$x^3 - x^2 - 2x \le 0$

$x(x^2 - x - 2) \le 0$

$x(x-2)(x+1) \le 0$.

1 mark Correct answer.




$\therefore\ x \le -1$ or $0 < x \le 2$.

or.

Consider $x - \frac{2}{x} = 1$. $x \ne 0$

$x^2 - 2 = x$

$x^2 - x - 2 = 0$

$(x-2)(x+1) = 0$.

1 mark Obtain the critical values

Critical values: $x = -1, 0, 2$.

1 mark Correct testing & answers.

① ② ③ ④

$-1$ $0$ $2$

Testing:

① when $x = -2$, LHS $= -2 + 1 = -1$ ✓

② when $x = -\frac{1}{2}$, LHS $= -\frac{1}{2} + 4 = 3\frac{1}{2}$ ×.

③ when $x = 1$, LHS $= 1 - 2 = -1$ ✓ 1

④ When $x = 3$, LHS $= 3 - \frac{2}{3} = 2\frac{1}{3}$ ×. $\therefore\ x \le -1$ or $0 < x \le 2$

| | | |
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| b). $\int_{-1}^{0} x\sqrt{1+x}\,dx$. | let $u=1+x$ <br> $du=dx$. | 1 mark <br> correct substitution |
| | when $x=-1$, $u=0$ | |
| $=\int_0^1 (u-1)\sqrt{u}\,du$. | when $x=0$, $u=1$. | |
| | $x=u-1$. | 1 mark. <br> correct integral |
| $=\int_0^1 (u^{\frac{3}{2}}-u^{\frac{1}{2}})\,du$ | | |
| $=\left[u^{\frac{5}{2}}\div\frac{5}{2}-u^{\frac{3}{2}}\div\frac{3}{2}\right]_0^1$ | | 1 mark <br> correct answer. |
| $=\frac{2}{5}-\frac{2}{3}$ | | |
| $=-\frac{4}{15}$. | | |
| c). $\tan(\frac{5\pi}{12})$ | | 1 mark <br> correctly apply $\tan(\alpha+\beta)$ |
| $=\tan(\frac{\pi}{6}+\frac{\pi}{4})$ | | |
| $=\dfrac{\tan\frac{\pi}{6}+\tan\frac{\pi}{4}}{1-\tan\frac{\pi}{6}\tan\frac{\pi}{4}}$ | | 1 mark. <br> get the exact value |
| $=\dfrac{\frac{1}{\sqrt{3}}+1}{1-\frac{1}{\sqrt{3}}}$ $\times\frac{\sqrt{3}}{\sqrt{3}}$. | | |
| $=\dfrac{1+\sqrt{3}}{\sqrt{3}-1}$ | | 1 mark <br> simplify to the final answer. |
| $=\dfrac{(1+\sqrt{3})(\sqrt{3}+1)}{(\sqrt{3})^2-1}$. | | |
| $=\dfrac{(\sqrt{3}+1)^2}{2}$ | | |
| $=\dfrac{4+2\sqrt{3}}{2}$ | | |
| $=2+\sqrt{3}$. | | |

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| d). (i). In sector OAB: $\overset{\frown}{AB} = r\theta$. | 1 mark. |
| | Use arc length: $l = r\theta$ |
| $AC = 2\overset{\frown}{AB} = 2r\theta$. | |
| In $\triangle AOC$, $\tan\theta = \frac{AC}{OA}$ | 1 mark |
| $= \frac{2r\theta}{r}$ | Correct proof. |
| $= 2\theta$. | |
| (ii). Let $f(\theta) = \tan\theta - 2\theta$. | 1 mark. |
| $f'(\theta) = \sec^2\theta - 2$. | correct $f(\theta)$ & $f'(\theta)$ |
| $\theta_1 = \theta_0 - \frac{f(\theta_0)}{f'(\theta_0)}$ | 1 mark |
| $= 1.2 - \frac{\tan(1.2) - 2(1.2)}{\sec^2(1.2) - 2}$ | Correct answer. |
| $= 1.1693\ldots$ | |
| $\doteqdot 1.17$ (2dp). | |

| | |
|---|---|
| 2). Let $\angle QPR = \alpha$ & $\angle QRP = \beta$. | 1 mark. |
| $\angle QPR = \angle PBA = \alpha$ (angle between the tangent & chord equals to the angles at the circumference subtended by the alternate segment). | Apply Alternate Segment |
| Similarly $\angle QRP = \angle RBH = \beta$. | 1 mark<br>Attempt to the final answer |
| $\therefore \angle PBR = \alpha + \beta$. | |
| In $\triangle QPR$, $\angle PQR + \alpha + \beta = 180^\circ$ (angle sum of a triangle) | 1 mark<br>Correct proof. |
| $\therefore \angle PQR = 180 - (\alpha + \beta)$. | |
| $\angle PBR + \angle PQR = 180 - (\alpha+\beta) + (\alpha+\beta)$<br>$= 180$. | |
| $\therefore$ PQRB is a cyclic quadrilateral. (Opposite angles are supplementary). | |

Q12.

(a). Let the statement be

$(1-\frac{2}{2\times3})(1-\frac{2}{3\times4})\cdots(1-\frac{2}{n(n+1)}) = (\frac{n+2}{3n})$ for $n\geq2$ $n\in\mathbb{Z}$.

1 mark
Correctly prove it's true for $n=2$.

Prove the statement is true for $n=2$.

$LHS = (1-\frac{2}{2\times3})$
$= 1-\frac{1}{3}$
$= \frac{2}{3}$.

$RHS = \frac{2+2}{3(2)}$
$= \frac{2}{3}$.

1 mark
Correct writing of assumption & the require to prove statement.

Assume the statement is true for $n=k$, $k\geq2$ $k\in\mathbb{Z}$.

ie $(1-\frac{2}{2\times3})(1-\frac{2}{3\times4})\cdots(1-\frac{2}{k(k+1)}) = \frac{k+2}{3k}$.

1 mark.
Correct proof.

To prove the statement is true for $n=k+1$.

ie $(1-\frac{2}{2\times3})(1-\frac{2}{3\times4})\cdots(1-\frac{2}{k(k+1)})(1-\frac{2}{(k+1)(k+2)}) = \frac{k+3}{3(k+1)}$.

$LHS = (1-\frac{2}{2\times3})(1-\frac{2}{3\times4})\cdots(1-\frac{2}{k(k+1)})(1-\frac{2}{(k+1)(k+2)})$
$= \frac{k+2}{3k}(1-\frac{2}{(k+1)(k+2)})$
$= \frac{k+2}{3k}(\frac{(k+1)(k+2)-2}{(k+1)(k+2)})$
$= \frac{k+2}{3k}(\frac{k^2+3k+2-2}{(k+1)(k+2)})$
$= \frac{k+2}{3k}(\frac{k^2+3k}{(k+1)(k+2)})$
$= \frac{k+2}{3k}(\frac{k(k+3)}{(k+1)(k+2)})$

$= \dfrac{k+3}{3(k+1)} = \text{RHS}.$

$\therefore$ The statement is true for $n=k+1$ if it is true for $n=k$.

Since the statement is true for $n=2$, it is true for $n=2+1=3$, $n=3+1=4$ .. By the concept of mathematical induction, it is true for all integers $n \geq 2$.

b) (i). $m_{OP} = \dfrac{ap^2-0}{2ap}$ similarly $m_{OQ} = \dfrac{q}{2}$ | 1 mark. Correct proof.

$= \dfrac{p}{2}.$

Since $OP \perp OQ$, $m_{OP} \times m_{OQ} = -1$.

$\dfrac{p}{2} \times \dfrac{q}{2} = -1.$

$pq = -4.$

(ii). Since OPRQ is a rectangle, the diagonals bisect each other. | 1 mark

$\therefore$ M is also the midpoint of OR. | Correct proof.

$\dfrac{0+x_R}{2} = a(p+q).$ $\dfrac{0+y_R}{2} = \frac{1}{2}a(p^2+q^2)$

$x_R = 2a(p+q).$ $y_R = a(p^2+q^2)$

$\therefore R(2a(p+q), a(p^2+q^2)).$

| | |
|---|---|
| iii) $x = 2a(p+q)$. | 1 mark |
| $\frac{x}{2a} = p+q$. | Make $p+q$ or $p^2 + q^2$ |
| $\frac{x^2}{4a^2} = p^2+q^2+2pq$. | in terms of $x$ and $y$. |
| $= p^2+q^2-8$ since $pq=-4$. | |
| $y = a(p^2+q^2)$ | 1 mark |
| $\frac{y}{a} = p^2+q^2$. | Correct equation. |
| $\therefore \frac{x^2}{4a^2} = \frac{y}{a} - 8$. | |
| $x^2 = 4a(y-8a)$. | |
| c) i). $\frac{dA}{dt} = \frac{dA}{dr} \times \frac{dr}{dt}$. | 1 mark. |
| $A = 4\pi r^2 \rightarrow \frac{dA}{dr} = 8\pi r$, $\frac{dA}{dt} = 1.5$ | Apply Chain Rule. |
| $\therefore 1.5 = 8\pi r \times \frac{dr}{dt}$ | 1 mark |
| $\frac{dr}{dt} = 1.5 \times \frac{1}{8\pi r}$ | Correct answer. |
| $= \frac{3}{16\pi r}$ mm/s | |
| ii) $\frac{dV}{dt} = \frac{dV}{dr} \times \frac{dr}{dt}$. | 1 mark |
| $V = \frac{4}{3}\pi r^3 \Rightarrow \frac{dV}{dr} = 4\pi r^2$. | Apply chain rule |
| $\frac{dV}{dt} = 4\pi r^2 \times \frac{3}{16\pi r}$ | 1 mark |
| $= \frac{3r}{4}$. | Correct answer. |

When $r=60$, $\frac{dv}{dt}=45\ mm^3/s$.

i). (i). $v^2=36-4x^2$ | 1 mark correct answer.

$\frac{1}{2}v^2=18-2x^2$

$\ddot{x}=\frac{d}{dx}(\frac{1}{2}v^2)$

$=-4x$.

$=-2^2x$.

$\therefore$ It's simple harmonic motion since $\ddot{x}=-n^2x$.

(ii). $\ddot{x}=-4x$. | 1 mark Get correct period or amplitude.

$n^2=4$

$n=2$ since $n>0$.

Period $T=\frac{2\pi}{2}$

$=\pi$ sec.

For amplitude, make $v=0$. | 1 mark correct answers

$36-4x^2=0$.

$x^2=9$

$x=\pm 3$.

since the centre of the motion is at $x=0$.

$\therefore$ Amplitude $a=3$.

(iii). $n=2$, $a=3$. | 1 mark Correct answer (both positive & negative are acceptable).

$x=3\sin 2t$ or $x=-3\sin 2t$.

13. a) i) $v = -x^2$

$v^2 = x^4$

$\frac{1}{2}v^2 = \frac{1}{2}x^4$

$\frac{d}{dx}\left(\frac{1}{2}v^2\right) = 2x^3$

When $x = 1$ $\quad \ddot{x} = 2 \times 1^3$

$= \boxed{2\ m/s^2}$

| | |
|---|---|
| 2 | correct sol$^n$ |
| 1 | one error in process |

ii) $\frac{dx}{dt} = -x^2$

$\int -\frac{dx}{x^2} = \int dt$

$\frac{1}{x} = t + c$

When $t = 0$ $\quad x = 1$

$\therefore c = 1$

$\therefore \frac{1}{x} = t + 1$

$\boxed{x = \frac{1}{t+1}}$

| | |
|---|---|
| 2 | Correct sol$^n$ |
| 1 | one error in process |

or $\frac{dx}{dt} = x^2$

$\therefore \frac{dt}{dx} = -\frac{1}{x^2}$

$\therefore t = \frac{1}{x} + c$

$t = 0, x = 1 \therefore c = -1$

$\therefore t = \frac{1}{x} - 1$

$\therefore \frac{1}{x} = t + 1$

$\therefore \boxed{x = \frac{1}{t+1}}$

b) i) $(7+3x)^{25} = 7^{25} + {}^{25}C_1\, 7^{24}(3x) + {}^{25}C_2\, 7^{23}(3x)^2 + \ldots (3x)^{25}$

$\therefore \boxed{t_k = {}^{25}C_k \times 7^{25-k} \times 3^k}$ or $\boxed{{}^{25}C_k . 7^{25-k} . 3^k}$

ii) $\frac{t_{k+1}}{t_k} = \frac{{}^{25}C_{k+1} \times 7^{25-(k+1)} \times 3^{k+1}}{{}^{25}C_k \times 7^{25-k} \times 3^k}$

$= \frac{\frac{25!}{(25-(k+1))!\,(k+1)!}}{\frac{25!}{(25-k)!\,k!}} \times 7^{25-k-1-25+k} \times 3^{k+1-k}$

$$= \frac{\cancel{25}!}{(24-k)!\,(k+1)!} \times \frac{(25-k)!\,k!}{\cancel{25}!} \times \frac{1}{7} \times 3$$

$$= \frac{3(25-k)}{7(k+1)} \quad \text{As req'd}$$

| | |
|---|---|
| 2 | sufficiently substantiated proof |
| 1 | partially substantiated proof |

iii) when $\frac{t_{k+1}}{t_k} > 1$ | $\frac{t_{k+1}}{t_k} < 1$ zero marks

$$\frac{3(25-k)}{7(k+1)} > 1$$

$$75 - 3k > 7k + 7 \qquad k+1 > 0$$

$$-10k > -68$$

$$k < 6.8$$

$$k = 6$$

$\therefore \quad k = 6 \quad T_{k+1} > T_k$

$\therefore \boxed{T_7 = {}^{25}C_7\, 7^{18} . 3^7}$

| | |
|---|---|
| 2 | correct solution |
| 1 | correct inequality and attempt to solve |

c) i) In $\triangle ACD$ $\tan 30^\circ = \frac{h}{AC}$

$$AC = \frac{h}{\tan 30^\circ}$$

$$AC = \sqrt{3}\,h$$

In $\triangle BCD$

$$\tan 60^\circ = \frac{h}{BC}$$

$$BC = \frac{h}{\tan 60^\circ} = \frac{h}{\sqrt{3}}$$

In $\triangle ABC$

$$7^2 = BC^2 + AC^2 - 2 \times BC \times AC \cos 60^\circ$$

$$49 = \left(\frac{h}{\sqrt{3}}\right)^2 + (\sqrt{3}h)^2 - 2 \times \frac{h}{\sqrt{3}} \times \sqrt{3}h \times \frac{1}{\cancel{2}}$$

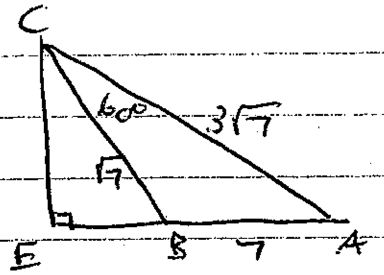
$$49 = \frac{h^2}{3} + 3h^2 - h^2$$

$$49 = \frac{7h^2}{3}$$

$$\boxed{h = \sqrt{21}\text{ m}}$$

| | |
|---|---|
| 1 | $AC = \sqrt{3}h$, $BC = \frac{h}{\sqrt{3}}$ or $h = \frac{AC}{\sqrt{3}}$, $h = \sqrt{3}BC$ or $AC = h\cot 30$, $BC = h\cot 60$ or $AC = \frac{h}{\tan 30}$, $BC = \frac{h}{\tan 60}$ |
| 2 | attempt sub in cos rule: $7^2 = BC^2 + AC^2 - 2BC.AC.\cos 60$ |
| 3 | $49 = \left(\frac{h}{\sqrt{3}}\right)^2 + (\sqrt{3}h)^2 - 2.\frac{h}{\sqrt{3}}.\sqrt{3}h.\frac{1}{2}$ <br> $\cos 60 = \frac{(\sqrt{3}h)^2 + \left(\frac{h}{\sqrt{3}}\right)^2 - 49}{2(\sqrt{3}h)\left(\frac{h}{\sqrt{3}}\right)}$ |
| 4 | correct $h = \sqrt{21}$ m |

ii) $h = \sqrt{21}$ $\therefore$ $AC = \sqrt{3} \times \sqrt{21}$ $= 3\sqrt{7}$ $\quad BC = \frac{\sqrt{21}}{\sqrt{3}}$ $= \sqrt{7}$




In $\triangle ABC$

$$\frac{\sin A}{\sqrt{7}} = \frac{\sin 60^\circ}{7}$$

$$\sin A = \frac{\sqrt{21}}{14}$$

$$A = 19.10660535$$

$\angle CBE = 60 + 19.10660535$

$\doteqdot 79.10660535$

$$\sin(\angle CBE) = \frac{CE}{\sqrt{7}}$$

$$CE = \sqrt{7}\sin(\angle CBE)$$

$= 2.598076211$

$\doteqdot \boxed{2.6 \text{ m}}$

| | |
|---|---|
| 1 | $CA = 3\sqrt{7}$ <br> $CB = \sqrt{7}$ <br> or <br> $CA = \sqrt{3}$ [ANS] <br> $CB = \frac{[ANS]}{\sqrt{3}}$ and <br> uses sine rule |
| 2 | correct soln |

14 a) $\sqrt{3}\sin x - \cos x = 1$

$$\sqrt{3} \times \frac{2t}{1+t^2} - \frac{1-t^2}{1+t^2} = 1$$

$$2\sqrt{3}t - 1 + t^2 = 1 + t^2$$

$$2\sqrt{3}t = 2$$

$$t = \frac{1}{\sqrt{3}}$$

$$\therefore \tan\frac{x}{2} = \frac{1}{\sqrt{3}}$$

$$\frac{x}{2} = \frac{\pi}{6},$$

$$x = \frac{\pi}{3},$$

1 correct sub into formula $\sqrt{3} \times \frac{2t}{1+t^2} - \frac{1-t^2}{1+t^2} = 1$

2 $x = \frac{\pi}{3}$ (ignored $x =$ ?

3. showed $x = \pi$ subs correctly to conclude

$\boxed{x = \frac{\pi}{3}, \pi}$

Check $x = \pi$ $\sqrt{3}\sin\pi - \cos\pi$

$= 0 - (-1)$

$= 1$

$\therefore$ $x = \pi$ is also a sol$^n$

$\therefore$ complete sol$^n$ $\boxed{x = \pi/3, \pi,}$

b) i) $\frac{dT}{dt} = k(T + 20)$

$t = 0$ $T = 100$, $T_0 = -20$

$100 = -20 + Ae^0$

$A = 120$

$\therefore T = -20 + 120e^{-kt}$

but $t = 3$, $T = 70$

$70 = -20 + 120e^{-3k}$

$e^{-3k} = \frac{3}{4}$

$\therefore -3k = \log_e\left(\frac{3}{4}\right)$

$\boxed{k = -\frac{1}{3}\log_e\left(\frac{3}{4}\right)}$

or $\boxed{k = \frac{1}{3}\log_e\frac{4}{3}}$

| | |
|---|---|
| 1 | $T = -20 + 120e^{-kt}$ |
| 2 | correct k |

ii) $T = 70 \quad T_0 = 20 \quad t = 0$

$\therefore 70 = 20 + Ae^{0}$

$A = 50$

$\therefore T = 20 + 50e^{-kt}$

at $k = \frac{1}{3}\log_e 3/4$

$\therefore T = 20 + 50e^{\left(-\frac{1}{3}\log_e \frac{3}{4}\right) \times 15}$

$T = 31.86523435$

$T \doteqdot 32^{0}$

$\boxed{\therefore \text{temperature} \doteqdot 32^{\circ}C}$

| | |
|---|---|
| 1 | $T = 20 + 50e^{-kt}$ |
| 2 | correct soln after subst. |

e) $\displaystyle\int_0^1 \frac{e^{\cos^{-1} x}}{\sqrt{1-x^2}}\,dx$

$u = \cos^{-1} x$

$du = -\dfrac{1}{\sqrt{1-x^2}}$

$= \displaystyle\int_{\pi/2}^{0} -e^{u}\,du$

when $x = 1 \quad u = \cos^{-1} 1$
$= 0$

$= \displaystyle\int_0^{\pi/2} e^{u}\,du$

$x = 0 \quad u = \cos^{-1} 0$
$= \pi/2$

$= \left[e^{u}\right]_0^{\pi/2}$

$= e^{\pi/2} - e^{0}$

$= \boxed{e^{\pi/2} - 1}$

| | |
|---|---|
| 2 | Correct Solution |
| 1 | substitution of all elements |

d) i) $x = Vt\cos 30^\circ$ — ①  $y = -5t^2 + Vt\sin 30^\circ$ — ②

From ①  $t = \dfrac{x}{V\cos 30^\circ}$

$= \dfrac{2x}{\sqrt{3}\,V}$  substitute into ②

$y = -5\left(\dfrac{2x}{\sqrt{3}\,V}\right)^2 + V\left(\dfrac{2x}{\sqrt{3}\,V}\right)\sin 30^\circ$ ✓

$= \dfrac{-20x^2}{3V^2} + \dfrac{2x}{\sqrt{3}} \times \dfrac{1}{2}$

$\boxed{y = \dfrac{-20x^2}{3V^2} + \dfrac{x}{\sqrt{3}}}$ As req'd ✓

| | |
|---|---|
| 2 | Correct Solutio |
| 1 | Making t the subject and substituting with one error |

ii) When $x = 14$  $y = -5$

$-5 = \dfrac{-20 \times 14^2}{3V^2} + \dfrac{14}{\sqrt{3}}$ ✓

| | |
|---|---|
| 2 | Correct solution |
| 1 | Just the substitut of $x = 14$ and $y = -$ |

$-15\sqrt{3}\,V^2 = -3920\sqrt{3} + 42V^2$

$V^2(42 + 15\sqrt{3}) = 3920\sqrt{3}$

$V = \sqrt{\dfrac{3920\sqrt{3}}{42 + 15\sqrt{3}}}$

$= 9.993792608$

$\boxed{V \doteqdot 10\ m/s}$

iii) Impact velocity $= \sqrt{\dot{x}^2 + \dot{y}^2}$

from i) $t = \dfrac{2x}{\sqrt{3}\,V}$ if $V \doteqdot 10$ from iii) then

When $x = 14$  $t = \dfrac{28}{10\sqrt{3}}$

$= \dfrac{14}{5\sqrt{3}}$

$\dot{x} = 10 \times \dfrac{\sqrt{3}}{2}$

$= 5\sqrt{3}$

$\dot{y} = \dfrac{-10 \times 14}{5\sqrt{3}} + 10 \times$

$= \dfrac{-28}{\sqrt{3}} + 5$

$$v = \sqrt{(5\sqrt{3})^2 + \left(5 - \frac{28}{\sqrt{3}}\right)^2}$$

$$= \sqrt{199.675258}$$

$$\doteq 14.13064959 \text{ m/s}$$

1 attempts to use $t = \frac{28}{10\sqrt{3}}$, $\dot{x} = 5\sqrt{3}$ and $\dot{y} = -\frac{28}{\sqrt{3}} + 5$ and sub into $v^2 = \dot{x}^2 + \dot{y}^2$

2 correct soln

Which is less than the threshold for padding requirement $\therefore$ no padding needed