Student's Name:____________________________ Teacher's Code : _________________________

Saint Ignatius' College, Riverview
Mathematics Assessment Task
2023

Year 12
Mathematics Extension 1
Task 4
Trial HSC
Date: Friday 18 August 2023

General Instructions:

- **Reading Time: 10 minutes**
- **Time Allowed: 2 hours**
- Write using black pen
- NESA approved calculators may be used
- Attempt all questions in the booklets provided.
- Use a new Answer Booklet for each of questions 11, 12, 13 and 14.
- Write **your name** and **your teacher's code** in the positions indicated on the Answer booklets
- Marks may not be awarded for missing or carelessly arranged working.

Teachers :

- Mr R Maxwell **REM**
- Mr D Reidy **DPR**
- Mr N Mushan **NHM**
- Mr P Collins **PPC**

Exam Sections:

Section A	**10 marks**
Multiple Choice	
Section B	
Short Answer	
Question 11	**15 marks**
Question 12	**15 marks**
Question 13	**15 marks**
Question 14	**15 marks**
Total	**70 Marks**

SECTION A: Multiple Choice

Answer Questions 1 to 10 on the Multiple Choice Answer Grid

1. What is the remainder when $P(x) = -2x^3 + 3x^2 - 6x + 4$ is divided by $x + 2$.

A. 44

B. -12

C. 12

D. 20

2. What is the derivative of $\tan^{-1}(4x-1)$?

A. $\dfrac{1}{16x^2 - 8x + 2}$

B. $\dfrac{16x-4}{8x^2 - 4x + 1}$

C. $\dfrac{4}{8x^2 - 4x + 1}$

D. $\dfrac{2}{8x^2 - 4x + 1}$

3. The domain for $f(x) = 3\cos^{-1} 4x$ is:

A. $\left[\dfrac{-3}{4}, \dfrac{3}{4}\right]$

B. $\left[\dfrac{-1}{3}, \dfrac{1}{3}\right]$

C. $\left[\dfrac{-1}{4}, \dfrac{1}{4}\right]$

D. $[-1, 1]$

4.	What is the value of $\sin 2x$ given that $\sin x = \frac{\sqrt{3}}{2}$ and x is obtuse? A. $-\frac{\sqrt{3}}{4}$ B. $-\frac{\sqrt{3}}{2}$ C. $\frac{\sqrt{3}}{4}$ D. $\frac{\sqrt{3}}{2}$
5.	A curve C has parametric equations $x = \cos^2 t$ and $y = 2\sin^2 t$ for $-\infty < t < \infty$. What is the Cartesian equation and domain of C? A. $y = 1 - x$ for $[0, 1]$ B. $y = 2 - 2x$ for $(-\infty, \infty)$ C. $y = 2 - 2x$ for $[0, 1]$ D. $y = 1 - x$ for $(-\infty, \infty)$
6.	Which of the following integrals is equivalent to $\int \cos^2 4x\, dx$? A. $\frac{1}{2}\int (x + \frac{1}{8}\sin 8x)\, dx$ B. $\frac{1}{2}\int (x - \frac{1}{8}\sin 8x)\, dx$ C. $\frac{1}{2}\int (1 + \cos 8x)\, dx$ D. $\frac{1}{2}\int (1 - \cos 8x)\, dx$

7. For the two vectors $\overrightarrow{AB}$ and $\overrightarrow{AC}$ it is known that $\overrightarrow{AB} \cdot \overrightarrow{AC} > 0$.
Which of the following statements must be true?

A. Either $\overrightarrow{AB}$ is positive and $\overrightarrow{AC}$ is positive, or, $\overrightarrow{AB}$ is negative and $\overrightarrow{AC}$ is negative.

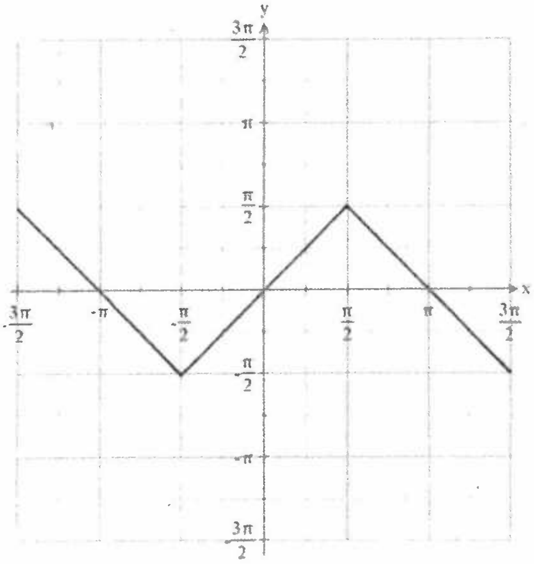
B. The angle between $\overrightarrow{AB}$ and $\overrightarrow{AC}$ is acute.

C. The product $\left|\overrightarrow{AB}\right|\left|\overrightarrow{AC}\right|$ is negative.

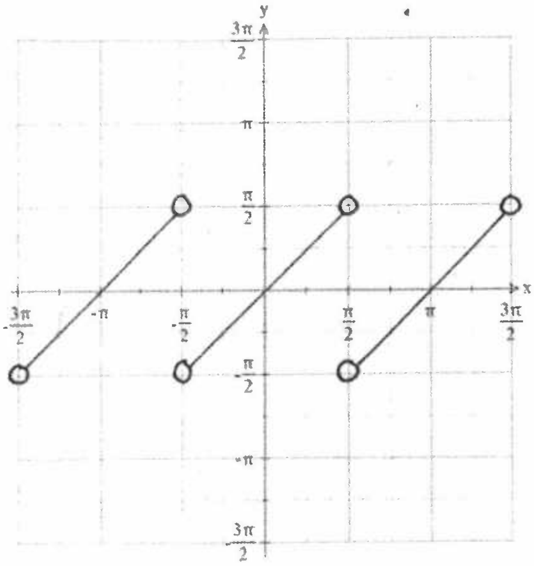
D. The points A, B and C are collinear.

8. Which graph represents the function $y = \tan^{-1}(\tan x)$?

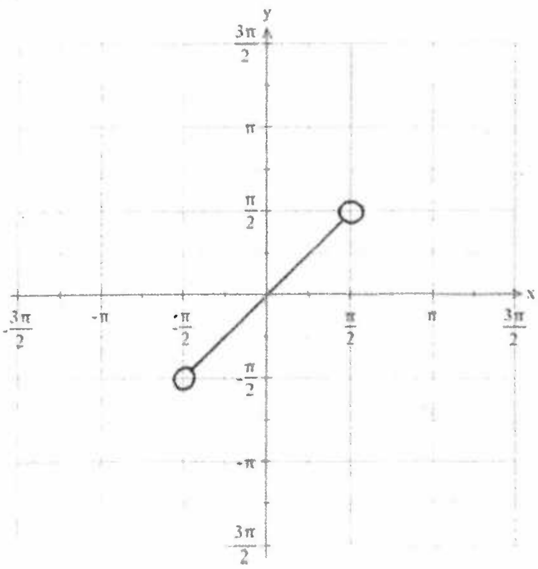
A.



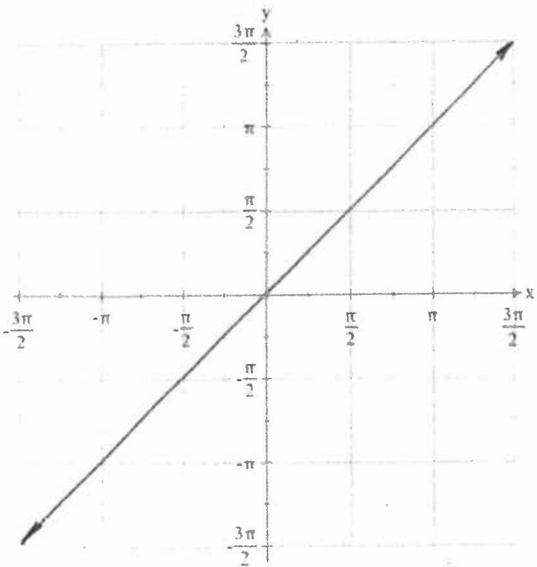
B.

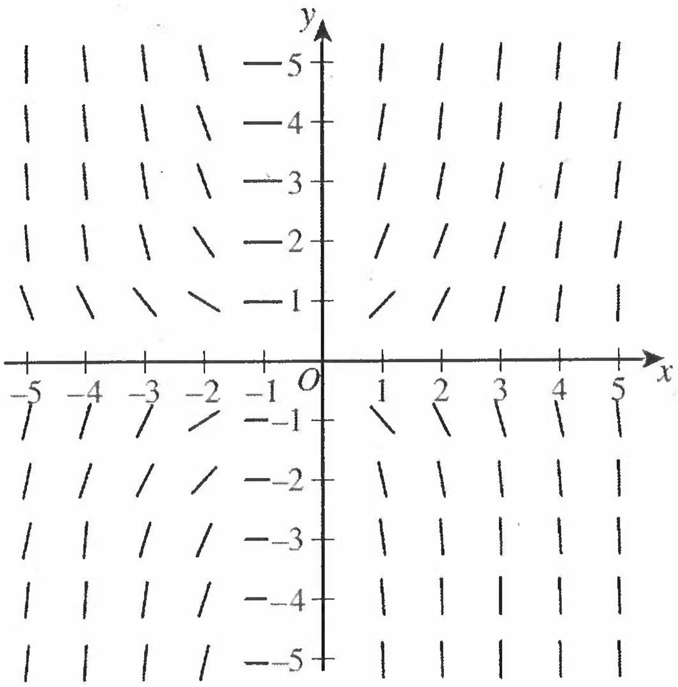


C.



D.

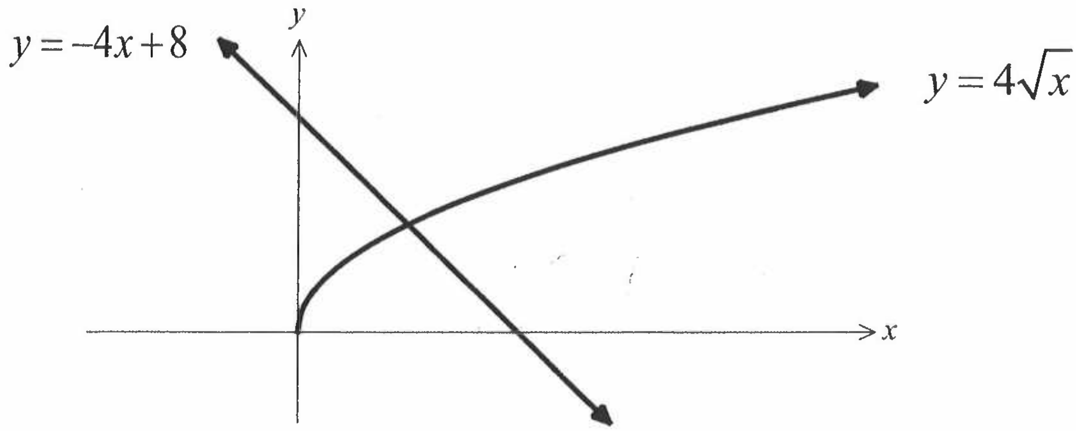


9.	Let $P(x) = x^2 + bx + c$ where b and c are constants. The zeros of $P(x)$ are α and $\alpha + 1$. What are the correct expressions for b and c in terms of α? A. $b = -(2\alpha + 1)$ and $c = \alpha^2 + \alpha$ B. $b = 2\alpha + 1$ and $c = \alpha^2 + \alpha$ C. $b = \alpha^2 + \alpha$ and $c = -(2\alpha + 1)$ D. $b = \alpha^2 + \alpha$ and $c = 2\alpha + 1$
10.	The direction (slope) field for a first order differential equation is shown.  Which of the following could be the differential equation represented? A. $\dfrac{dy}{dx} = (x+1)^3$ B. $\dfrac{dy}{dx} = x(y+1)$ C. $\dfrac{dy}{dx} = (x-1)y$ D. $\dfrac{dy}{dx} = (x+1)y$

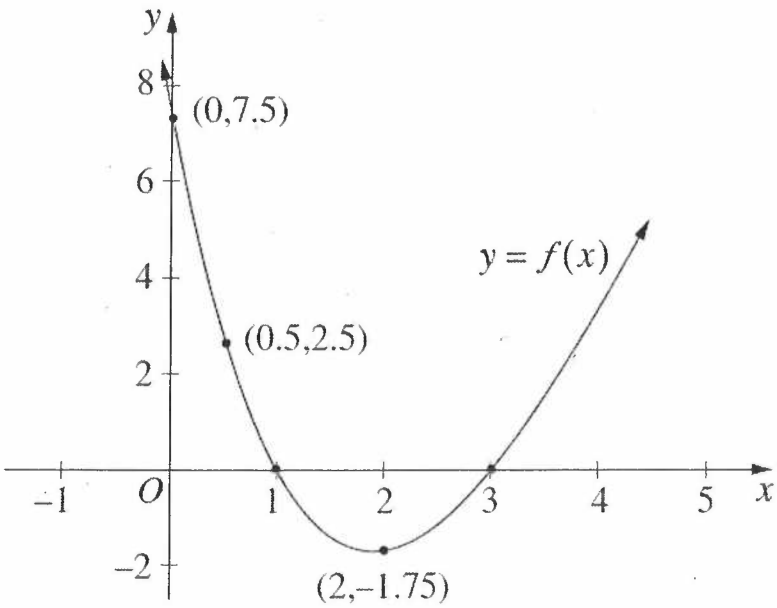
QUESTION 11 Use a new writing booklet (15 marks)

		Marks
a.	Find the anti-derivative of $\dfrac{1}{\sqrt{4-9x^2}}$	2
b.	**i.** Find the exact value of $\sin\dfrac{23\pi}{12}$. (Express answer as a single fraction with a rational denominator)	2
	ii. Find the exact value of $\sin^{-1}(\sin\dfrac{23\pi}{12})$	1
c.	Consider the vectors $\underset{\sim}{u}=\underset{\sim}{i}-2\underset{\sim}{j}$ and $\underset{\sim}{v}=a\underset{\sim}{i}+2\underset{\sim}{j}$ **i.** Find the value of a if $\underset{\sim}{u}$ and $\underset{\sim}{v}$ are perpendicular.	1
	ii. Find the values of a if the angle between $\underset{\sim}{u}$ and $\underset{\sim}{v}$ is $60°$. Give answers correct to 1 decimal place	3
d.	Given $\underset{\sim}{u} = \begin{pmatrix}4\\2\end{pmatrix}$ and $\underset{\sim}{v} = \begin{pmatrix}6\\-8\end{pmatrix}$ **i.** Find $\text{proj}_{\underset{\sim}{u}}\underset{\sim}{v}$	2
	ii. Find the direction of $\text{proj}_{\underset{\sim}{u}}\underset{\sim}{v}$ Give answer to nearest degree.	1
e.	Consider the cubic equation $6x^3-9x^2+4x-3=0$ with roots α, β and γ. Find the value of: **i.** $\alpha+\beta+\gamma$	1
	ii. $\alpha\beta+\alpha\gamma+\beta\gamma$	1
	iii. $\alpha^2+\beta^2+\gamma^2$	1

QUESTION 12 Use a new writing booklet (15 marks)

		Marks
a.	The area enclosed by the curve $y = 4\sqrt{x}$, the line $y = -4x + 8$ and the x axis is rotated about the x axis. 	
	i. Find the coordinates of the point where $y = 4\sqrt{x}$ and $y = -4x + 8$ intersect.	1
	ii. Find the exact volume of the solid of revolution.	3
b.	Solve: $\dfrac{3x}{x+3} \le -2$	3
c.	Use the substitution $u = 1 + 2\tan x$ to evaluate $\displaystyle\int_0^{\frac{\pi}{4}} \frac{1}{(1+2\tan x)^2 \cos^2 x}\,dx$	3
d.	**i.** Express $\cos x - \sqrt{3}\sin x$ in the form $R\cos(x+\alpha)$	1
	ii. Hence, state the greatest value of the function: $f(x) = \cos x - \sqrt{3}\sin x + 10$	1
e.	Consider the function $y = 2x(x+6)^6$. Find the derivative of the inverse function. Give answer in terms of y.	3

QUESTION 13 Use a new writing booklet (15 marks)

		Marks
a.	The diagram below is a sketch of the graph $y = f(x)$. 	
	i. Sketch the graph of $y = f(\lvert x \rvert)$ showing all important features, together with the location of the points corresponding to the labelled points on the original sketch.	2
	ii. Sketch the graph of $y = \dfrac{1}{f(x)}$ showing all important features, together with the location of the points corresponding to the labelled points on the original sketch.	2
b.	Solve the differential equation $f'(x) = 4x^3y^2$ given $f(2) = 1$	3
c.	Use mathematical induction to prove $7^{2n} + 7^n + 4$ is divisible by 6 for all integers $n \geq 0$.	3
d.	A metal is heated to $200°C$ and then left to cool, which it does according to the formula $\dfrac{dT}{dt} = -0.05(T - 23)$ where T is the temperature in $°C$ and t is time in minutes.	
	i. Solve the differential equation. (Make T the subject)	3
	ii. Find the temperature after 20 minutes. (Answer to 2 decimal places)	1
	iii. Find the time when the temperature reaches $40°C$. (Answer to 2 decimal places)	1

QUESTION 14 Use a new writing booklet (15 marks)

		Marks
a.	A particle is projected from a point O on level ground with a speed of 25 ms^{-1} at an angle θ to the horizontal. At time T seconds, the particle passes through the point $B(15, 2)$. The displacement vector $R(t)$ for the motion of the particle is: $R(t) = (Vt\cos\theta)\underset{\sim}{i} + (-\frac{1}{2}gt^2 + Vt\sin\theta)\underset{\sim}{j}$ (Do Not Prove this!) Where t is the time in seconds after projection, $\underset{\sim}{i}$ and $\underset{\sim}{j}$ are unit vectors of magnitude 1metre and g is the acceleration due to gravity = 10 ms^{-2}.	
	i. By considering the horizontal component of the particle's motion, show that $T = \frac{3}{5}\sec\theta$.	1
	ii. By considering the vertical component of the particle's motion and, using the result from part **i**, show that $9\tan^2\theta - 75\tan\theta + 19 = 0$.	2
	iii. Find the particle's least possible flight time from O to B. Give your answer correct to 2 decimal places.	2
b.	Two unbiased dice are rolled.	
	i. Find the probability that the difference of the numbers on the dice is 1. (answer to 2 decimal places)	1
	ii. The two dice are rolled 60 times. For the binomial distribution of rolling a difference of 1, find the mean and the variance. (answer to 2 decimal places)	2
c.	**i.** If $t = \tan\frac{A}{2}$, express $3\sin A - 4\cos A - 4$ in terms of t	2
	ii. Hence solve $3\sin A - 4\cos A = 4$ for $0 \le A \le 2\pi$ (answer to 2 decimal places)	2
d.	Given $\sqrt{\dfrac{1-\sin 2x}{1+\sin 2x}} = \dfrac{1-\tan x}{1+\tan x}$ Show that $\tan\frac{\pi}{8} = \sqrt{2} - 1$	3

END OF TASK

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Suggested Solutions

Exam Sections:		**Marker**
Section A	10 marks	
Multiple Choice		JPN
Section B: Short Answer		
Question 11	15 marks	PPC
Question 12	15 marks	NHM
Question 13	15 marks	DPR
Question 14	15 marks	REM
Total	**70 marks**	

Multiple Choice

1. A
2. D
3. C
4. B
5. C
6. C
7. B
8. B
9. A
10. D

Multiple Choice Solutions

1. $P(-2) = -2(-2)^3 + 3(-2)^2 - 6(-2) + 4$
$= 44$

ANSWER A

2. from formula sheet

$$\tan^{-1}(4x-1) = \frac{4}{(1-4x)^2+1}$$
$$= \frac{4}{16x^2-8x+1+1}$$
$$= \frac{4}{16x^2-8x+2}$$
$$= \frac{2}{8x^2-4x+1}$$

ANSWER D

3. $f(x) = 3\cos^{-1} 4x$

Domain is affected by the 4

$\therefore$ Domain $\left[-\frac{1}{4}, \frac{1}{4}\right]$ ANSWER C

4. $\sin x = \frac{\sqrt{3}}{2}$ x is obtuse $\therefore$ $\cos x = -\frac{1}{2}$

$$\sin 2x = 2\sin x\cos x$$
$$= 2 \times \frac{\sqrt{3}}{2} \times -\frac{1}{2}$$
$$= -\frac{\sqrt{3}}{2}$$

ANSWER B

5.
$$2x + y = 2\cos^2 t + 2\sin^2 t$$
$$= 2(\cos^2 t + \sin^2 t)$$
$$y = 2 - 2x$$

Since $x = \cos^2 t$ $0 \leq x \leq 1$

$\therefore$ $y = 2 - 2x$ $[0, 1]$ ANSWER C

6. from reference sheet

$$\cos^2 4x = \frac{1}{2}(1+\cos 8x)$$

$$\therefore \int \cos^2 4x = \int \frac{1}{2}(1+\cos 8x)\,dx$$

$$= \frac{1}{2}\int (1+\cos 8x)\,dx$$

ANSWER (C)

7. $\underset{\sim}{u} \cdot \underset{\sim}{v} = |\underset{\sim}{u}||\underset{\sim}{v}| \cos\theta$

Since $\overrightarrow{AB} \cdot \overrightarrow{AC} > 0$ and $|\overrightarrow{AB}| \times |\overrightarrow{AC}| > 0$

$\therefore \cos\theta > 0$

$\therefore$ θ must be acute. ANSWER (B)

8. $\tan^{-1}(\tan x)$ is undefined when $x = \pm\frac{\pi}{2}, \pm\frac{3\pi}{2}$, etc

but defined for all other values

ANSWER (B)

9. $\alpha + \beta = -\frac{b}{a}$ $\qquad$ $\alpha\beta = \frac{c}{a}$

$\alpha + \alpha + 1 = -b$ $\qquad$ $\alpha(\alpha+1) = c$

$2\alpha + 1 = -b$ $\qquad$ $c = \alpha^2 + \alpha$

$\therefore b = -(2\alpha+1)$

ANSWER (A)

10. ANSWER (D)

Test numerous points and eliminate

eg. $x = -5$ $y = -1$ gradient positive $\therefore$ not A

$x = -3$ $y = -1$ gradient zero $\therefore$ not B

$x = 1$ $y = 1$ gradient zero $\therefore$ not C

Answer (D)

QUESTION 11 | Marker's Comments

(a) $\int \frac{1}{\sqrt{4-9x^2}}\,dx$

$= \int \frac{1}{\sqrt{9\left(\frac{4}{9}-x^2\right)}}\,dx$

$= \int \frac{1}{3\sqrt{\frac{4}{9}-x^2}}\,dx$

$= \frac{1}{3} \times \int \frac{1}{\sqrt{\left(\frac{2}{3}\right)^2-x^2}}\,dx$

$= \boxed{\frac{1}{3}\sin^{-1}\frac{3x}{2}+c}$ ✓

OR FROM REF. SHEET.

$\int \frac{f'(x)}{\sqrt{a^2-(f(x))^2}}\,dx = \sin^{-1}\frac{f(x)}{a}+c$

$\therefore \frac{1}{3}\int \frac{3\times 1}{\sqrt{2^2-(3x)^2}} = \frac{1}{3}\sin^{-1}\frac{3x}{2}+c$. ✓

Many missed the $\frac{1}{3}$ out front or put 3 out front!

(b)(i) $\sin\frac{23\pi}{12}$

$= \sin\left(\frac{5\pi}{3}+\frac{\pi}{4}\right)$

$= \sin\frac{5\pi}{3}\cos\frac{\pi}{4}+\cos\frac{5\pi}{3}\sin\frac{\pi}{4}$

$= -\frac{\sqrt{3}}{2}\times\frac{1}{\sqrt{2}} + \frac{1}{2}\times\frac{1}{\sqrt{2}}$

$= \frac{1-\sqrt{3}}{2\sqrt{2}}$

$= \boxed{\frac{\sqrt{2}-\sqrt{6}}{4}}$ ✓

✓ OR correct similar

The New Fx 8200 will do it for you

(b)(ii) $\sin^{-1}\left(\sin\frac{23\pi}{12}\right)$

$= \sin^{-1}\left(\sin\frac{-\pi}{12}\right)$

$= \boxed{-\frac{\pi}{12}}$ ✓

Range of $y=\sin^{-1}$

$-\frac{\pi}{2} \le y \le \frac{\pi}{2}$

$\therefore$ Not defined for $\frac{23\pi}{12}$ but is for $-\frac{\pi}{12}$.

(c)(i) $\underset{\sim}{u}\cdot\underset{\sim}{v} = x_1x_2+y_1y_2$

$\underset{\sim}{u}\cdot\underset{\sim}{v} = 1\times a - 2\times 2$

$= a-4$

perpendicular $\therefore a-4=0$

$\therefore \boxed{a=4}$ ✓

(i) Very well done

(ii) $|\underset{\sim}{u}| = \sqrt{1+(-2)^2}$ $\quad |\underset{\sim}{v}| = \sqrt{a^2+2^2}$

$= \sqrt{5}$ $\quad = \sqrt{a^2+4}$

$\underset{\sim}{u}\cdot\underset{\sim}{v} = |\underset{\sim}{u}||\underset{\sim}{v}|\cos\theta$

$a-4 = \sqrt{5}\times\sqrt{a^2+4}\times\cos 60^\circ$ ✓

$2a-8 = \sqrt{5a^2+20}$

$4a^2-32a+64 = 5a^2+20$

$0 = a^2+32a-44$ ✓

$a = \frac{-32\pm\sqrt{32^2-4(1)(-44)}}{2\times 1}$

$\boxed{a = 1.3 \text{ or } -33.3}$ ✓

Many problems with algebraic manipulations.

QUESTION 11 | Marker's Comments

(d) (i) $\underset{\sim}{u} \cdot \underset{\sim}{v} = 4 \times 6 + 2 \times -8$

$= 24 - 16$

$= 8$

$|\underset{\sim}{u}| = \sqrt{4^2 + 2^2}$

$= \sqrt{20}$

$\text{proj}_{\underset{\sim}{u}} \underset{\sim}{v} = \dfrac{\underset{\sim}{u} \cdot \underset{\sim}{v}}{|\underset{\sim}{u}|^2} \underset{\sim}{u}$

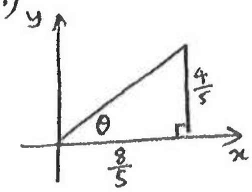
This formula is not on the Reference sheet. You need to learn it, and apply it, accurately.

$= \dfrac{8}{20} \times \begin{pmatrix} 4 \\ 2 \end{pmatrix}$

$= \begin{pmatrix} \frac{8}{5} \\ \frac{4}{5} \end{pmatrix}$

also accept $\dfrac{2}{5}\begin{bmatrix} 4 \\ 2 \end{bmatrix}$

(ii)



$\tan\theta = \frac{4}{5} \div \frac{8}{5}$

$\tan\theta = \frac{1}{2}$

$\theta = 26.565...^\circ$

Direction $= \theta = \boxed{27^\circ}$

Many who were in-correct in part (i) still managed part (ii) as the projection and $\underset{\sim}{u}$ have the same angle!

(e) $6x^3 - 9x^2 + 4x - 3 = 0$

Page 1 of Reference sheet!

(i) $\alpha + \beta + \gamma = \dfrac{-b}{a} = \dfrac{9}{6} = \boxed{\dfrac{3}{2}}$

(ii) $\alpha\beta + \alpha\gamma + \beta\gamma = \dfrac{c}{a} = \dfrac{4}{6} = \boxed{\dfrac{2}{3}}$

(i)(ii) Generally well done

(iii) $(\alpha+\beta+\gamma)^2 = \alpha^2 + \alpha\beta + \alpha\gamma + \alpha\beta + \beta^2 + \beta\gamma + \alpha\gamma + \beta\gamma + \gamma^2$

$= \alpha^2 + \beta^2 + \gamma^2 + 2(\alpha\beta + \alpha\gamma + \beta\gamma)$

$\therefore \alpha^2 + \beta^2 + \gamma^2 = (\alpha+\beta+\gamma)^2 + 2(\alpha\beta + \alpha\gamma + \beta\gamma)$

(iii) No so well done

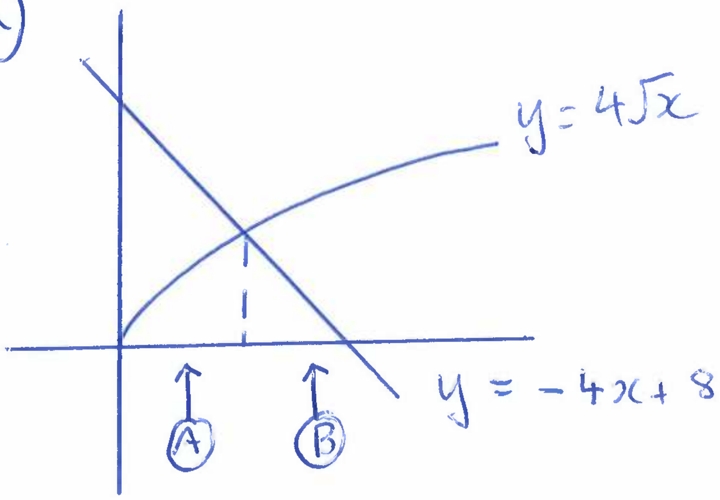
$= \left(\frac{3}{2}\right)^2 - 2\left(\frac{2}{3}\right)$

$= \frac{9}{4} - \frac{4}{3}$

$= \boxed{\frac{11}{12}}$

QUESTION 12

a)



Fairly well done.

Some careless errors.

i) $y = 4\sqrt{x}$

$y = -4x + 8$

$4\sqrt{x} = -4x + 8$

$16x = 16x^2 - 64x + 64$

$0 = 16x^2 - 80x + 64$

$= x^2 - 5x + 4$

$= (x-1)(x-4)$

$\therefore x = 1 \quad x = 4$

$y = 4 \quad y = -8$

$\therefore$ Point of Intersection

$(1, 4)$ ← 1mk

ii) $V = \pi \int_a^b y^2 \, dx$

Volume (A) $V = \pi \int_0^1 16x \, dx$

$= \pi \left[8x^2 \right]_0^1$

$= 8\pi$ units^3 ← 1mk

Volume (B)

$$V = \pi \int_1^2 (-4x+8)^2 \, dx$$

$$= \pi \int_1^2 (16x^2 - 64x + 64) \, dx$$

$$= \pi \left[\frac{16x^3}{3} - 32x^2 + 64x \right]_1^2$$

$$= \pi \left[\left(\frac{128}{3} \right) - \left(\frac{112}{3} \right) \right]$$

$$= \frac{16\pi}{3} \text{ units}^3$$

1mk

Many need more time revising the Volumes topic.

$$\text{Volume Total} = 8\pi + \frac{16\pi}{3}$$

$$= \frac{40\pi}{3} \; u^3$$

1mk

4

b) $$\frac{3x}{x+3} \le -2$$

$$3x(x+3) \le -2(x+3)^2$$

$$2(x+3)^2 + 3x(x+3) \le 0$$

$$(x+3)(2(x+3)+3x) \le 0$$

$$(x+3)(5x+6) \le 0$$

$$-3 < x \le -\frac{6}{5}$$

Too many had forgotten method.

$$-3 < x \le -\frac{6}{5}$$

1mk 1mk 1mk

3

c)

$$\int_0^{\frac{\pi}{4}} \frac{1}{(1+2\tan x)^2 \cos^2 x}\, dx$$

$u = 1+2\tan x$

$\frac{du}{dx} = 2\sec^2 x$

$du = 2\sec^2 x\, dx$

$\frac{1}{2} du = \frac{1}{\cos^2 x} dx$

$x = 0 \quad u = 1$

$x = \frac{\pi}{4} \quad u = 3$

$$\therefore \int_0^{\frac{\pi}{4}} \frac{1}{(1+2\tan x)^2 \cos^2 x}\, dx = \frac{1}{2}\int_1^3 \frac{1}{u^2}\, du$$

$$= \frac{1}{2}\int_1^3 u^{-2}\, du$$

$$= \frac{1}{2}\left[-\frac{1}{u}\right]_1^3$$

$$= \frac{1}{2}\left[\left(-\frac{1}{3}\right)-(-1)\right]$$

$$= \frac{1}{3}.$$

Poorly done

1mk

1mk for correct integral

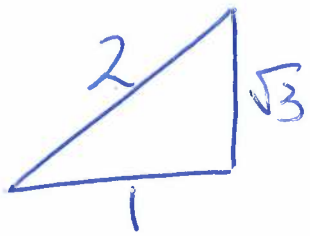
1mk.

3

d) i) $\cos x - \sqrt{3}\sin x$

$= R\cos(x+\alpha)$

Well done



$R^2 = 1^2 + (\sqrt{3})^2$ $\quad$ $\tan\alpha = \frac{\sqrt{3}}{1}$

$R = 2$ $\quad$ $\alpha = \frac{\pi}{3}$

$\therefore \cos x - \sqrt{3}\sin x = 2\cos(x + \frac{\pi}{3})$ ← 1mk

ii) $f(x) = \cos x - \sqrt{3}\sin x + 10$

$= \underbrace{2\cos(x+\frac{\pi}{3})}_{max = 2} + 10$

Well done

$\therefore$ Max $= 12$ ← 1mk

2

e)

$$y = 2x(x+6)^6$$

$$x = 2y(y+6)^6$$ 1mk

$$\frac{dx}{dy} = 2y\left[6(y+6)^5\right] + [y+6]^6 \cdot 2$$ 1mk for Correct use of Product rule

$$= (y+6)^5 \cdot (12y + 2(y+6))$$

$$= (y+6)^5 \cdot (14y + 12)$$

$$= 2(y+6)^5(7y+6)$$

$$\therefore \frac{dy}{dx} = \frac{1}{2(y+6)^5(7y+6)}$$ 1mk

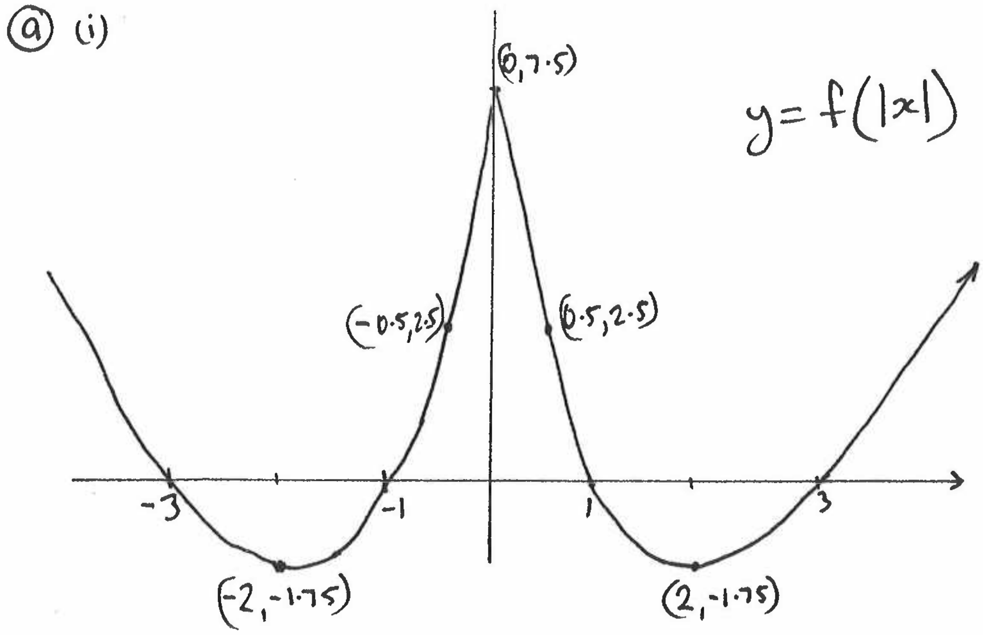
Desirable to go to $\frac{dy}{dx} =$

but no penalty if left at <u>CORRECT</u> $\frac{dx}{dy}$

QUESTION 13

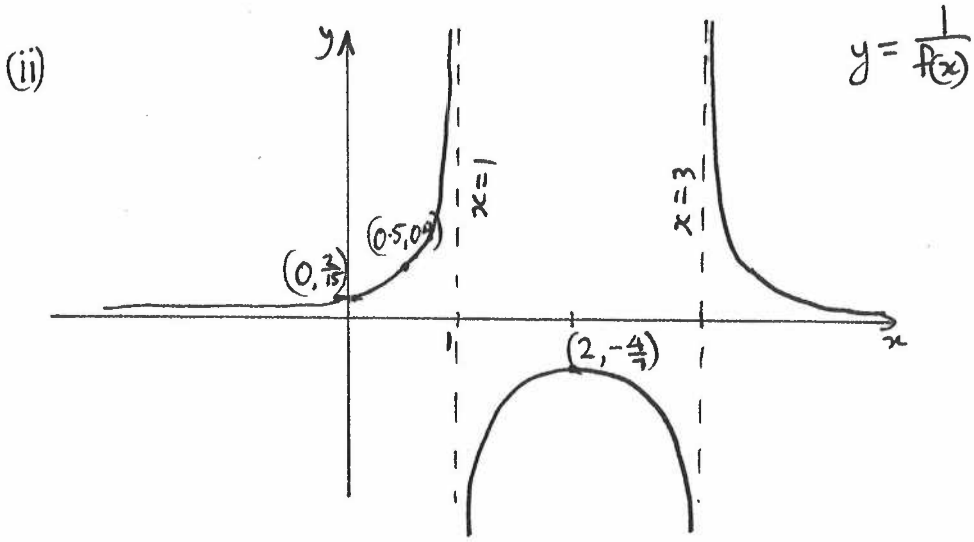
Marker's Comments

(a) (i)



* Well answered.
* 1 mark for correct shape.
* 1 mark for including coordinates of intercepts and turning points.

(ii)



* Well answered
* 1 mark for correct shape
* 1 mark for including coordinates of intercepts and turning points.

(b) $\frac{dy}{dx} = 4x^3y^2$

$\frac{1}{y^2}\,dy = 4x^3\,dx$

$\int y^{-2}\,dy = \int 4x^3\,dx$

$\frac{y^{-1}}{-1} + c_1 = x^4 + c_2$

$-\frac{1}{y} = x^4 + k$

when $x=2$, $y=1$

$-\frac{1}{1} = 2^4 + k$

$-1 = 16 + k$

$k = -17$

$\therefore -\frac{1}{y} = x^4 - 17$

$\boxed{y = \frac{1}{17 - x^4}}$

* Well answered.
* some students did not calculate the constant correctly but received carry on marks.
* some students did not set up the differentials correctly at the start but received carry on marks afterwards.

QUESTION 13

Marker's Comments

ⓒ Prove $7^{2n}+7^n+4$ is divisible by 6 for all integers $n\geq 0$

Step 1: Prove true for $n=0$

$7^{2\times 0}+7^0+4=1+1+4=6$ which is divisible by 6

$\therefore$ true for $n=0$

Step 2: Assume true for $n=k$

ie $7^{2k}+7^k+4=6M$ where M is a positive integer

ie $7^{2k}=6M-7^k-4$

Step 3: Prove true for $n=k+1$

ie $7^{2(k+1)}+7^{(k+1)}+4=6P$ where P is a positive integer

$LHS=7^{2k+2}+7^{k+1}+4$

$=7^{2k}\times 7^2+7^{k+1}+4$

$=49(6M-7^k-4)+7\times 7^k+4$ ... from assumption in step2

$=294M-49\times 7^k-196+7\times 7^k+4$

$=294M-42\times 7^k-192$

$=6(49M-7\times 7^k-32)$

$=6P$ since M and k are integers.

If true for $n=k$, then true for $n=k+1$

Since it is true for $n=0$, then it is true for $n=1$.

If true for $n=1$, then it is true for $n=2$ and so on

for all posit integers $n\geq 0$

* Well answered.

* Some students substituted n=1 for the first step and got penalised.

QUESTION 13

Marker's Comments

(d) (i)
$$\frac{dT}{dt} = -0.05(T-23)$$
$$\frac{dt}{dT} = \frac{-1}{0.05(T-23)}$$
$$= \frac{-20}{T-23}$$
$$t = -20\int \frac{1}{T-23}\, dT$$
$$= -20\ln|T-23| + c$$
when $t=0$ $T=200$
$$0 = -20\ln|200-23| + c$$
$$c = 20\ln 177$$
$$\therefore\ t = -20\ln|T-23| + 20\ln 177$$
$$t = 20(\ln 177 - \ln|T-23|)$$
$$t = 20\ln\frac{177}{T-23}$$
$$\frac{t}{20} = \ln\frac{177}{T-23}$$
$$e^{\frac{t}{20}} = e^{\ln\frac{177}{T-23}}$$
$$\frac{177}{T-23} = e^{\frac{t}{20}}$$
$$177 = (T-23)e^{\frac{t}{20}}$$
$$177e^{-\frac{t}{20}} = T-23$$
$$T = 23 + 177e^{-\frac{t}{20}}$$
$$\boxed{T = 23 + 177e^{-0.05t}}$$

* Well answered

* Some students incorporated the ~~power~~ constant of integration into the exponential power when calculating and lost marks

(ii) when $t = 20$
$$T = 23 + 177e^{-0.05\times 20}$$
$$\boxed{T = 88.11^\circ C}$$

* Well answered

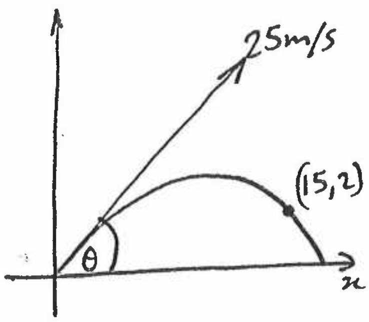
(iii) when $T = 40$
$$40 = 23 + 177e^{-0.05t}$$
$$17 = 177e^{-0.05t}$$
$$\frac{17}{177} = e^{-0.05t}$$
$$\ln\frac{17}{177} = -0.05t$$
$$t = \frac{\ln\frac{17}{177}}{-0.05} = \boxed{46.86\text{ minutes}}$$

46 minutes 52 seconds

* Well answered.

QUESTION 14 | Marker's Comments

(a)



When $t=T$
$x=15$

(i) $x = Vt\cos\theta$

$15 = 25t\cos\theta$

$T = \frac{15}{25\cos\theta}$ → 1 Mark

$T = \frac{3}{5}\sec\theta$ (as required)

When $t=T$
$y=2$

(ii) $y = Vt\sin\theta - \frac{1}{2}\times 10\times t^2$

$2 = 25\times\frac{3}{5}\sec\theta\sin\theta - 5\left(\frac{3}{5}\sec\theta\right)^2$

$2 = 15\tan\theta - 5\times\frac{9}{25}\sec^2\theta$ → 1 Mark

$2 = 15\tan\theta - \frac{9}{5}\sec^2\theta$

$10 = 75\tan\theta - 9(1+\tan^2\theta)$ → 1 Mark.

$10 = 75\tan\theta - 9 - 9\tan^2\theta$

$9\tan^2\theta - 75\tan\theta + 19 = 0$ (as required)

(iii) $\tan\theta = \frac{75 \pm \sqrt{75^2 - 4(9)(19)}}{2\times 9}$

$= \frac{75\pm\sqrt{4941}}{18}$

$\tan\theta = 0.2615418\ldots$ or $\tan\theta = 8.07179\ldots$ → 1 Mark.

$\theta = 14.6569\ldots^\circ$ or $\theta = 82.9377\ldots^\circ$

$\therefore T = \frac{3}{5\cos 14.656\ldots^\circ}$ $\quad T = \frac{3}{5\cos 82.9377\ldots^\circ}$

$= 0.62$ seconds (2d.p) $\quad = 4.88$ seconds

$\therefore$ Least possible flight time from O to B is $\boxed{0.62 \text{ seconds}}$ — Correct Answer.

QUESTION 14

Marker's Comments

(b) (i)

2nd Die \ 1st Die (Difference)	1	2	3	4	5	6
1	0	1	2	3	4	5
2	1	0	1	2	3	4
3	2	1	0	1	2	3
4	3	2	1	0	1	2
5	4	3	2	1	0	1
6	5	4	3	2	1	0

$P(\text{difference} = 1) = \frac{10}{36}$

$= \boxed{\frac{5}{18}}$ OR $0.28 \longrightarrow$ 1 Mark

(ii) mean $= E(X) = 60 \times \frac{5}{18}$

$= \frac{50}{3} = 16\frac{2}{3} = \boxed{16.\dot{6}}$ OR $60 \times 0.28 \longrightarrow$ ✓

$= 16.67$

$= 16.80 \longrightarrow$ ✓

(iii) Variance $= Var(X) = np(1-p)$

$= 60 \times \frac{5}{18}\left(1 - \frac{5}{18}\right)$ OR $60 \times 0.28 \times (1 - 0.28) \longrightarrow$ ✓

$= \frac{325}{27}$ $\doteqdot$ $12.10 \longrightarrow$ ✓

$= \boxed{12.04}$ (2 d.p)

QUESTION 14

Marker's Comments

(c) (i) $3\sin A - 4\cos A - 4$

$= 3\times\frac{2t}{1+t^2} - \frac{4\times(1-t^2)}{1+t^2} - 4$ → 1 Mark

$= \frac{6t - 4 + 4t^2 - 4 - 4t^2}{1+t^2}$

$= \boxed{\frac{6t-8}{1+t^2}}$ → 1 Mark

(ii) $3\sin A - 4\cos A = 4$

$3\sin A - 4\cos A - 4 = 0$

$\therefore \frac{6t-8}{1+t^2} = 0$

$6t - 8 = 0$

$6t = 8$

$t = \frac{4}{3}$

1 Mark

* NOTE: Need to Check $A=\pi$ as a solution ✓

$LHS = 3\sin\pi - 4\cos\pi$
$= 0 + 4 = 4$
$= RHS$
$\therefore A = \pi$ is a solution

$\tan\frac{A}{2} = \frac{4}{3}$

$0 \le A \le 2\pi$

$\therefore 0 \le \frac{A}{2} \le \pi$ (only 1st Q)

$\frac{A}{2} = 0.9272$

$\boxed{A = 1.85}$ (2d.p) → 1 Mark Correct answer

(d) $\sqrt{\frac{1-\sin 2x}{1+\sin 2x}} = \frac{1-\tan x}{1+\tan x}$

let $x = \frac{\pi}{8}$ $\therefore 2x = \frac{\pi}{4}$ (since $\tan\frac{\pi}{4} = 1$)

$\sqrt{\frac{1-\sin\frac{\pi}{4}}{1+\sin\frac{\pi}{4}}} = \frac{1-\tan\frac{\pi}{8}}{1+\tan\frac{\pi}{8}}$ → 1 Mark

$\sqrt{\frac{1-\frac{1}{\sqrt2}}{1+\frac{1}{\sqrt2}}} = \frac{\tan\frac{\pi}{4} - \tan\frac{\pi}{8}}{1+\tan\frac{\pi}{4}\tan\frac{\pi}{8}}$ → 1 Mark

$\sqrt{\frac{\sqrt2-1}{\sqrt2+1}} = \tan\left(\frac{\pi}{4}-\frac{\pi}{8}\right)$

$\sqrt{\frac{\sqrt2-1}{\sqrt2+1}\times\frac{\sqrt2-1}{\sqrt2-1}} = \tan\frac{\pi}{8}$

$\sqrt{\frac{(\sqrt2-1)^2}{2-1}} = \tan\frac{\pi}{8}$

$\sqrt2 - 1 = \tan\frac{\pi}{8}$ as required.

1 Mark for correct simplification of right hand side.

Alternatively!

$\sqrt2 - 1 = \frac{1-\tan\frac{\pi}{8}}{1+\tan\frac{\pi}{8}}$

$\sqrt2 + \sqrt2\tan\frac{\pi}{8} - 1 - \tan\frac{\pi}{8} = 1 - \tan\frac{\pi}{8}$ ✓

$\sqrt2\tan\frac{\pi}{8} = 2 - 2\sqrt2$

$\tan\frac{\pi}{8} = \frac{2-\sqrt2}{\sqrt2}$ ✓

$= \sqrt2 - 1$

* Can square both sides & form a quadratic in $\tan\frac{\pi}{8}$