Student's Name: ________________________ Teacher's Code: ________________

Saint Ignatius' College, Riverview

Mathematics Assessment Task

2025

Year 12
Mathematics (Extension One)
Task 4
Trial HSC Examination
Date: Wednesday 13th August 2025

General Instructions:

- **Reading time: 10 minutes**
- **Time Allowed: 2 hours.**
- Write using a black pen.
- Calculators approved by NESA may be used.
- Attempt all questions in the booklets provided.
- Write **your name** and **your teacher's code** in the positions indicated.
- Marks may not be awarded for missing or carelessly arranged work.

Teachers:

- Mr R Maxwell **REM**
- Mr D Reidy **DPR**
- Mr S Maher **SJM**
- Mr P Collins **PPC**

Topics Examined:

Section A	**10 Marks**
Multiple Choice	
Section B	
Short Answer	
Question 11	**15 Marks**
Question 12	**15 Marks**
Question 13	**15 Marks**
Question 14	**15 Marks**
Total	**70 Marks**

BLANK PAGE

SECTION A: Multiple Choice Questions 1 - 10: **MARKS 10**

1. The multiplicity of the negative root of the polynomial $y = (x - 3)^3(x - 2)(x + 4)^4$ is

A. 1

B. 2

C. 3

D. 4

2. The magnitude of the projection of the vector $\underset{\sim}{\mathbf{a}} = \underset{\sim}{\mathbf{i}} + 4\underset{\sim}{\mathbf{j}}$ in the direction of the vector $\underset{\sim}{\mathbf{b}}$ is $\dfrac{11}{\sqrt{13}}$.

Which of the following could be equal to $\underset{\sim}{\mathbf{b}}$?

A. $2\underset{\sim}{\mathbf{i}} + 3\underset{\sim}{\mathbf{j}}$

B. $3\underset{\sim}{\mathbf{i}} + 2\underset{\sim}{\mathbf{j}}$

C. $3\underset{\sim}{\mathbf{i}} + 4\underset{\sim}{\mathbf{j}}$

D. $4\underset{\sim}{\mathbf{i}} + 3\underset{\sim}{\mathbf{j}}$

3. What is the exact value of $\sin 75°$

A. $\dfrac{1+\sqrt{3}}{2\sqrt{2}}$

B. $\dfrac{1-\sqrt{3}}{2\sqrt{2}}$

C. $\dfrac{\sqrt{2}-\sqrt{6}}{2}$

D. $\dfrac{\sqrt{2}+\sqrt{6}}{2}$

4. The value of $cos^{-1}\left(-\frac{1}{2}\right)$ is:

A. $\frac{\pi}{3}$

B. $-\frac{\pi}{3}$

C. $\frac{2\pi}{3}$

D. $\frac{4\pi}{3}$

5. Using the trigonometric product to sum result, the value of $\int sin\ 3x sin x dx$ is:

A. $\frac{1}{4}sin\ 2x - \frac{1}{8}sin\ 4x\ + c$

B. $\frac{1}{8}sin\ 2x - \frac{1}{4}sin\ 4x\ + c$

C. $\frac{1}{4}cos\ 2x - \frac{1}{8}cos\ 4x\ + c$

D. $\frac{1}{8}cos\ 4x - \frac{1}{4}cos\ 2x\ + c$

6. Consider the equation $x^2 + 2x + 3 = 0$. The sum of each of the reciprocal roots of this equation are:

A. $\frac{2}{3}$

B. $-\frac{2}{3}$

C. $\frac{1}{2}$

D. $-\frac{1}{2}$

7. If $f(x) = \dfrac{3 + e^{2x}}{5}$, the expression for $f^{-1}(x)$ is:

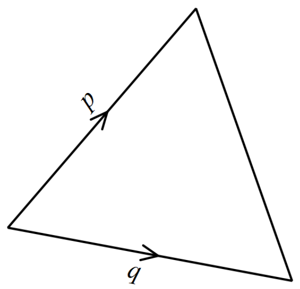
A. $ln(5x - 3)$

B. $\frac{1}{2}ln(5x - 3)$

C. $ln\ 5x - ln\ 3x$

D. $\frac{1}{2}(ln\ 5x - ln\ 3x)$

8. An equilateral Δ of side length 3 units is shown below.



Vectors $\underset{\sim}{\mathbf{p}}$ and $\underset{\sim}{\mathbf{q}}$ are represented in the diagram, the value of $\underset{\sim}{\mathbf{p}} \cdot \underset{\sim}{\mathbf{q}}$ is:

A. 9

B. $\dfrac{9}{\sqrt{2}}$

C. 0

D. $\dfrac{9}{2}$

9. Given that $x = 40t$ and $y = 56t - 16t^2$, then the expression for $\frac{dy}{dx}$ is given by:

A. $\frac{dy}{dx} = \frac{5}{7-4t}$

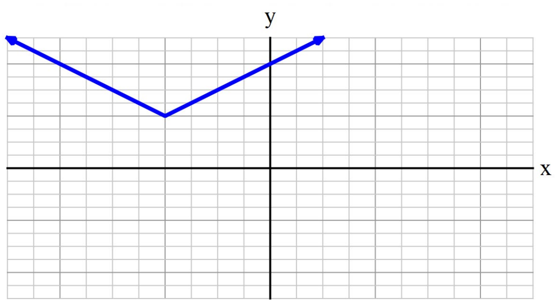
B. $\frac{dy}{dx} = \frac{7-4t}{5}$

C. $\frac{dy}{dx} = 5 \times (7-4t)$

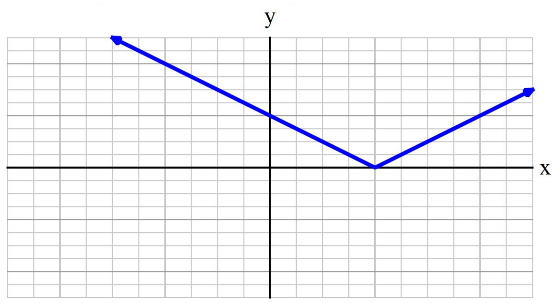
D. $\frac{dy}{dx} = \frac{1}{5(7-4t)}$

10. The graph of $y = |2 - x|$ is:

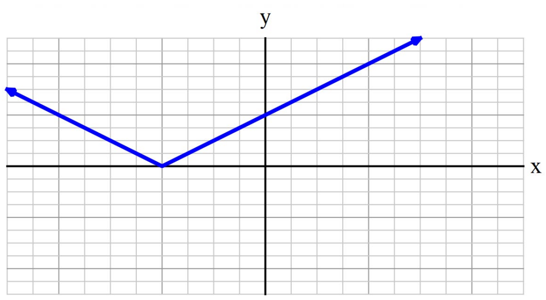
A



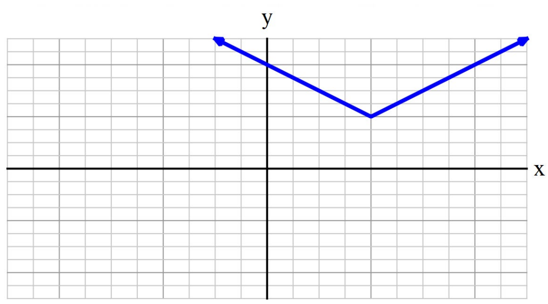
B



C



D



END OF SECTION A

SECTION B: QUESTIONS 11 TO 14. (15 MARKS EACH.)

PLEASE ANSWER EACH QUESTION ON SEPARATE BOOKLETS.

Question 11 Start on a new answer booklet **15 Marks**

(a) Solve for x, given $\dfrac{1-2x}{1+x} \geq 1$ 3

(b) When a polynomial $P(x)$ is divided by $x^2 - 16$ the remainder is $3x - 1$.
What is the remainder when $P(x)$ is divided by $(x - 4)$. 2

(c) Find $\dfrac{d}{dx}(e^x tan^{-1} x)$ 2

(d) The equation $I(t) = 4sin\ 0.02t + 8cos\ 0.02t$ measures the current, I amperes, in a particular circuit after t seconds.

(i) Express the function in the form $I(t) = Asin(at + b)$. for $b = \left[0, \dfrac{\pi}{2}\right]$ 2

(ii) When does the current first reach its maximum value to the nearest second? 2

(e) Find the equation of the curve $r = r(\theta)$, which satisfies the differential equation
$\dfrac{dr}{d\theta} = \dfrac{r^2}{\theta}$, with initial conditions $r(1) = 2$. 2

(f) Sketch $y = 2sin^{-1}x + \dfrac{\pi}{2}$. Showing all intercepts. 2

End of Question 11

Question 12 Start on a new answer booklet **15 Marks**

(a) Factorise and solve $2x^3 + x^2 - 7x - 6 = 0$. 2

(b) Show that $tan\left(cos^{-1}\left(\frac{2}{3}\right) + tan^{-1}\left(-\frac{3}{4}\right)\right) = \frac{4\sqrt{5} - 6}{3\sqrt{5} + 8}$. 3

(c) Prove by the principal of Mathematical Induction for all integers $n \geq 1$.

$$x + x^3 + x^5 + \ldots + x^{2n-1} = \frac{x(x^{2n} - 1)}{(x-1)(x+1)}.$$ 3

(d) $\Delta\ ABC$ has vertices $A(-4,7)$, $B(3,6)$ and $C(-5,0)$.

(i) Express $\overrightarrow{AB}$ and $\overrightarrow{AC}$ in column vector form. 2

(ii) Find the magnitude of $\overrightarrow{AB}$ and $\overrightarrow{AC}$. 2

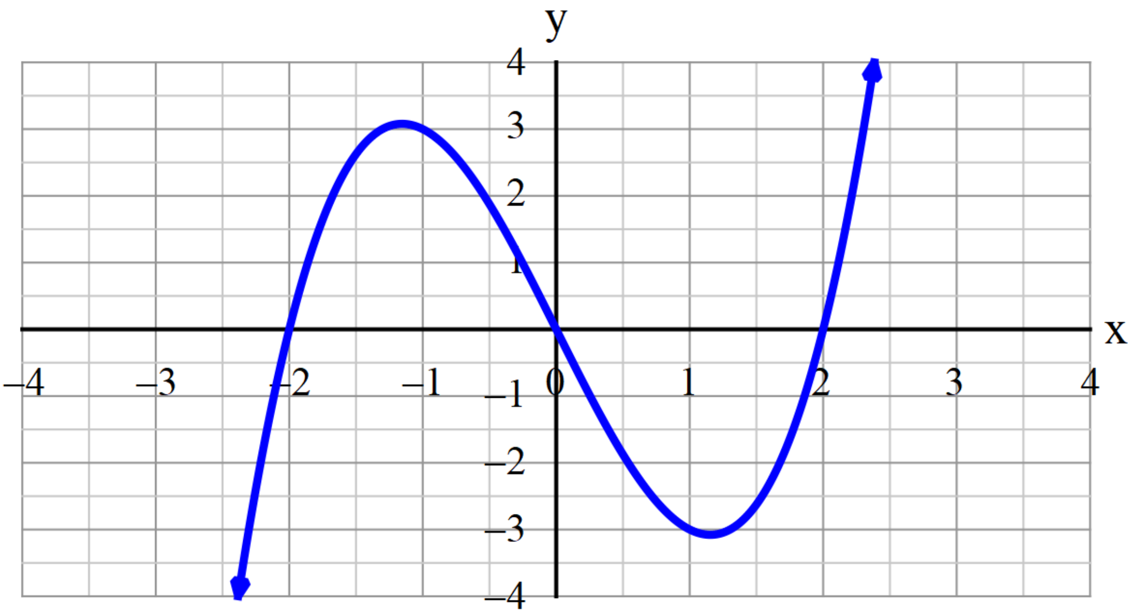
(iii) Find $\angle BAC$. 2

(iv) Hence calculate the area of Δ ABC. 1

End of Question 12

Question 13 Start on a new answer booklet **15 Marks**

(a) The diagram below shows the graph of a function $y = f(x)$.
The function has zeros at $x = -2$, $x = 0$ and $x = 2$ and a y-intercept of 0 as shown.



(i) Sketch the graph of $y = f(x)$ and $y = |f(x)|$ on the same axis.
Showing all essential features: intercepts, asymptotes. 1

(ii) Sketch the graph of $y = f(x)$ and $y = \dfrac{1}{f(x)}$ on the same axis. 3
Showing all essential features: intercepts, asymptotes.
Your graph in both cases should be at least half a page each.

(b) The graph of the function $y = sin^{-1}(x - 3)$ is transformed by being dilated by a factor 3 horizontally and then translated to the right by 3 units.
Find the equation of the transformed path. 2

(c) (i) Find $\dfrac{d}{dx}\left[\dfrac{2x}{4+x^2} + tan^{-1}\dfrac{x}{2}\right]$,

write your answer in the form $\dfrac{A}{(4+x^2)^2}$ 2

(ii) Hence evaluate $\displaystyle\int_0^2 \frac{dx}{(4+x^2)^2}$. 2

(*d*) A fast food chain has estimated that if they spend x on advertising their new product. It will attract a proportion $y(x)$ of the potential customers for the product.

where $\dfrac{dy}{dx} = ay(1-y)$. $a > 0$ is a given constant.

(*i*) Show that $\dfrac{1}{y} + \dfrac{1}{1-y} = \dfrac{1}{y(1-y)}$. 1

(*ii*) Using part (*i*) or otherwise,

show that $y(x) = \dfrac{1}{ke^{-ax} + 1}$ for some constant $k > 0$. 3

(*iii*) The fast food chain knows that if they do not spend the money on advertising the product then the number of customers will be one-fifth of the potential customers. Hence find the value of the constant k in part (*ii*). 1

End of Question 13

Question 14 Start on a new answer booklet **15 Marks**

(a) Find the exact value of $tan\left(2cos^{-1}\frac{12}{13}\right)$ 2

Showing all appropriate calculations.

(b) (i) Use the substitution $t = tan\frac{x}{2}$ to show that cosec $x + cot\, x = cot\frac{x}{2}$. 2

(ii) Hence or otherwise evaluate $\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} (cosec\, x + cot\, x)dx$.

Answer in the simplest exact form. 3

(c) The points P, Q and R have position vectors $\underset{\sim}{\mathbf{p}}$, $\underset{\sim}{\mathbf{q}}$ and $\underset{\sim}{\mathbf{r}}$ respectively.

λ and μ are non-zero numbers such that $\lambda \underset{\sim}{\mathbf{p}} + \mu \underset{\sim}{\mathbf{q}} - \underset{\sim}{\mathbf{r}} = 0$ and $\lambda + \mu = 1$

(i) Show that points P, Q and R are collinear. 2

(ii) It is known that $\left|\underset{\sim}{\mathbf{p}}\right| = 2$, the angle between $\underset{\sim}{\mathbf{p}}$ and $\underset{\sim}{\mathbf{q}}$ is acute and the area of $\Delta\ OPQ$ is k $units^2$.

Show that $\left(\underset{\sim}{\mathbf{p}}.\underset{\sim}{\mathbf{q}}\right)^2 = 4\left(\left|\underset{\sim}{\mathbf{q}}\right|^2 - k^2\right)$. 3

Point S lies on the line segment PR and has a position vector $\underset{\sim}{\mathbf{s}}$.

(iii) Given that $k = 6$, $\left|\underset{\sim}{\mathbf{q}}\right| = 10$ and $\angle POS = 90°$. Find $\underset{\sim}{\mathbf{s}}$ in terms of $\underset{\sim}{\mathbf{p}}$ and $\underset{\sim}{\mathbf{q}}$. 3

End of Question 14

End of Task 4

2025

BLANK PAGE

Student's Name: ________________________ Teacher's Code: ________________

SUGGESTED SOLUTIONS

Saint Ignatius' College, Riverview

Mathematics Assessment Task

2025

Year 12
Mathematics (Extension One)
Task 4
Trial HSC Examination
Date: Wednesday 13th August 2025

General Instructions:

- **Reading time: 10 minutes**
- **Time Allowed: 2 hours.**
- Write using a black pen.
- Calculators approved by NESA may be used.
- Attempt all questions in the booklets provided.
- Write **your name** and **your teacher's code** in the positions indicated.
- Marks may not be awarded for missing or carelessly arranged work.

Teachers:

- Mr R Maxwell **REM**
- Mr D Reidy **DPR**
- Mr S Maher **SJM**
- Mr P Collins **PPC**

Topics Examined:

Section A Multiple Choice	**10 Marks**
Section B Short Answer	
Question 11	**15 Marks**
Question 12	**15 Marks**
Question 13	**15 Marks**
Question 14	**15 Marks**
Total	**70 Marks**

Start your answer here.

1. $y = (x-3)^3(x-2)(x+4)^4$

Roots: $x=3$ +, multiplicity $=2$

$x=2$ +, " $=1$

$x=-4$ −, " $=4$

∴ D.

2. For scalar projection $\frac{\bar{a}\cdot\bar{b}}{|b|} = \frac{11}{\sqrt{13}}$

$|\bar{b}|$: $A=\sqrt{13}$, $B=\sqrt{13}$, $C=\sqrt{25}$, $D=\sqrt{25}$

∴ Either A or B.

Check A: $\bar{a}\cdot\bar{b} = (1)(2)+(4)(3) = 14$ ✗

B: $\bar{a}\cdot\bar{b} = (1)(3)+(4)(2) = 11$ ✓

∴ B.

3. $\sin 75^\circ = \sin(45+30)$

$= \sin 45 \cdot \cos 30 + \cos 45 \cdot \sin 30$

$= \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \cdot \frac{1}{2}$

$= \frac{\sqrt{3}}{4} + \frac{1}{2\sqrt{2}}$

$= \frac{1+\sqrt{3}}{2\sqrt{2}}$

∴ A

4. $\cos^{-1}\left(-\frac{1}{2}\right) = \pi - \cos^{-1}\left(\frac{1}{2}\right)$

$= \pi - \frac{\pi}{3}$

$= \frac{2\pi}{3}$

$\therefore$ C

5. $\int \sin 3x \sin x \, dx$ use formula $\sin A \sin B$

$= \frac{1}{2}\left[\cos(3x-x) - \cos(3x+x)\right]$

$= \frac{1}{2} \cos 2x - \cos 4x$

$\therefore = \frac{1}{2}\int \cos 2x - \cos 4x \, dx$

$= \frac{1}{2} \frac{\sin 2x}{2} - \frac{1}{2} \frac{\sin 4x}{4} + c$

$= \frac{\sin 2x}{4} - \frac{\sin 4x}{8} + c$

$\therefore$ A

6. $x^2 + 2x + 3 = 0$

$\alpha + \beta = -2 \quad \alpha\beta = 3$

$\therefore \frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha+\beta}{\alpha\beta} = \frac{-2}{3}$

$\therefore$ B

Additional writing space on back page.

y

7. $y = \frac{3+e^{2x}}{5}$

$\therefore x = \frac{3+e^{2y}}{5}$

$3+e^{2y} = 5x$

$e^{2y} = 5x-3$

$2y = \ln|5x-3|$

$y = \frac{1}{2}\ln|5x-3|$

$\therefore B$

8. $\bar{p}\cdot\bar{q} = |\bar{p}|\cdot|\bar{q}|\cos\theta = 3\times3\times\cos60^0$

$= \frac{9}{2}$

$\therefore D$

9. $x = 40t$, $y = 56t-16t^2$

$dx = 40dt$, $dy = (56-32t)dt$

$\therefore \frac{dy}{dx} = \frac{(56-32t)\cancel{dt}}{40\cancel{dt}} = \frac{\cancel{8}(7-4t)}{\cancel{8}\times5}$

$\therefore B$

10. Graph: $x=2$, $y=0$

$\therefore B$

You may ask for an extra Writing Booklet if you need more space to answer this question.

Student's Name: Solutions Teacher's Code: Ext1: Trials 2025

Question Number	Marker use only
	Q11

(a)	
(b)	
(c)	
(d)	
(e)	
(f)	
(g)	
(h)	

Saint Ignatius' College
RIVERVIEW

Mathematics Extension 1

Writing Booklet

Instructions

- Start each question in a new booklet.
- Except for multiple choice answers, you should show relevant mathematical reasoning and/or calculations.
- You may ask for an extra Writing Booklet to answer this question if you need more space.
- If you do not attempt a question you must still hand in a Writing Booklet, with your Name, your Teacher's Name, the Question Number the words 'NOT ATTEMPTED' written clearly on the front cover.
- Write the number of each section part inside the margin at the beginning of each answer.
- Write using black or blue pen.
- Write on the ruled pages only.

You may not remove any Writing Booklets, used or unused, from the examination room.

Start your answer here.

✓ (a) $\dfrac{1-2x}{1+x} \ge 1$ $\qquad 1+x \ne 0 \quad \therefore x \ne -1$

$\therefore \dfrac{(1-2x)(1+x)}{(1+x)^2} \ge 1$

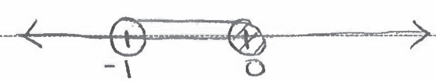
$(1-2x)(1+x) \ge (1+x)^2$

$(1-2x)(1+x) - (1+x)^2 \ge 0$

$(1+x)[1-2x-1-x] \ge 0$

$(1+x)(-3x) \ge 0$

$3x(1+x) \le 0$



$-1 < x \le 0 \quad$ or $\quad (-1, 0]$

✓ (b) $(x^2-16)\cdot Q(x) + (3x-1)$

$P(4) = 0 \cdot Q(x) + 3 \times 4 - 1$

$= 11$

✓ (c) $\dfrac{d}{dx}\left(e^x \tan^{-1} x\right)$ $\qquad u = e^x \quad v = \tan^{-1} x$

$\qquad u' = e^x \quad v' = \dfrac{1}{1+x^2}$

$\therefore e^x \tan^{-1} x + e^x \times \dfrac{1}{1+x^2}$

or $\quad e^x \left(\tan^{-1} x + \dfrac{1}{1+x^2}\right)$

~~(d)~~ 11 (d)

(i) $I(t) = 4\sin(0{\cdot}02t) + 8\cos(0{\cdot}02t)$

$A = \sqrt{4^2+8^2} = \sqrt{80} = 4\sqrt{5}$

$\tan\theta = \frac{8}{4}$ $\quad \theta = \tan^{-1}\left(\frac{8}{4}\right) = \tan^{-1} 2$

$I(t) = 4\sqrt{5}\sin(0{\cdot}02t + \tan^{-1}2)$

(ii) Max. Value at $\sin\theta = \frac{\pi}{2}$

$\therefore\ 0{\cdot}02t + \tan^{-1}(2) = \frac{\pi}{2}$

$0{\cdot}02t = \frac{\pi}{2} - \tan^{-1}2$

$t = \dfrac{\frac{\pi}{2} - \tan^{-1}2}{0{\cdot}02} = 23$ Seconds.

(1st max)

Additional writing space on back page.

Start your answer here.

11 (e) $\dfrac{dr}{d\theta} = \dfrac{r^2}{\theta}$

$\therefore \displaystyle\int \frac{1}{r^2}\,dr = \int \frac{d\theta}{\theta}$

$-\dfrac{1}{r} = \ln\theta + c \qquad \theta = 1,\ r = 2$

$\therefore -\dfrac{1}{2} = \ln 1 + c \qquad c = -\dfrac{1}{2}$

$\therefore -\dfrac{1}{r} = \ln|\theta| - \dfrac{1}{2}$

$\therefore \dfrac{1}{r} = \dfrac{1}{2} - \ln|\theta|$

$\dfrac{1}{r} = \dfrac{1 - 2\ln|\theta|}{2}$

$r = \dfrac{2}{1 - 2\ln|\theta|}$

OR.

$\dfrac{1}{r^2}\,dr = \dfrac{1}{\theta}\,d\theta$

$\therefore \displaystyle\int_2^r \frac{1}{r^2}\,dr = \int_1^\theta \frac{1}{\theta}\,d\theta.$

$\left[-\dfrac{1}{r}\right]_2^r = \left[\ln|\theta|\right]_1^\theta$

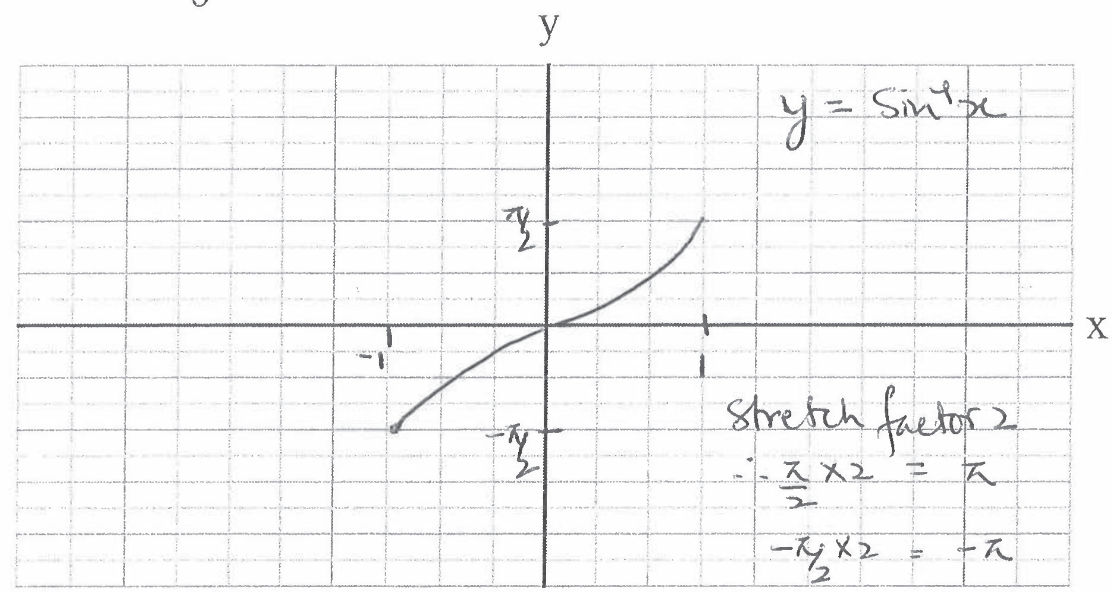
$-\dfrac{1}{r} + \dfrac{1}{2} = \ln|\theta| - \ln 1$

$\dfrac{1}{r^2} = \dfrac{1}{2} - \ln|\theta|$

$\dfrac{1}{r} = \dfrac{1 - 2\ln|\theta|}{2}$

$\therefore\ r = \dfrac{2}{1 - 2\ln|\theta|}$

(f) $y = 2\sin^{-1}x + \pi/2$



$y = \sin^{-1}x$

stretch factor 2

$\therefore \frac{\pi}{2} \times 2 = \pi$

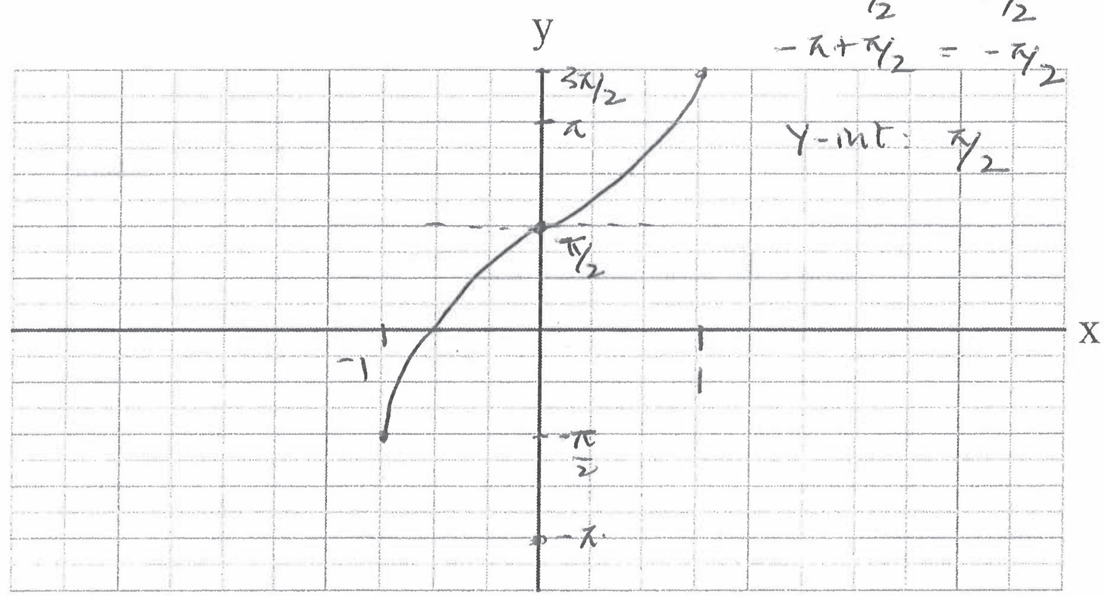
$-\pi/2 \times 2 = -\pi$

Up $= \pi/2$

$\therefore \pi + \pi/2 = 3\pi/2$

$-\pi + \pi/2 = -\pi/2$

Y-int: $\pi/2$



Student's Name: Q12 Teacher's Code: Solutions

Question Number	Marker use only
	Q12

(a)	
(b)	
(c)	
(d)	
(e)	
(f)	
(g)	
(h)	

QUANTUM POTES TANTUM AUDE

Saint Ignatius' College
RIVERVIEW

Mathematics Extension 1

Writing Booklet

Instructions

- Start each question in a new booklet.
- Except for multiple choice answers, you should show relevant mathematical reasoning and/or calculations.
- You may ask for an extra Writing Booklet to answer this question if you need more space.
- If you do not attempt a question you must still hand in a Writing Booklet, with your Name, your Teacher's Name, the Question Number the words 'NOT ATTEMPTED' written clearly on the front cover.
- Write the number of each section part inside the margin at the beginning of each answer.
- Write using black or blue pen.
- Write on the ruled pages only.

You may not remove any Writing Booklets, used or unused, from the examination room.

Start your answer here. Question 12

12 (a) $2x^3+x^2-7x-6=0$

$P(-1)=0 \quad \therefore \quad (x+1)$ is a factor

$2x^3+2x^2-x^2-x-6x-6$

$2x^2(x+1)-x(x+1)-6(x+1)$

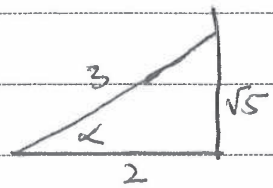
$(x+1)(2x^2-x-6)$

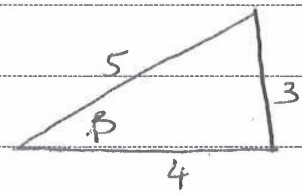
$(x+1)(2x+3)(x-2)=0$

$\therefore x=-1, \quad x=-\frac{3}{2}, \quad x=2$

12 (b) $\tan\left[\cos^{-1}\left(\frac{2}{3}\right)+\tan^{-1}\left(\frac{-3}{4}\right)\right]$

$$\tan(\alpha+\beta) = \frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$$





$\tan\alpha = \frac{\sqrt{5}}{2}$ $\qquad$ $\tan\beta = -\frac{3}{4}$ (II, IV Q)

$$\therefore \tan(\alpha+\beta) = \frac{\frac{\sqrt{5}}{2}+\left(\frac{-3}{4}\right)}{1-\frac{\sqrt{5}}{2}\times\left(\frac{-3}{4}\right)}$$

$$= \frac{\frac{2\sqrt{5}-3}{4}}{\frac{8+3\sqrt{5}}{8}} = \frac{2\sqrt{5}-3}{4}\times\frac{8}{8+3\sqrt{5}}$$

$$= \frac{2(2\sqrt{5}-3)}{8+3\sqrt{5}}$$

OR $$= \frac{4\sqrt{5}-6}{3\sqrt{5}+8}$$

You may ask for an extra Writing Booklet if you need more space to answer this question.

12 (c) Induction:

$$x+x^3+x^5+\cdots+x^{2n-1}=\frac{x(x^{2n}-1)}{(x-1)(x+1)}$$

Prove for $n=1$

LHS: $x^{2(1)-1}=x$

RHS: $\frac{x(x^{2(1)}-1)}{(x-1)(x+1)}=\frac{x(x^2-1)}{(x^2-1)}=x$

$\therefore$ LHS $=$ RHS.

Assume true for $n=k$.

$$\therefore x+x^3+x^5+\cdots+x^{2k-1}=\frac{x(x^{2k}-1)}{(x-1)(x+1)}$$

Prove for $n=k+1$.

$$\therefore x+x^3+x^5+\cdots+x^{2k-1}+x^{2k+1}=\frac{x(x^{2(k+1)}-1)}{(x-1)(x+1)}=\frac{x(x^2\cdot x^{2k}-1)}{(x^2-1)}$$

By Assumption LHS

$$\frac{x(x^{2k}-1)}{(x^2-1)}+x^{2k+1}$$

$$\frac{x(x^{2k}-1)+x\cdot x^{2k}(x^2-1)}{(x^2-1)}$$

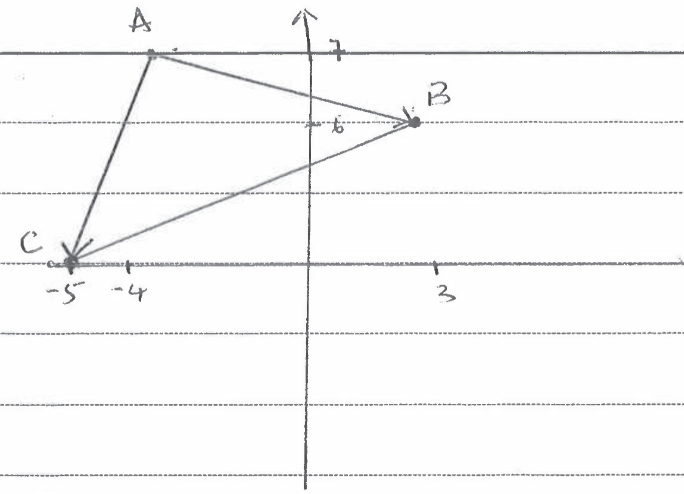
$$\frac{\cancel{x\cdot x^{2k}}-x+x^3\cdot x^{2k}-\cancel{x\cdot x^{2k}}}{(x^2-1)}$$

$$=\frac{x(x^2\cdot x^{2k}-1)}{(x^2-1)}=\text{RHS}$$

$\therefore$ True by mathematical induction.

Additional writing space on back page.

d.



(i) $\overline{AB} = \overline{OB} - \overline{OA}$

$$= \begin{pmatrix} 3 \\ 6 \end{pmatrix} - \begin{pmatrix} -4 \\ 7 \end{pmatrix} = \begin{pmatrix} 7 \\ -1 \end{pmatrix}$$

$\overline{AC} = \overline{OC} - \overline{OA}$

$$= \begin{pmatrix} -5 \\ 0 \end{pmatrix} - \begin{pmatrix} -4 \\ 7 \end{pmatrix} = \begin{pmatrix} -1 \\ -7 \end{pmatrix}$$

(ii) $|\overline{AB}| = \sqrt{7^2 + (-1)^2} = \sqrt{50} = 5\sqrt{2}$

$|\overline{AC}| = \sqrt{(-1)^2 + (-7)^2} = \sqrt{50} = 5\sqrt{2}$

(iii) $\overline{AB} \cdot \overline{AC} = |\overline{AB}||\overline{AC}| \cos\theta$

$(7)(-1) + (-1)(-7) = 5\sqrt{2} \times 5\sqrt{2} \cos\theta$

$-7 + 7 = 50 \cos\theta$

$0 = 50 \cos\theta$

$\therefore \cos\theta = 0 \qquad \theta = \frac{\pi}{2}$

$\angle BAC = \frac{\pi}{2}$

Additional writing space on back page.

(iv) Area ΔABC (since rt. Δ)

$\frac{1}{2} b \times h$

$= \frac{1}{2} \times 5\sqrt{2} \times 5\sqrt{2}$

$= 25u^2$

OR: $A = \frac{1}{2} ab \sin C$ (Formula)

$= \frac{1}{2} \times 5\sqrt{2} \times 5\sqrt{2} \sin \frac{\pi}{2}$

$= 25u^2$.

You may ask for an extra Writing Booklet if you need more space to answer this question.

Student's Name: Solutions Teacher's Code: Q13

Question Number	Marker use only
	Q 13

(a)	
(b)	
(c)	
(d)	
(e)	
(f)	
(g)	
(h)	

RIVERVIEW

QUANTUM POTES TANTUM AUDE

Saint Ignatius' College

RIVERVIEW

Mathematics Extension 1

Writing Booklet

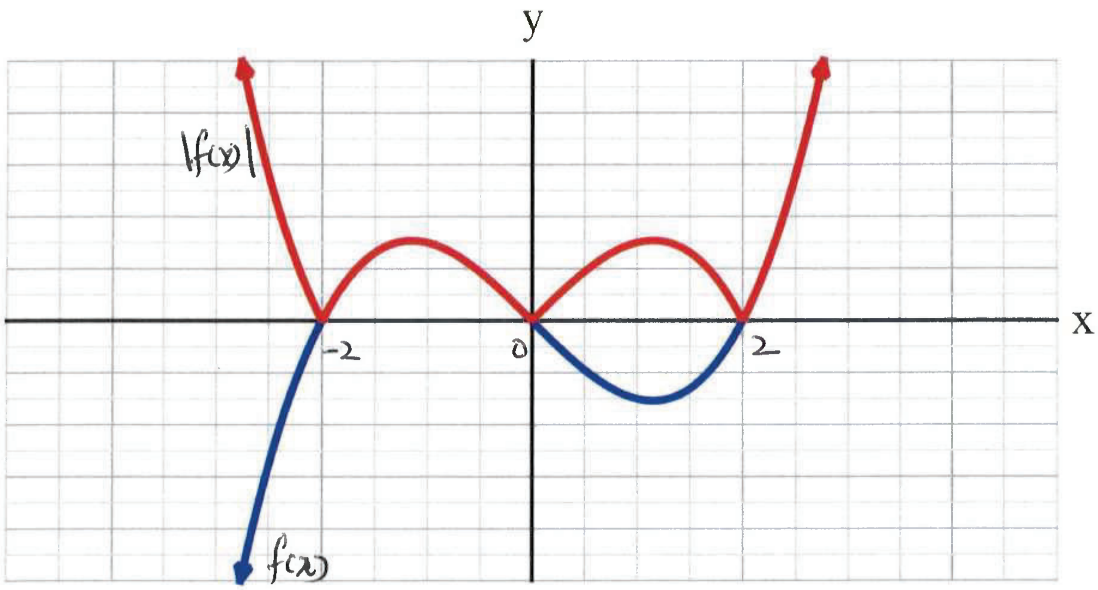
Instructions

- Start each question in a new booklet.
- Except for multiple choice answers, you should show relevant mathematical reasoning and/or calculations.
- You may ask for an extra Writing Booklet to answer this question if you need more space.
- If you do not attempt a question you must still hand in a Writing Booklet, with your Name, your Teacher's Name, the Question Number the words 'NOT ATTEMPTED' written clearly on the front cover.
- Write the number of each section part inside the margin at the beginning of each answer.
- Write using black or blue pen.
- Write on the ruled pages only.

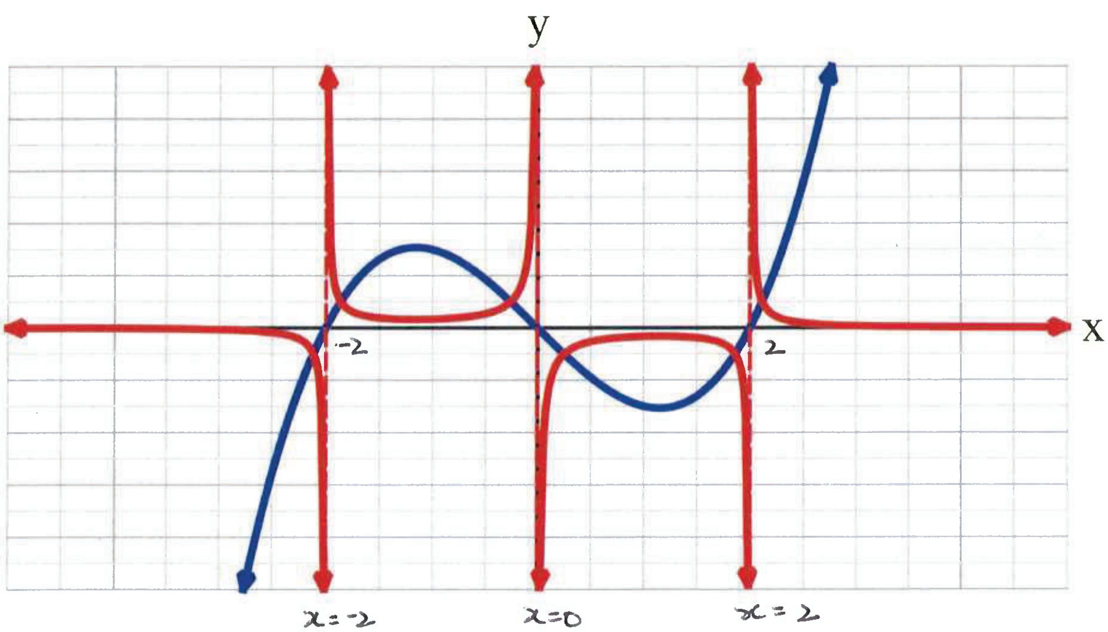
You may not remove any Writing Booklets, used or unused, from the examination room.

Question 13 (a)

(i)



(ii)



Asymptotes at $x = \pm 2$, $x = 0$ and $y = 0$

Start your answer here.

13 (b)

$y = \sin^4 (x-3)$

Dilated by a factor 3 from the y-axis

$x \rightarrow \frac{x}{3}$

$\therefore \; y = \sin^4 \left(\frac{x}{3} - 3\right) = \sin^4\left(\frac{x-9}{3}\right)$

Translate to the right by 3

$x \rightarrow x-3$

$\therefore \; y = \sin^4 \left[\frac{(x-3) - 9}{3}\right]$

$= \sin^4 \left(\frac{x-3-9}{3}\right)$

$y = \sin^4 \left(\frac{x-12}{3}\right)$ Final Ans.

13 (b) (i)

$$\frac{d}{dx}\left[\frac{2x}{4+x^2}\right] \qquad u = 2x \quad v = 4+x^2 \qquad u' = 2 \quad v' = 2x$$

$$= \frac{2(4+x^2) - 2x(2x)}{(4+x^2)^2}$$

$$= \frac{8+2x^2-4x^2}{(4+x^2)^2}$$

$$= \frac{8-2x^2}{(4+x^2)^2}$$

$$\frac{d}{dx}\left[\tan^{-1}\frac{x}{2}\right] = \frac{1}{1+\left(\frac{x}{2}\right)^2} \times \frac{1}{2}$$

$$= \frac{1}{\frac{4+x^2}{4}} \times \frac{1}{2}$$

$$= \frac{2}{4+x^2}$$

$$\therefore \frac{d}{dx}\left[\frac{2x}{4+x^2} + \tan^{-1}\frac{x}{2}\right] = \frac{8-2x^2}{(4+x^2)^2} + \frac{2}{4+x^2}$$

$$= \frac{8-2x^2+2(4+x^2)}{(4+x^2)^2}$$

$$= \frac{8-2x^2+8+2x^2}{(4+x^2)^2}$$

$$= \frac{16}{(4+x^2)^2} = \frac{A}{(4+x^2)^2} \text{ Form.}$$

Additional writing space on back page.

(ii) $$\int_0^2 \frac{dx}{(4+x^2)^2} = \frac{1}{16}\int \frac{16}{(4+x^2)^2}\,dx$$

$$= \frac{1}{16}\left[\frac{2x}{4+x^2} + \tan^{-1}\frac{x}{2}\right]_0^2$$

$$= \frac{1}{16}\left[\frac{4}{4+4} + \tan^{-1}1 - 0 - \tan^{-1}0\right]$$

$$= \frac{1}{16}\left[\frac{4}{8} + \frac{\pi}{4}\right]$$

$$= \frac{1}{32} + \frac{\pi}{64}$$

13(d)

(i) $\frac{1}{y} + \frac{1}{y+1} =$

$\frac{1}{y} + \frac{1}{1-y} = \frac{1-y+y}{y(1-y)} = \frac{1}{y(1-y)}$

(ii) $\frac{dy}{dx} = ay(1-y)$

$\therefore \frac{dy}{y(1-y)} = a\,dx$

$\therefore \int\left(\frac{1}{y} + \frac{1}{1-y}\right)dy = \int a\,dx$ from (i)

$\ln|y| - \ln|(1-y)| = ax + c$

$-ax - c = \ln|1-y| - \ln|y|$

$= \ln\left|\frac{1-y}{y}\right|$

$e^{-ax-c} = \frac{1-y}{y}$

$e^{-ax-c} = \frac{1}{y} - 1$

$e^{-ax-c} + 1 = \frac{1}{y}$

$y = \frac{1}{e^{-ax-c} + 1}$ $\qquad e^{-ax} \cdot e^{-c}$ $\qquad Ke^{-ax}$

$y = \frac{1}{Ke^{-ax} + 1}$ As reqd.

Additional writing space on back page.

(iv)

$$y(x) = \frac{1}{Ke^{-ax}+1}$$

Given $x = 0$, $y(x) = \frac{1}{5}$

$\therefore \frac{1}{5} = \frac{1}{K+1}$

$K+1 = 5$

$K = 4$

You may ask for an extra Writing Booklet if you need more space to answer this question.

Student's Name: Question 14 Teacher's Code: Solutions

Question Number	Marker use only
	Q14

(a)	
(b)	
(c)	
(d)	
(e)	
(f)	
(g)	
(h)	

Saint Ignatius' College

RIVERVIEW

Mathematics Extension 1

Writing Booklet

Instructions

- Start each question in a new booklet.
- Except for multiple choice answers, you should show relevant mathematical reasoning and/or calculations.
- You may ask for an extra Writing Booklet to answer this question if you need more space.
- If you do not attempt a question you must still hand in a Writing Booklet, with your Name, your Teacher's Name, the Question Number the words 'NOT ATTEMPTED' written clearly on the front cover.
- Write the number of each section part inside the margin at the beginning of each answer.
- Write using black or blue pen.
- Write on the ruled pages only.

You may not remove any Writing Booklets, used or unused, from the examination room.

14 (a)

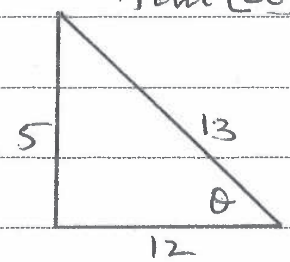
$$\tan\left[2\cos^{-1}\frac{12}{13}\right]$$

$$= \frac{2\tan\left[\cos^{-1}\frac{12}{13}\right]}{1-\tan^{2}\left[\cos^{-1}\frac{12}{13}\right]}$$

$$= \frac{2\left(\frac{5}{12}\right)}{1-\left(\frac{5}{12}\right)^{2}} \qquad \cancel{\frac{144}{169}}$$

$$= \frac{120}{144-25}$$

$$= \frac{120}{119} \checkmark$$



$\tan[2\theta]$ formula.

$$= \frac{2\tan\theta}{1-\tan^{2}\theta}$$

Additional writing space on back page.

Start your answer here.

Q14 (b) $t = \tan\frac{x}{2}$

(i) $\therefore$ LHS: $\operatorname{cosec} x + \cot x$

$\frac{1+t^2}{2t} + \frac{1-t^2}{2t}$

$= \frac{1+t^2+1-t^2}{2t}$

$= \frac{2\cancel{t}}{2t} = \frac{1}{t}$

$= \cot\frac{x}{2} = $ RHS.

(ii) $\int_{\pi/3}^{\pi/2} (\operatorname{cosec} x + \cot x)\,dx$

$= \int_{\pi/3}^{\pi/2} \cot\frac{x}{2}\,dx = 2\int_{\pi/3}^{\pi/2} \frac{\frac{1}{2}\cos\frac{x}{2}}{\sin\frac{x}{2}}\,dx$

$= 2\left[\ln\left(\sin\frac{x}{2}\right)\right]_{\pi/3}^{\pi/2}$

$= 2\left[\ln\left(\sin\frac{\pi}{4}\right) - \ln\sin\left(\frac{\pi}{6}\right)\right]$

$= 2\left[\ln\frac{1}{\sqrt{2}} - \ln\frac{1}{2}\right] = 2\ln\left(\frac{1/\sqrt{2}}{1/2}\right)$

$= 2\ln 2^{1/2} = \ln 2.$

(c)

(i) $\overline{PQ} = \underset{\sim}{q} - \underset{\sim}{p}$, $\overline{PR} = \underset{\sim}{r} - \underset{\sim}{p}$

$= \lambda \underset{\sim}{p} + \mu \underset{\sim}{q} - \underset{\sim}{p}$

$= (\lambda - 1)\underset{\sim}{p} + \mu \underset{\sim}{q}$

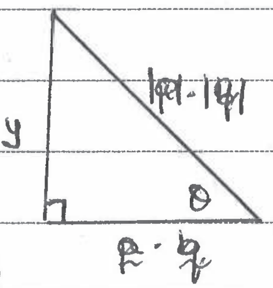
$= -\mu \underline{p} + \mu \underline{q}$

$= \mu(\underset{\sim}{q} - \underset{\sim}{p})$

$\therefore \overline{PR} = \mu \overline{PQ}$

$\therefore$ P, Q, R are Collinear.

(ii)



$y^2 = (|\underline{p}||\underline{q}|)^2 - (\underline{p}\cdot\underline{q})^2$

$K = \frac{1}{2}|\underline{p}||\underline{q}| \sin\theta$ (Given)

$\therefore K = \frac{1}{2}|\underline{p}||\underline{q}| \times \frac{\sqrt{(|p||q|)^2 - (\underline{p}\cdot\underline{q})^2}}{|p||q|}$

Sq. $4K^2 = (|\underline{p}||\underline{q}|)^2 - (\underline{p}\cdot\underline{q})^2$

$(\underline{p}\cdot\underline{q})^2 = (|\underline{p}||\underline{q}|)^2 - 4K^2$ $|p| = 2$

$= 4|\underline{q}|^2 - 4K^2$

$= 4(|\underline{q}|^2 - K^2)$

You may ask for an extra Writing Booklet if you need more space to answer this question.

Start your answer here.

(c)

(iii) $(\underset{\sim}{p}\cdot\underset{\sim}{q})^2 = 4\left[|\underset{\sim}{q}|^2 - r^2\right]$

$= 4\left[10^2 - 6^2\right]$

$= 256$

$\therefore \underset{\sim}{p}\cdot\underset{\sim}{q} = 16$

Since D lies on $\overline{PQ}$, then

$\overline{OD} = \overline{OA} + \lambda\overline{AB} \quad \lambda \in R$

$\underset{\sim}{s} = \underset{\sim}{p} + \lambda(\underset{\sim}{q} - \underset{\sim}{p})$

and $\angle POS = 90^\circ \quad \therefore \underset{\sim}{p}\cdot\underset{\sim}{s} = 0$

$\underset{\sim}{p}\left[\underset{\sim}{p} + \lambda(\underset{\sim}{q} - \underset{\sim}{p})\right] = 0$

$|\underset{\sim}{p}|^2 + \lambda\underset{\sim}{p}\cdot\underset{\sim}{q} - \lambda|\underset{\sim}{p}|^2 = 0$

$|\underset{\sim}{p}|^2(1-\lambda) + \lambda\underset{\sim}{p}\cdot\underset{\sim}{q} = 0$

$4(1-\lambda) + 16\lambda = 0$

$\lambda = \frac{1}{3}$

$\underset{\sim}{s} = \underset{\sim}{p} - \frac{1}{3}(\underset{\sim}{q} - \underset{\sim}{p})$

$\underset{\sim}{s} = \frac{4}{3}\underset{\sim}{p} - \frac{1}{3}\underset{\sim}{q}$

Start your answer here.

<u>Alternate Soln</u>

~~14F~~

14(c)

(i) $\overline{PQ} = \underline{q} - \underline{p}$

$\overline{PR} = \underline{r} - \underline{p}$

$\overline{PR} = \lambda \underline{p} + \mu \underline{q} - \underline{p}$

$= (\lambda - 1)\underline{p} + \mu \underline{q}$

$= -\mu \underline{p} + \mu \underline{q}$

$\therefore \overline{PR} = \mu(\underline{q} - \underline{p})$

$\therefore P, Q, R$ collinear.

Given:

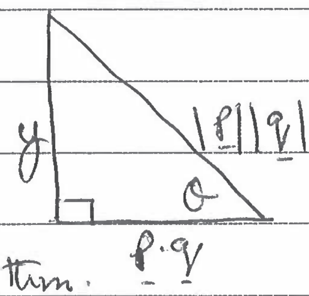
$\lambda \underline{p} + \mu \underline{q} - \underline{r} = 0$

$\lambda + \mu = 1$

$\therefore \mu = 1 - \lambda$

$\boxed{-\mu = \lambda - 1}$

(ii)



$\therefore \underline{p} \cdot \underline{q} = |\underline{p}||\underline{q}| \cos\theta$

or $y^2 = (|\underline{p}||\underline{q}|)^2 - (\underline{p} \cdot \underline{q})^2$ By thm.

$K = \frac{1}{2}|\underline{p}||\underline{q}| \sin\theta$ Area of Δ

$$K = \frac{1}{2}\cancel{|\underline{p}||\underline{q}|} \times \frac{\sqrt{(|\underline{p}||\underline{q}|)^2 - (\underline{p} \cdot \underline{q})^2}}{\cancel{|\underline{p}| \cdot |\underline{q}|}}$$

$$4K^2 = (|\underline{p}||\underline{q}|)^2 - (\underline{p} \cdot \underline{q})^2$$

$$(\underline{p} \cdot \underline{q})^2 = (|\underline{p}||\underline{q}|)^2 - 4K^2$$

$\underline{p} = 2$

$$= 4|\underline{q}|^2 - 4K^2$$

$$= 4(|\underline{q}|^2 - K^2)$$

(iii) $(\underline{p}\cdot\underline{q})^2 = 4\left(|\underline{q}|^2 - K^2\right)$

$= 4\left(10^2 - 6^2\right)$

$= 256$

$\therefore \underline{p}\cdot\underline{q} = 16$

Since $\underline{S}$ lies on $\overline{PR}$,

then $\overline{OS} = \overline{OP} + \lambda\overline{PQ}$

$\circledast$ $\underline{s} = \underline{p} + \lambda(\underline{q} - \underline{p}) \quad \lambda \in \mathbb{R}.$

If $\angle POS = 90^\circ$ then $\underline{p}\cdot\underline{s} = 0$

$\therefore \underline{p}\cdot(\underline{p} + \lambda(\underline{q} - \underline{p})) = 0$

$|\underline{p}|^2 + \lambda\underline{p}\cdot\underline{q} - \lambda|\underline{p}|^2 = 0 \qquad \underline{p}\cdot\underline{p} = |\underline{p}|^2$

$|\underline{p}|^2(1-\lambda) + \lambda\underline{p}\cdot\underline{q} = 0 \qquad \underline{p}\cdot\underline{q} = 16$

$4(1-\lambda) + 16\lambda = 0 \qquad \underline{p} = 2$

$4 - 4\lambda + 16\lambda = 0 \qquad \cancel{-4\lambda =}$

$\cancel{+12} \; 12\lambda + 4 = 0 \qquad \lambda = -\frac{1}{3}$

$\therefore \underline{s} = \underline{p} + \lambda(\underline{q} - \underline{p})$

$= \underline{p} - \frac{1}{3}(\underline{q} - \underline{p})$

or $\frac{4}{3}\underline{p} - \frac{1}{3}\underline{q}$

Additional writing space on back page.