Section I

10 marks

Attempt Questions 1 – 10

Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1 – 10.

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1 Which of the following expressions is equivalent to $cosx + sinx$?

(A) $\sqrt{2}\cos\,(x + \frac{\pi}{4})$

(B) $\sqrt{2}\cos\,(x - \frac{\pi}{4})$

(C) $\sqrt{2}\sin\,(x - \frac{\pi}{4})$

(D) $\sqrt{2}\sin\,(x + \frac{\pi}{2})$

2 Which of the following is the correct expansion of $(3x - 1)^3$?

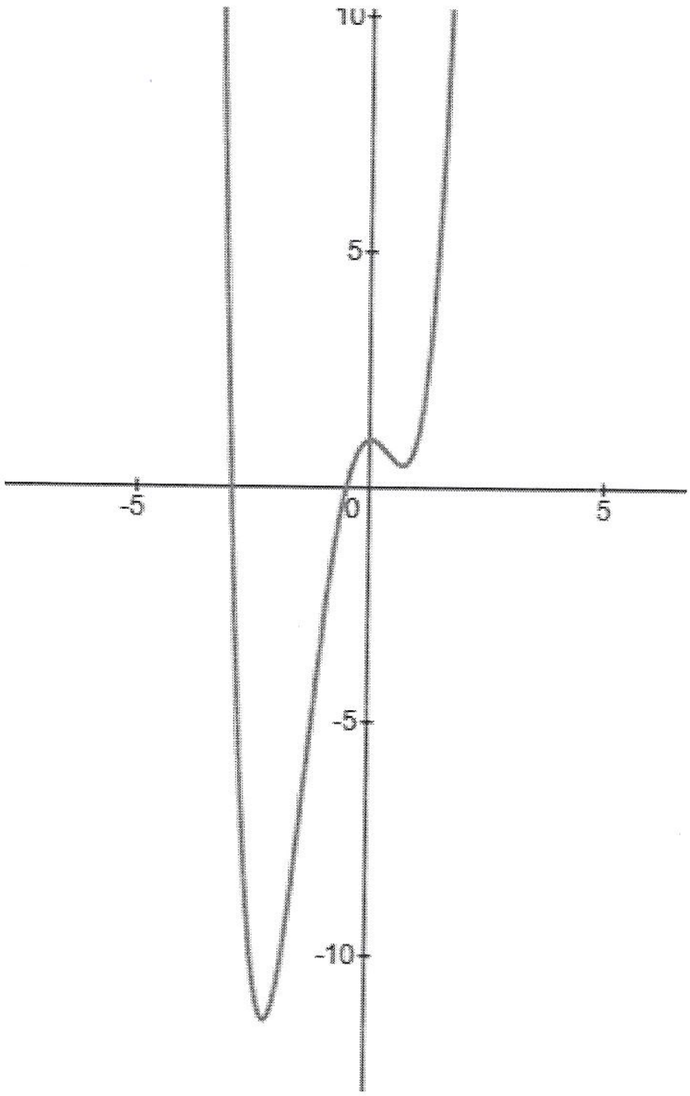
(A) $27x^3 + 27x^2 + 9x + 1$

(B) $9x^3 - 27x^2 + 12x - 1$

(C) $27x^3 - 27x^2 + 9x - 1$

(D) $27x^3 - 27x^2 + 12x - 9$

3 The graph of a polynomial $y = P(x)$ is shown below.



What is the minimum possible degree of the polynomial $(P(x))^3$?

(A) 3

(B) 6

(C) 9

(D) 12

4 Which of the following is the derivative of $y = 3x\cos^{-1}(3x)$?

(A) $3\cos^{-1}(3x) + \frac{9x}{\sqrt{1-9x^2}}$

(B) $3\cos^{-1}(3x) - \frac{9x}{\sqrt{1-9x^2}}$

(C) $3x\cos^{-1}(3x) + \frac{9x}{\sqrt{1-9x^2}}$

(D) $3x\cos^{-1}(3x) - \frac{9x}{\sqrt{1-9x^2}}$

5 What is the angle between the vectors $\begin{pmatrix} 1 \\ -1 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$?

(A) 30°

(B) 45°

(C) 60°

(D) 75°

6 The polynomial $x^3 + 3x^2 - 10x - 24$ has zeros $-2, 3$ and a.

What is the value of a ?

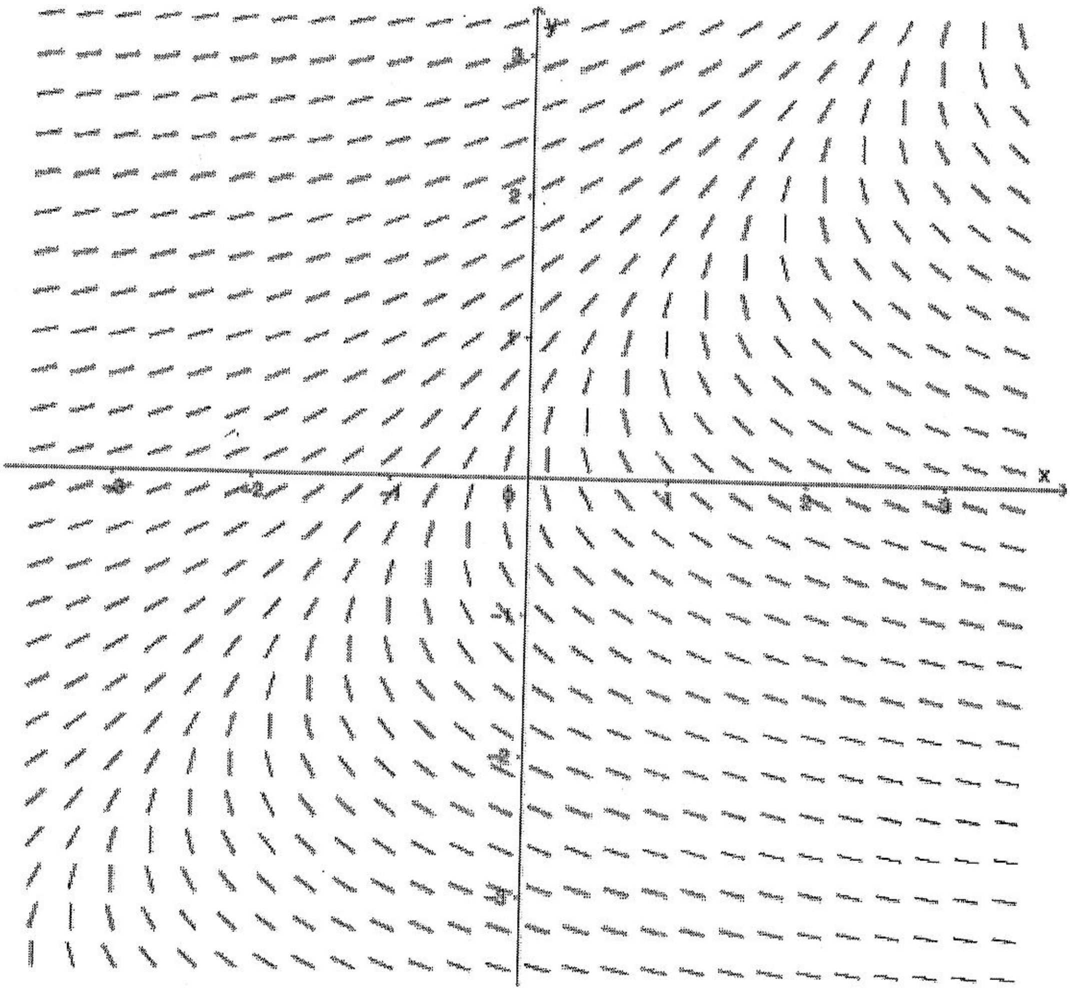
(A) 4

(B) -4

(C) 3

(D) 6

7 A differential equation has a direction field shown below.



A solution curve to the differential equation includes $(1, -3)$

Which one of the following points will the solution curve also include?

(A) $(-1, \ -0.5)$

(B) $(-1, \ 0)$

(C) $(0, \ 0.75)$

(D) $(1, \ 2)$

8 A bag contains 6 identical red marbles, 9 identical white marbles and 4 identical black marbles. Three marbles are drawn at random.

Which expression below gives the correct probability that exactly two white marbles are drawn?

(A) $\frac{{}^{9}C_{2}}{{}^{19}C_{3}}$

(B) $\frac{{}^{9}C_{2}\times{}^{10}C_{1}}{{}^{19}C_{3}}$

(C) $\frac{{}^{9}C_{2}\times{}^{6}C_{1}\times{}^{4}C_{1}}{{}^{19}C_{3}}$

(D) $\frac{6}{19}\times\frac{5}{18}\times\frac{13}{17}$

9 Which of the following is the value of $\int_0^1 \frac{dx}{4+4x^2}$?

(A) $\frac{1}{4}$

(B) $\frac{\pi}{2}$

(C) $\frac{\pi}{4}$

(D) $\frac{\pi}{16}$

10 The population, P, of turkeys at Brookvale oval is unfortunately increasing such that $\frac{dP}{dt} = P(20\,000 - P)$, where t is time.

The initial population is 8 000 Turkeys.

Which of the following is true?

(A) $P = 20\,000 - 12\,000e^{20\,000t}$

(B) The population is increasing most rapidly when $P = 10\,000$

(C) The population is increasing most rapidly when $t = 10\,000$

(D) The maximum population is $P = 10\,000$

Section II

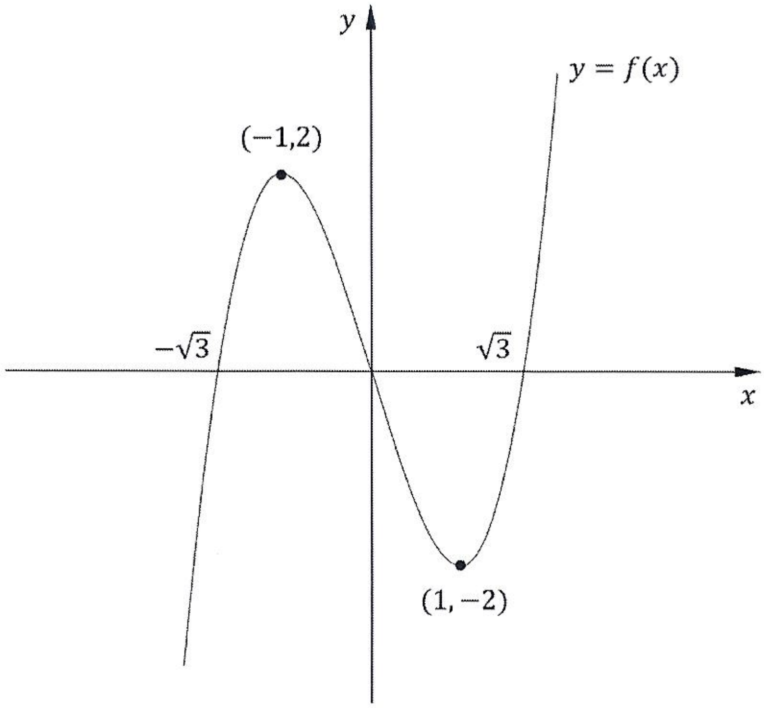
60 marks
Attempt Questions 11-14
Allow about 1 hour and 45 minutes for this section

Answer each question in the appropriate writing booklet. Extra writing booklets are available.

For questions in Section II, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Start a new writing booklet

(a) The diagram shows the graph of $y = f(x)$, where $f(x) = x^3 - 3x$. **3**



Sketch $y = \frac{1}{f(x)}$, showing all turning points and asymptotes.

Question 11 continues on next page

Question 11 (continued)

(b) Using the substitution $u = 5x^3 - 1$, or otherwise, find **2**

$$\int x^2(5x^3 - 1)^5 \, dx.$$

(c) Evaluate **3**

$$\int_0^{\frac{\pi}{3}} \sin^2 3x \, dx.$$

(d) (i) In how many ways can the letters of the word SUCCESS be arranged in a line if there are no restrictions? **1**

(ii) In how many ways can the letters of the word SUCCESS be arranged in a line if the two Cs are NOT adjacent? **2**

(e) Find the angle between the vectors $\begin{pmatrix} -12 \\ 5 \end{pmatrix}$ and $\begin{pmatrix} 8 \\ 6 \end{pmatrix}$, correct to the nearest degree. **2**

(f) The volume, V, of a weather balloon in cubic metres after t minutes is modelled by the function $V = 1.26 + Ae^{0.1t}$. **2**

The balloon initially has a volume of 2.5 cubic metres.

Find the volume of the weather balloon after ten minutes, correct to two decimal places.

End of Question 11

Question 12 (14 marks) Start a new writing booklet

(a) Solve the equation $4\sin x + 3\cos x = 2.5$ for $0 \leq x \leq 2\pi$ (correct to 2 d.p.) **3**

(b) Find the coefficient of x^8 in the expansion of $(2x + 2)^{10}$. **2**

(c) Use mathematical induction to prove for all integers $n \geq 1$ that $3^{2n} - 1$ is divisible by 8. **3**

(d) Given the polynomial $P(x) = x^3 + 2x^2 + 3x + 10$, find the polynomial $Q(x)$, such that $P(x) = (x + 2)Q(x) + R$. **3**

(e) When a fair coin is tossed, the chance that it will come up tails is exactly 50%. **3**

The coin is tossed 100 times.

What is the probability that it will NOT come up tails exactly 50 times?
Leave your answer in unsimplified form

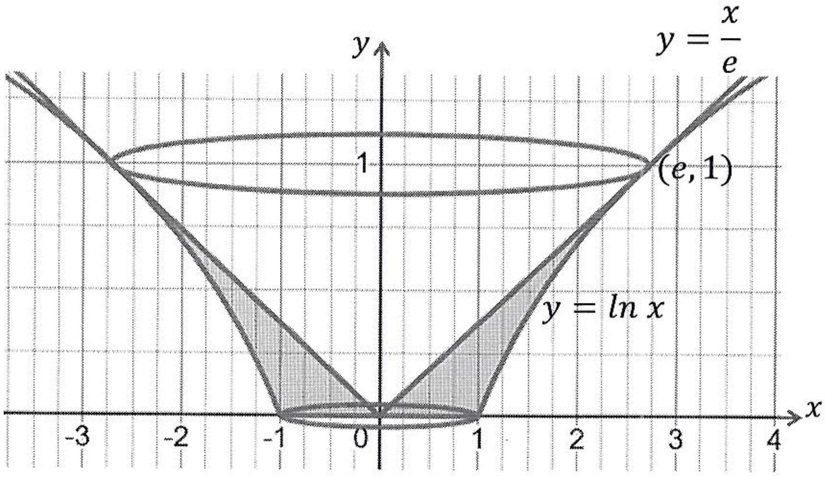
End of Question 12

Question 13 (15 marks) Start a new writing booklet

(a) Using $t = \tan\frac{x}{2}$, or otherwise, prove that **2**

$$\frac{2}{1+\cos x} = \sec^2\left(\frac{x}{2}\right).$$

(b) A three dimensional shape is modelled by rotating the region S between the curves $y = \ln x$ and the line $y = \frac{x}{e}$ for $0 \leq y \leq 1$, about the y axis. **4**



Find the exact volume of the shape.

(c) A curve is described by the parametric equations given below: **3**

$$x = 12q$$
$$y = 6(q+1)^2$$

Find the gradient of the tangent to the curve at the point where $q = 1\frac{1}{2}$

Question 13 continues on next page

(d) *You may use the information page after Q14 to answer this question.* **3**

A bus driver passes through the same set of traffic lights forty times every day.

Each time the driver approaches the traffic lights there is a 30% chance that the lights are green.

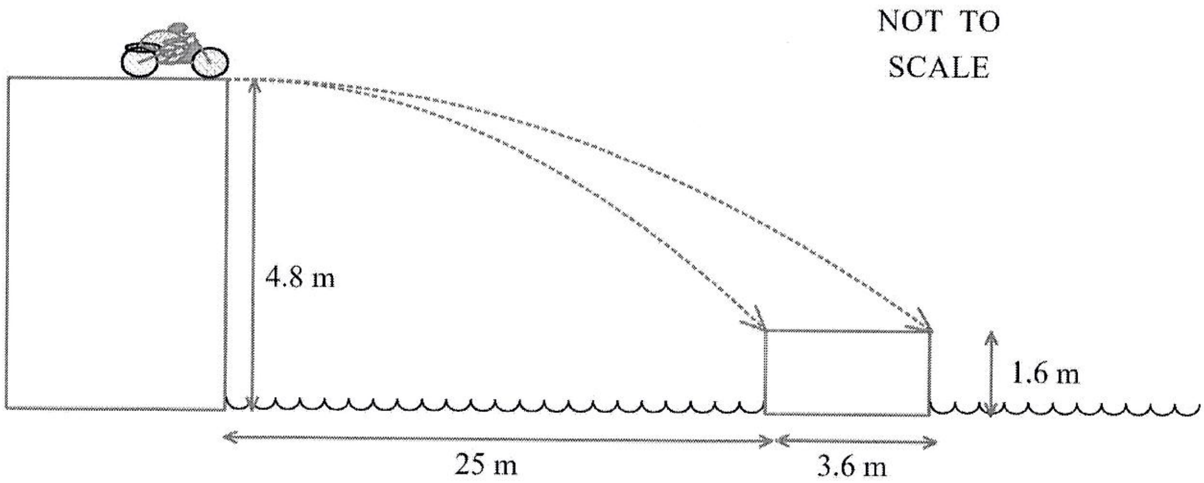
Use the standard normal distribution to approximate the probability that, on a particular day, the lights are green at least fifteen times. Answer as a percentage correct to one decimal place.

(e) Find the curve, in the form $y = f(x)$, which satisfies the differential equation $\frac{dy}{dx} = cosec\ y$ and passes through the point $(-\frac{\pi}{3}, \frac{\pi}{2})$. **3**

End of Question 13

Question 14 (16 marks) Start a new writing booklet

(a) A motorcycle stunt rider is positioned on top of a 4.8 m high wharf and will ride off the end of the wharf at a constant speed with the intention of landing on a barge positioned in the water 25 m from the wharf. The barge is 3.6 m wide and sits 1.6 m above the water level.



For safety, she aims to land in the centre of the barge.

For our calculations, we will use the point at which she leaves the wharf as the origin, gravity as $g = 10\ \text{ms}^{-2}$ and treat the bike and rider as a single particle.

Using t as the time in seconds after leaving the edge of the wharf, the position vector of the particle $r(t)$ is given by:

$\underset{\sim}{r}(t) = \begin{pmatrix} Vt \\ -5t^2 \end{pmatrix}$ [DO NOT PROVE THIS]

where V is the velocity in ms^{-1} when she leaves the wharf.

(i) Find the initial velocity required to land exactly in the centre of the barge. **2**

(ii) At what angle, to the nearest degree, will she hit the barge? **2**

Question 14 continues on next page

Question 14 (continued)

(b) The cubic polynomial $P(x) = x^3 + x^2 - 8x - 12$ has roots $\alpha,\ \beta$ and γ. **2**

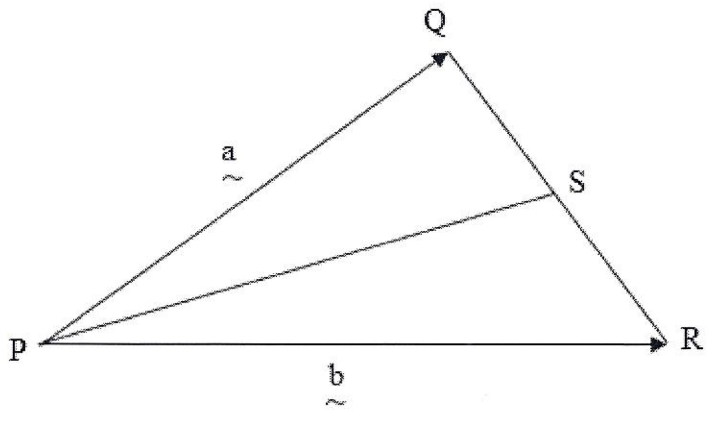
Find $\alpha^2 + \beta^2 + \gamma^2$.

(c) It is given that **3**

$$\tan(3\theta) = \frac{3\tan\theta - \tan^3\theta}{1 - 3\tan^2\theta}.$$

By letting $x = \tan\theta$, solve $x^3 - 3x^2 - 3x + 1 = 0$.

(d)



PQR is a triangle and S is the midpoint of QR. **3**

Let $\overrightarrow{PQ} = \underset{\sim}{a}$, $\overrightarrow{PR} = \underset{\sim}{b}$.

Show that $2\left(\left|\overrightarrow{PS}\right|^2 + \left|\overrightarrow{QS}\right|^2\right) = \left|\overrightarrow{PQ}\right|^2 + \left|\overrightarrow{PR}\right|^2$.

Question 14 continues on next page

Question 14 (continued)

(e) The curved surface area, A, of a cone with a fixed height 12 cm is increasing at the rate of 18π $\text{cm}^2 s^{-1}$. **4**

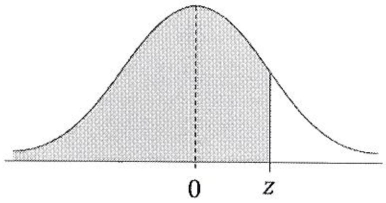
The curved surface of a cone with radius r and slant height l has area given by $A = \pi r l$.

Find the rate at which its radius is increasing when its slant height is 15 cm?

End of paper

You may use the information below to answer Question 13 (e).

Table of values $P(Z < z)$ for the normal distribution $N(0, 1)$



z	0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.00	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.10	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.20	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.30	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.40	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.50	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.60	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.70	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.80	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.90	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.00	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.10	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.20	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.30	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.40	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.50	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.60	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.70	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.80	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.90	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.00	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.10	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.20	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.30	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.40	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.50	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.60	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.70	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.80	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.90	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.00	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990

6.

12 Ext 1 Trial 2025

BCDBB
BCBDB

1. $\cos x + \sin x = R\cos(x - \alpha)$

$R = \sqrt{1^2 + 1^2}$ $\qquad \tan\alpha = \frac{1}{1}$

$= \sqrt{2}$ $\qquad \alpha = \frac{\pi}{4}$

B

2. $(3x-1)^3 = 1(3x)^3 - 3(3x)^2 + 3(3x) - 1$

$= 27x^3 - 27x^2 + 9x - 1$

C

3. $P(x)$ degree 4 at least

$(P(x))^3$ degree 12 at least

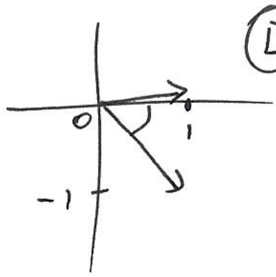
D

4. $y = 3x\cos^{-1}(3x)$

$y' = \cos^{-1}(3x) \times 3 + 3x \times \frac{-3}{\sqrt{1-(3x)^2}}$

B

5. $\theta = 45^\circ$



B

6. $-2 + 3 + a = -3$

$a = -4$

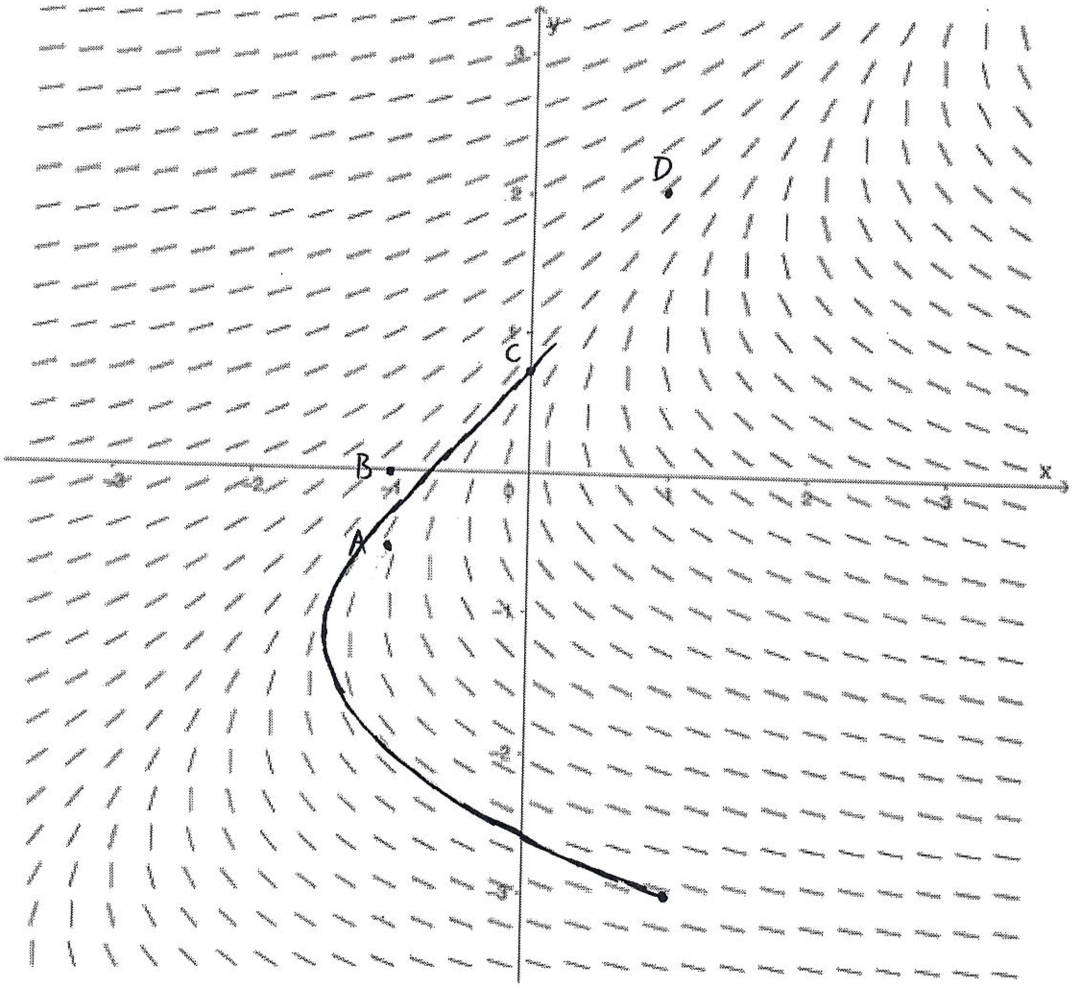
B

OR $\quad -2 \times 3 \times a = 24$

$-6a = 24$

$a = -4$

7 A differential equation has a direction field shown below.



A solution curve to the differential equation includes $(1, -3)$

Which one of the following points will the solution curve also include?

(A) $(-1, -0.5)$

(B) $(-1, 0)$

(C) $(0, 0.75)$

(D) $(1, 2)$

7. By obs. (C)

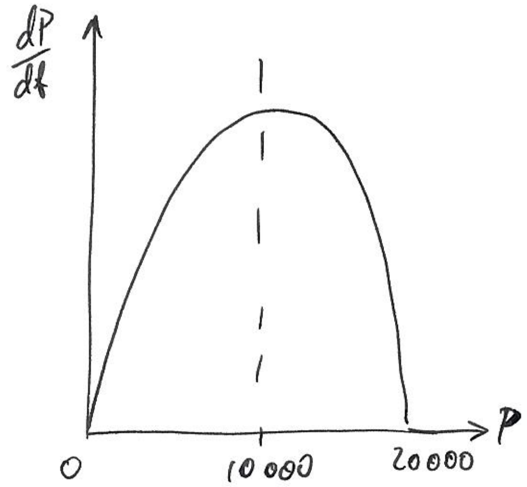
8. $\dfrac{{}^{9}C_2 \times {}^{10}C_1}{{}^{19}C_3}$ (B)

9. $\displaystyle\int_0^1 \frac{1}{4+4x^2}\,dx = \frac{1}{4}\left[\tan^{-1}x\right]_0^1$

$= \frac{1}{4} \times \left(\frac{\pi}{4} - 0\right)$ (D)

$= \frac{\pi}{16}$

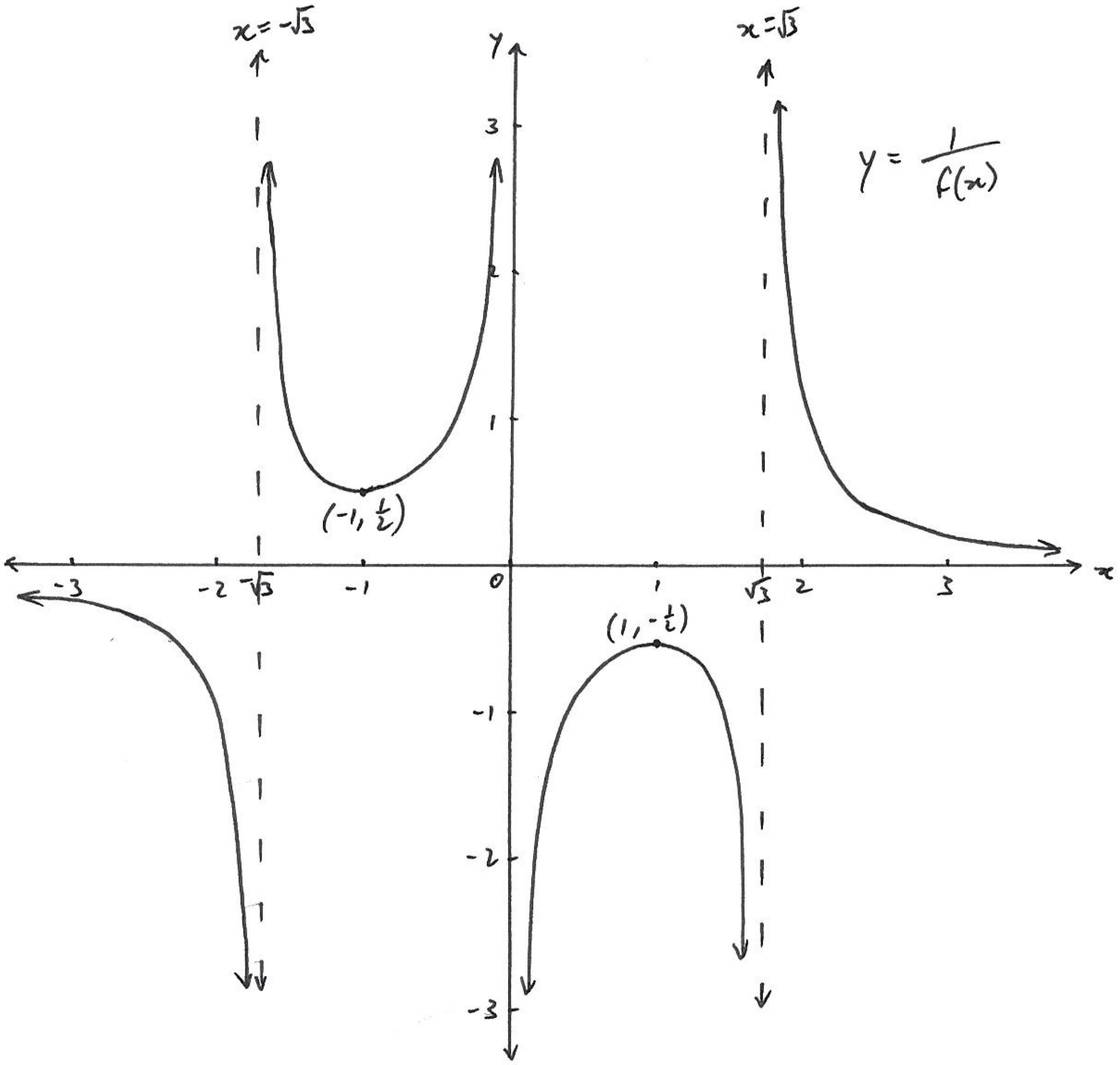
10.



(B)

$\frac{dP}{dt}$ max when $P = 10\,000$

11a.



b. $\int x^2(5x^3-1)^5\,dx$

$u = 5x^3 - 1$

$\frac{du}{dx} = 15x^2$

$\frac{du}{15} = x^2\,dx$

$= \int u^5 \cdot \frac{du}{15}$

$= \frac{1}{15}\frac{u^6}{6} + c$

$= \frac{(5x^3-1)^6}{90} + c$

<u>OR</u> $\int x^2(5x^3-1)^5\,dx = \frac{1}{15}\int 15x^2(5x^3-1)^5\,dx$

$= \frac{1}{15} \times \frac{(5x^3-1)^6}{6} + c$ (slurpee)

11c. $\int_0^{\frac{\pi}{3}} \sin^2 3x \, dx = \frac{1}{2}\int_0^{\frac{\pi}{3}} 1 - \cos 6x \, dx$

$$= \frac{1}{2}\left[x - \frac{\sin 6x}{6}\right]_0^{\frac{\pi}{3}}$$

$$= \frac{1}{2}\left[\frac{\pi}{3} - \frac{\sin 2\pi}{6} - \left(0 - \frac{\sin 0}{6}\right)\right]$$

$$= \frac{\pi}{6}$$

d. i)

S U C E
S   C
S

No. arr $= \frac{7!}{3! \times 2!}$

$= 420$

ii) If the C's are together: $\frac{6! \times 2!}{3! \times 2!} = 120$

$\therefore$ If C's not together: $420 - 120 = 300$

e. $\underset{\sim}{u} \cdot \underset{\sim}{v} = |\underset{\sim}{u}||\underset{\sim}{v}| \cos\theta$

$$\cos\theta = \frac{-12 \times 8 + 5 \times 6}{\sqrt{12^2+5^2} \times \sqrt{6^2+8^2}}$$

$$= \frac{-66}{130}$$

$$\theta = 120.5102374$$

$= 121°$ (nearest deg)

f. when $t = 0$, $V = 2.5$

ie. $2.5 = 1.26 + Ae^{0}$

$1.24 = A$

when $t = 10$,

$V = 1.26 + 1.24e^{0.1 \times 10}$

$= 4.630669467$

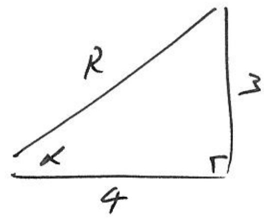
$= 4.63 \text{ m}^3 \text{ (2dp)}$

12. a. Let $4\sin x + 3\cos x = R\sin(x+\alpha)$

$= R\sin x\cos\alpha + R\sin\alpha\cos x$

$\therefore R\cos\alpha = 4$ $\quad$ $R\sin\alpha = 3$

$\cos\alpha = \frac{4}{R}$ $\quad$ $\sin\alpha = \frac{3}{R}$



$\therefore R = 5$

$\tan\alpha = \frac{3}{4}$

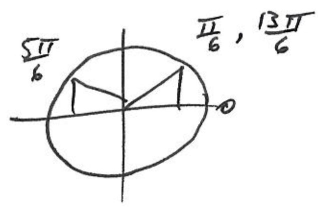
$\alpha = 0.6435011088$

$0 \leq x \leq 2\pi$

$0.64... \leq x + 0.64... \leq 2\pi + 0.64...$

$\therefore 5\sin(x+\alpha) = 2.5$

$\sin(x+\alpha) = \frac{1}{2}$



$x + \alpha = \frac{5\pi}{6}, \frac{13\pi}{6}$

$x = 1.974492769, \; 6.163282974$

$= 1.97 \text{ (2dp)}, \quad 6.16 \text{ (2dp)}$

b. $(2x+2)^{10} = {}^{10}C_0\,(2x)^{10} + {}^{10}C_1\,(2x)^9 \times 2 + \dots + {}^{10}C_r\,(2x)^{10-r}(2)^r + \dots$

for x^8, $r = 2$

$\therefore$ coeff of x^8 is ${}^{10}C_2\; 2^8 \times 2^2 = 45 \times 1024$

$= 46080$

c. when $n=1$, $3^{2n} - 1 = 3^2 - 1$

$= 8$

which is divisible by 8

$\therefore$ true for $n=1$

Assume true for $n=k$

ie $3^{2k} - 1 = 8P$ where P is an integer

Prove true for $n = k+1$

ie RTP: $3^{2(k+1)} - 1 = 8Q$ where Q is an integer

$$\begin{aligned}
LHS &= 3^{2(k+1)} - 1 \\
&= 3^{2k+2} - 1 \\
&= 3^2 \times 3^{2k} - 1 \\
&= 9(8P+1) - 1 \quad \text{from assumption} \\
&= 72P + 9 - 1 \\
&= 72P + 8 \\
&= 8(9P+1) \\
&= 8Q
\end{aligned}$$

where $Q = 9P+1$ and must be an integer since P is an integer.

$\therefore$ true for $n = k+1$

$\therefore$ by induction, the statement is true for all integers $n \geq 1$

d.

$$x^3 + 2x^2 + 3x + 10 = x^2(x+2) + 3(x+2) + 4$$

$$= (x+2)(x^2+3) + 4$$

ie $Q(x) = x^2 + 3$

e. $P(50 \text{ tails}) = {}^{100}C_{50} \left(\frac{1}{2}\right)^{50}\left(\frac{1}{2}\right)^{50}$

$\therefore P(\text{not } 50T) = 1 - {}^{100}C_{50} \frac{1}{2^{100}}$

13a. $LHS = \dfrac{2}{1 + \cos x}$

$$= \frac{2}{1 + \frac{1-t^2}{1+t^2}}$$

$$= 2 \div \left(\frac{1+t^2+1-t^2}{1+t^2}\right)$$

$$= \frac{2(1+t^2)}{2}$$

$$= 1 + t^2$$

$$= 1 + \tan^2\left(\frac{x}{2}\right)$$

$$= \sec^2\left(\frac{x}{2}\right)$$

b. $$V = \pi \int_0^1 (e^y)^2 - (ey)^2 \, dy$$

$y = \ln x \qquad y = \frac{x}{e}$

$x = e^y \qquad x = ey$

$$= \pi \int_0^1 e^{2y} - e^2 y^2 \, dy$$

$$= \pi \left[\frac{e^{2y}}{2} - \frac{e^2 y^3}{3} \right]_0^1$$

$$= \pi \left[\frac{e^2}{2} - \frac{e^2}{3} - \left(\frac{e^0}{2} - 0 \right) \right]$$

$$= \pi \left[\frac{e^2}{6} - \frac{1}{2} \right]$$

$$= \frac{\pi}{6} \left[e^2 - 3 \right] \quad \text{cu. units.}$$

c. $x = 12q \qquad y = 6(q+1)^2$

$$\frac{dx}{dq} = 12 \qquad \frac{dy}{dq} = 12(q+1)$$

$$\therefore \frac{dy}{dx} = \frac{dy}{dq} \times \frac{dq}{dx}$$

$$= \frac{12(q+1)}{12}$$

$$= q+1$$

When $q = \frac{3}{2}$, $m_{TANG} = \frac{3}{2} + 1$

$$= \frac{5}{2}$$

d. $n = 40$, $p = 0.3$ $q = 0.7$

$$\mu = np$$
$$= 40 \times 0.3$$
$$= 12$$

$$\sigma = \sqrt{npq}$$
$$= \sqrt{40 \times 0.3 \times 0.7}$$
$$= \sqrt{8.4}$$
$$= 2.898275349$$

$$z = \frac{15 - 12}{\sqrt{8.4}}$$
$$= 1.035098339$$
$$= 1.04 \text{ (2dp) for table}$$

$$P(X \geq 15) = P(z \geq 1.04)$$
$$= 1 - 0.8508$$
$$= 0.1492$$
$$= 14.9\% \text{ (1dp)}$$

e. $$\frac{dy}{dx} = \operatorname{cosec} y$$
$$= \frac{1}{\sin y}$$
$$\therefore \sin y \, dy = dx$$
$$\int \sin y \, dy = \int dx$$

$$-\cos y = x + c$$

at $\left(-\frac{\pi}{3}, \frac{\pi}{2}\right)$, $\quad -\cos\frac{\pi}{2} = -\frac{\pi}{3} + c$

$$0 = -\frac{\pi}{3} + c$$

$$c = \frac{\pi}{3}$$

$$\therefore -\cos y = x + \frac{\pi}{3}$$

$$\cos y = -\left(x + \frac{\pi}{3}\right)$$

$$y = \cos^{-1}\left(-\left(x+\frac{\pi}{3}\right)\right)$$ and, since $\cos^{-1}(-x) = \pi - \cos^{-1}x$

$$y = \pi - \cos^{-1}\left(x + \frac{\pi}{3}\right)$$

14. i) centre of barge is at $(26.8, -3.2)$

$\therefore Vt = 26.8$ and $-5t^2 = -3.2$

$$t^2 = 0.64$$

$$t = 0.8 \quad (t>0)$$

when $t = 0.8$

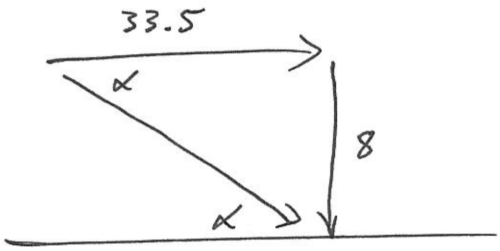
$$0.8V = 26.8$$

$$V = \frac{26.8}{0.8}$$

$$= 33.5 \text{ m/s}$$

ii) $\underset{\sim}{r} = \begin{pmatrix} Vt \\ -5t^2 \end{pmatrix}$

$$\underset{\sim}{v} = \begin{pmatrix} V \\ -10t \end{pmatrix}$$



$$\tan \alpha = \frac{8}{33.5}$$

$$\alpha = 13.43102887$$

$$= 13^\circ \text{ (nearest deg)}$$

b. $$\alpha^2 + \beta^2 + \gamma^2 = (\alpha + \beta + \gamma)^2 - 2(\alpha\beta + \beta\gamma + \gamma\alpha)$$

$$= (-1)^2 - 2 \times -8$$

$$= 1 + 16$$

$$= 17$$

c. $$x^3 - 3x^2 - 3x + 1 = 0$$

ie $$\tan^3\theta - 3\tan^2\theta - 3\tan\theta + 1 = 0$$

$$-3\tan^2\theta + 1 = 3\tan\theta - \tan^3\theta$$

$$1 = \frac{3\tan\theta - \tan^3\theta}{1 - 3\tan^2\theta}$$

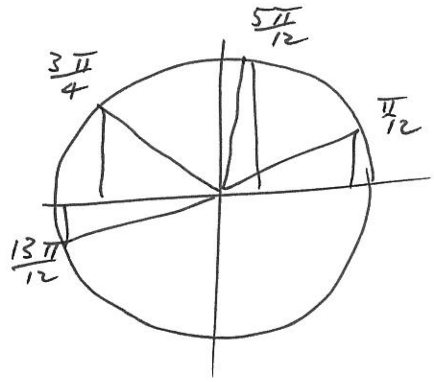
$$\therefore \tan 3\theta = 1$$

$$3\theta = \frac{\pi}{4}, \frac{5\pi}{4}, \frac{9\pi}{4}, \frac{13\pi}{4}, \ldots$$

$$\theta = \frac{\pi}{12}, \frac{5\pi}{12}, \frac{3\pi}{4}, \frac{13\pi}{12}, \ldots$$

$\therefore$ soln's to the eq'n are:

$$x = \tan\frac{\pi}{12}, \quad x = \tan\frac{5\pi}{12}, \quad x = \tan\frac{3\pi}{4}, \quad x = \tan\frac{13\pi}{12} \ldots$$



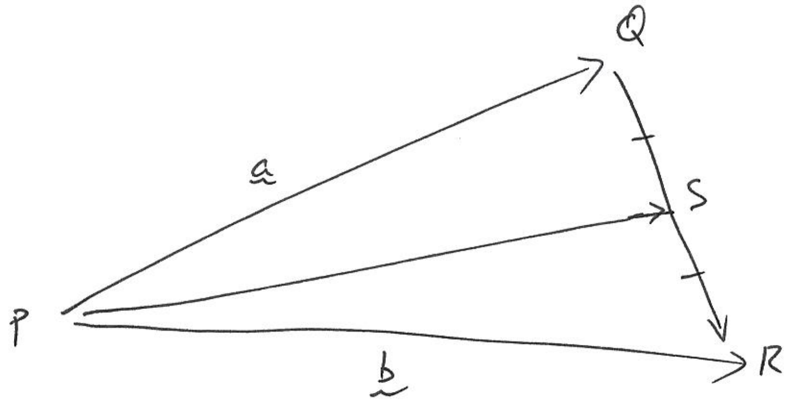
but, $\tan\frac{13\pi}{12} = \tan\frac{\pi}{12}$

and further soln's will repeat similarly

$\therefore$ the 3 distinct soln's are:

$$x = \tan\frac{\pi}{12}, \quad x = \tan\frac{5\pi}{12} \quad \text{and} \quad x = -1$$

d.



$$\vec{PQ} + \vec{QR} = \vec{PR}$$

$$\vec{QR} = \vec{PR} - \vec{PQ}$$

$$= \underset{\sim}{b} - \underset{\sim}{a}$$

$$\therefore \quad \vec{QS} = \frac{1}{2}\underset{\sim}{b} - \frac{1}{2}\underset{\sim}{a}$$

$$\vec{PQ} + \vec{QS} = \vec{PS}$$

$$\vec{PS} = \underset{\sim}{a} + \frac{1}{2}\underset{\sim}{b} - \frac{1}{2}\underset{\sim}{a}$$

$$= \frac{1}{2}\underset{\sim}{a} + \frac{1}{2}\underset{\sim}{b}$$

Now, $2\left(|\vec{PS}|^2 + |\vec{QS}|^2\right) = 2\left(\vec{PS}.\vec{PS} + \vec{QS}.\vec{QS}\right)$

$$= 2\left(\frac{1}{4}(\underset{\sim}{a}+\underset{\sim}{b})\cdot(\underset{\sim}{a}+\underset{\sim}{b}) + \frac{1}{4}(\underset{\sim}{b}-\underset{\sim}{a})\cdot(\underset{\sim}{b}-\underset{\sim}{a})\right)$$

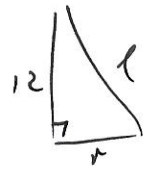
$$= \frac{1}{2}\left(\underset{\sim}{a}\cdot\underset{\sim}{a} + 2\underset{\sim}{a}\cdot\underset{\sim}{b} + \underset{\sim}{b}\cdot\underset{\sim}{b} + \underset{\sim}{b}\cdot\underset{\sim}{b} - 2\underset{\sim}{a}\cdot\underset{\sim}{b} + \underset{\sim}{a}\cdot\underset{\sim}{a}\right)$$

$$= \frac{1}{2}\left(2\underset{\sim}{a}\cdot\underset{\sim}{a} + 2\underset{\sim}{b}\cdot\underset{\sim}{b}\right)$$

$$= \left(|\underset{\sim}{a}|^2 + |\underset{\sim}{b}|^2\right)$$

$$= |\vec{PQ}|^2 + |\vec{PR}|^2$$

c. $\frac{dA}{dt} = 18\pi \qquad \frac{dr}{dt} = ? \qquad A = \pi r l$



$$r^2 + 12^2 = l^2$$

$$l = \sqrt{r^2+144} \qquad (l > 0)$$

$$\therefore A = \pi r\sqrt{r^2+144}$$

$$= \pi r\left(r^2+144\right)^{\frac{1}{2}}$$

$$\frac{dA}{dr} = \left(r^2+144\right)^{\frac{1}{2}} \times \pi + \pi r \times \frac{1}{2}\left(r^2+144\right)^{-\frac{1}{2}} \times 2r$$

$$= \pi\sqrt{r^2+144} + \frac{\pi r^2}{\sqrt{r^2+144}}$$

$$= \frac{\pi\left(r^2+144\right) + \pi r^2}{\sqrt{r^2+144}}$$

$$= \frac{2\pi r^2 + 144\pi}{\sqrt{r^2 + 144}}$$

when $l = 15$, $r^2 = 15^2 - 12^2$

$= 81$

$r = 9 \qquad (r > 0)$

$$\frac{dr}{dt} = \frac{dr}{dA} \times \frac{dA}{dt}$$

$$= \frac{\sqrt{81 + 144}}{2\pi \times 81 + 144\pi} \times 18\pi$$

$$= \frac{15 \times 18\pi}{306\pi}$$

$$= \frac{15}{17}$$

ie. radius is increasing at $\frac{15}{17}$ cm/s when $l = 15$ cm