Student Number: ______________________ Class Teacher: ____________________

St George Girls High School

Trial Higher School Certificate Examination

2017

Mathematics
Extension 1

General Instructions

- Reading time – 5 minutes
- Working time – 2 hours
- Write using black pen
- Board-approved calculators may be used
- A reference sheet is provided
- In Questions 11 – 15, show relevant mathematical reasoning and/or calculations

Section I	/10
Section II	
Question 11	/12
Question 12	/12
Question 13	/12
Question 14	/12
Question 15	/12
Total	**/70**

Total Marks – 70

Section 1 Pages 3 – 6

10 marks

- Attempt Questions 1 – 10
- Allow about 15 minutes for this section
- Answer on the multiple choice answer sheet provided at the back of this paper

Section II Pages 7 – 11

60 marks

- Attempt Questions 11 – 15
- Allow about 1 hour and 45 minutes for this section
- Begin each question in a new writing booklet

Students are advised that this is a Trial Examination only and does not necessarily reflect the content or format of the Higher School Certificate Examination.

Section I

10 marks
Attempt Questions 1 – 10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1-10

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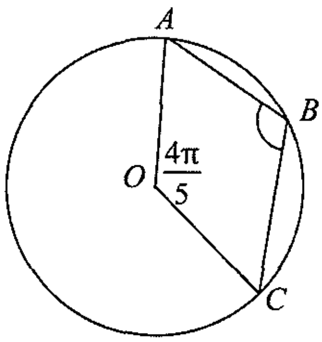
1 The points A, B and C lie on a circle with centre O, as shown in the diagram.

The size of $\angle AOC$ is $\frac{4\pi}{5}$ radians.



Not to scale

What is the size of $\angle ABC$ in radians?

(A) $\frac{3\pi}{10}$

(B) $\frac{\pi}{2}$

(C) $\frac{3\pi}{5}$

(D) $\frac{4\pi}{5}$

2 Which of the following is the exact value of $\int_{\frac{3}{\sqrt{2}}}^{3} \frac{4\ dx}{\sqrt{9-x^2}}$?

(A) $-\pi$

(B) $-\frac{\pi}{4}$

(C) $\frac{\pi}{4}$

(D) π

3 What are the coordinates of the point that divides the interval joining $P\ (2, 1)$ and $Q\ (2, 8)$ internally in the ratio 3: 4?

(A) (1, 7)

(B) (2, 4)

(C) (2, 7)

(D) (4, 2)

4 An oil slick is in the shape of a circle. Its surface area is increasing at a rate of $10\ m^2/s$. Let r metres be the radius of the oil slick in t seconds.

The rate of increase of r in m/s, is given by

(A) $\frac{5}{\pi r}$

(B) $\frac{20}{\pi r}$

(C) $\frac{10}{\pi r^2}$

(D) $\frac{1}{20\pi r}$

5 Let $f(x) = \frac{2}{x-3} + 1.$

The equations of the asymptotes of the graph of the inverse function $f^{-1}(x)$ are

(A) $x = 1$ and $y = 3$

(B) $x = 1$ and $y = -3$

(C) $x = 3$ and $y = 1$

(D) $x = -3$ and $y = -1$

6 A particle moves in a straight line such that it's displacement from the origin is x metres.

The velocity of the particle at any point is given by $v = 2x^2 - 3$ m/s.

Find the acceleration of the particle when it is 2 units to the right of the origin.

(A) $-\frac{2}{3}$ m/s^2

(B) 5 m/s^2

(C) 8 m/s^2

(D) 40 m/s^2

7 The function $f(x) = \sin x - \frac{2x}{3}$ has a real root close to $x = 1.5$.

Let $x = 1.5$ be a first approximation to the root.

What is the second approximation to the root using Newton's method?

(A) 1.495

(B) 1.496

(C) 1.503

(D) 1.504

8 If $\sqrt{3}\tan x = -1$ which expression gives all the possible values of x, where n is an integer?

(A) $x = 2n\pi \pm \frac{\pi}{6}$

(B) $x = n\pi - \frac{5\pi}{6}$

(C) $x = n\pi + \frac{\pi}{6}$

(D) $x = n\pi - \frac{\pi}{6}$

9 What is the domain and range of of $y = 3\sin^{-1}(2x)$?

(A) Domain: $-\frac{1}{2} \leq x \leq \frac{1}{2}$. Range $-\frac{1}{3} \leq y \leq \frac{1}{3}$

(B) Domain : $-2 \leq x \leq 2$. Range $-\frac{1}{3} \leq y \leq \frac{1}{3}$

(C) Domain : $-\frac{1}{2} \leq x \leq \frac{1}{2}$. Range $-\frac{3\pi}{2} \leq y \leq \frac{3\pi}{2}$

(D) Domain : $-2 \leq x \leq 2$. Range $-\frac{3\pi}{2} \leq y \leq \frac{3\pi}{2}$

10 The roots of $2x^3 - 6x^2 - 8x + 12 = 0$ are α, β and γ.

What is the value of $(\alpha + 2)(\beta + 2)(\gamma + 2)$?

(A) -12

(B) -6

(C) 6

(D) 12

End of Section I

Section II

60 marks

Attempt Questions 11 – 15

Allow about 1 hour and 45 minutes for this section

Answer each question in the appropriate writing booklet.

Your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (12 marks) Use a separate writing booklet **Marks**

(a) Find the size of the acute angle between the lines $x - y - 4 = 0$ and $3x - y + 4 = 0$. Answer to the nearest degree. **2**

(b) Differentiate $y = \log_e(\sin^{-1}x)$ **2**

(c) Find $\int \frac{1}{9+x^2} \, dx$ **2**

(d) Evaluate $\lim_{x \to 0} \frac{3x}{\sin 2x}$ **2**

(e) Show that $(x + 3)$ is a factor of $x^3 - 3x^2 - 10x + 24$ and hence factorise $x^3 - 3x^2 - 10x + 24$ fully. **4**

End of question 11

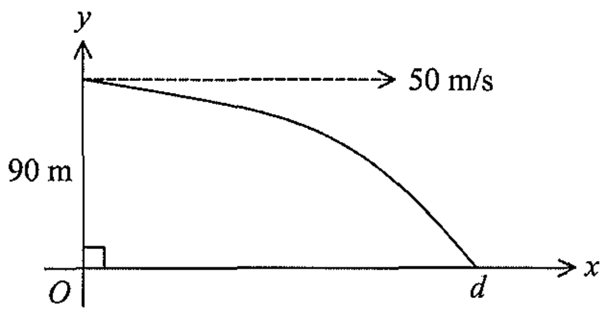
Question 12 (12 marks) Use a separate writing booklet **Marks**

(a) Evaluate $\int_{-1}^{0} x\sqrt{1+x}\,dx$, using the substitution $u = 1 + x$ **3**

(b) Solve the inequality $\frac{2x}{x-1} \geq 1$ **3**

(c) Find the exact value of $\sin(2\tan^{-1}\frac{1}{2})$ **2**

(d) The diagram below shows the trajectory of a ball thrown horizontally, at a speed of 50 ms^{-1}, from the top of a tower 90 metres above ground level.



The ball strikes the ground d metres from the base of the tower.

(i) Show that the equations describing the trajectory of the ball are: **2**

$x = 50t$ and $y = 90 - \frac{1}{2}gt^2$

where g is the acceleration due to gravity and t is the time in seconds.

(ii) Prove that the ball strikes the ground at time $t = 6\sqrt{\frac{5}{g}}$ seconds. **1**

(iii) How far from the base of the tower does the ball strike the ground? **1**

End of question 12

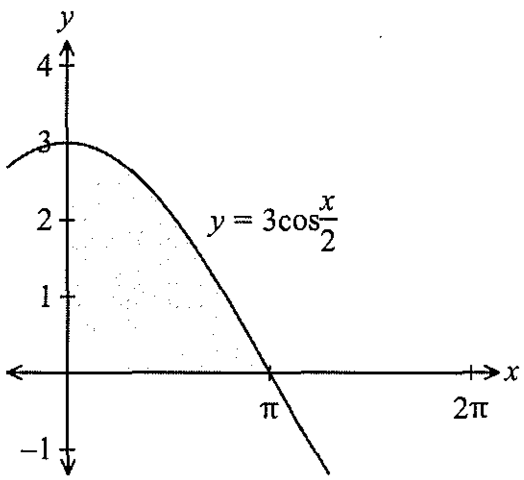
Question 13 (12 marks) Use a separate writing booklet **Marks**

(a) (i) Express $\sqrt{3}\sin t + \cos t$ in the form $R\sin(t+\alpha)$ where α is in radians, and $0 \le \alpha \le \frac{\pi}{2}$ **2**

(ii) Hence, or otherwise, find the solutions of the equation **2**

$$\sqrt{3}\sin t + \cos t = \sqrt{3} \quad \text{for} \quad 0 \le t \le 2\pi$$

(b) The region bounded by the graph and the x-axis between and is rotated about the x-axis to form a solid **3**



Find the exact volume of the solid.

(c) Newton's law of cooling states that when an object at temperature $T\,°\text{C}$ is placed in an environment at temperature $T_0°\text{C}$, the rate of the temperature loss is given by the equation $\frac{dT}{dt} = -k(T - T_0)$ where t is the time in minutes and k is a positive constant.

(i) Show that $T = T_0 + Ae^{-kt}$ satisfies the above equation. **2**

(ii) An object whose initial temperature is 60°C is placed in a room in which the internal temperature is maintained at 12°C. After 25 minutes, the temperature of the object is 30°C. How long will it take for the object's temperature to reduce to 15°C? **3**

End of question 13

Question 14 (12 marks) Use a separate writing booklet **Marks**

(a) $P(2at, at^2)$ is any point on the parabola $x^2 = 4ay$. The line d is parallel to the tangent at P and passes through the focus S of the parabola.

(i) Show the equation of the tangent at P is $y = tx - at^2$ **1**

(ii) Find the equation of the line d. **1**

(iii) The line d intersects the x-axis at the point R.
Find the coordinates of the midpoint, M, of the interval RS. **2**

(iv) Find the equation of the locus of M. **1**

(b) Simplify $\cos(2\cos^{-1}x)$ and hence evaluate $\int_0^{\frac{1}{2}} \cos(2\cos^{-1}x)\, dx$. **2**

(c) A particle is moving in a straight line performing Simple Harmonic Motion.

At time t seconds it has a displacement x metres from a fixed point O on the line given by

$$x = 1 + 2\cos(2t - \frac{\pi}{3})$$

(i) Show that $\ddot{x} = -4(x - 1)$ **1**

(ii) Find the centre of the motion and the time taken for the particle to first reach maximum speed. **2**

(iii) Find the first time the particle is at rest and the amplitude of the motion **2**

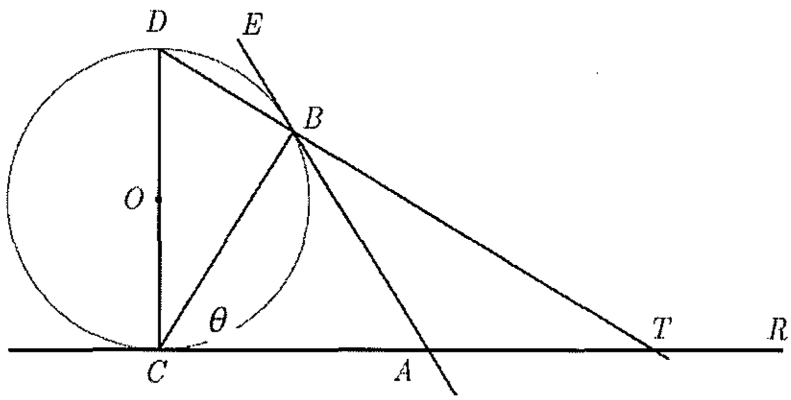
End of question 14

Marks

Question 15 (12 marks) Use a separate writing booklet

(a) Use Mathematical Induction to show that $5^n > 4^n + 3^n$ for all integers $n \geq 3$ **3**

(b)



In the diagram, CD is the diameter of the circle, centre O, and CR is a tangent to the circle C.

The line DT intersects the circle at B and CR at T.

The tangent to the circle at B intersects CR at A and $\angle BCA = \theta$.

Copy this diagram into your examination booklet.

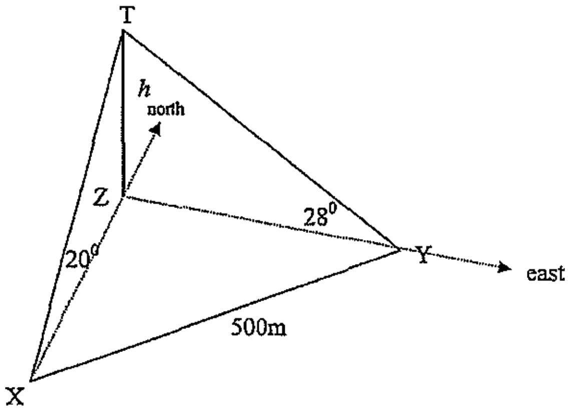
(i) Prove that $\angle ABT = \frac{\pi}{2} - \theta$. **2**

(ii) Prove that $AC = AT$ **2**

Question 15 continues on next page

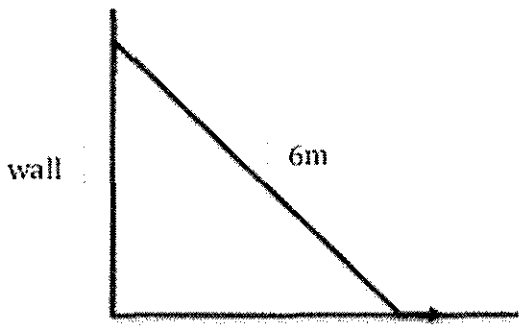
Question 15 continued

(c) 2



A person observes the angle of elevation of the top of a tree, which is h metres tall, from two positions.From a ponit X, due south of the tree,it is 20° and from the point Y, due east of the tree, it is 28°. The distance XY is 500m.

(d) 3



A ladder 6 metres long has its upper end against a vertical wall and its lower end on the horizontal floor.

The ladder is initially parallel to the wall, with the lower end at the origin.

The lower end moves away from the wall at a constant speed of 2m/s.

Find the speed at which the upper end moves down the wall two seconds after the lower end has left the wall.

End of paper

MATHEMATICS EXTENSION I – QUESTION

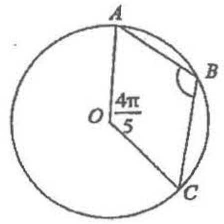
2017 Mathematics Extension 1

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS

Multiple choice.

1\. The points A, B and C lie on a circle with centre O, as shown in the diagram.

The size of $\angle AOC$ is $\frac{4\pi}{5}$ radians.



Not to scale

What is the size of $\angle ABC$ in radians?

reflex angle $\angle AOC = 2\pi - \frac{4\pi}{5}$

$= \frac{10\pi}{5} - \frac{4\pi}{5}$

$= \frac{6\pi}{5}$

$\angle ABC = \frac{1}{2} \times \frac{6\pi}{5}$

$= \frac{3\pi}{5}$ Answer C.

2\. $\displaystyle\int_{\frac{3}{\sqrt{2}}}^{3} \frac{4\,dx}{\sqrt{9-x^2}}$

$= \displaystyle\int_{\frac{3}{\sqrt{2}}}^{3} \frac{4\,dx}{\sqrt{3^2-x^2}}$

$= 4\left[\sin^{-1}\left(\frac{x}{3}\right)\right]_{\frac{3}{\sqrt{2}}}^{3}$

$= 4\left[\sin^{-1}\left(\frac{3}{3}\right) - \sin^{-1}\left(\frac{3}{\sqrt{2}} \div 3\right)\right]$

$= 4\left[\sin^{-1} 1 - \sin^{-1}\left(\frac{1}{\sqrt{2}}\right)\right]$

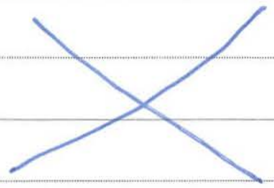
$= 4\left[\frac{\pi}{2} - \frac{\pi}{4}\right]$

$= \pi$ Answer D.

MATHEMATICS EXTENSION I – QUESTION

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS

3. $P(2,1)$ $\qquad$ $Q(2,8)$



$3 \quad : \quad 4$

$= \left[\dfrac{4(2)+3(2)}{3+4}, \dfrac{4(1)+3(8)}{3+4}\right]$

$= \left(\dfrac{8+6}{7}, \dfrac{4+24}{7}\right)$

$= (2, 4)$ $\qquad$ Answer B.

4. $A = \pi r^2$

$\dfrac{dA}{dr} = 2\pi r$

$\dfrac{dA}{dt} = \dfrac{dA}{dr} \cdot \dfrac{dr}{dt}$

$10 = 2\pi r \cdot \dfrac{dr}{dt}$

$\dfrac{10}{2\pi r} = \dfrac{dr}{dt}$

$\therefore \dfrac{dr}{dt} = \dfrac{5}{\pi r}$ $\qquad$ Answer A.

5. $f(x) = \dfrac{2}{x-3} + 1$ $\quad$ has $x = 3$ & $y = 1$ $\Big\}$ $f(x)$ has as asymptotes

$\therefore$ the inverse has asymptotes $y = 3$ & $x = 1$.

Answer A.

MATHEMATICS EXTENSION I – QUESTION

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS

6. $a = \frac{d}{dx}\ \frac{1}{2}v^2$

$= \frac{d}{dx}\frac{[2x^2-3]^2}{2}$

$= \frac{d}{dx}\left[\frac{4x^4-12x^2+9}{2}\right]$

$= \frac{d}{dx}[2x^4-6x^2+4.5]$

$= 8x^3-12x$

when $x=2$

$a = 8(2)^3 - 12(2)$

$= 64-24$

$= 40\ m/s^2$ Answer D.

7. $f(x) = \sin x - \frac{2x}{3}$

$f'(x) = \cos x - \frac{2}{3}$.

formula sheet says

$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$

$f(1.5) = \sin(1.5) - \frac{2\times1.5}{3}$

$= \sin(1.5) - 1$

$f'(1.5) = \cos(1.5) - \frac{2}{3}$

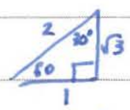
$x_1 = 1.5$

$\therefore x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$

$= 1.5 - \frac{[\sin(1.5)-1]}{\cos 1.5 - \frac{2}{3}}$

$= 1.496$ (3 dp). Answer B.

8) $\frac{\sqrt{3}\tan x}{\sqrt{3}} = \frac{-1}{\sqrt{3}}$



$\tan\frac{\pi}{6} = \frac{1}{\sqrt{3}}$

$\tan x = -\frac{1}{\sqrt{3}}$

formula sheet says....

$\tan\theta = \alpha$

$\therefore x = n\pi + \tan^{-1}\left(-\frac{1}{\sqrt{3}}\right)$

$\theta = n\pi + \tan^{-1}\alpha$.

$= n\pi - \frac{\pi}{6}$. Answer D.

MATHEMATICS EXTENSION I – QUESTION

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS

9. $y = 3\sin^{-1}(2x)$

Background

If $y = \sin^{-1} x$ $\quad -1 \le x \le 1$

$-\frac{\pi}{2} \le y \le \frac{\pi}{2}$

If $y = 3\sin^{-1}(2x)$ $\quad -1 \le 2x \le 1$

$\frac{y}{3} = \sin^{-1}(2x)$ $\quad -\frac{1}{2} \le x \le \frac{1}{2}$

$-\frac{\pi}{2} \le \frac{y}{3} \le \frac{\pi}{2}$

$-\frac{3\pi}{2} \le y \le \frac{3\pi}{2}$

Answer C.

10. $2x^3 - 6x^2 - 8x + 12 = 0$. $a=2$ $b=-6$ $c=-8$ $d=12$

$$\left.\begin{aligned} \alpha + \beta + \gamma &= -\frac{b}{a} \\ \alpha\beta + \alpha\gamma + \beta\gamma &= \frac{c}{a} \\ \alpha\beta\gamma &= -\frac{d}{a} \end{aligned}\right\}.$$

$\alpha + \beta + \gamma = \frac{6}{2} = 3$

$\alpha\beta + \alpha\gamma + \beta\gamma = \frac{-8}{2} = -4$

$\alpha\beta\gamma = \frac{-12}{2} = -6$.

$(\alpha+2)(\beta+2)(\gamma+2)$

$= (\alpha+2)(\beta\gamma + 2\beta + 2\gamma + 4)$

$= \alpha\beta\gamma + 2\alpha\beta + 2\alpha\gamma + 4\alpha + 2\beta\gamma + 4\beta + 4\gamma + 8$

$= \alpha\beta\gamma + 2(\alpha\beta + \alpha\gamma + \beta\gamma) + 4(\alpha + \beta + \gamma) + 8$

$= -6 + 2(-4) + 4(3) + 8$

$= -6 - 8 + 12 + 8$

$= -14 + 20$

$= 6$. Answer C.

TRIAL EXAM- MATHEMATICS EXTENSION 1 – QUESTION 11		
SUGGESTED SOLUTIONS	**MARKS**	**MARKER'S COMMENTS**
(a) $x-y-4=0$ $\quad$ $3x-y+4=0$	2	provides correct solution
$y=x-4$ $\quad$ $y=3x+4$		
$\frac{dy}{dx}=1$ $\quad$ $\frac{dy}{dx}=3$	1	finds gradient of the lines OR shows some understanding.
$\therefore m_1=1$ $\quad$ $\therefore m_2=3$		
$\tan\theta=\left\lvert\frac{m_1-m_2}{1+m_1m_2}\right\rvert$ ← (NOTE correct formula)		
• wrong formula, no marks!		
$=\left\lvert\frac{1-3}{1+(1)(3)}\right\rvert$		
$=\left\lvert\frac{-2}{4}\right\rvert$		
$\tan\theta=\frac{1}{2}$		
$\theta=26.56505118$		
$\therefore\theta=27^\circ$		
(b) $y=\ln(\sin^{-1}x)$	2	provides correct solution
$\frac{dy}{dx}=\frac{1}{\sin^{-1}x}\times\frac{1}{\sqrt{1-x^2}}$		
$=\frac{1}{\sin^{-1}x\sqrt{1-x^2}}$	1	demonstrates understanding of ~~differentiating~~ a log, ie $\frac{1}{f(x)}\times f'(x)$
• Correct rule on Reference sheet!	where	$f(x)=\sin^{-1}x$
	and	$f'(x)=\frac{1}{\sqrt{1-x^2}}$

TRIAL EXAM- MATHEMATICS EXTENSION 1 – QUESTION 11		
SUGGESTED SOLUTIONS	**MARKS**	**MARKER'S COMMENTS**
(c) $\int \frac{1}{9+x^2}\,dx$	2	provides correct solution
$= \int \frac{1}{3^2+x^2}\,dx$		1 for $\frac{1}{3}$ out the front
$= \frac{1}{3}\tan^{-1}\frac{x}{3} + C$		1 ~~for $\tan^{-1}\frac{x}{3}+C$~~
• correct rule on Reference sheet!		
(d) $\lim_{x \to 0} \frac{3x}{\sin 2x}$	2	provides correct solution
$= \frac{3}{2} \lim_{x \to 0} \frac{2x}{\sin 2x}$	1	demonstrates progress towards answer
$= \frac{3}{2} \times 1$ $\left\{ \text{since } \lim_{x \to 0} \frac{\sin x}{x} = 1 \right.$		
$= \frac{3}{2}$		

TRIAL EXAM- MATHEMATICS EXTENSION 1 – QUESTION 11

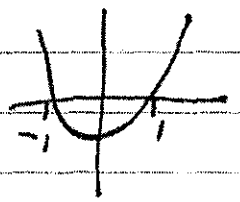
SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
(e) let $P(x)=x^3-3x^2-10x+24$		
$P(-3)=(-3)^3-3(-3)^2-10(-3)+24$		
$=-27-27+30+24$		
$=0$		
Since $P(-3)=0$ then $(x+3)$ is		
a factor of $P(x)$. —	1	for "show that" with working.
x^2-6x+8 — 1		or equivalent
$x+3\overline{)x^3-3x^2-10x+24}$		
$-\ \underline{x^3+3x^2}$		
$-6x^2-10x$		
$-\ \underline{-6x^2-18x}$		
$8x+24$		
$-\ \underline{8x+24}$		
0		
$\therefore P(x)=(x+3)(x^2-6x+8)$		
$=(x+3)(x-4)(x-2)$		
1 1		for each factor

• If the question says "show that", then it is asking you to completely justify the given answer by showing every step of working.

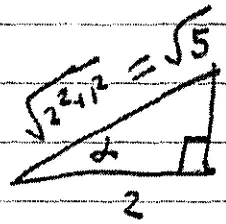
TRIAL EXAM- MATHEMATICS EXTENSION 1 – QUESTION 11

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
OR (e)		
let $P(x) = x^3 - 3x^2 - 10x + 24$		
$\qquad x^2 - 6x + 8$ — ①	OR	equivalent
$x+3\,)\,\overline{x^3 - 3x^2 - 10x + 24}$		
$-\ \underline{x^3 + 3x^2}$		
$-6x^2 - 10x$		
$-\ \underline{-6x^2 - 18x}$		
$8x + 24$		
$-\ \underline{8x + 24}$		
0		
Since there is a zero remainder,		
then $(x+3)$ is a factor of $P(x)$. —		① for "show that" with working
$P(x) = (x+3)(x^2 - 6x + 8)$		
$= (x+3)\underline{(x-4)}\,\underline{(x-2)}$		
① ①	for each factor.	

MATHEMATICS EXTENSION I – QUESTION 12

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
a) $\int_{-1}^{0} x\sqrt{1+x}\,dx = \int_{0}^{1}(u-1)\sqrt{u}\,du$ $u = 1+x$, $\therefore x = u-1$	1 for	correct substitution
$\frac{dx}{du} = 1$	1 for	correct limits of integration
$dx = du$		
$= \int_{0}^{1} u\sqrt{u} - \sqrt{u}\,du$ when $x = -1$, $u = 0$		
when $x = 0$, $u = 1$		
$= \int_{0}^{1} u^{\frac{3}{2}} - u^{\frac{1}{2}}\,du$		
$= \left[\frac{2u^{5/2}}{5} - \frac{2u^{\frac{3}{2}}}{3}\right]_{0}^{1}$		
$= \frac{2}{5} - \frac{2}{3}$		
$= \frac{-4}{15}$	1	
b) $\frac{2x}{x-1} \geq 1$ $x \neq 1$		
$(x-1)^2\frac{2x}{x-1} \geq (x-1)^2$	1	
$2x(x-1) \geq (x-1)^2$		
$2x(x-1) - (x-1)^2 \geq 0$		
$(x-1)[2x - (x-1)] \geq 0$		
$(x-1)(x+1) \geq 0$	1	
Sketch $y = (x-1)(x+1)$ 		Recognising that $x \neq 1$ was an integral part of this question; you could not earn the 3rd mark without it
$\therefore -1 \leq x,\ x > 1$	1	

MATHEMATICS EXTENSION I – QUESTION 12

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
c) $\sin\left(2\tan^{-1}\frac{1}{2}\right)$		
let $\alpha = \tan^{-1}\frac{1}{2}$ $\therefore \tan\alpha = \frac{1}{2}$ 	1	
$\sin\left(2\tan^{-1}\frac{1}{2}\right) = \sin 2\alpha$ $= 2\sin\alpha\cos\alpha$ $= 2 \times \frac{1}{\sqrt{5}} \times \frac{2}{\sqrt{5}}$ $= \frac{4}{5}$	1	
d) i $\ddot{x} = 0$ $\dot{x} = \int 0\,dt$ $= C_1$ when $t=0$, $\dot{x} = 50$ $50 = C_1$ $\therefore \dot{x} = 50$ $x = \int 50\,dt$ $= 50t + C_2$ when $t=0$, $x=0$ $0 = 50(0) + C_2$ $C_2 = 0$ $\therefore x = 50t$	1	Note that your working <u>must</u> begin at $\ddot{x}=0$, and the constant of integration <u>must</u> be calculated at each step.
$\ddot{y} = -g$ $\dot{y} = \int -g\,dt$ $= -gt + C_3$ when $t=0$, $\dot{y} = 0$ $0 = -g(0) + C_3$		

MATHEMATICS EXTENSION I – QUESTION 12

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
$\therefore C_3 = 0$ $\therefore \dot{y} = -gt$ $y = \int -gt \, dt$ $= -\frac{1}{2}gt^2 + C_4$ when $t=0, y=90$ $\therefore 90 = -\frac{1}{2}g(0)^2 + C_4$ $\therefore C_4 = 90$ $\therefore y = -\frac{1}{2}gt^2 + 90$ $= 90 - \frac{1}{2}gt^2$	1	
ii The ball strikes the ground when $y=0$ $90 - \frac{1}{2}gt^2 = 0$ $\frac{1}{2}gt^2 = 90$ $gt^2 = 180$ $t^2 = \frac{180}{g}$ $t = \pm\sqrt{\frac{180}{g}}$ $= \sqrt{36} \times \sqrt{\frac{5}{g}}$ ($t>0$, time) $t = 6\sqrt{\frac{5}{g}}$	1	
iii When $t = 6\sqrt{\frac{5}{g}}$, $x = 50 \times 6\sqrt{\frac{5}{g}}$ $= 300\sqrt{\frac{5}{g}}$ $\therefore$ lands $300\sqrt{\frac{5}{g}}$ metres from tower	1	

MATHEMATICS EXTENSION I – QUESTION 13		
SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
(a) i) $\sqrt{3}\sin t + \cos t \equiv R\sin(t+\alpha)$		
$= R\sin t\cos\alpha + R\cos t\sin\alpha$		This part of the question was mostly done well.
$= R\cos\alpha . \sin t + R\sin\alpha . \cos t$		
Equating coefficients		
$R\cos\alpha = \sqrt{3}$ --- (1)		
$R\sin\alpha = 1$ --- (2)		
$(1)^2 + (2)^2$		
$R^2\cos^2\alpha + R^2\sin^2\alpha = (\sqrt{3})^2 + 1$		
$R^2(\cos^2\alpha + \sin^2\alpha) = 3+1$		
$R^2 = 4$		
$R = 2$ since $R>0$	1	
and (2) ÷ (1)		
$\frac{\sin\alpha}{\cos\alpha} = \frac{1}{\sqrt{3}}$	$\frac{1}{2}$	
$\tan\alpha = \frac{1}{\sqrt{3}}$		
$\alpha = \frac{\pi}{6}$.	$\frac{1}{2}$	
$\therefore \sqrt{3}\sin t + \cos t = 2\sin(t+\frac{\pi}{6})$		
ii) From (i)		
$2\sin(t+\frac{\pi}{6}) = \sqrt{3}$		
$\sin(t+\frac{\pi}{6}) = \frac{\sqrt{3}}{2}$.		
$t+\frac{\pi}{6} = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{13\pi}{3}, \ldots$	1	
$t = \frac{\pi}{6}, \frac{\pi}{2}, \frac{25\pi}{6}, \ldots$		Some students did not consider the domain that $0 \le t \le 2\pi$.
but $0 \le t \le 2\pi$		
$\therefore t = \frac{\pi}{6}$ or $\frac{\pi}{2}$	1	

MATHEMATICS EXTENSION I – QUESTION 13

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
b) $V=\pi\int_0^{\pi} 3^2\cos^2\frac{x}{2}\,dx$	$\frac{1}{2}$	
$=\pi\int_0^{\pi} 9\cos^2\frac{x}{2}\,dx$		
$=9\pi\int_0^{\pi}\frac{1}{2}(1+\cos x)\,dx$	1	
$=\frac{9\pi}{2}\int_0^{\pi} 1+\cos x\,dx$		
$=\frac{9\pi}{2}\left[x+\sin x\right]_0^{\pi}$	$\frac{1}{2}$	
$=\frac{9\pi}{2}\left[(\pi+\sin\pi)-(0+\sin 0)\right]$	$\frac{1}{2}$	
$=\frac{9\pi^2}{2}\ u^3$	$\frac{1}{2}$	3
c) i) Let $T=T_0+Ae^{-kt}$ --- (1)		
$\frac{dT}{dt}=-kAe^{-kt}$	1	
$=-k\left[Ae^{-kt}+T_0-T_0\right]$	1	2
$=-k\left[T-T_0\right]$ from (1)		
OR From (1) $T-T_0=Ae^{-kt}$ --- (2)	1	
$\frac{dT}{dt}=-kAe^{-kt}$	1	
$=-k\left[T-T_0\right]$ from (2)		Students who did not show where Ae^{-kt} came from received only 1 mark.
ii)		

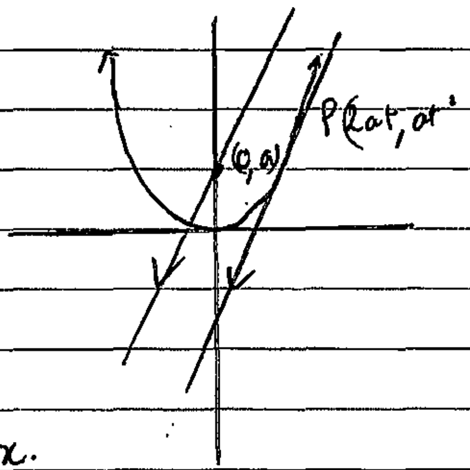
MATHEMATICS EXTENSION I – QUESTION 13		
SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
d) ii) At $t=0$, $T_0=12$, $T=60$		
$t=25$, $T=30$		
$t=?$, $T=15°C$		
Using $T=T_0+Ae^{-kt}$		
when $t=0$, $T_0=12$, $T=60$		
$60=12+Ae^{-k(0)}$		
$48=Ae^0$		
$A=48$	1	
$\therefore T=12+48e^{-kt}$		
when $t=25$, $T=30$		
$30=12+48e^{-k(25)}$		
$18=48e^{-25k}$		
$\frac{3}{8}=e^{-25k}$		
$-25k=\ln\left\|\frac{3}{8}\right\|$		
$k=-\frac{\ln\left\|\frac{3}{8}\right\|}{25}$ --- (1)	1	
when $T=15$,		
$15=12+48e^{-kt}$		
$3=48e^{-kt}$		3
$\frac{1}{16}=e^{-kt}$		
$-kt=\ln\left\|\frac{1}{16}\right\|$		Some students
$t=\frac{\ln\left\|\frac{1}{16}\right\|}{-k}$		rounded down to 70 minutes
$=\frac{\ln\frac{1}{16}}{\frac{\ln\left\|3/8\right\|}{25}} = 70\cdot66$	1	when it should have been
$\doteqdot 71$ minutes		rounded up to 71 min. Marks were still awarded

MATHEMATICS EXTENSION I – QUESTION 14

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
(a) $x^2 = 4ay$	1.	some students
(i) $y = \frac{x^2}{4a}$		found $\frac{dx}{dt}$
$\frac{dy}{dx} = \frac{2x}{4a}$		and $\frac{dy}{dt}$
$= \frac{x}{2a}$		used $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$
when $x = 2at$		
$\frac{dy}{dx} = \frac{2at}{2a}$		Well done.
$= t$. ← must prove		
$\therefore$ gradient of tangent at $(2at, at^2)$ is t.		
equation of tangent at $(2at, at^2)$		
$y - y_1 = m(x - x_1)$		
$y - at^2 = t(x - 2at)$		
$y - at^2 = tx - 2at^2$		
$y = tx - 2at^2 + at^2$		
$y = tx - at^2$		
(ii) line d parallel to tangent at P through focus	1	. Some students
for parallel lines $m_1 = m_2$		incorrectly
$\therefore m = t$ focus $(0, a)$		stated the focus.
$y - y_1 = m(x - x_1)$		
$y - a = t(x - 0)$		. well done.
$y - a = tx$		
$y = tx + a$		

MATHEMATICS EXTENSION I – QUESTION 14

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
pg 2 Q 14		
iii) $y = tx + a$		well done.
for x intercept, $y = 0$.		
$0 = tx + a$		
$tx = -a$		
$x = \frac{-a}{t}$		
$\therefore R\left(\frac{-a}{t}, 0\right)$ $\quad S = (0, a)$	R(1)	
$M = \text{Midpoint}_{RS}$		
$= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$		
$= \left(\frac{\frac{-a}{t} + 0}{2}, \frac{0 + a}{2}\right)$		
$= \left(\frac{-a}{2t}, \frac{a}{2}\right)$	M (1)	0
iv) $M = \left(\frac{-a}{2t}, \frac{a}{2}\right)$	(1)	only 1 student noted $x \neq 0$.
$x = \frac{-a}{2t}$		
$y = \frac{a}{2}$		
$\therefore$ y is independent of x.		
x has a domain of all x except $x = 0$		
$\therefore$ equation of locus is $y = \frac{a}{2}$ $(x \neq 0)$.		
$x = 0$, (this is because if at vertex, line through focus parallel to tangent will have no x intercept.		



MATHEMATICS EXTENSION I – QUESTION

14 SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
(b) $\cos\left(2\cos^{-1}x\right)$		
		difficulties
let $\alpha = \cos^{-1}x$	1	encountered
$\cos\alpha = x$		using
		$\cos 2\alpha = 1 - 2\sin^2\alpha$.
$\therefore \cos\left(2\cos^{-1}x\right) = \cos 2\alpha$		etc
$= 2\cos^2\alpha - 1$		
$= 2x^2 - 1$		
hence...		
$\int_0^{\frac{1}{2}} \cos\left(2\cos^{-1}x\right)dx$	1	
$= \int_0^{\frac{1}{2}} 2x^2 - 1 \, dx$		
$= \left[\frac{2x^3}{3} - x\right]_0^{\frac{1}{2}}$		
$= \frac{1}{12} - \frac{1}{2} - 0$		
$= \frac{-5}{12}$		
		Students needed to
c (i) $x = 1 + 2\cos\left(2t - \frac{\pi}{3}\right)$	1.	show all
$\dot{x} = -4\sin\left(2t - \frac{\pi}{3}\right)$		steps for
$\ddot{x} = -8\left(\cos\left(2t - \frac{\pi}{3}\right)\right)$		full marks
$= -\cancel{8}4\left[2\cos\left(2t - \frac{\pi}{3}\right) + 1 - 1\right]$		
$= -4\left[\quad x - 1\right]$ *		
since * $x = 1 + 2\cos\left(2t - \frac{\pi}{3}\right)$		
(ii) Centre of motion. $\ddot{x} = 0$	1.	
$0 = -4(x-1)$		

$\therefore x = 1$ is the centre of the motion if $\sin(2t - \frac{\pi}{3}) = \mp 1$

$2t - \frac{\pi}{3} = \frac{\pi}{2} \mp n\pi$ n integer

$2t = \frac{5\pi}{6} \pm n\pi$

$t = \frac{5\pi}{12} + \frac{n\pi}{2}$

$\therefore$ first time is when $n = 0$

$\therefore$ first time max^m speed is reached is $t = \frac{5\pi}{12}$ seconds 1

(iii) if at rest, $v = 0$. $t = ?$ first time $v = 0$. 1

$\dot{x} = -4\sin(2t - \frac{\pi}{3})$

$0 = \sin(2t - \frac{\pi}{3})$

$\therefore 2t - \frac{\pi}{3} = n\pi$ n integer

$2t = \frac{\pi}{3} + n\pi$ n integer

$t = \frac{\pi}{6} + \frac{n\pi}{2}$

$\therefore$ first time will be when $n = 0$

$\therefore t = \frac{\pi}{6}$ seconds 1

amplitude = $\underline{2}$. 1

MATHEMATICS EXTENSION I – QUESTION 15 (1)

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
a) Show that $5^n > 4^n + 3^n$ by mathematical induction.		
Step 1		
Show that the result is true for $n=3$	$\frac{1}{2}$	The majority of students got this mark.
$5^n > 4^n + 3^n$		
LHS $= 5^3$		
$= 125$		
RHS $4^3 + 3^3$		
$= 91$		
Since $125 > 91$		
$5^n > 4^n + 3^n$ for $n = 3$		
Step 2		
Assume the result is true for $n=k$ (an integer ≥ 3)	$\frac{1}{2}$	Well done
ie. Assume $5^k > 4^k + 3^k$		
Step 3		
Prove the result is true for $n=k+1$, assuming the statement is true for $n=k$	$\frac{1}{2}$	The majority of students know the correct layout for mathematical induction problems.

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
Prove $5^{k+1} > 4^{k+1} + 3^{k+1}$ — *		This part of the proof was not attempted as expected.
if $5^k > 4^k + 3^k$		
LHS $= 5^{k+1}$		
$= 5(5^k)$		
$> 5(4^k + 3^k)$ using the assumption	1/2	
$= 5.4^k + 5.3^k$		Common mistakes made by students include:
$> 4.4^k + 3.3^k$	1/2	• Modifying the statement * without using the assumption.
$= 4^{k+1} + 3^{k+1}$		• Using the inequation as an equation by substituting $5^k = 4^k + 3^k$ directly into the statement *.
$\therefore 5^{k+1} > 4^{k+1} + 3^{k+1}$		
Step 4		
Since the result is true for $n=3$, it is also true for $n=k+1$, ie, $n=3+1$, and so on for $n=3,4,5,\cdots$.	1/2	
Therefore, by the Principle of Mathematical Induction it holds true for all $n \geq 3$		

MATHEMATICS EXTENSION I – QUESTION

3

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
i) $\angle ABT = \pi/2 - \theta$		
In $\triangle ABC$,		Students
$AB = AC$ (tangents from an external	½	were not
common point to a		penalised
circle are equal)		for not
		using the
$\therefore \angle ABC = \angle ACB = \theta$ (angles opposite	½	appropriate
equal sides of a		terminology
triangle are equal)		for the
		circle
In $\triangle BCD$, $\angle CBD = \pi/2$ (angle subtended	½	geometry
by the diameter is a		theorems
right angle).		but it is
		highly
$\therefore \angle CBT = \pi/2$ ($\angle DBT$ is a	½	recommended
straight angle)		that
$\therefore \angle ABT = \pi/2 - \theta$		everyone
		re-visits
		these
		theorems.
		For example,
		the majority
		of students
		used
		alternate
		segment
		theorem in
		short.
		Students
		using alternate
		methods

MATHEMATICS EXTENSION I – QUESTION

4

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
		were awarded ½ mk for each theorem. leading towards the proof.
c) $\tan 20° = \dfrac{h}{xz}$		Quite well done by the majority of students
$xz = \dfrac{h}{\tan 20°}$ or $\cot 20°$		
$\tan 28° = h/yz$	½	
$yz = h/\tan 28°$ or $\cot 28°$		
$xz^2 + yz^2 = 500^2$ Pythagoras' Theorem.		
$\left(\dfrac{h}{\tan 20}\right)^2 + \left(\dfrac{h}{\tan 28}\right)^2 = 500^2$	½	
$h^2\left[\left(\dfrac{1}{\tan 20°}\right)^2 + \left(\dfrac{1}{\tan 28}\right)^2\right] = 500^2$		
$h^2\left[\dfrac{\tan^2 25 + \tan^2 20}{\tan^2 20° \tan^2 28°}\right] = 25000$		Some students made mistakes in their calculations ~~at this~~ stage.
$h^2 = \dfrac{250000 \times \tan^2 20 \times \tan^2 28}{\tan^2 28 + \tan^2 20}$	½	
$= \sqrt{22551.44}$		
$= 150$ metres (nearest metre)	½	

MATHEMATICS EXTENSION I – QUESTION

SUGGESTED SOLUTIONS	MARKS	MARKER'S COMMENTS
d) From Pythagoras, $x^2 + y^2 = 36$		
$y = (36 - x^2)^{1/2}$		
$\frac{dy}{dx} = \frac{1}{2}(36 - x^2)^{-1/2} \times -2x$	½	
$= \frac{-x}{\sqrt{36-x^2}}$	½	
From the chain rule, and given that the rate at which the bottom of the ladder slides to the right $\frac{dx}{dt}$ is 2 m/s		
$\frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt}$		
$= \frac{-x}{\sqrt{36-x^2}} \times 2$		
$= \frac{-2x}{\sqrt{36-x^2}}$	½	
When $t = 2$, $x = 4$ m	½	
$\frac{dy}{dt} = \frac{-8}{\sqrt{36-16}}$	½	Some students left the
$= \frac{-8}{\sqrt{20}}$		speed as
$= \frac{-4}{\sqrt{5}}$ m/s	½	$\frac{-4}{\sqrt{5}}$. No
∴ the speed at which the top falls is $\frac{4}{\sqrt{5}}$ m/s		marks were deducted. Speed, being a scalar quantity ~~should be~~ left as positive.