Student Number:
Teacher:

St George Girls High School

Mathematics Extension 1

2024 Trial HSC Examination

General Instructions

- Reading time – 10 minutes
- Working time – 2 hours
- Write using black pen
- Calculators approved by NESA may be used
- A reference sheet is provided
- For questions in **Section I**, use the Multiple-Choice answer sheet provided

  For questions in **Section II:**
  - Answer the questions in the booklets provided
  - Start each question in a new writing booklet
  - Show relevant mathematical reasoning and/or calculations
  - Marks may not be awarded for incomplete or poorly presented solutions, or where multiple solutions are provided

Total marks: 70

Section I – 10 marks (pages 3 – 7)

- Attempt Questions 1– 10
- Allow about 15 minutes for this section

Section II – 60 marks (pages 8 –12)

- Attempt Questions 11–15
- Allow about 1 hour and 45 minutes for this section

Q1-10	/10
Q11	/12
Q12	/12
Q13	/12
Q14	/13
Q15	/11
TOTAL	**/70**
	%

Section I

10 marks
Attempt Questions 1 – 10
Allow about 15 minutes for this section
Use the multiple-choice answer sheet provided for Questions 1 to 10.

1. A section of a proof by mathematical induction is shown below.

Test $n = 1$

LHS $= n$	RHS $= \frac{1}{6} \times 1 \times (1+1) \times (2+1)$
$= 1$	$= \frac{1}{6} \times 1 \times 2 \times 3$
	$= 1$

$\therefore$ true for $n = 1$

What is the name of this section of the proof?

A. Initial statement

B. Base case

C. Inductive step

D. Conclusion

2. What is the domain of the function $f(x) = 3\cos^{-1}\left(1 + \frac{x}{2}\right)$?

A. $[-4, 0]$

B. $[-4, 4]$

C. $[-2, 2]$

D. $[-1, 1]$

Section I continued

3. What is the Cartesian equation of the curve with the parametric equations below?

$$x = 3\sin\theta + 1$$
$$y = 3\cos\theta$$

A. $x^2 + (y-1)^2 = 3$

B. $(x-1)^2 + y^2 = 3$

C. $x^2 + (y-1)^2 = 9$

D. $(x-1)^2 + y^2 = 9$

4. In ΔAOB, $OA = 10\text{cm}$, $OB = 10\text{cm}$, and $\angle AOB = \theta$ radians. θ is increasing at a constant rate of 0.01 radians/second. What, correct to 2 decimal places, is the rate at which the area of the triangle is changing when $\theta = 1$?

A. $0.27\text{cm}^2/\text{s}$

B. $0.28\text{cm}^2/\text{s}$

C. $0.49\text{cm}^2/\text{s}$

D. $0.50\text{cm}^2/\text{s}$

5. Ten people, including Gordon and David, wish to board a plane. The plane has only one door, so only one person can board at a time. Given David has to board before Gordon, in how many ways can the 10 people board the plane?

A. 181 440

B. 362 880

C. 1 814 400

D. 3 628 800

Section I continued

6. Consider the vector $\boldsymbol{u} = 5\boldsymbol{i} - 12\boldsymbol{j}$. Which of the following is a vector which is perpendicular to $\boldsymbol{u}$ and has magnitude 26 units?

A. $\boldsymbol{v} = -12\boldsymbol{i} - 5\boldsymbol{j}$

B. $\boldsymbol{v} = 12\boldsymbol{i} + 5\boldsymbol{j}$

C. $\boldsymbol{v} = 24\boldsymbol{i} - 10\boldsymbol{j}$

D. $\boldsymbol{v} = -24\boldsymbol{i} - 10\boldsymbol{j}$

7. A bag contains 5 blue balls, 7 green balls, 8 yellow balls, 9 orange balls, and 10 red balls. Balls are drawn at random one at a time without replacement from the bag. What is the least number of balls that needs to be drawn from the bag in order to be certain that 8 balls of the same colour are amongst those selected?

A. 9

B. 34

C. 35

D. 36

8. Which of the following is an expression for $\int \frac{1}{ax^2 + a^2}\,dx$, where a is a constant ?

A. $\frac{1}{\sqrt{a}}\tan^{-1}\left(\frac{x}{\sqrt{a}}\right) + C$

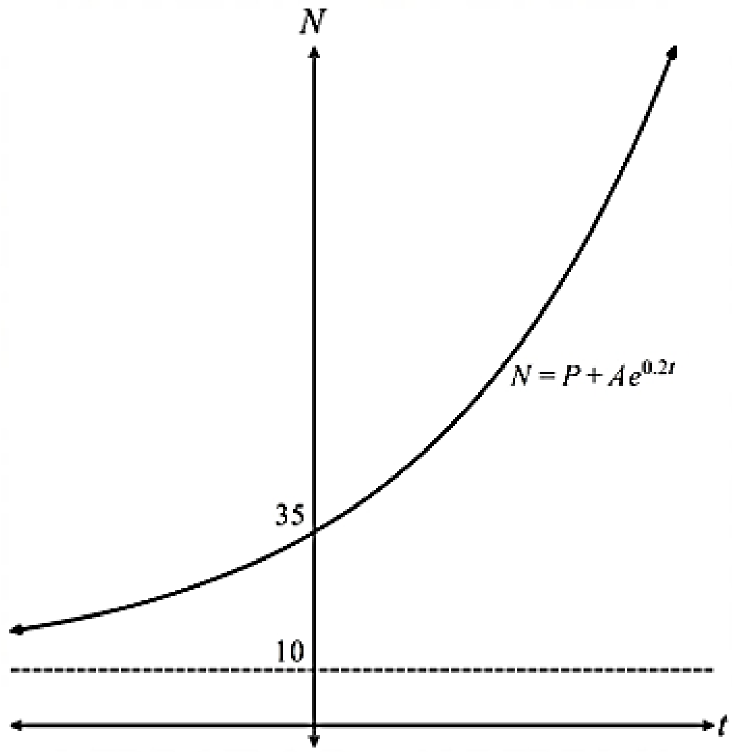
B. $\frac{1}{a}\tan^{-1}\left(\frac{x}{\sqrt{a}}\right) + C$

C. $\frac{1}{a\sqrt{a}}\tan^{-1}\left(\frac{x}{\sqrt{a}}\right) + C$

D. $\frac{1}{a^2}\tan^{-1}\left(\frac{x}{\sqrt{a}}\right) + C$

Section I continued

9. The quantity N increases exponentially over time according to the equation $N = P + Ae^{0.2t}$. The graph below shows the change in N over time (t).



Which equation describes the rate of change of N with respect to t?

A. $\dfrac{dN}{dt} = 0.2(N - 10)$

B. $\dfrac{dN}{dt} = 0.2(N - 25)$

C. $\dfrac{dN}{dt} = 10(N - 0.2)$

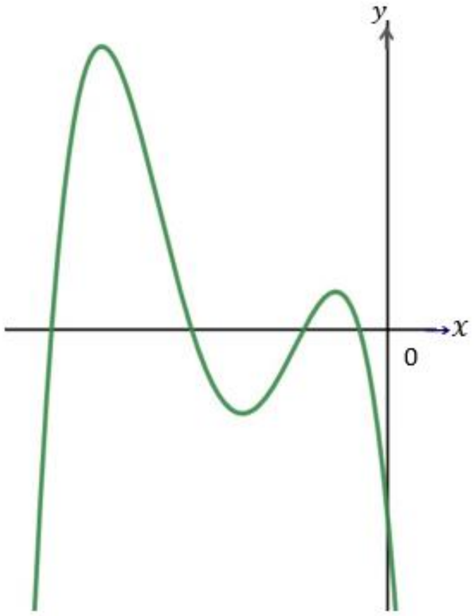
D. $\dfrac{dN}{dt} = 25(N - 10)$

Section I continued

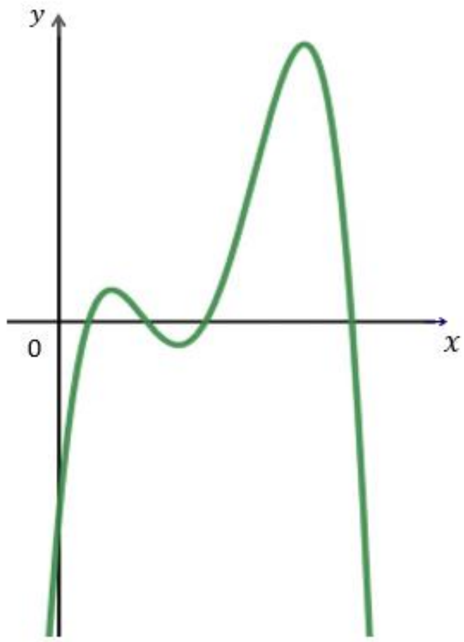
10. Consider the polynomial $P(x) = ax^4 + bx^3 + cx + d$, where the constant a is negative and the constant b is positive.

    Which graph could represent $y = P(x)$?

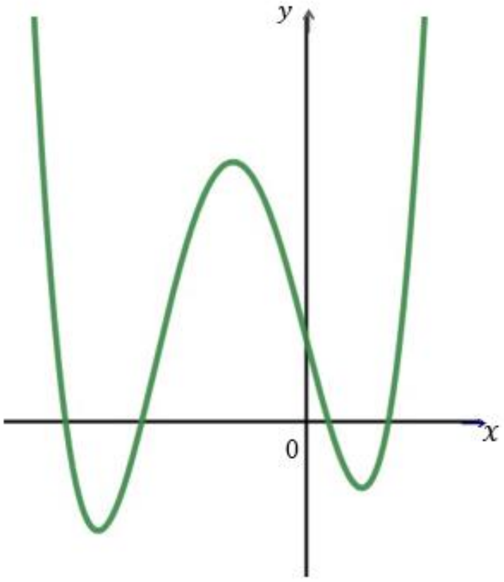
A.



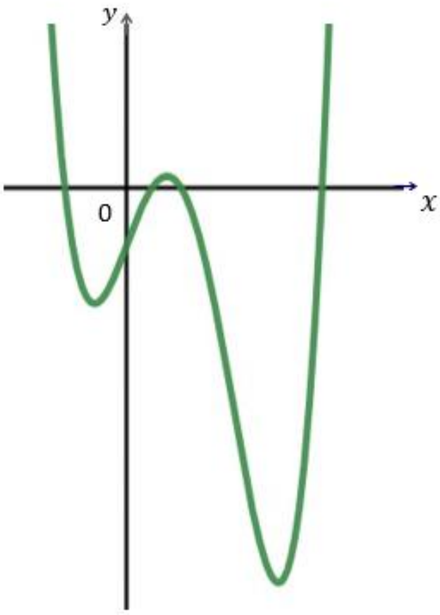
B.



C.



D.



End of Section I

Section II
60 marks
Attempt Questions 11 – 15
Allow about 1 hour and 45 minutes for this section

In Questions 11-15, your responses should include relevant mathematical reasoning and/or calculations

Question 11 (12 marks) **Marks**

(a) Evaluate $\int_0^{\pi} \cos^2 \theta \ d\theta$. **3**

(b) Sketch the curve $y = \tan^{-1} x$, showing all important features. **2**

(c) Consider the polynomial $P(x) = x^4 - 3x^3 - 6x^2 + 28x - 24$. It is known that $P(x)$ has a triple root at $x = \alpha$.

(i) By using differentiation, find the value of α. **2**

(ii) Hence express $P(x)$ as a product of linear factors. **1**

(iii) Hence solve $x^4 - 3x^3 - 6x^2 + 28x - 24 \geq 0$. **1**

(d) Use the t-formulae to solve $2\cos x - \sqrt{3}\sin x = -2$, for $0 \leq x \leq 2\pi$. **3**

Give your answer correct to 2 decimal places where necessary.

Question 12 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) (i) Express $\sqrt{12}\sin x + 2\cos x$ in the form $R\sin(x-\alpha)$, where $R > 0$ and $0 \le \alpha < 2\pi$. **3**

(ii) Hence, or otherwise, solve $\sqrt{12}\sin x + 2\cos x = 2$, for $0 \le x \le 2\pi$. **1**

(b) Use mathematical induction to prove that $9^n - 4^n$ is divisible by 5 for all integers $n \ge 1$. **3**

(c) Find the coefficient of x^3 in the expansion of $\left(2x^3 - \frac{1}{x^2}\right)^{11}$. **2**

(d) By using the substitution $u = x + 1$, or otherwise, evaluate $\int_0^1 x\sqrt{x+1}\,dx$, correct to 2 decimal places. **3**

Question 13 (12 marks) Use a SEPARATE writing booklet. **Marks**

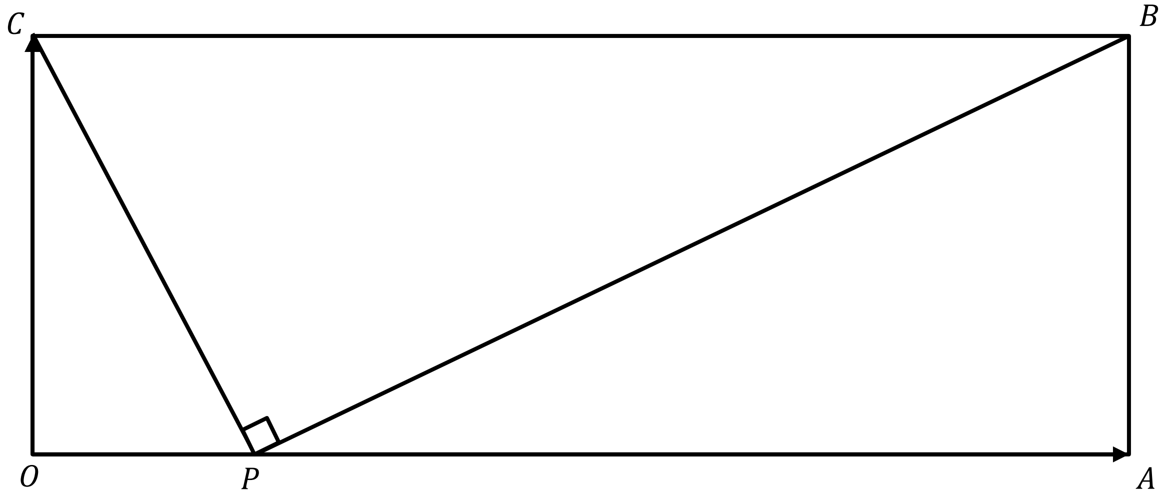
(a) Greta Hayes hits a hockey ball at an angle of $30°$ above the horizontal, at an initial velocity of $35\ ms^{-1}$. Gravity acts downward at $9.8\ ms^{-2}$. **2**

By integrating $\ddot{y} = -9.8$, show that the ball reaches its maximum height before $t = 2$.

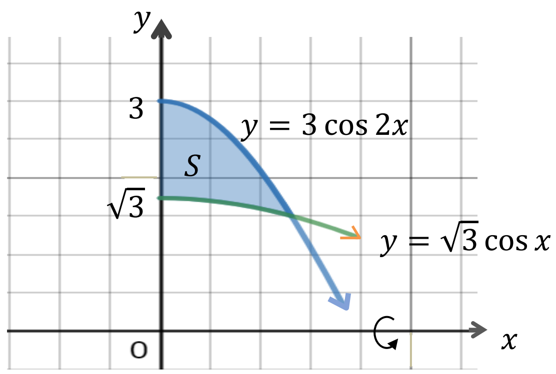
(b) Solve $\dfrac{3}{x} > x + 2$. **2**

(c) In the diagram, $OABC$ is a rectangle in which $\overrightarrow{OA} = \frac{5}{2}\underset{\sim}{i}$ and $\overrightarrow{OC} = \underset{\sim}{j}$. **3**
P is a point on OA such that $\overrightarrow{OP} = \lambda\underset{\sim}{i}$, for some scalar parameter λ.

Use vector methods, and the fact that $\angle CPB = 90°$, to show that $2\lambda^2 - 5\lambda + 2 = 0$, and hence find any values of λ.



(d) Consider the curves $y = 3\cos 2x$ and $y = \sqrt{3}\cos x$, and the shaded region between them and the y-axis, as shown in the diagram below.



(i) By solving the equations simultaneously, show that the two curves meet when $x = \frac{\pi}{6}$. **2**

(ii) Find the volume of the solid of revolution formed when the shaded region is rotated about the x-axis. **3**

Question 14 (13 marks) Use a SEPARATE writing booklet. **Marks**

(a) Use Mathematical Induction to show that for all integers $n \geq 1$. **3**

$$\left(1 \times 1! + \frac{0}{1!}\right) + \left(2 \times 2! + \frac{1}{2!}\right) + \left(3 \times 3! + \frac{2}{3!}\right) + \cdots + \left(n \times n! + \frac{n-1}{n!}\right) = (n+1)! - \frac{1}{n!}$$

(b) Consider the curve $x = \sqrt{4 - y^2}$, and the two vectors $\boldsymbol{u} = \boldsymbol{i} + \sqrt{3}\boldsymbol{j}$ and $\boldsymbol{v} = \boldsymbol{i} - \sqrt{3}\boldsymbol{j}$.

(i) Sketch the curve, as well as $\boldsymbol{u}$ and $\boldsymbol{v}$ as position vectors. **1**

(ii) Find the angle between the two vectors $\boldsymbol{u}$ and $\boldsymbol{v}$. Give your answer exactly, in radians. **1**

(iii) Hence, or otherwise, find the exact area enclosed between the curve $x = \sqrt{4 - y^2}$ and $x = 1$. **2**

(c) The velocity of a particle is given by $v = \tan t$, for $0 \leq t \leq \frac{\pi}{2}$, where t is the time in seconds.

(i) Use differentiation from first principles to find an expression for a, the acceleration. You may use the fact that $\lim_{h \to 0} \frac{\tan h}{h} = 1$. **3**

(ii) Find when the acceleration is $2m/s^2$, correct to 2 decimal places. **1**

(iii) Find how far the particle has travelled in the first second in exact form. **2**

Question 15 (11 marks) Use a SEPARATE writing booklet. **Marks**

(a) Find $\displaystyle\int_0^{\frac{1}{2}} \frac{\sin^{-1} x}{\sqrt{1-x^2}}\,dx$. **2**

(b) O is at the base of a ramp inclined at an angle $\beta = \tan^{-1}\frac{1}{2}$ above the horizontal.

A stone is projected from point O with speed $V\ ms^{-1}$ at an angle $\alpha = \tan^{-1}\frac{4}{3}$ above the horizontal. The stone moves in a vertical plane above the ramp under gravity where the acceleration due to gravity is $g\ ms^{-2}$. At time t seconds the horizontal and vertical displacement of the stone relative to O is given by:

$$x = Vt\cos\alpha$$

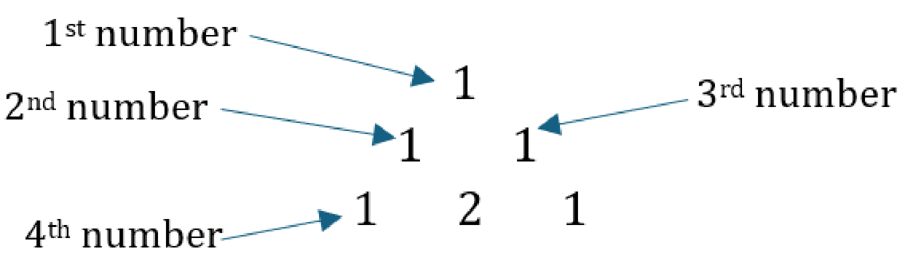
$$y = Vt\sin\alpha - \frac{1}{2}gt^2$$

(Do **not** prove these results.)

(i) Show that the stone hits the ramp at time $t = \frac{V}{g}$ seconds. **3**

(ii) Hence or otherwise, find the angle of impact, θ, between the stone and the ramp. **3**

(c) The first rows of Pascal's triangle are shown below. Consider an ordering of the numbers, counting down and across the rows, as given in the diagram. **3**



Find the sum of the first 496 numbers of Pascal's triangle.

END OF EXAMINATION

12MAT AT4 2024 Q1-10 **MARKER'S COMMENTS**

1. B

2. $-1 < 1+\frac{x}{2} < 1$

$-2 < \frac{x}{2} < 0$

$-4 < x < 0 \quad \therefore A$

3. $x = 3\sin\theta + 1$

$x - 1 = 3\sin\theta \quad (1)$

$y = 3\cos\theta \quad (2)$

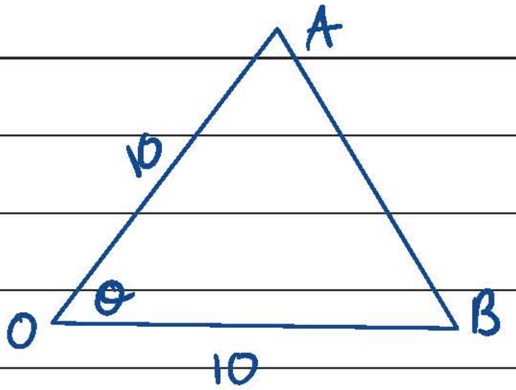
$(1)^2 + (2)^2$

$(x-1)^2 + y^2 = 9\sin^2\theta + 9\cos^2\theta$

$= 9(\sin^2\theta + \cos^2\theta)$

$= 9 \quad \therefore D$

4.



$\frac{d\theta}{dt} = 0.01$

$A = \frac{1}{2}ab\sin C$

$= \frac{1}{2} \times 10 \times 10 \times \sin\theta$

$= 50\sin\theta$

$\frac{dA}{d\theta} = 50\cos\theta$

when $\theta = 1$, $\frac{dA}{d\theta} = 50\cos 1$

$\therefore \frac{dA}{dt} = \frac{dA}{d\theta} \times \frac{d\theta}{dt}$

$= 50(\cos 1) \times 0.01$

$= 0.27015\ldots \quad \therefore A$

Made with Goodnotes

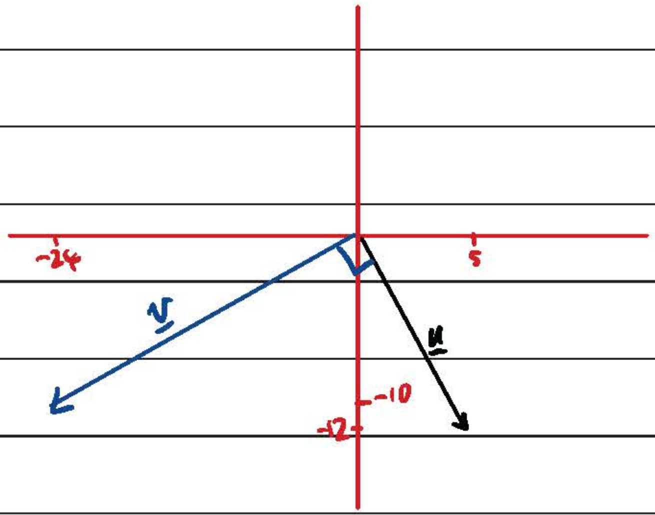
12MAT AT4 2024 Q1-10 **MARKER'S COMMENTS**

(5) 10 people can board in 10! ways.

In half of those ways, David boards before Gordon.

$\therefore \frac{1}{2} \times 10! = 1814400 \quad \therefore C$

(6)



$\therefore D$

(7) In the worst case, you might draw 5 blue, 7 green, 7 yellow, 7 orange, and 7 red balls $\therefore$ 33 balls without 8 of one colour. But the 34th ball must make a group of 8 with one colour. $\therefore B$

(8)
$$\int \frac{1}{ax^2 + a^2}\,dx = \frac{1}{a}\int \frac{1}{x^2 + (\sqrt{a})^2}\,dx$$
$$= \frac{1}{a}\left[\frac{1}{\sqrt{a}}\tan^{-1}\frac{x}{\sqrt{a}}\right] + C$$
$$= \frac{1}{a\sqrt{a}}\tan^{-1}\frac{x}{\sqrt{a}} + C \quad \therefore C$$

(9)
$$N = P + Ae^{0.2t}$$
$$\frac{dN}{dt} = 0.2Ae^{0.2t}$$
$$= 0.2(N - P)$$
$$= 0.2(N - 10) \quad \therefore A$$
since $P = 10$

(10) Negative quartic $\therefore$ not C, not D

Sum of roots $= \frac{-b}{a}$

> 0 (since $a < 0$, $b > 0$)

$\therefore B$

Question 11

MARKER'S COMMENTS

a) $\int_0^{\pi} \cos^2\theta \, d\theta$

$= \int_0^{\pi} \frac{1}{2}(1+\cos 2\theta)\, d\theta$ — 1 mark

$= \frac{1}{2}\int_0^{\pi} 1 + \cos 2\theta \, d\theta$

$= \frac{1}{2}\left[\theta + \frac{1}{2}\sin 2\theta\right]_0^{\pi}$ — 1 mark

$= \frac{1}{2}\left[\left(\pi + \frac{1}{2}\sin 2\pi\right) - \left(0 + \frac{1}{2}\sin 2(0)\right)\right]$

$= \frac{1}{2}\left[\left(\pi + \frac{1}{2}\times 0\right) - \left(0 + \frac{1}{2}\times 0\right)\right]$

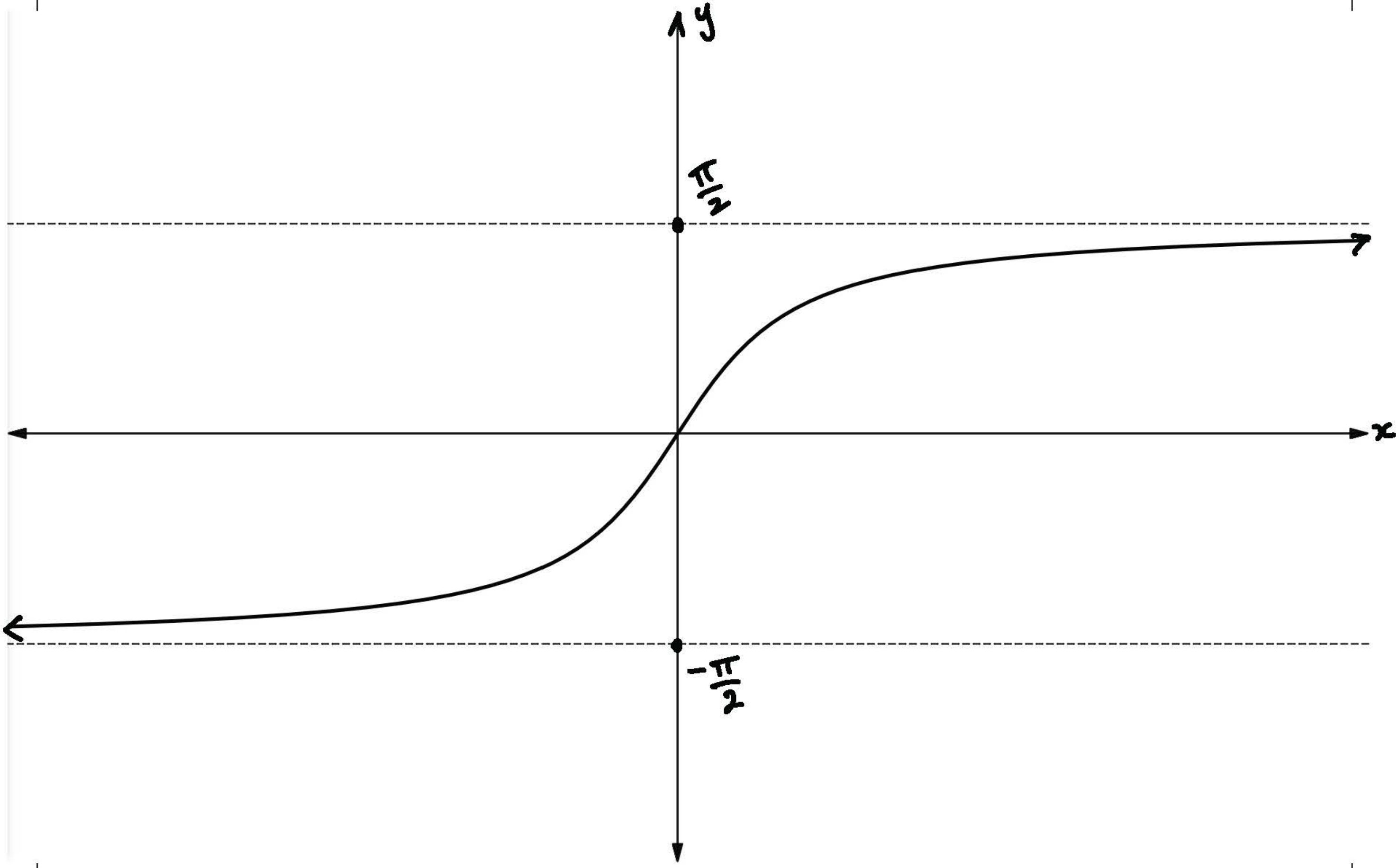
$= \frac{\pi}{2}$ — 1 mark (Exact answers only, CFPE allowed.)

*Students could receive 1 mark if they used the wrong formula but showed the last skill of substitution.

Question 11

MARKER'S COMMENTS

b)



$\frac{1}{2}$ mark – shape approaching asymptotes correctly

$\frac{1}{2}$ mark – shape through origin and centre smooth and correct.

$\frac{1}{2}$ mark – asymptotes shown

$\frac{1}{2}$ mark – values of asymptotes shown.

Zero marks for wrong graph.

Question 11

MARKER'S COMMENTS

c) i) $P(x) = x^4 - 3x^3 - 6x^2 + 28x - 24$

$P'(x) = 4x^3 - 9x^2 - 12x + 28$

$P''(x) = 12x^2 - 18x - 12$

For a triple root $x = \alpha$,

$P(\alpha) = P'(\alpha) = P''(\alpha) = 0$

Let $P''(\alpha) = 0$

$12\alpha^2 - 18\alpha - 12 = 0$ – 1 mark

$6(2\alpha^2 - 3\alpha - 2) = 0$

$6(2\alpha + 1)(\alpha - 2) = 0$

$\therefore \alpha = -\frac{1}{2}$ or 2 – $\frac{1}{2}$ mark

$P(2) = 2^4 - 3(2)^3 - 6(2)^2 + 28(2) - 24 = 0$

$P'(2) = 4(2)^3 - 9(2)^2 - 12(2) + 28 = 0$

$\therefore \alpha = 2$ – $\frac{1}{2}$ mark

c) ii) Method 1 – Quickest

Using trial and error, $P(-3) = 0$

$\therefore P(x) = (x-2)^3(x+3)$ – 1 mark

Question 11

MARKER'S COMMENTS

c) ii) continued

<u>Method 2</u> - Long way

$$(x-2)^3 = (x^2-4x+4)(x-2)$$

$$= x^3-4x^2+4x-2x^2+8x-8$$

$$= x^3-6x^2+12x-8$$

$$x+3$$

$$x^3-6x^2+12x-8 \enclose{longdiv}{x^4-3x^3-6x^2+28x-24}$$

$$\underline{x^4-6x^3+12x^2-8x}$$

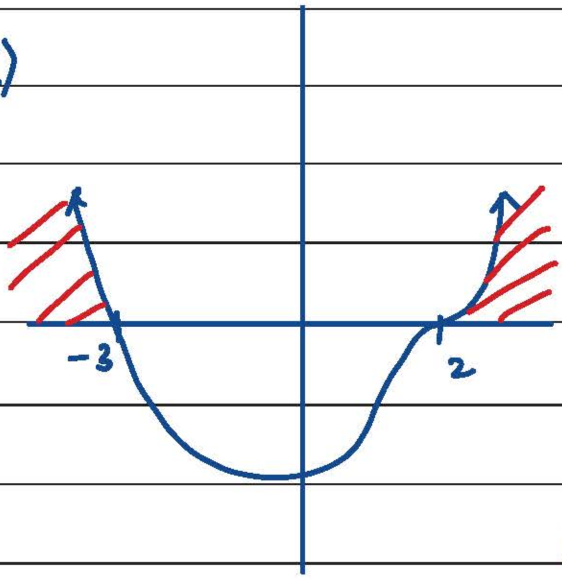
$$3x^3-18x^2+36x-24$$

$$\underline{3x^3-18x^2+36x-24}$$

$$0$$

$\therefore\ P(x) = (x-2)^3(x+3)$ - 1 mark

c) iii)



Solution to

$x^4-3x^2-6x^2+28x-24 \geq 0$

is when

$(x-2)^3(x+3) \geq 0$

Answer: $x \leq -3$ or $x \geq 2$.

$\frac{1}{2}$ mark each

Question 11

MARKER'S COMMENTS

d) $2\cos x - \sqrt{3}\sin x = -2$

Let $t = \tan\frac{x}{2}$

$\therefore \cos x = \frac{1-t^2}{1+t^2}$ and $\sin x = \frac{2t}{1+t^2}$

$\therefore 2\left(\frac{1-t^2}{1+t^2}\right) - \sqrt{3}\left(\frac{2t}{1+t^2}\right) = -2$ $-\frac{1}{2}$ mark for substitution.

Multiply both sides by $1+t^2$:

$2(1-t^2) - \sqrt{3}(2t) = -2(1+t^2)$ $-\frac{1}{2}$ mark

a few students lost this negative.

$2 - \cancel{2t^2} - 2\sqrt{3}t = -2 - \cancel{2t^2}$

$-2\sqrt{3}t = -4$

$t = \frac{2}{\sqrt{3}}$ $-\frac{1}{2}$ mark

$\therefore \tan\frac{x}{2} = \frac{2}{\sqrt{3}}$

$\frac{x}{2} = \tan^{-1}\left(\frac{2}{\sqrt{3}}\right)$

$x = 2 \times \tan^{-1}\left(\frac{2}{\sqrt{3}}\right)$

$x = 1.71$ (2 decimal places) $-\frac{1}{2}$ mark

Next we must also test $x = \pi$ $\longrightarrow$ next page

MARKER'S COMMENTS

SUB $x = \pi$ into LHS of the equation:

$$\begin{aligned} LHS &= 2\cos\pi - \sqrt{3}\sin\pi \\ &= 2(-1) - \sqrt{3}(0) \\ &= -2 - 0 \\ &= -2 \\ &= RHS \end{aligned}$$

$-\frac{1}{2}$ mark

$\therefore x = 1.71$ or π. $-\frac{1}{2}$ mark

MARKER'S COMMENTS

Question 12

a) $\sqrt{12}\sin x + 2\cos x = R\sin(x-\alpha)$ where
$(R>0$ and $0 \le \alpha \le 2\pi)$

$= R(\sin x\cos\alpha - \cos x\sin\alpha)$

$R\cos\alpha = \sqrt{12}$ ---① $\quad R\sin\alpha = -2$ ---②

square ① and ② and add them together:

$R^2\cos^2\alpha + R^2\sin^2\alpha = 12 + 4$

$R^2(\sin^2\alpha + \cos^2\alpha) = 16$

$R^2 \times 1 = 16$

$R^2 = 16$

$R = \pm 4$ (But $R>0$)

$\therefore R = 4$ — 1mk

From ① $\cos\alpha = \frac{\sqrt{12}}{4}$ $\quad$ From ② $\sin\alpha = \frac{-2}{4}$

$= \frac{\sqrt{4}\sqrt{3}}{4}$ $\quad = -\frac{1}{2}$

$= \frac{\sqrt{3}}{2}$

$\therefore \tan\alpha = -\frac{1}{2} \div \frac{\sqrt{3}}{2}$

$= \frac{-1}{2} \times \frac{2}{\sqrt{3}}$

$= \frac{-1}{\sqrt{3}}$

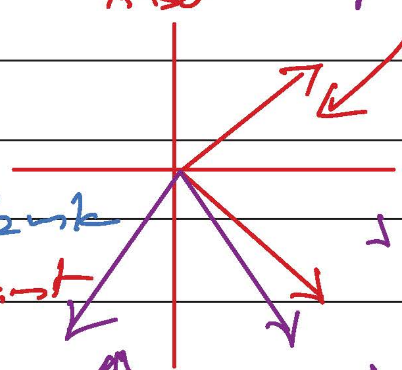
$\therefore$ Related angle $= \frac{\pi}{6}$ — ½ mk

$\therefore$ overlap is quadrant 4

$\therefore \alpha = \frac{11\pi}{6}$ — 1 mk

as $0 \le \alpha \le 2\pi$

Also



MARKER'S COMMENTS

$\therefore \sqrt{12}\sin x + 2\cos x = 4\sin\left(x - \frac{11\pi}{6}\right)$

$-\frac{1}{2}$ mk

Markers' Comments

Many students had difficulty in finding the value of α.

ii) $\sqrt{12}\sin x + 2\cos x = 2$

$0 \le x \le 2\pi$

$-\frac{11\pi}{6} \le x - 11\pi/6 \le \pi/6.$

$4\sin\left(x - 11\pi/6\right) = 2$

$\sin\left(x - \frac{11\pi}{6}\right) = \frac{1}{2}$

Related angle $= \pi/6$

$x - \frac{11\pi}{6} = \pi/6, \frac{5\pi}{6}, \frac{-7\pi}{6}, \frac{-11\pi}{6}$

$x = 2\pi, \frac{17\pi}{6} (> 2\pi), \frac{2\pi}{3}, 0$

$= 0, \frac{2\pi}{3}, 2\pi$

Must have at least 2 of the x - values for full marks.

Markers' Comments

1.2 b

MARKER'S COMMENTS

① Initial statement

Prove that $9^n - 4^n$ is divisible by 5

② Base case or initial case

Prove that the statement is true for: $n = 1$

$LHS = 9^n - 4^n$ ½mk

$= 9^1 - 4^1$

$= 5$ which is divisible by 5

$\therefore$ proved true for $n = 1$

③ Inductive hypothesis or Assumption

Assume that the statement is true for $n = k$

That is, $9^k - 4^k = 5m$, where m is any integer.

$\therefore 9^k = 5m + 4^k$ ½mk

MARKER'S COMMENTS

(4) Prove that the statement is true for $n = k+1$

That is, $9^{k+1} - 4^{k+1} = 5p$, where p is any integer.

$$\begin{aligned} LHS &= 9^{k+1} - 4^{k+1} \\ &= 9^k \cdot 9^1 - 4^k \cdot 4^1 \\ &= 9(5m + 4^k) - 4 \cdot 4^k \quad \text{— 1 mk} \\ &= 45m + 9 \cdot 4^k - 4 \cdot 4^k \\ &= 45m + 5 \cdot 4^k \\ &= 5(9m + 4^k) \quad \text{— ½ mk} \\ &= 5p, \text{ where } p = 9m + 4^k \text{ is an integer.} \end{aligned}$$

(5) Conclusion

Therefore, by the principle of mathematical induction, the initial statement holds true for all positive integers $n \geq 1$ ½ mk

Marker's Comments

Mostly well done.

MARKER'S COMMENTS

12 c)

$T_{k+1} = {}^nC_k \, a^{n-k} \, b^k$

$= {}^{11}C_k \, (2x^3)^{11-k} \, (-x^{-2})^k$

$= {}^{11}C_k \, 2^{11-k} \, x^{33-3k} \, (-1)^k \, x^{-2k}$

$\therefore = {}^{11}C_k \, 2^{11-k} \, (-1)^k \, x^{33-5k}$ — ½ mk

For the coefficient of x^3

$33 - 5k = 3$

$5k = 30$

$\therefore k = 6$ — ½ mk

$\therefore$ coefficient of x^3 is:

${}^{11}C_6 \, 2^{11-6} \, (-1)^6$

$= {}^{11}C_6 \, 2^5$ or ${}^{11}C_5 \, 2^5$

— 1mk

Marker's Comments

- Students who wrote in the following way $(-x^{-2})^k = (-1)^k (x^{-2k})$ experienced difficulty.
- Many students just found the value of k without any working. Full marks were awarded in this case.

MARKER'S COMMENTS

12 d) $\int_0^1 x\sqrt{x+1}\,dx$

Let $u = x+1 \Rightarrow x = u-1$

$\frac{du}{dx} = 1$

$du = dx$

When $x=0$
$u = x+1$
$= 0+1$
$= 1$

When $x=1$
$u = x+1$
$= 1+1$
$= 2$ — 1mk

$\int_1^2 (u-1)u^{1/2}\,du$ — 1mk

$\int_1^2 u^{3/2} - u^{1/2}\,du$

$= \left[\frac{2u^{5/2}}{5} - \frac{2}{3}u^{3/2}\right]_1^2$

$= \left(\frac{2}{5}(2)^{5/2} - \frac{2}{3}(2)^{3/2}\right) - \left(\frac{2}{5} - \frac{2}{3}\right)$ — ½mk

$= 0.6437902833\ldots$
$= 0.64$ (2 dp). — ½mk

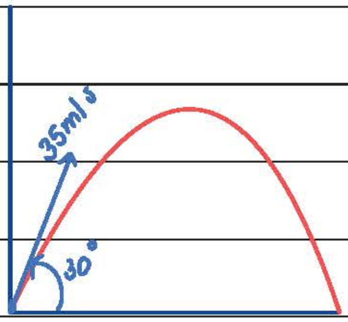
Marker's Comments:

Generally well done. Some students need to review index laws

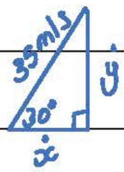
QUESTION 13

MARKER'S COMMENTS

a)

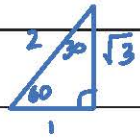


Initial Conditions



$\dot{x} = 35\cos 30^\circ$
$= \dfrac{35\sqrt{3}}{2}$

$\dot{y} = 35\sin 30^\circ$
$= \dfrac{35}{2}$



$\ddot{y} = -9.8$ Max height occurs $\dot{y} = 0$

$\dot{y} = -9.8t + c$

$\dot{y} = \frac{35}{2},\ t=0$ $\quad \frac{35}{2} = -9.8(0) + c$

$c = \frac{35}{2}$

$\therefore\ \dot{y} = -9.8t + \frac{35}{2}$ 1 — ① mark to establish $\dot{y}$

Max height occurs when $\dot{y} = 0$

$0 = -9.8t + \frac{35}{2}$ ½ — (½) mark to let $\dot{y}=0$

$0 = -19.6t + 35$

$t \doteq 1.79\text{s} < 2$ ½ — (½) mark for answer that was < 2

$\therefore$ The ball reaches its max height before $t=2$

NOTE: More explanation to state $t \doteq 1.79$s

$\therefore t < 2$

$\therefore$ ball reaches max height before $t = 2$.

Q13 cont ...

MARKER'S COMMENTS

b) METHOD 1

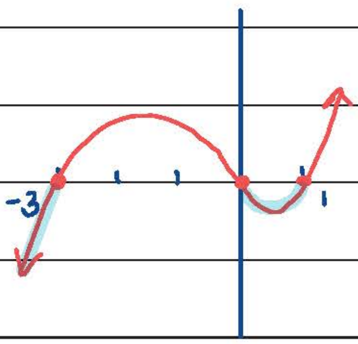
$x^2 \times \frac{3}{x} > (x+2) \times x^2$

$3x > x^3 + 2x^2$

$x^3 + 2x^2 - 3x < 0$

$x(x^2 + 2x - 3) < 0$

$x(x-1)(x+3) < 0$



½ only awarded for a minor error such as transcrib a number / sign

1 mark - Correctly multiplies through by x^2 to obtain correct cubic equation

½ mark $x < -3$

½ mark $0 < x < 1$

$x < -3$ or $0 < x < 1$

METHOD 2

$\frac{3}{x} = x + 2 \qquad x \neq 0$

$3 = x^2 + 2x$

$x^2 + 2x - 3 = 0$

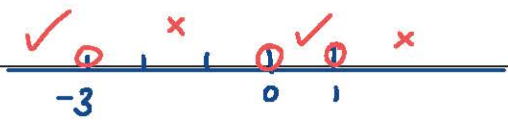
$(x-1)(x+3) = 0$

$x = 1, -3$

1 mark solves for equality

½ mark $x < -3$

½ mark $0 < x < 1$



test $x = -4 \qquad x = -1 \qquad x = \frac{1}{2} \qquad x = 2$

$-\frac{3}{4} > -2 \qquad -3 > 1 \qquad 6 > 2\frac{1}{2} \qquad 1\frac{1}{2} > 4$

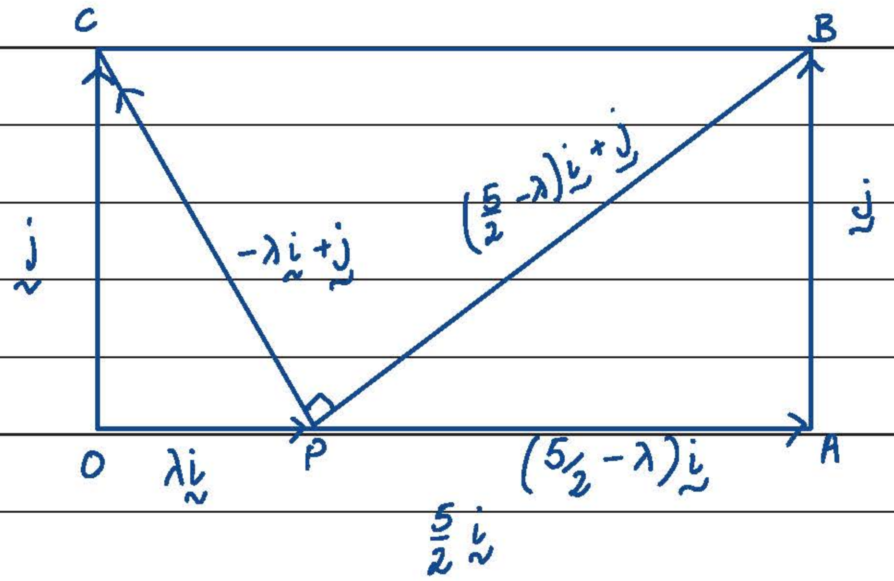
✓ ✗ ✓ ✗

$\therefore\ x < -3$ or $0 < x < 1$

Q13 cont ...

MARKER'S COMMENTS

c)



$\vec{PC} = \vec{PO} + \vec{OC}$

$= -\vec{OP} + \vec{OC}$

$= -\lambda\underset{\sim}{i} + \underset{\sim}{j}$

$\vec{PB} = \vec{PA} + \vec{AB}$

$= \left(\frac{5}{2}-\lambda\right)\underset{\sim}{i} + \underset{\sim}{j}$

$\vec{PA} = \vec{OA} - \vec{OP}$

$= \frac{5}{2}\underset{\sim}{i} - \lambda\underset{\sim}{i}$

$= \left(\frac{5}{2}-\lambda\right)\underset{\sim}{i}$

$\vec{AB} = \vec{OC}$ (given OABC is a rectangle $\therefore$ opp sides $=$ and $\parallel$)

$\vec{PC} \cdot \vec{PB} = 0$ (given $\angle CPB = 90°$)

$(-\lambda\underset{\sim}{i}+\underset{\sim}{j}) \cdot \left[\left(\frac{5}{2}-\lambda\right)\underset{\sim}{i} + \underset{\sim}{j}\right] = 0$

$-\lambda\left(\frac{5}{2}-\lambda\right) + 1 \times 1 = 0$

$-\frac{5}{2}\lambda + \lambda^2 + 1 = 0$

$-5\lambda + 2\lambda^2 + 2 = 0$

$2\lambda^2 - 5\lambda + 2 = 0$

$(2\lambda-1)(\lambda-2) = 0$

$\lambda = \frac{1}{2}, 2$

$2\lambda \quad -1$
$\lambda \quad -2$

1 to apply dot product that is mostly correct

1½ correct reasoning and setting out (some half marks lost for lack of reasoning and setting out)

½ finding correct values for λ.

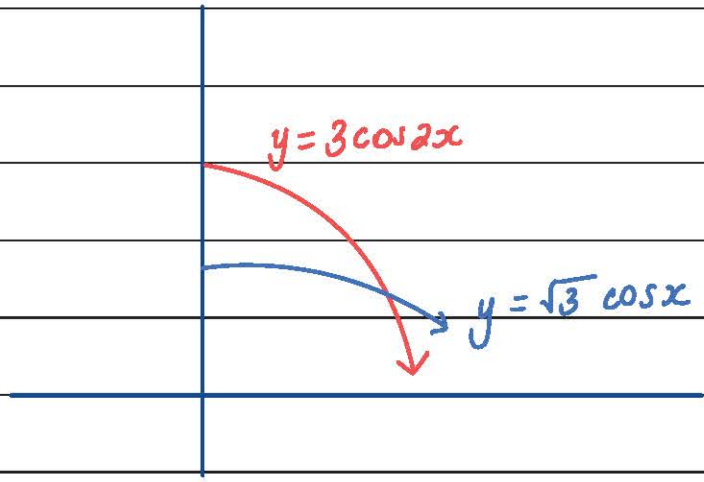
MARKER'S COMMENTS

This question was very poorly set out. It is a show question, so ensure all reasoning is included. Students who simply solved the quadratic without showing where it came from using vector methods received only ½ a mark.

Q13 cont...

MARKER'S COMMENTS

d) $y = 3\cos 2x \qquad y = \sqrt{3}\cos x$



(i) $3\cos 2x = \sqrt{3}\cos x$

$\cos 2x = \cos^2 x - \sin^2 x$

$= \cos^2 x - (1 - \cos^2 x)$

$= 2\cos^2 x - 1$

$3(2\cos^2 x - 1) = \sqrt{3}\cos x$

$6\cos^2 x - \sqrt{3}\cos x - 3 = 0$

let $m = \cos x$

$6m^2 - \sqrt{3}m - 3 = 0$

$m = \dfrac{-(-\sqrt{3}) \pm \sqrt{3 - 4(6)(-3)}}{12}$

$m = \dfrac{\sqrt{3} \pm \sqrt{75}}{12}$

$m = \dfrac{\sqrt{3} \pm 5\sqrt{3}}{12}$

$m = \dfrac{6\sqrt{3}}{12}$ or $m = \dfrac{-4\sqrt{3}}{12}$

$m = \dfrac{\sqrt{3}}{2} \qquad = -\dfrac{\sqrt{3}}{3}$

1 mark for correct quadratic equation

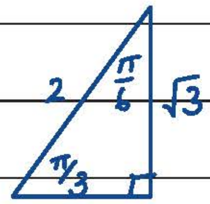
1 mark for correctly establishing $\cos x = \frac{\sqrt{3}}{2}$ and hence justifying the $x = \frac{\pi}{6}$ value for the intersection. Some half marks lost here for unsimplified values for cosx to enable use of exact triangle.

$/2$

Q13 cont ...

MARKER'S COMMENTS

$\therefore \cos x = \frac{\sqrt{3}}{2}$ or $\cos x = \frac{-\sqrt{3}}{3}$



quad 1, 4 quad 2, 3

quad 1 $x = \frac{\pi}{6}$ $x \doteq \pi - 0.9553,\ \pi + 0.9553$

$\doteq 0.5236$ $\doteq 2.186,\ 4.097$

$\therefore$ first time curves meet is at $x = \frac{\pi}{6}$

Some students left $\cos x$ unsimplified and then simply stated $x = \pi/6$. It needed to be simplified to $\frac{\sqrt{3}}{2}$ at least so the exact triangle could imply $x = \pi/6$

(ii) $V = \pi \int_0^{\pi/6} \left[(3\cos 2x)^2 - (\sqrt{3}\cos x)^2 \right] dx$

① mark for correct integral to find the volume.

$= \pi \int_0^{\pi/6} \left(9\cos^2 2x - 3\cos^2 x \right) dx$

$\cos 2x = 2\cos^2 x - 1$

$\cos^2 x = \frac{1}{2}(\cos 2x + 1)$

$\therefore \cos^2 2x = \frac{1}{2}(\cos 4x + 1)$

$= \pi \int_0^{\pi/6} \left[9 \times \frac{1}{2}(\cos 4x + 1) - 3 \times \frac{1}{2}(\cos 2x + 1) \right] dx$

① mark for using double angle correctly.

$= \pi \int_0^{\pi/6} \left(\frac{9}{2}\cos 4x + \frac{9}{2} - \frac{3}{2}\cos 2x - \frac{3}{2} \right) dx$

$= \pi \int_0^{\pi/6} \left(\frac{9}{2}\cos 4x - \frac{3}{2}\cos 2x + 3 \right) dx$

$= \pi \left[\frac{9}{8}\sin 4x - \frac{3}{4}\sin 2x + 3x \right]_0^{\pi/6}$

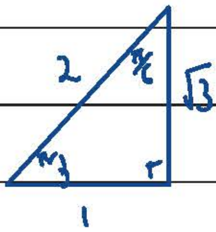
$= \pi \left(\frac{9}{8}\sin(4 \times \pi/6) - \frac{3}{4}\sin\frac{\pi}{3} + \frac{3\pi}{6} - 0 \right)$

Q13 contin....

MARKER'S COMMENTS

$= \pi \left(\frac{9}{8} \sin\frac{2\pi}{3} - \frac{3}{4}\sin\frac{\pi}{3} + \frac{\pi}{2} \right)$

$= \frac{\pi}{8}\left(9\sin\frac{\pi}{3} - 6\sin\frac{\pi}{3} + 4\pi \right)$



$= \frac{\pi}{8}\left(9\times\frac{\sqrt{3}}{2} - 6\times\frac{\sqrt{3}}{2} + 4\pi \right) \longleftarrow$ ① mark to obtain correct answer after evaluating integral. Students needed to evaluate $\sin\frac{\pi}{3}$ and $\sin\frac{2\pi}{3}$. Some half marks lost for expansion errors and other minor evaluation errors. Various forms accepted. Please keep exact.

$= \frac{\pi}{8}\left(\frac{3\sqrt{3}}{2} + 4\pi \right)$

or $\frac{3\sqrt{3}}{16}\pi + \frac{\pi^2}{2}$ or $\frac{3\sqrt{3}\,\pi + 8\pi^2}{16}$

From:

$V = \pi \int_0^{\pi/6} \left[9\times\frac{1}{2}(\cos 4x + 1) - 3\times\frac{1}{2}(\cos 2x + 1) \right] dx$ ① mark

$= \frac{9\pi}{2}\int_0^{\pi/6} (\cos 4x + 1 \;\; dx) - \frac{3\pi}{2}\int_0^{\pi/6} (\cos 2x + 1)\, dx$ ① mark

$= \frac{9\pi}{2}\left[\frac{1}{4}\sin 4x + x \right]_0^{\pi/6} - \frac{3\pi}{2}\left[\frac{1}{2}\sin 2x + x \right]_0^{\pi/6}$

$= \frac{9\pi}{2}\left(\frac{1}{4}\sin\frac{2\pi}{3} + \frac{\pi}{6} - 0 \right) - \frac{3\pi}{2}\left[\frac{1}{2}\sin\frac{\pi}{3} + \frac{\pi}{6} - 0 \right]$

$= \frac{9\pi}{2}\left(\frac{\sqrt{3}}{8} + \frac{\pi}{6} \right) - \frac{3\pi}{2}\left(\frac{\sqrt{3}}{4} + \frac{\pi}{6} \right)$

etc... ① mark

Lots of minor algebraic and substitution errors in this question.

MARKER'S COMMENTS

Question 14

i) Initial statement

$$\left(1\times 1! + \frac{0}{1!}\right) + \left(2\times 2! + \frac{1}{2!}\right) + \left(3\times 3! + \frac{2}{3!}\right)$$

$$+ \cdots + \left(n\times n! + \frac{n-1}{n!}\right) = (n+1)! - \frac{1}{n!}$$

I) Base case or initial case

Prove the statement is true for $n=1$.

$$\text{LHS} = n\times n! + \frac{n-1}{n!}$$

$$= 1\times 1! + \frac{1-1}{1!}$$

$$= 1! + \frac{0}{1!}$$

$$= 1$$

$$\text{RHS} = (n+1)! - \frac{1}{n!}$$

$$= (1+1)! - \frac{1}{1!}$$

$$= 2! - \frac{1}{1!}$$

$$= 2 - 1$$

$$= 1$$

$\therefore$ LHS = RHS

$\therefore$ Proved true for $n=1$

— $\frac{1}{2}$ k

3) Inductive hypothesis or assumption

Assume the statement is true for $n = k$

$-\frac{1}{2}$ mk

That is,

$$\left(1 \times 1! + \frac{0}{1!}\right) + \left(2 \times 2! + \frac{1}{2!}\right) + \left(3 \times 3! + \frac{2}{3!}\right) + \dots + k \times k! + \frac{k-1}{k!} = (k+1)! - \frac{1}{k!}$$

4) Prove the statement is true for $n = k+1$

That is,

$$\underbrace{\left(1 \times 1! + \frac{0}{1!}\right) + \left(2 \times 2! + \frac{1}{2!}\right) + \left(3 \times 3! + \frac{2}{3!}\right) + \dots + k \times k! + \frac{k-1}{k!}}_{\text{from inductive hypothesis}} + (k+1) \times (k+1)! + \frac{k+1-1}{(k+1)!}$$

$$= (k+2)! - \frac{1}{(k+1)!}$$

MARKER'S COMMENTS

$$= \underbrace{(k+1)! - \frac{1}{k!}}_{\frac{1}{2}\text{mk for using the inductive hypothesis}} + (k+1)\times(k+1)! + \frac{k+1-1}{(k+1)!}$$

$$= (k+1)! - \frac{1}{k!} + (k+1)(k+1)! + \frac{k}{(k+1)!}$$

$$= \underbrace{(k+1)! + (k+1)(k+1)!}\; - \frac{1}{k!} + \frac{k}{(k+1)!}$$

$$= (k+1)!\left[1 + k+1\right] - \frac{1}{k!} + \frac{k}{(k+1)!}$$

$$= \underbrace{(k+1)!\left[k+2\right]}_{\frac{1}{2}\text{mk}} - \frac{1}{k!} + \frac{k}{(k+1)!}$$

$$= (k+2)! + \underbrace{\left(-\frac{1}{k!} + \frac{k}{(k+1)!}\right)}$$

$$= (k+2)! + \frac{-k-1+k}{(k+1)!}$$

$$= (k+2)! - \frac{1}{(k+1)!}$$

$= RHS$ ∴ Proved true.

$-\frac{1}{2}$mk

MARKER'S COMMENTS

MARKER'S COMMENTS

Conclusion

By the principle of mathematical induction the initial statement holds true for all integers $n \geq 1$ $-\frac{1}{2}$ mk

Marker's Comments

This question was poorly attempted by the majority of students. A number of students made mistakes with the inductive step. Also, students made errors when transcribing the signs like multiplication and addition symbols. More care needs to be taken when transcribing a question as simplifying the question earns no marks.

MARKER'S COMMENTS

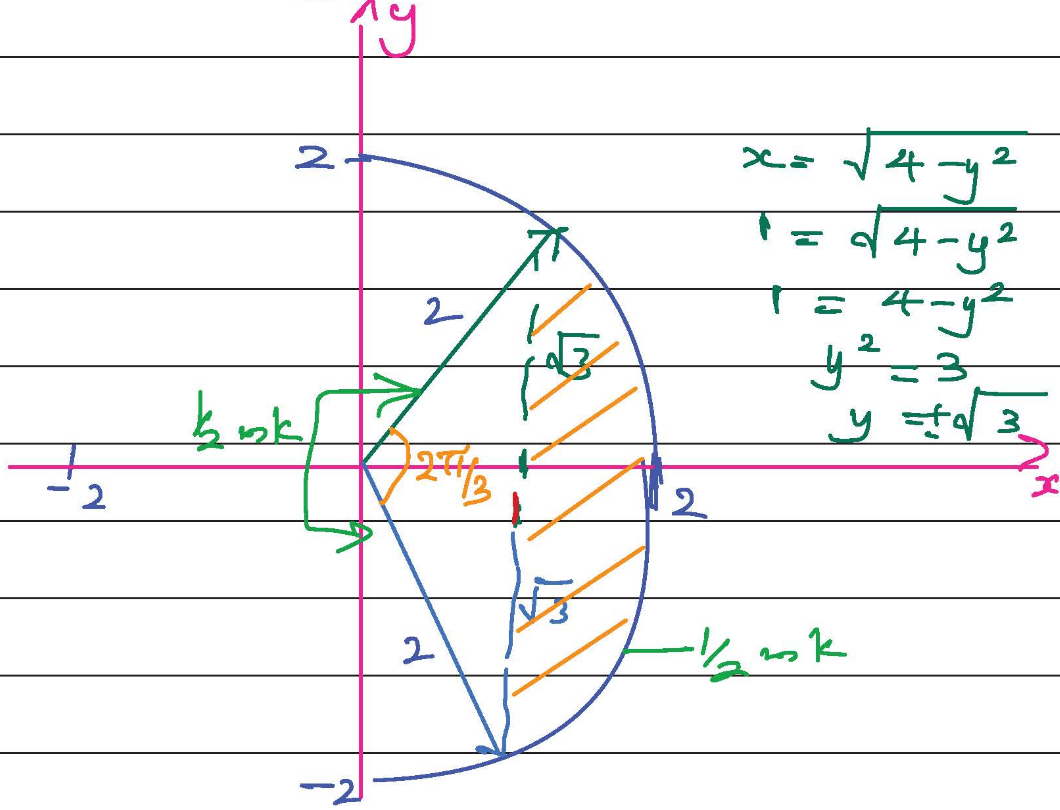
4b) $x = \sqrt{4 - y^2}$

$\underset{\sim}{u} = \underset{\sim}{i} + \sqrt{3}\underset{\sim}{j}$ and $\underset{\sim}{v_s} = \underset{\sim}{i} - \sqrt{3}\underset{\sim}{j}$

i) $x = \sqrt{4-y^2}$

Now, $y = \sqrt{r^2 - x^2}$ is a semi-circle with radius 2 units.

So, $y = \sqrt{4-x^2}$ is the inverse relation of $x = \sqrt{4-y^2}$



Marker's Comments

This question was poorly attempted.

A number of students did not realise that the 2 vectors met the semi-circle.

MARKER'S COMMENTS

iii) $\cos\theta = \dfrac{\underset{\sim}{u}\cdot\underset{\sim}{v}}{|\underset{\sim}{u}|\,|\underset{\sim}{v}|}$

$= \dfrac{1\times 1-\sqrt{3}\times\sqrt{3}}{\sqrt{1+3}\,\sqrt{1+3}}$

$= \dfrac{1-3}{4}$

$= -\frac{1}{2}$ — ½ mk

$\cos\theta$ is negative

$\therefore$ θ is an obtuse angle.

$\theta = \cos^{-1}\left(\frac{1}{2}\right)$

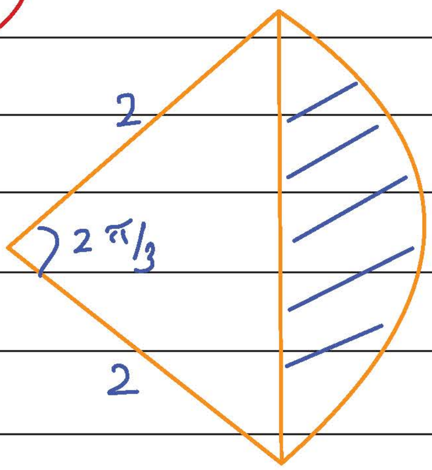
$= \pi/3$

But θ is obtuse $\therefore = 2\pi/3$

— ½ mk

Quite well done. If an extra solution was provided, ½ a mark was deducted.

iii)



Area of segment

$= \frac{1}{2}r^2(\theta - \sin\theta)$ 1 mk

$= \frac{1}{2}\,2^2\left(\frac{2\pi}{3} - \sin\frac{2\pi}{3}\right)$

$= 2\left(\frac{2\pi}{3} - \sin\frac{2\pi}{3}\right)u^2$

1 mk

MARKER'S COMMENTS

Marker's Comments.

This question was poorly attempted by the majority of students as they failed to realise that the area between the curve and $x = 1$ represented the actual area of the segment of a circle.

Students are encouraged to write down the appropriate formulae and then make the substitution.

Incorrect substitution into the formulae was not given any marks as the question was simplified.

MARKER'S COMMENTS

c) $r = \tan t \qquad 0 \le t \le \pi/2$

$$\frac{dr}{dt} = \lim_{h \to 0} \frac{f(t+h) - f(t)}{h}$$

$$= \lim_{h \to 0} \frac{\tan(t+h) - \tan t}{h} \quad \text{½ mk}$$

$$= \lim_{h \to 0} \frac{\frac{\tan t + \tan h}{1 - \tan t \tan h} - \tan t}{h} \quad \text{½ mk}$$

$$= \lim_{h \to 0} \frac{\frac{\cancel{\tan t} + \tan h - \cancel{\tan t} + \tan^2 t \tan h}{1 - \tan t \tan h}}{h}$$

$$= \lim_{h \to 0} \frac{\tan h + \tan^2 t \tan h}{h(1 - \tan t \tan h)}$$

½ mk

$$= \lim_{h \to 0} \frac{\tan h\,(1 + \tan^2 t)}{h\,(1 - \tan t \tan h)}$$

$$= \lim_{h \to 0} \frac{\tan h}{h} \times \lim_{h \to 0} \frac{1 + \tan^2 t}{1 - \tan t \tan h} \quad \text{½ mk}$$

$$= 1 \times \lim_{h \to 0} \frac{1 + \tan^2 t}{1 - \tan t \tan h}$$

½ mk ½ mk

$$= \frac{1 + \tan^2 t}{1 - \tan t \tan 0} = 1 + \tan^2 t \text{ or } \sec^2 t$$

MARKER'S COMMENTS

Marker's Comments

This question was poorly attempted by the majority of students as students had forgotten how to use the first principles. Students need to re-visit these concepts before HSC exams.

MARKER'S COMMENTS

14 c ii) $a = 2$

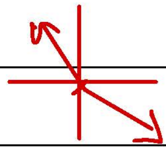
$1 + \tan^2 t = 2$

$\tan^2 t = 1$

$\tan t = \pm 1$ — ½ mk

$\tan t = 1$ or $\tan t = -1$

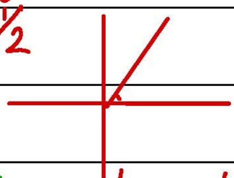
$t = \pi/4$'s as $0 \le t \le \pi/2$



— ½ mk

or $\sec^2 t = 2$

$\frac{1}{\cos^2 t} = 2$ $\quad 0 \le t \le \pi/2$



$2\cos^2 t = 1$

$\cos t = \pm \frac{1}{\sqrt{2}}$ — ½ mk $\quad \cos t = \frac{1}{\sqrt{2}}$

$\therefore t = \pi/4$'s

— ½ mk

Marker's Comments

This question was poorly attempted as a number of students forgot to consider the positivi and negativi scenarios when taking the square root of a number. Also, ~~some~~ students forgot to consider $0 \le t \le \pi/2$ their final answer. Students are encouraged to consider restrictions given in the question.

MARKER'S COMMENTS

iii) $x = \int_0^1 \tan t \, dt$ — ½ mk

$= -\int \frac{-\sin t}{\cos t} \, dt$

$= -\left[\ln |\cos t| \right]_0^1$ — ½ mk

$\therefore = -\left[\ln |\cos 1| - \ln |\cos 0| \right]$

$= -\left[\ln \cos 1 - \ln |1| \right]$

$= -\ln \cos 1 - 0$

$= (-\ln \cos 1)\, m$ — 1 mk

Marker's Comments

Quite well done. students are encouraged to write dt. They are also encouraged to visit $\int \tan t \, dt$ for HSC

12MAX 2024 AT4 Q15 **MARKER'S COMMENTS**

a) $$\int_0^{\frac{1}{2}} \frac{\sin^{-1}x}{\sqrt{1-x^2}}\,dx = \int_0^{\frac{1}{2}} \frac{1}{\sqrt{1-x^2}} \times \sin^{-1}x \, dx$$

$$= \left[\frac{(\sin^{-1}x)^2}{2}\right]_0^{\frac{1}{2}}$$

$$= \frac{1}{2}\left[\left(\sin^{-1}\tfrac{1}{2}\right)^2 - \left(\sin^{-1}0\right)^2\right]$$

$$= \frac{1}{2}\left[\left(\frac{\pi}{6}\right)^2 - 0^2\right]$$

$$= \frac{1}{2} \times \frac{\pi^2}{36}$$

$$= \frac{\pi^2}{72}$$

2 marks Complete solution

1 mark Finds correct primitive

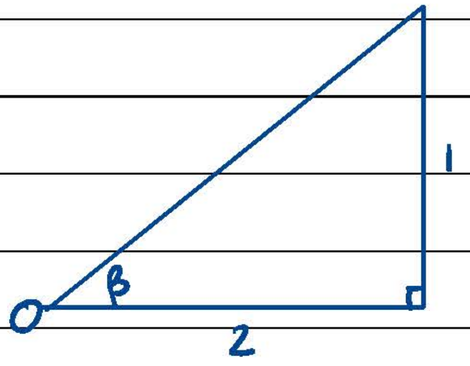
Many students need to revise the reverse chain rule!

Integration by substitution could also be used to evaluate this integral, but many students need to revise the correct notation for changing variables; inventing your own notation (eg. $0 \sim \frac{1}{2}$) is not usually a good idea. See HSC solutions for the correct notation.

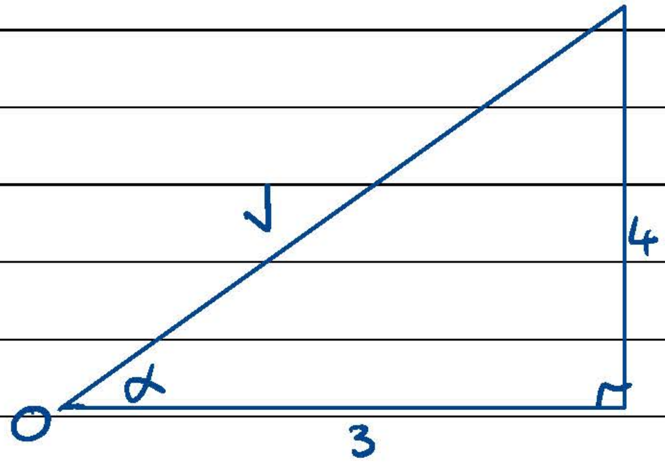
Also note that in Extension 1, you will always be given the substitution, if it is required. If the exam doesn't give you a substitution, think if an easier way might be possible.

12MAX 2024 AT4 Q15 **MARKER'S COMMENTS**

b) i



Ramp



Stone

Stone hits ramp when $x:y = 2:1$

$$\text{ie } \frac{x}{y} = \frac{2}{1}$$

$$x = 2y$$

$$\text{ie } Vt\cos\alpha = 2\left(Vt\sin\alpha - \frac{1}{2}gt^2\right)$$

$$gt^2 + Vt\cos\alpha - 2Vt\sin\alpha = 0$$

$$gt^2 + t\left(V\cos\alpha - 2V\sin\alpha\right) = 0$$

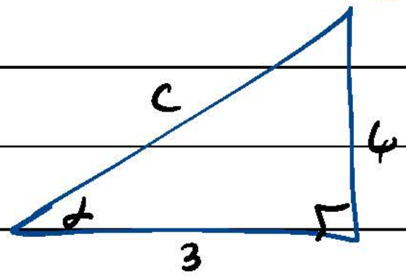
$$t\left(gt + \left(V\cos\alpha - 2V\sin\alpha\right)\right) = 0$$

$$\therefore t = 0 \quad gt + \left(V\cos\alpha - 2V\sin\alpha\right) = 0$$

launch

$$gt = 2V\sin\alpha - V\cos\alpha$$

$$t = \frac{V}{g}\left(2\sin\alpha - \cos\alpha\right)$$



$c^2 = 3^2 + 4^2$ (Pythagoras's Theorem)
$= 25$

$c = \pm\sqrt{25}$
$= 5$ ($c > 0$, length)

$\therefore \sin\alpha = \frac{4}{5}$, $\cos\alpha = \frac{3}{5}$

$$= \frac{V}{g}\left(2\left(\frac{4}{5}\right) - \frac{3}{5}\right)$$

$$= \frac{V}{g}\left(\frac{8}{5} - \frac{3}{5}\right)$$

$$= \frac{V}{g}$$

12MAX 2024 AT4 Q15 MARKER'S COMMENTS

3 marks	Complete solution
2 marks	Solves quadratic up to $t = \frac{2V\sin\alpha - V\cos\alpha}{g}$, or equivalent merit
1 mark	Clearly states that the stone hits the ramp when $x:y = 2:1$

This question was poorly done. Many students failed to read the question, and made the erroneous assumption that the stone lands when $y=0$. This fundamental error prevented them from demonstrating any of the required skills.

Also, this is a "show" question: don't leave anything out!

b) ii $x = Vt\cos\alpha$

$\dot{x} = V\cos\alpha$

$= \frac{3}{5}V$

$y = Vt\sin\alpha - \frac{1}{2}gt^2$

$\dot{y} = V\sin\alpha - gt$

when $t = \frac{V}{g}$, $\dot{y} = V\sin\alpha - g\left(\frac{V}{g}\right)$

$= \frac{4}{5}V - V$

$= -\frac{1}{5}V$

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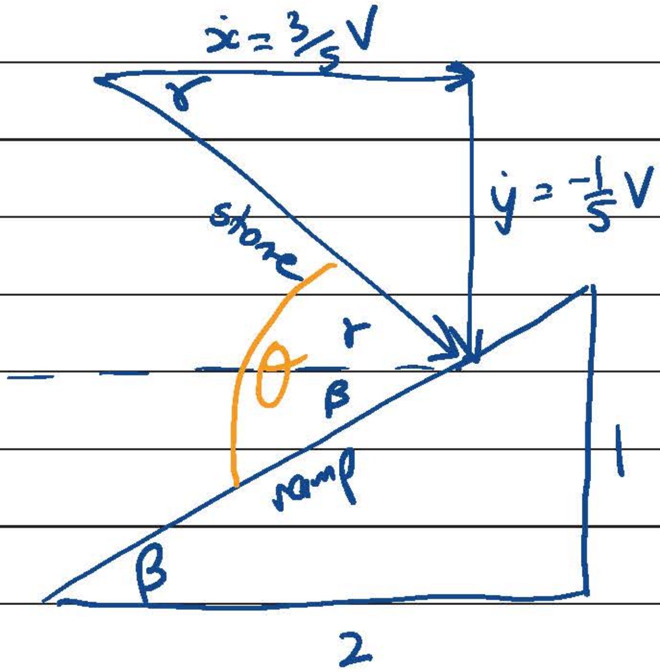
12MAX 2024 AT4 Q15 **MARKER'S COMMENTS**



$\tan r = \dfrac{\left(\frac{1}{5}\right)}{\left(\frac{3}{5}\right)}$

$= \frac{1}{3}$

$r = \tan^{-1}\left(\frac{1}{3}\right)$

$= 26.565\ldots^\circ$

$\tan\beta = \frac{1}{2}$

$\beta = \tan^{-1}\left(\frac{1}{2}\right)$

$= 18.434\ldots^\circ$

$\therefore$ Angle of impact between stone and ramp

$\theta = r + \beta$

$= 26.565\ldots + 18.434\ldots$

$= 45^\circ$

3 marks	Complete solution
2 marks	Correctly finds angle between stone and horizontal (18.4°)
1 mark	Correctly evaluates $\dot{x} = \frac{3}{5}V$ or $\dot{y} = -\frac{1}{5}V$

Note that only $\dot{x}$ and $\dot{y}$ are useful here; any calculations concerning x and y are irrelevant.
Marks were not awarded simply for finding the angle of the ramp.

12MAX 2024 AT4 Q15 **MARKER'S COMMENTS**

How many rows is 496 numbers?

$1+2+3+\dots+n=496$

$\frac{n}{2}(1+n)=496$

$n+n^2=992$

$n^2+n-992=0$

$(n+32)(n-31)=0$

$\therefore n=-32,\ n=31$

$\therefore n=31 \quad (n>0)$

$\therefore$ 31 rows

What is the sum of the first 31 rows?

$\underbrace{2^0+2^1+2^2+\dots+2^{30}}_{\text{G.P. with } a=1,\ r=2,\ n=31}$

$S_{31}=\dfrac{1(2^{31}-1)}{2-1}$

$=2^{31}-1$

$=2\ 147\ 483\ 647$

3 marks Complete solution

2 marks Correctly identifies 31 rows

1 mark Correctly establishes $1+2+3+\dots+n=496$

Marks were not awarded for quoting $S_n=2^n-1$, only for using it to get the answer.

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