Student Number:
Teacher:

St George Girls High School

Mathematics Extension 1

2025 Trial HSC Examination

General Instructions

- Reading time – 10 minutes
- Working time – 2 hours
- Write using black pen
- Calculators approved by NESA may be used
- A reference sheet is provided
- For questions in **Section I**, use the Multiple-Choice answer sheet provided

  For questions in **Section II:**
  - Answer the questions in the booklets provided
  - Start each question in a new writing booklet
  - Show relevant mathematical reasoning and/or calculations
  - Marks may not be awarded for incomplete or poorly presented solutions, or where multiple solutions are provided

Total marks: 70

Section I – 10 marks (pages 3 – 6)

- Attempt Questions 1– 10
- Allow about 15 minutes for this section

Section II – 60 marks (pages 7 –13)

- Attempt Questions 11–15
- Allow about 1 hour and 45 minutes for this section

Q1-10	/10
Q11	/14
Q12	/13
Q13	/12
Q14	/12
Q15	/9
TOTAL	**/70**
	%

Section I

10 marks
Attempt Questions 1 – 10
Allow about 15 minutes for this section
Use the multiple-choice answer sheet provided for Questions 1 to 10.

1. Consider $\underset{\sim}{a} = 8\underset{\sim}{i} + 2\underset{\sim}{j}$.

   Which of the following vectors is parallel to $\underset{\sim}{a}$?

   A. $12\underset{\sim}{i} + 4\underset{\sim}{j}$

   B. $4\underset{\sim}{i} + \underset{\sim}{j}$

   C. $6\underset{\sim}{i} + \underset{\sim}{j}$

   D. $2\underset{\sim}{i} + \underset{\sim}{j}$

2. Which of the following represents $\int \frac{dx}{\sqrt{100-x^2}}$?

   A. $\cos^{-1}\left(\frac{x}{10}\right) + C$

   B. $\sin^{-1}\left(\frac{x}{10}\right) + C$

   C. $\cos^{-1}\left(\frac{x}{100}\right) + C$

   D. $\sin^{-1}\left(\frac{x}{100}\right) + C$

3. A group of 9 people are to be arranged into two groups to sit around two circular tables. One of the groups must contain 5 people.
   In how many ways can they sit around the two tables?

   A. 36288

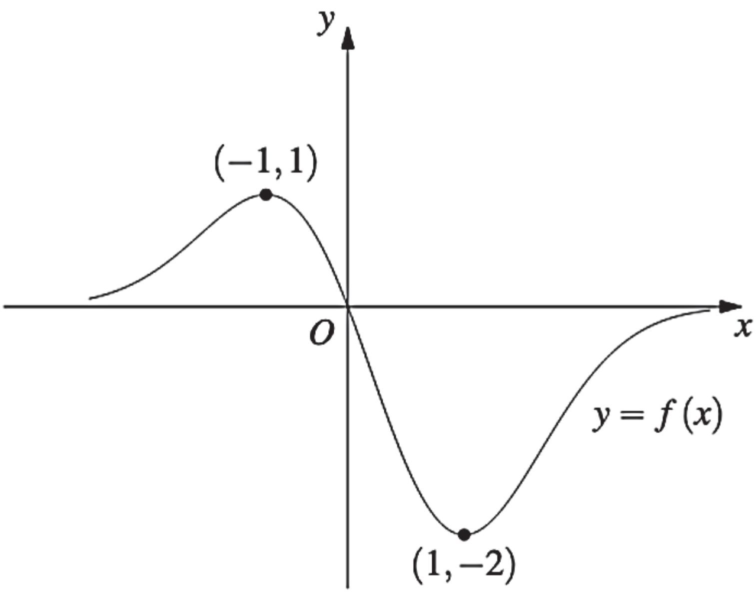
   B. 90 720

   C. 362 880

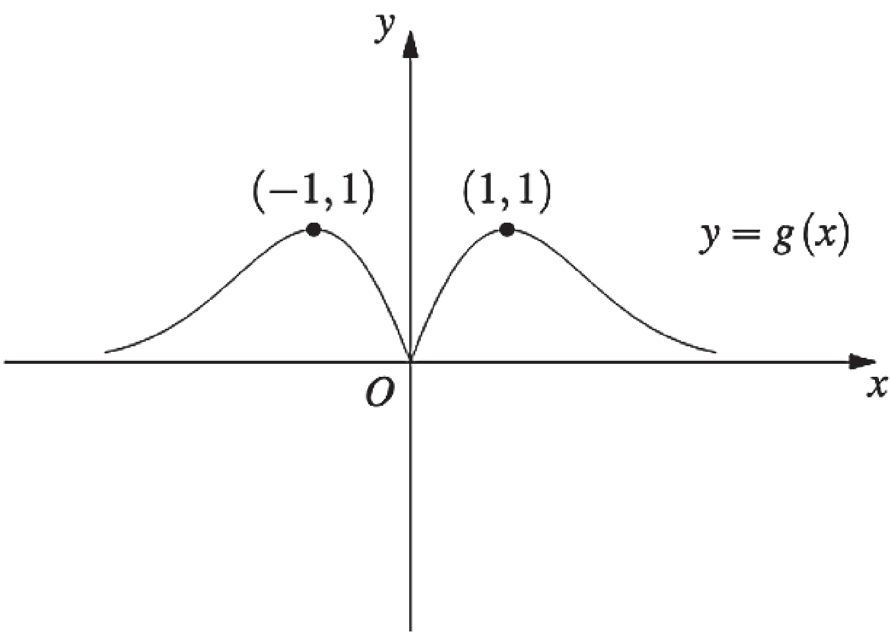
   D. 144

Section I continued

4. The graph of the function $y = f(x)$ is shown below.



The graph of $f(x)$ was transformed to get the graph of $g(x)$ as shown below.



What transformation was applied?

A. $g(x) = f(|-x|)$

B. $g(x) = |f(-|x|)|$

C. $g(x) = \sqrt{f(-x)}$

D. $[g(x)]^2 = -f(x)$

Section I continued

5. Students from seven different schools are eligible for the district netball training squad.

What is the **minimum** number of students in the training squad if it is known that there must be at least 3 students in the training squad from at least one of the schools?

A. 14

B. 15

C. 21

D. 22

6. If $y = e^{2x}\cos^{-1}(3x)$, which of the following is the correct expression for $\frac{dy}{dx}$?

A. $e^{2x}\left[2\cos^{-1}(3x) + \dfrac{3}{\sqrt{1-9x^2}}\right]$

B. $2\,e^{2x}\cos^{-1}(3x) - \dfrac{3e^{2x}}{\sqrt{1-9x^2}}$

C. $e^{2x}\left[2\cos^{-1}(3x) - \dfrac{3}{\sqrt{1-3x^2}}\right]$

D. $2\,e^{2x}\cos^{-1}(3x) + \dfrac{3e^{2x}}{\sqrt{1-3x^2}}$

7. Which of the following is an expression for $\int_0^b \sin 2x \sin 3x \, dx$?

A. $\dfrac{\cos 5b}{10} - \dfrac{\cos b}{2}$

B. $-\dfrac{\cos 5b}{10} + \dfrac{\cos b}{2}$

C. $\dfrac{\sin 5b}{10} - \dfrac{\sin b}{2}$

D. $-\dfrac{\sin 5b}{10} + \dfrac{\sin b}{2}$

Section I continued

8. Which expression is equivalent to $\sin\left(\frac{\pi}{4} - x\right)\cos\left(\frac{\pi}{4} - x\right)$?

A. $\frac{1}{2}\sin 2x$

B. $2\cos 2x$

C. $\frac{1}{2}\cos 2x$

D. $\frac{1}{2}\sin\left(\frac{\pi}{4} - 2x\right)$

9. Consider the vectors $\underset{\sim}{p} = \sin 3\lambda\, \underset{\sim}{i} - \cos 3\lambda\, \underset{\sim}{j}$ and $\underset{\sim}{q} = \cos\lambda\, \underset{\sim}{i} + \sin\lambda\, \underset{\sim}{j}$, where λ is a constant such that $0 \le \lambda \le \frac{\pi}{4}$.

If the scalar projection of $\underset{\sim}{p}$ onto $\underset{\sim}{q}$ is $\frac{\sqrt{3}}{2}$, which of the following represents $\underset{\sim}{p}$?

A. $\underset{\sim}{p} = 0\,\underset{\sim}{i} - \underset{\sim}{j}$

B. $\underset{\sim}{p} = \frac{\sqrt{3}}{2}\underset{\sim}{i} - \underset{\sim}{j}$

C. $\underset{\sim}{p} = \underset{\sim}{i} + \frac{1}{2}\underset{\sim}{j}$

D. $\underset{\sim}{p} = \underset{\sim}{i} - 0\,\underset{\sim}{j}$

10. What is the domain and range of the function $f(x) = \tan\,(\tan^{-1}x) - x$?

A. Domain: $(-\infty, \infty)$ and Range: $[0,0]$

B. Domain: $(-\infty, \infty)$ and Range: $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

C. Domain: $(-1, 1)$ and Range: $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

D. Domain: $(-\infty, \infty)$ and Range: $(-\infty, \infty)$

End of Section I

<u>Section II</u>
60 marks
Attempt Questions 11 – 15
Allow about 1 hour and 45 minutes for this section

In Questions 11-15, your responses should include relevant mathematical reasoning and/or calculations

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Question 11 (14 marks) **Marks**

(a) Solve the inequality: **2**

$$\frac{x+3}{x-1} > 1.$$

(b) Find the constant term in the expansion of $\left(x^2 + \frac{2}{x}\right)^6$. **2**

(c) Use the substitution $u = x - 2$ to show that **3**

$$\int_2^4 2x\sqrt{x-2}\,dx = \frac{128\sqrt{2}}{15}.$$

(d) (i) Express $\cos\theta + \sqrt{3}\sin\theta$ in terms of t, where $t = \tan\frac{\theta}{2}$. **1**

(ii) Hence, solve the equation: **3**

$\cos\theta + \sqrt{3}\sin\theta = 1$ for $0 \leq \theta \leq 2\pi$.

(e) Consider the polynomial $P(x) = 2x^3 - 3ax + 4b^3$, where $a \neq b \neq 0$. **3**

This polynomial has $x = 2$ as a root of multiplicity 2.

Find the values of a and b.

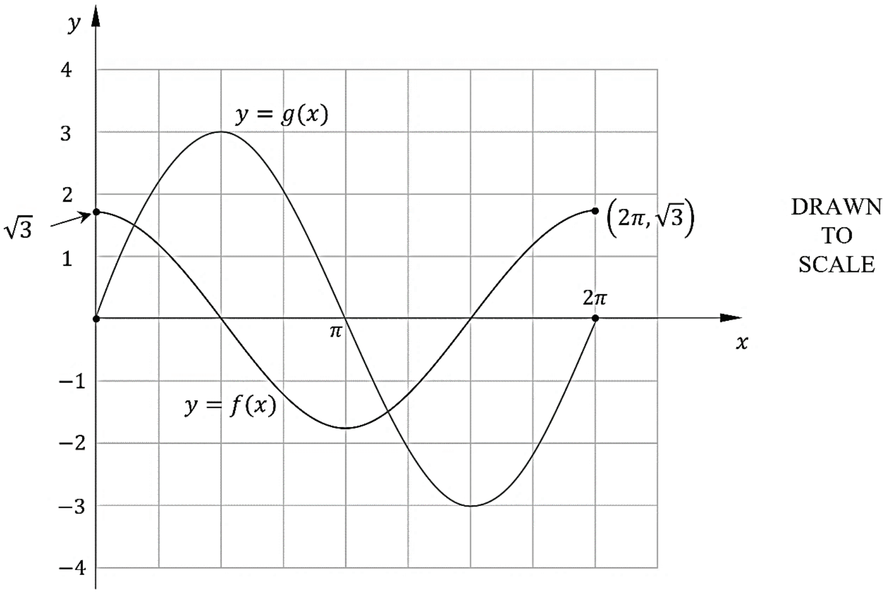
Question 12 (13 marks) Use a SEPARATE writing booklet. **Marks**

(a) The graphs of $y = f(x)$ and $y = g(x)$ are represented below, **3**

where $f(x)$ is a cosine function and $g(x)$ is a sine function.

It is given that $f(x) + g(x) = R\cos(x - \theta)$, where $0 < \theta < \frac{\pi}{2}$.

Find R and θ, where θ is in radians.



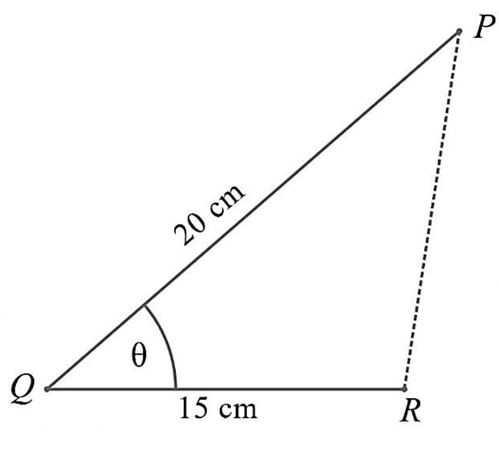
(b) Can we use mathematical induction to prove that $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$? **1**

Justify your answer.

(c) The triangle PQR, shown below, has two sides of fixed length: $PQ = 20$ cm and **3**
$QR = 15$ cm.

$\angle PQR = \theta$, which is increasing at a rate of $\dfrac{\pi}{6}$ radians/hour.

Find the **exact** rate at which the area of the triangle is changing at the time when $\theta = \dfrac{\pi}{3}$.



Question 12 continued

(d) (i) Show that $\frac{d}{d\theta}\left(\cos^2\theta\right) = -\sin 2\theta$. **1**

(ii) Hence, or otherwise, show that **2**

$$\int \tan 2\theta \cos 2\theta \, d\theta = -\cos^2\theta + c.$$

(e) Use mathematical induction to prove that $4^{2n} - 1$ is divisible by 3 for all integers $n \geq 1$. **3**

Question 13 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) A ball is kicked with velocity 20 m/s at an angle of 45° to the ground and its displacement at time t seconds is given by the following equation:

$$\underset{\sim}{r}(t) = \frac{20t}{\sqrt{2}}\underset{\sim}{i} + (-5t^2 + \frac{20t}{\sqrt{2}})\underset{\sim}{j} \quad \text{(Do NOT prove this.)}$$

(i) Find the maximum height reached by the ball. **2**

(ii) Find what time the ball lands on the ground. **1**

(b) Prove the following statement using mathematical induction. **3**

$$r + r^2 + r^3 + \cdots + r^n = \frac{1-r^{n+1}}{1-r} - 1 \text{ for integer values of } n \geq 1.$$

(c) A curve is defined parametrically by $x = \cos t$, $y = \cos^2 t$. **3**

Find the exact area enclosed by this curve, for $0 \leq t \leq \frac{\pi}{2}$, above the x-axis.

(d) Let $\underset{\sim}{u} = \begin{bmatrix} 4 \\ 8 \end{bmatrix}$ and $\underset{\sim}{v} = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$. **3**

Find the component of $\underset{\sim}{u}$ perpendicular to $\underset{\sim}{v}$.

Question 14 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) A carton of milk is removed from a fridge and placed on a table. **3**

The temperature of this carton of milk after t minutes is given by the following equation:

$$T(t) = P + Ae^{kt}$$

The temperature in the fridge is 3°C, and the temperature in the room is 24°C.

After sitting on the table for five minutes, the temperature of the milk is 15°C.

Find the total number of minutes that the milk can be left on the table before it reaches a temperature of 20°C.

Give your answer to the nearest whole number.

(b) It is given that **3**

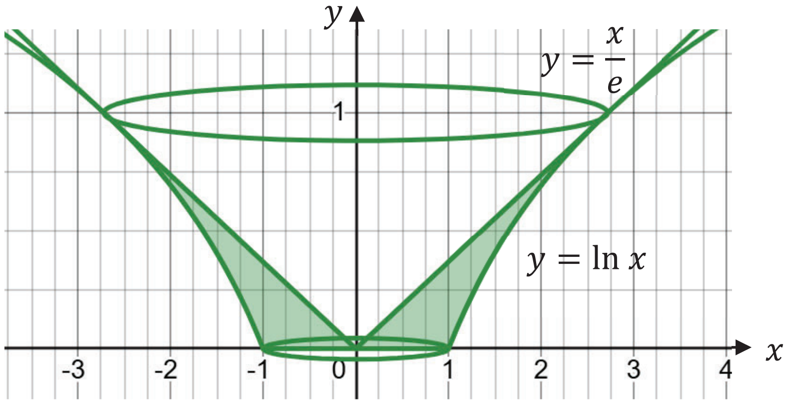
$$\tan 3\theta = \frac{3\tan\theta - \tan^3\theta}{1 - 3\tan^2\theta}.$$

By letting $x = \tan\theta$, find all the distinct solutions of the following equation.

$$x^3 - 3x^2 - 3x + 1 = 0$$

(c) A three-dimensional shape is modelled by rotating the region between **3**

the curve $y = \ln x$ and the line $y = \dfrac{x}{e}$ for $0 \le y \le 1$, about the y axis.



Find the **exact volume** of the shape.

(d) Given that $\sin^{-1}p + \cos^{-1}p = \frac{\pi}{2}$, solve the following equation: **3**

$$2\sin^{-1}(1-x) - \cos^{-1}(x-1) = -\frac{\pi}{6}.$$

(Leave your answer in **exact form**)

Question 15 (9 marks) Use a SEPARATE writing booklet. **Marks**

(a) $ABCDEF$ is a regular hexagon with centre O.

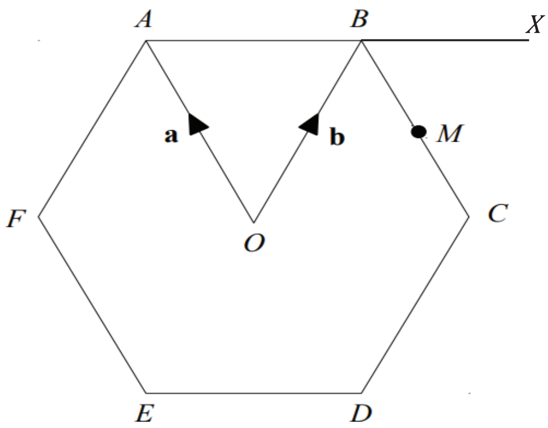
$\overrightarrow{OA} = \underset{\sim}{a}$

$\overrightarrow{OB} = \underset{\sim}{b}$

$\overrightarrow{OC} = \underset{\sim}{c}$

M is the midpoint of BC and AO is parallel to BC.

AB is extended to point X, such that $AB : BX = 3 : 2$.



(Not to scale)

(i) Show that $\overrightarrow{OM} = \underset{\sim}{b} - \frac{1}{2}\underset{\sim}{a}$. **1**

(ii) Prove that $\overrightarrow{BX} = \frac{2}{3}(\underset{\sim}{b} - \underset{\sim}{a})$. **1**

(iii) Hence or otherwise, justify that E, M and X lie on the same straight line. **3**

Question 15 continued

(b) Two projectiles A and B are launched simultaneously at $t = 0$.

At some point after launch, the two projectiles collide.

Projectile A is launched upwards towards the right, from ground level and has an initial velocity(in m/s) of $\underset{\sim}{u}_A = 10\underset{\sim}{i} + 10\underset{\sim}{j}$.

Projectile B is launched upwards towards the left, from a ledge 15 m above the ground and 30 m horizontally away from where Projectile A is launched.

Projectile B has an initial velocity(in m/s) of $\underset{\sim}{u}_B = -5\underset{\sim}{i} + 2.5\underset{\sim}{j}$.

It is given that:

$\underset{\sim}{r}_A(t) = 10t\underset{\sim}{i} + (-4.9t^2 + 10t)\underset{\sim}{j}$. (Do NOT prove this)

$\underset{\sim}{r}_B(t) = (-5t + 30)\underset{\sim}{i} + (-4.9t^2 + 2.5t + 15)\underset{\sim}{j}$. (Do NOT prove this)

Let acceleration due to gravity be $\ddot{\underset{\sim}{r}}_A(t) = \ddot{\underset{\sim}{r}}_B(t) = 0\underset{\sim}{i} - 9.8\underset{\sim}{j}$

Assume there is no air resistance.

(i) Find the time when the two projectiles collide. **1**

(ii) Find the coordinates of the point of collision. **1**

(iii) Find the speed of Projectile A at the point of collision. **2**
Leave your answer correct to 3 significant figures.

END OF EXAMINATION

HSC TRIAL EXAMINATION 2025

MATHEMATICS EXTENSION 1

Q NO.	SOLUTIONS	
1.	Two vectors $\underset{\sim}{a}$ and $\underset{\sim}{b}$ are parallel if $\underset{\sim}{b} = \lambda \underset{\sim}{a}$, where λ is a constant. $8\underset{\sim}{i} + 2\underset{\sim}{j} = 2(4\underset{\sim}{i} + \underset{\sim}{j})$ so, the vectors $8\underset{\sim}{i} + 2\underset{\sim}{j}$ and $4\underset{\sim}{i} + \underset{\sim}{j}$ must be parallel. Hence, the correct option is **B**.	**B**
2.	$\int \frac{dx}{\sqrt{100-x^2}}$? From the reference sheet $$\int \frac{f'(x)}{\sqrt{a^2-[f(x)]^2}} dx = \sin^{-1}\frac{f(x)}{a} + c$$ $a = 10 \ \ and \ f(x) = x$ Since $RHS = \ \sin^{-1}\left(\frac{\boldsymbol{f(x)}}{\boldsymbol{a}}\right) + \boldsymbol{c}$ **Therefore, the answer is:** $\sin^{-1}\left(\frac{\boldsymbol{x}}{\boldsymbol{10}}\right) + \boldsymbol{c}$ Hence, the correct option is **B**.	**B**
3.	Initially the group must be divided into a group of 5 and a group of 4. There are $\binom{9}{5}$ ways of selecting a group of 5, leaving a corresponding group of 4. So, there will be $\binom{9}{5} = 126$ ways of dividing the group into a group of 5 and a group of 4. The group of 5 can be arranged around a circular table in $4! = 24$ ways and the group of 4 can be arranged around a circular table in $3! = 6$ ways. The two groups can arrange themselves around the 2 tables in 2 different ways. Therefore, the total number of arrangements possible will be $126 \times 24 \times 6 \times 2 =$ 36288 Hence, the correct option is **A**.	**A**
4.	If the graph represents the graph of $y = f(\lvert x \rvert)$, we delete everything to the left of the y —axis and replace it by duplicating everything to the right of the y —axis. However, the best answer has $y = f(-\lvert x \rvert)$, which involves deleting everything to the right of the y —axis and replacing it by duplicating everything to the left of the y —axis. Since this section of the graph is not negative as everything lies above the x-axis then $g(x) = \lvert f(-\lvert x \rvert)\rvert$ when applied does not produce any further change. You can also substitute appropriate values to verify the correct transformation .	**B**

5.	$2 \times 7 + 1 = 15$ Explanation: With fourteen students in the training squad, we could have two from each of the seven schools, so no school with 3 or more students in the squad. As soon as we add one more student at least one school must have at least three students in the squad. Hence, the correct option is **B.**	**B**
6.	Using the product rule: $\frac{dy}{dx} = \frac{d}{dx}(e^{2x})\cos^{-1}(3x) + e^{2x}\frac{d}{dx}[\cos^{-1}(3x)]$ $= 2e^{2x}\cos^{-1}(3x) + e^{2x} \times \frac{-3}{\sqrt{1-(3x)^2}}.$ Hence, the correct option is **B.**	**B**
7.	$\int_0^b \sin 2x \sin 3x dx$ $= \frac{1}{2}\int_0^b \cos(2x - 3x) - \cos(2x + 3x) dx$ $= \frac{1}{2}\int_0^b \cos(-x) - \cos 5x \ dx$ As $\cos(-x) = \cos\ x$, then $= \frac{1}{2}\int_0^b \cos\ x - \cos 5x \ dx$ $= \frac{1}{2}\left[\sin x - \frac{1}{5}\sin 5x\right]_0^b$ $= \frac{1}{2}(\sin b - \frac{\sin 5b}{5}) - 0$ $= -\frac{\mathbf{\sin 5b}}{\mathbf{10}} + \frac{\mathbf{\sin b}}{\mathbf{2}}$ Hence, the correct option is **D.**	**D**

8.	$\sin\left(\frac{\pi}{4}-x\right)\cos\left(\frac{\pi}{4}-x\right)$ Using compound angle formula, we get: $=\frac{1}{2}\sin 2\left(\frac{\pi}{4}-x\right)=\frac{1}{2}\sin\left(\frac{\pi}{2}-2x\right)=\frac{1}{2}\cos 2x.$ Hence, the correct option is **C**. **Alternative method** $sin\left(\frac{\pi}{4}-x\right)=\sin\frac{\pi}{4}\cos x-\cos\frac{\pi}{4}sinx$ $=\frac{\cos x}{\sqrt{2}}-\frac{\sin x}{\sqrt{2}}$ $cos\left(\frac{\pi}{4}-x\right)=\cos\frac{\pi}{4}\cos x+\sin\frac{\pi}{4}sinx$ $=\frac{\cos x}{\sqrt{2}}+\frac{\sin x}{\sqrt{2}}$ So, $sin\left(\frac{\pi}{4}-x\right)cos\left(\frac{\pi}{4}-x\right)$ $=\left(\frac{\cos x}{\sqrt{2}}-\frac{\sin x}{\sqrt{2}}\right)\left(\frac{\cos x}{\sqrt{2}}+\frac{\sin x}{\sqrt{2}}\right)$ $=\frac{\cos^2 x}{2}-\frac{\sin^2 x}{2}$ $=\frac{1}{2}(\cos^2 x-\sin^2 x)=\frac{1}{2}\cos 2x$ Hence, the correct option is **C**.	**C**
9.	The scalar projection, l, of $\underset{\sim}{p}$ onto $\underset{\sim}{q}$, is $l=\frac{\underset{\sim}{p}\cdot\underset{\sim}{q}}{\lvert\underset{\sim}{q}\rvert}=\frac{\sin 3\lambda\ cos\lambda-\cos 3\lambda\ sin\lambda}{\sqrt{cos^2\lambda+sin^2\lambda}}$ Note: $cos^2\lambda+sin^2\lambda=1$ and $\sin 3\lambda\ cos\lambda-\cos 3\lambda\ \sin\lambda=\sin(3\lambda-\lambda)=\sin 2\lambda.$ So, $\ell=\sin 2\lambda$. Also, we are given that $\ell=\frac{\sqrt{3}}{2}$, this indicates that $\sin 2\lambda=\frac{\sqrt{3}}{2}$. This means $2\lambda=\frac{\pi}{3},\frac{2\pi}{3},\ldots$ that is, $\lambda=\frac{\pi}{6},\frac{\pi}{3},\ldots$ but as $0\le\lambda\le\frac{\pi}{4}$. This shows that only $\lambda=\frac{\pi}{6}$ is valid. Now, by substituting $\lambda=\frac{\pi}{6}$ into p, we get $\underset{\sim}{p}=sin\ \frac{\pi}{2}\underset{\sim}{i}-cos\ \frac{\pi}{2}\underset{\sim}{j}=\underset{\sim}{i}-0\underset{\sim}{j}$ So, $\underset{\sim}{p}=\underset{\sim}{i}-0\underset{\sim}{j}$. Hence, the correct option is **D**.	**D**
10.	The domain of $y=\tan^{-1}x$ and $y=-x$ is $(-\infty,\infty)$. So, the domain of $f(x)=\tan(\tan^{-1}x)-x$ is also $(-\infty,\infty)$. Now, *as* $\tan(\tan^{-1}x)=x$ then $f(x)=\tan(\tan^{-1}x)-x=x-x=0$ Therefore, the range of *is* $[0,0]$ Hence, the correct option is **A**.	**A**

QUESTION 11

MARKER'S COMMENTS

a) METHOD 1

$$\frac{x+3}{x-1} > 1$$

$$(x-1)^2 \times \frac{x+3}{x-1} > 1(x-1)^2$$

① mark multiply through by the square

$$(x-1)(x+3) > (x-1)^2$$

$$(x-1)(x+3) - (x-1)^2 > 0$$

$$(x-1)[x+3-(x-1)] > 0$$

$$(x-1)(x+3-x+1) > 0$$

$$4(x-1) > 0$$

$$4x > 4$$

$$x > 1$$

① mark correct final answer

$\boxed{\not 2}$

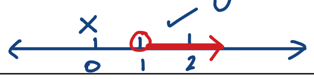
METHOD 2

$$\frac{x+3}{x-1} = 1 \qquad x \neq 1$$

$$x+3 \neq x-1$$

$\therefore$ $x=1$ is the only critical point

① mark some correct working towards final answer



Test $x=0$ $\qquad$ $x=2$

$$\frac{0+3}{0-1} > 1 \qquad \frac{2+3}{2-1} > 1$$

$$-3 > 1 \ \times \qquad 5 > 1 \ \checkmark$$

$$\therefore x > 1$$

① mark correct final answer

NOTE : Well done by most students.

MARKER'S COMMENTS

b) $\left(x^2 + \frac{2}{x}\right)^6$

METHOD 1

$${}^6C_0 (x^2)^6 + {}^6C_1 (x^2)^5\left(\frac{2}{x}\right) + {}^6C_2 (x^2)^4\left(\frac{2}{x}\right)^2 + {}^6C_3 (x^2)^3\left(\frac{2}{x}\right)^3 + {}^6C_4 (x^2)^2\left(\frac{2}{x}\right)^4 + \cdots$$

$$= x^{12} + {}^6C_1\, c^6 + {}^6C_2\, 2^2 x^6 + {}^6C_3\, 2^3 x^3 + {}^6C_4\, 2^4 + \cdots$$

1 mark for correct expansion to help find the constant term.

$\therefore$ The constant term is ${}^6C_4 \times 2^4$

$= 240$

1 mark correct constant term

$\frac{\ }{2}$

METHOD 2

$$T_{k+1} = {}^nC_k\, a^{n-k}\, b^k$$

$$= {}^6C_k\, (x^2)^{6-k}\left(\frac{2}{x}\right)^k$$

$$= {}^6C_k\, x^{12-2k} \times \frac{2^k}{x^k}$$

1 mark for correct use of formula and obtain correct indice for x

$$= {}^6C_k\, 2^k\, x^{12-2k-k}$$

$$= {}^6C_k\, 2^k\, x^{12-3k}$$

For the term independent of x

$12 - 3k = 0$

$3k = 12$

$k = 4$

$\therefore\ {}^6C_k\, 2^k$

$= {}^6C_4\, 2^4$

$= 240.$

OR

$${}^6C_k\left(\frac{2}{x}\right)^{6-k}(x^2)^k$$

$$= {}^6C_k\, 2^{6-k}\cdot x^{k-6}\cdot x^{2k}$$

$$= {}^6C_k\, 2^{6-k}\cdot x^{3k-6}$$

$3k - 6 = 0$

$\therefore k = 2$

$\therefore\ {}^6C_k\, 2^{6-k} = {}^6C_2\, 2^{6-2} = 240$

1 mark correctly equating indice with zero and correct constant term

NOTE: Well done by most students

MARKER'S COMMENTS

c)

$$LHS = \int_2^4 2x\sqrt{x-2}\,dx$$

$u = x-2 \longrightarrow x = u+2$

$\frac{du}{dx} = 1$

$du = dx$

When $x = 4 \longrightarrow u = 4-2 = 2$

$x = 2 \longrightarrow u = 2-2 = 0$

$$= \int_0^2 2(u+2)u^{\frac{1}{2}}\,du$$

$$= 2\int_0^2 u^{\frac{3}{2}} + 2u^{\frac{1}{2}}\,du$$

$$= 2\left[\frac{2}{5}u^{\frac{5}{2}} + 2\times\frac{2}{3}u^{\frac{3}{2}}\right]_0^2$$

① mark to correctly change bounds

① mark to express integral entirely in terms of u.

$$= 2\left[\frac{2}{5}(\sqrt{u})^5 + \frac{4}{3}(\sqrt{u})^3\right]_0^2$$

$$= 2\left[\frac{2}{5}(\sqrt{2})^5 + \frac{4}{3}(\sqrt{2})^3 - \left(\frac{2}{5}(\sqrt{0})^5 + \frac{4}{3}(\sqrt{0})^3\right)\right]$$

$$= 2\left[\frac{2}{5}\times 4\sqrt{2} + \frac{4}{3}\times 2\sqrt{2}\right]$$

$$= \frac{16\sqrt{2}}{5} + \frac{16\sqrt{2}}{3}$$

$$= \frac{48\sqrt{2} + 80\sqrt{2}}{15}$$

$$= \frac{128\sqrt{2}}{15}$$

① mark to correctly show final result.

$= RHS$

NOTE: Some students wasted time keeping their answers in index notation. Much easier to change to a surd.

$\boxed{/3}$

MARKER'S COMMENTS

d) (i) $t = \tan\frac{\theta}{2}$ then $\cos\theta = \frac{1-t^2}{1+t^2}$ $\sin\theta = \frac{2t}{1+t^2}$

$\therefore \cos\theta + \sqrt{3}\sin\theta$

$= \frac{1-t^2}{1+t^2} + \frac{\sqrt{3}(2t)}{1+t^2}$

$= \frac{1-t^2+2\sqrt{3}t}{1+t^2}$

① correct answer

$-\frac{1}{2}$ for very minor numerical error ?

(ii) $\cos\theta + \sqrt{3}\sin\theta = 1$ $0 \le \theta \le 2\pi$

$\therefore \frac{1-t^2+2\sqrt{3}t}{1+t^2} = 1$

$1-t^2+2\sqrt{3}t = 1+t^2$

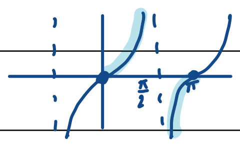
$2t^2 - 2\sqrt{3}t = 0$

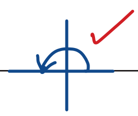
$2t(t-\sqrt{3}) = 0$

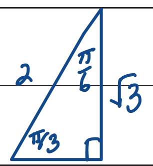
① mark to correctly find the two equations for $\tan\frac{\theta}{2}$

$\therefore t = 0$ or $t = \sqrt{3}$

$\therefore \tan\frac{\theta}{2} = 0$ $\tan\frac{\theta}{2} = \sqrt{3}$ $0 \le \theta \le 2\pi$







$0 \le \frac{\theta}{2} \le \pi$

Check $\theta = \pi$ is a solution

$\frac{\theta}{2} = 0, \pi$ $\frac{\theta}{2} = \frac{\pi}{3}$

$\cos\theta + \sqrt{3}\sin\theta$

$\theta = 0, 2\pi$ $\theta = \frac{2\pi}{3}$

$= \cos\pi + \sqrt{3}\sin\pi$

① mark to solve $\tan\frac{\theta}{2} = 0$ ($\frac{1}{2}$ for each solution)

① mark to solve $\tan\frac{\theta}{2} = \sqrt{3}$

$= -1 + \sqrt{3}(0)$

$\therefore \theta = 0, \frac{2\pi}{3}, 2\pi$

$= -1$

$\neq \sqrt{3}$ $\therefore \theta = \pi$ is NOT a solution

NOTE: Many students forgot to include the solution of 2π. More care required for solutions to be within the given domain. Reminder to check $\theta = \pi$ in original equation.

$\boxed{3}$

MARKER'S COMMENTS

e) $P(x) = 2x^3 - 3ax + 4b^3$

$(x-2)^2$ is a factor of $P(x)$

$\therefore P(2) = 0$

$\therefore 0 = 2(2)^3 - 3a(2) + 4b^3$

$0 = 16 - 6a + 4b^3$

$2b^3 - 3a = -8$

1 mark to use $P(2)=0$ to establish equation in terms of a and b.

Also $p'(2) = 0$

$p'(x) = 6x^2 - 3a$

$\therefore 0 = 6(2)^2 - 3a$

$0 = 24 - 3a$

$3a = 24$

$a = 8$

1 mark to find the value of a using $p'(2)=0$

Sub $a = 8$ into $2b^3 - 3a = -8$ 24

$2b^3 - 3(8) = -8$

$2b^3 = 16$

$b^3 = 8$

$b = 2$

1 mark to find the value of b.

$\therefore a = 8, \quad b = 2$

3/3

NOTE: Question was well done.

12 MAX MARKER'S COMMENTS 2025 AT4

Q12

a) $f(x) = \sqrt{3}\cos x$ and $g(x) = 3\sin x$ — 1

Now $f(x) + g(x) = R\cos(x - \theta)$

i.e $\sqrt{3}\cos x + 3\sin\theta = R\cos(x-\theta)$ - - - (1)

$= R\cos x \cos\theta + R\sin x \sin\theta$

$= R\cos\theta . \cos x + R\sin\theta . \sin x$

Equating coefficients

$\sqrt{3} = R\cos\theta$ ... (2) and $3 = R\sin\theta$ ... (3)

$(2)^2 + (3)^2$

$R^2\cos^2\theta + R^2\sin^2\theta = (\sqrt{3})^2 + 3^2$

$R^2(\cos^2\theta + \sin^2\theta) = 3 + 9$

$R^2 = 12$

$R = \sqrt{12}$, $R > 0$ — 1

$R = 2\sqrt{3}$ sub in (2) + (3)

$\sqrt{3} = 2\sqrt{3}\cos\theta$ and $3 = 2\sqrt{3}\sin\theta$

$\cos\theta = \frac{1}{2}$

$\sin\theta = \frac{3}{2\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{2}$

θ overlaps in quadrant 1

So now, $\frac{\sin\theta}{\cos\theta} = \frac{\sqrt{3}/2}{1/2}$

$\tan\theta = \sqrt{3}$

$\therefore \theta = \frac{\pi}{3}$ — 1

$\therefore R = 2\sqrt{3}$ and $\theta = \pi/3$

This question was very well done, however some students wrote $g(x) = \sin x$ instead of $3\sin x$. $2\frac{1}{2}$ marks was given for those students.

MARKER'S COMMENTS

b)

We cannot use mathematical induction to prove $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ as this statement applies to all possible values of θ, because θ is continuous (and not discrete).

Note: Mathematical Induction, in this syllabus, only applies to positive integer values of 'n'.

— 1

This question was not answered well.
Please review.

MARKER'S COMMENTS

c) $PQ = 20$, $QR = 15\text{cm}$ and $\frac{d\theta}{dt} = \frac{\pi}{6}$

Area of ΔABC, $A = \frac{1}{2}ab\sin C$

$$= \frac{1}{2} \times 20 \times 15 \times \sin\theta$$ ——— ½mk

$$A = 150\sin\theta$$

$$\frac{dA}{d\theta} = 150\cos\theta$$ ——— ½ mk

Using the chain rule

$$\frac{dA}{dt} = \frac{dA}{d\theta} \times \frac{d\theta}{dt}$$

$$= 150\cos\theta \times \frac{\pi}{6}$$

$$= 25\pi\cos\theta$$ ——— 1 mk

When $\theta = \frac{\pi}{3}$

$$\frac{dA}{dt} = 25\pi\cos\left(\frac{\pi}{3}\right)$$

$$= 25\pi \times \frac{1}{2}$$

$$\therefore \frac{dA}{dt} = \frac{25\pi}{2} \text{ cm}^2/\text{h}$$ ——— 1

This question was answered very well overall. Some students though need to take more care when writing the units for rates, such as for $\frac{dA}{dt}$.

The units for $\frac{dA}{dt}$ should be written as cm^2/h

MARKER'S COMMENTS

d) i) $\frac{d}{d\theta}(\cos^2\theta) = \frac{d}{d\theta}(\cos\theta)^2$

$= 2.\cos\theta^1 \times (-\sin\theta)$ — 1mk

$= -2\sin\theta\cos\theta$

$= -\sin 2\theta$

or

$\frac{d}{d\theta}(\cos^2\theta) = \frac{d}{d\theta}\left(\frac{1}{2}(1+\cos 2\theta)\right)$

$= \frac{d}{d\theta}\left(\frac{1}{2} + \frac{1}{2}\cos 2\theta\right)$ — 1 mk

$= -2\times\frac{1}{2}\sin 2\theta$ [A few students left this step out, however good to include in a 'show' question]

$= -\sin 2\theta$

This part was well done overall.

ii) $\int \tan 2\theta \cos 2\theta \, d\theta$

$= \int \frac{\sin 2\theta}{\cos 2\theta} . \cos 2\theta \, d\theta$ — 1 mk

$= \int \sin 2\theta \, d\theta$ — $\frac{1}{2}$ mk

$= -\int -\sin 2\theta \, d\theta$ — $\frac{1}{2}$ mk

$= -\cos^2\theta + c$ from (i)

Overall, this question was well answered, however there were still many students who had difficulty connecting part (i) to this question. If students followed to proceed to prove this statement with the use of the result from part (i), but did not mention it or apply it anywhere, then ½ mark was lost.

MARKER'S COMMENTS

e) Let the statement $S(n)$ be
$4^{2n}-1$ is divisible by 3.

Prove $S(n)$ true for $n=1$

$S(1)$: $4^{2(1)}-1 = 4^2-1$
$= 16-1$
$= 15$
$= 3\times 5$ 1
$\therefore$ divisible by 3
$\therefore$ statement is true for $n=1$

Assume $S(n)$ is true for some integer k, $k\geq 1$
ie Suppose
$4^{2k}-1 = 3M$ where M is an integer
$4^{2k} = 3M+1$ ---- (*)

Prove $S(n)$ true for $n=k+1$
ie Prove that
$4^{2(k+1)}-1 = 3P$ where P is an integer

Consider the LHS.

$4^{2(k+1)}-1 = 4^{2k+2}-1$
$= 4^{2k}.4^2-1$
$=(3M+1).4^2-1$ (from the inductive hypothesis) (*) — 1
$= 16(3M+1)-1$
$= 48M+16-1$
$= 48M+15$
$= 3(16M+5)$ — 1
which is divisible by 3

$\therefore$ true for $n=k+1$ since it is true for $n=k$

Hence by the principle of mathematical induction the statement $S(n)$ is true for all integers $n\geq 1$.

MARKER'S COMMENTS

- 3 marks for correct solution showing all steps clearly
- 2 marks for the base case, inductive hypothesis and for showing some correct lines of working in the inductive step
- 1 mark for proving the base case.

The majority of the students proved this statement well by mathematical induction, however a handful of students did not write some important lines in the proof and they are highlighted in the solution. Please include these statements when you revise for the HSC. No marks were lost here, but I can't guarantee that in the HSC.

s

Alternative Solution to the Inductive step

$$
\begin{aligned}
4^{2k+2}-1 &= 4^2.4^{2k}-1 \\
&= 4^2.4^{2k}-4^2+4^2-1 \\
&= 4^2(4^{2k}-1)+4^2-1 \\
&= 16(3M)+16-1 \quad \text{(from the inductive hypothesis)} \\
&= 16(3M)+15 \\
&= 3(16M+5) \\
&= 3P
\end{aligned}
$$

$\therefore$ true for $n=k+1$ since it is true for $n=k$

Hence, by the principle of mathematical induction the statement $S(n)$ is true for all integer n, $n \geq 1$.

12MAX AT4 2025 Q13 MARKER'S COMMENTS

a) i $y = -5t^2 + 10\sqrt{2}t$

$\therefore \dot{y} = -10t + 10\sqrt{2}$

Maximum height when $\dot{y} = 0$

$-10t + 10\sqrt{2} = 0$

$10t = 10\sqrt{2}$

$t = \sqrt{2}$

when $t = \sqrt{2}$, $y = -5(\sqrt{2})^2 + 10\sqrt{2}(\sqrt{2})$

$= -10 + 20$

$= 10\text{ m}$

2 marks Complete solution

1 mark Finds $t = \sqrt{2}$

A large number of students wasted time deriving the equations of motion - the question gave you the full equation for displacement!

ii lands at twice the time for maximum height ie $2\sqrt{2}$

OR lands when $y = 0$

$-5t^2 + 10\sqrt{2}t = 0$

$5t(2\sqrt{2} - t) = 0$

$t = 2\sqrt{2}$

Students that gave two answers could not receive full marks.

b) *Prove true for $n=1$

LHS $= r$

$$\text{RHS} = \frac{1-r^{n+1}}{1-r} - 1$$

$$= \frac{1-r^2}{1-r} - 1$$

$$= \frac{(1-r)(1+r)}{1-r} - 1$$

$$= 1+r-1$$

$$= r$$

$$= \text{LHS}$$

$\therefore$ true for $n=1$

* Suppose true for $n=k$

That is, $r + r^2 + r^3 + \dots + r^k = \dfrac{1-r^{k+1}}{1-r} - 1$

* Prove true for $n=k+1$

That is, $r + r^2 + r^3 + \dots + r^k + r^{k+1} = \dfrac{1-r^{k+1+1}}{1-r} - 1$

LHS $= r + r^2 + r^3 + \dots + r^k + r^{k+1}$

$$= \frac{1-r^{k+1}}{1-r} - 1 + r^{k+1} \quad \text{(By inductive hypothesis)}$$

$$= \frac{1-r^{k+1}}{1-r} + \frac{(1-r)r^{k+1}}{1-r} - 1$$

$$= \frac{1 - r^{k+1} + r^{k+1} - r.r^{k+1}}{1-r} - 1$$

$$= \frac{1 - r.r^{k+1}}{1-r} - 1$$

$$= \frac{1 - r^{k+1+1}}{1-r} - 1$$

$= RHS$

$\therefore$ true for $n=k+1$ if it is true for $n=k$.
$\therefore$ By the principle of mathematical induction, true for all n.

3 marks Complete solution.
2 marks Correctly proves base case AND Correctly uses hypothesis.
1 mark Correctly proves base case OR Correctly uses hypothesis.

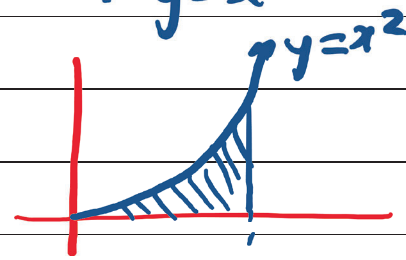
Mathematical induction questions are very predictable; this is not the place to get creative! Follow the procedure exactly. Many proofs broke down because of poor setting out: when this breaks the logic of your argument you should expect to lose marks.
Also, there is plenty of paper: stop cramming seven equals signs on each line!

12MAX AT4 2025 Q13 MARKER'S COMMENTS (continued)

c) $x = \cos t, \quad y = \cos^2 t \quad \therefore y = x^2$

when $t = 0$, $x = 1$

when $t = \frac{\pi}{2}$, $x = 0$



$\therefore A = \int_0^1 x^2\,dx$

$= \left[\frac{x^3}{3}\right]_0^1$

$= \frac{1}{3}$ units^2

3 marks Complete solution

$2\frac{1}{2}$ marks Finds Area as $\left|\int_1^0 x^2\,dx\right| = \left|-\frac{1}{3}\right| = \frac{1}{3}$

2 marks Correctly states $A = \int_0^1 x^2\,dx$

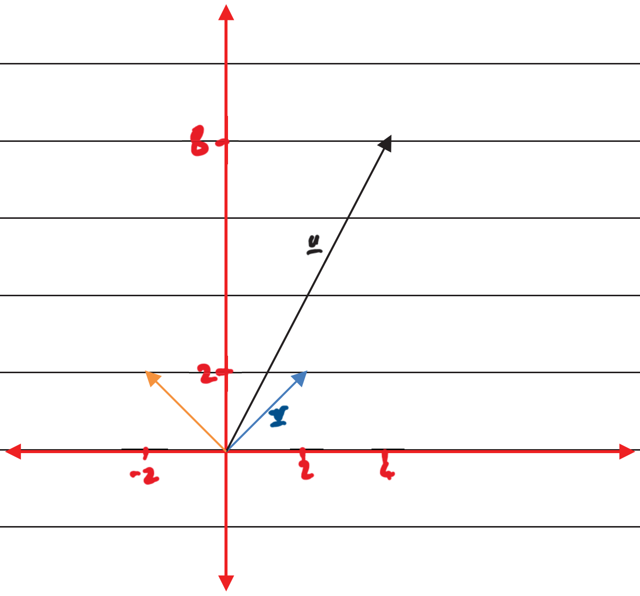
1 mark Correctly states $y = x^2$

or $A = \int_0^1 \cos^2(\cos^{-1}x)\,dx$

No marks were awarded for just integrating any old expression you felt like.

Overall not well done. Better responses found the Cartesian equation immediately and never looked back. Also, if you get a negative area, don't just slap absolute value signs on it - go back and see what's going on.

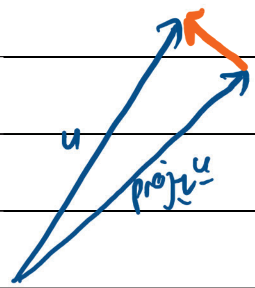
d)



$$\text{proj}_{\binom{-2}{2}}\underset{\sim}{u} = \frac{4\times-2+8\times2}{-2\times-2+2\times2}\begin{pmatrix}-2\\2\end{pmatrix}$$
$$= \frac{-8+16}{8}\begin{pmatrix}-2\\2\end{pmatrix}$$
$$= \begin{pmatrix}-2\\2\end{pmatrix}$$

OR

$$\text{proj}_{\underset{\sim}{v}}\underset{\sim}{u} = \frac{8+16}{4+4}\begin{pmatrix}2\\2\end{pmatrix}$$
$$= \frac{24}{8}\begin{pmatrix}2\\2\end{pmatrix}$$
$$= \begin{pmatrix}6\\6\end{pmatrix}$$



$$\therefore \begin{pmatrix}4\\8\end{pmatrix} - \begin{pmatrix}6\\6\end{pmatrix} = \begin{pmatrix}-2\\2\end{pmatrix}$$

3 marks Complete solution

2 marks Correctly finds $\text{proj}_{\underset{\sim}{v}}\underset{\sim}{u} = \begin{pmatrix}6\\6\end{pmatrix}$

1 mark Correctly states $\text{proj}_{\underset{\sim}{v}}\underset{\sim}{u} = \dfrac{4\times2+8\times2}{2\times2+2\times2}\begin{pmatrix}2\\2\end{pmatrix}$

Note: If your projection is bigger than your original vector, you need to go back and check what went wrong. CFPE could not be awarded in such clearly incorrect situations. Revise this as it is a common question!

MARKER'S COMMENTS

14a) $T(t) = P + Ae^{kt}$

$P = 24$ (ambient temperature)

$T(t) = 24 + Ae^{kt}$ $-\frac{1}{2}$

SUB $t=0$, $T=3$

$3 = 24 + Ae^{0}$

$A = -21$

$T = 24 - 21e^{kt}$ $-\frac{1}{2}$ (no CFPE)

Many students did not know to let $P = 24$.

SUB $t=5$, $T=15$

$15 = 24 - 21e^{5k}$

$21e^{5k} = +9$

$e^{5k} = \frac{+9}{21} = \frac{3}{7}$ $-\frac{1}{2}$ (CFPE allowed)

$5k = \log_e\left(\frac{3}{7}\right)$

$k = \frac{1}{5}\log_e\left(\frac{3}{7}\right)$ $-\frac{1}{2}$ (CFPE allowed)

Find t when $T = 20$.

$20 = 24 - 21e^{kt}$

$21e^{kt} = 4$

$e^{kt} = \frac{4}{21}$ $-\frac{1}{2}$ (CFPE allowed)

$kt = \log_e\frac{4}{21}$

$t = \frac{\log_e\frac{4}{21}}{k} = 9.7853904$ $\frac{1}{2}$

$\doteqdot 10$ minutes (number required)

Made with Goodnotes

MARKER'S COMMENTS

14b) Given $\tan 3\theta = \dfrac{3\tan\theta - \tan^3\theta}{1 - 3\tan^2\theta}$

Equation:

$$x^3 - 3x^2 - 3x + 1 = 0$$

SUB $x = \tan\theta$

$$\tan^3\theta - 3\tan^2\theta - 3\tan\theta + 1 = 0$$

$$-3\tan^2\theta + 1 = 3\tan\theta - \tan^3\theta$$

Divide by $1 - 3\tan^2\theta$

$$1 = \frac{3\tan\theta - \tan^3\theta}{1 - 3\tan^2\theta}$$

$$1 = \tan 3\theta \quad - 1$$

$$3\theta = \frac{\pi}{4}, \frac{5\pi}{4}, \frac{9\pi}{4}, \ldots$$

$$\theta = \frac{\pi}{12}, \frac{5\pi}{12}, \frac{9\pi}{12} \quad - 1 \text{ (if at least 2 correct)}$$

Many forgot the following step.

$$x = \tan\theta = \tan\frac{\pi}{12}, \tan\frac{5\pi}{12}, \tan\frac{9\pi}{12} \quad - 1$$

(lose $\frac{1}{2}$ if not exactly 3 solutions.)

OR

$x = \tan\theta = 2 - \sqrt{3}, 2 + \sqrt{3}, -1$ (new calculator)

Made with Goodnotes

MARKER'S COMMENTS

14 c) $y = \frac{x}{e} \longrightarrow x = ey$ $\frac{1}{2}$

$y = \log_e x \longrightarrow x = e^y$ $\frac{1}{2}$

$$V = \pi \int_0^1 x_2^2 - x_1^2 \, dy$$

$$V = \pi \int_0^1 (e^y)^2 - (ey)^2 \, dy$$

$$= \pi \int_0^1 e^{2y} - e^2 y^2 \, dy \quad -1$$

(lose $\frac{1}{2}$ for each mistake in formula)

$$= \pi \left[\frac{1}{2} e^{2y} - \frac{e^2 y^3}{3} \right]_0^1$$

$$= \pi \left[\frac{1}{2} e^2 - \frac{e^2 (1)^3}{3} - \left(\frac{1}{2} e^0 - \frac{e^2 (0)^3}{3} \right) \right]$$

$$= \pi \left[\frac{1}{2} e^2 - \frac{e^2}{3} - \frac{1}{2} + 0 \right] \quad \frac{1}{2}$$

$$= \frac{\pi (3e^2 - 2e^2 - 3)}{6}$$

$$= \frac{\pi (e^2 - 3)}{6} \text{ units}^3 \quad \frac{1}{2}$$ (or for equivalent)

A number of students switched x_1 and x_2, forgot π, or put 2π.

MARKER'S COMMENTS

14d) <u>Method 1</u>

$\sin^{-1}p + \cos^{-1}p = \frac{\pi}{2}$ (Given)

$2\sin^{-1}(1-x) - \cos^{-1}(x-1) = -\frac{\pi}{6}$

$2 \times -\sin^{-1}(x-1) - \cos^{-1}(x-1) = -\frac{\pi}{6}$ 1

$-2\sin^{-1}(x-1) - \cos^{-1}(x-1) = -\frac{\pi}{6}$

$-\sin^{-1}(x-1) - \left[\sin^{-1}(x-1) + \cos^{-1}(x-1)\right] = -\frac{\pi}{6}$

$-\sin^{-1}(x-1) - \frac{\pi}{2} = -\frac{\pi}{6}$ $\frac{1}{2}$ for substitution

$-\sin^{-1}(x-1) = \frac{\pi}{3}$

$\sin^{-1}(x-1) = -\frac{\pi}{3}$ $\frac{1}{2}$

$x - 1 = \sin\left(-\frac{\pi}{3}\right)$ $\frac{1}{2}$

$x = 1 + \sin\left(-\frac{\pi}{3}\right)$

$= 1 - \frac{\sqrt{3}}{2}$ $\frac{1}{2}$ for exact

OR $= \frac{2-\sqrt{3}}{2}$ (new calculator)

CFPE allowed for last 3 steps.

MARKER'S COMMENTS

14d) Method 2

$\sin^{-1} p + \cos^{-1} p = \frac{\pi}{2}$ (Given)

$\therefore \sin^{-1}(x-1) + \cos^{-1}(x-1) = \frac{\pi}{2}$ ① $\frac{1}{2}$

$2\sin^{-1}(1-x) - \cos^{-1}(x-1) = -\frac{\pi}{6}$

$2 \times \underline{-\sin^{-1}(x-1)} - \cos^{-1}(x-1) = -\frac{\pi}{6}$ 1

$-2\sin^{-1}(x-1) - \cos^{-1}(x-1) = -\frac{\pi}{6}$ ②

① + ② : $-\sin^{-1}(x-1) = \frac{\pi}{2} - \frac{\pi}{6}$

$-\sin^{-1}(x-1) = \frac{\pi}{3}$

$\sin^{-1}(x-1) = -\frac{\pi}{3}$ $\frac{1}{2}$

$x - 1 = \sin\left(-\frac{\pi}{3}\right)$ $\frac{1}{2}$

$x = 1 + \sin\left(-\frac{\pi}{3}\right)$

$= 1 - \frac{\sqrt{3}}{2}$ $\frac{1}{2}$ for exact

OR $= \frac{2-\sqrt{3}}{2}$ (new calculator)

CFPE allowed for last 3 steps.

Method 3

MARKER'S COMMENTS

14d) $\sin^{-1}p + \cos^{-1}p = \frac{\pi}{2}$ (Given)

$\therefore \sin^{-1}(x-1) + \cos^{-1}(x-1) = \frac{\pi}{2}$ ①

$2\sin^{-1}(1-x) - \cos^{-1}(x-1) = -\frac{\pi}{6}$

$2 \times \underline{-\sin^{-1}(x-1)} - \cos^{-1}(x-1) = -\frac{\pi}{6}$ 1

$-2\sin^{-1}(x-1) - \cos^{-1}(x-1) = -\frac{\pi}{6}$ ②

$-2\left[\frac{\pi}{2} - \cos^{-1}(x-1)\right] - \cos^{-1}(x-1) = -\frac{\pi}{6}$ $\frac{1}{2}$

$-\pi + 2\cos^{-1}(x-1) - \cos^{-1}(x-1) = -\frac{\pi}{6}$

$+\cos^{-1}(x-1) - \pi = -\frac{\pi}{6}$

$+\cos^{-1}(x-1) = \frac{5\pi}{6}$ $\frac{1}{2}$

$x-1 = \cos\left(\frac{5\pi}{6}\right)$ $\frac{1}{2}$

CFPE allowed in last 3 steps

$x = 1 + \cos\frac{5\pi}{6}$

$= 1 - \frac{\sqrt{3}}{2}$ $\frac{1}{2}$ for exact

OR $= \frac{2-\sqrt{3}}{2}$ (new calculator)

Method 4

MARKER'S COMMENTS

14d) $\sin^{-1} p + \cos^{-1} p = \frac{\pi}{2}$ (Given)

$\therefore \sin^{-1}(1-x) + \cos^{-1}(1-x) = \frac{\pi}{2}$

$\therefore \sin^{-1}(1-x) = \frac{\pi}{2} - \cos^{-1}(1-x)$ ①

$2\sin^{-1}(1-x) - \cos^{-1}(x-1) = -\frac{\pi}{6}$

$2\sin^{-1}(1-x) - \left[\pi - \cos^{-1}(1-x)\right] = -\frac{\pi}{6}$ 1

①

$2\left[\frac{\pi}{2} - \cos^{-1}(1-x)\right] - \pi + \cos^{-1}(1-x) = -\frac{\pi}{6}$ $\frac{1}{2}$

$\pi - 2\cos^{-1}(1-x) - \pi + \cos^{-1}(1-x) = -\frac{\pi}{6}$

$-\cos^{-1}(1-x) = -\frac{\pi}{6}$

$\cos^{-1}(1-x) = \frac{\pi}{6}$ $\frac{1}{2}$

$1 - x = \cos\frac{\pi}{6}$ $\frac{1}{2}$

$x = 1 - \cos\frac{\pi}{6}$

$x = 1 - \frac{\sqrt{3}}{2}$ $\frac{1}{2}$ for exact

OR $x = \frac{2-\sqrt{3}}{2}$

CFPE allowed in last 3 steps.

MARKER'S COMMENTS

The following errors were made by some students

* $\sin^{-1}(1-x) = \sin^{-1}(x-1)$ ✗

instead of

$\sin^{-1}(1-x) = -\sin^{-1}(x-1)$ ✓

* $\cos^{-1}(x-1) = -\cos^{-1}(1-x)$ ✗

instead of

$\cos^{-1}(x-1) = \pi - \cos^{-1}(1-x)$ ✓

* Didn't notice the difference of $1-x$ and $x-1$

* Addition and subtraction errors with constants terms in π

* Transcription errors.

MARKER'S COMMENTS

Mathematics Extension 1 HSC Trial Solutions 2025

a i) $\overrightarrow{OA} = \underset{\sim}{a}$, $\overrightarrow{OB} = \underset{\sim}{b}$, $\overrightarrow{OC} = \underset{\sim}{c}$ and $\overrightarrow{BC} = -\underset{\sim}{a}$

M is the mid-point of BC, therefore

$\overrightarrow{OM} = \overrightarrow{OB} + \overrightarrow{BM}$

$= \overrightarrow{OB} + \frac{1}{2}\overrightarrow{BC}$ — 1mk — no ½ mk

$= \underset{\sim}{b} - \frac{1}{2}\underset{\sim}{a}$

Quite well done

a ii) $AB : BX = 3 : 2$

$\overrightarrow{AB} = \underset{\sim}{b} - \underset{\sim}{a}$

$\frac{\overrightarrow{AB}}{3} = \frac{\overrightarrow{BX}}{2}$

$\frac{2\overrightarrow{AB}}{3} = \overrightarrow{BX}$ — 1mk → no ½ mk

$\therefore \overrightarrow{BX} = \frac{2}{3}(\underset{\sim}{b} - \underset{\sim}{a})$

or

$\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB}$

$= -\underset{\sim}{a} + \underset{\sim}{b}$

$= \underset{\sim}{b} - \underset{\sim}{a}$

$\overrightarrow{AB} = \frac{3}{2}\overrightarrow{BX}$ (since $AB : BX = 3 : 2$)

$\therefore \overrightarrow{BX} = \frac{2}{3}\overrightarrow{AB}$ — 1mk — no ½ mk

$= \frac{2}{3}(\underset{\sim}{b} - \underset{\sim}{a})$

Quite well attempted

MARKER'S COMMENTS

iii) $\overrightarrow{EM} = \overrightarrow{EO} + \overrightarrow{OM}$

$= \underset{\sim}{b} + \underset{\sim}{b} - \frac{1}{2}\underset{\sim}{a}$ ← from (i)

$= 2\underset{\sim}{b} - \frac{1}{2}\underset{\sim}{a}$

$= \frac{1}{2}(4\underset{\sim}{b} - \underset{\sim}{a})$ — 1mk

$\overrightarrow{MX} = \overrightarrow{MB} + \overrightarrow{BX}$

$= \frac{1}{2}\underset{\sim}{a} + \frac{2}{3}(\underset{\sim}{b} - \underset{\sim}{a})$ — from (ii)

$= -\frac{1}{6}\underset{\sim}{a} + \frac{2}{3}\underset{\sim}{b}$

$= \frac{1}{6}(4\underset{\sim}{b} - \underset{\sim}{a})$ — 1mk

$\overrightarrow{EM} = 3 \times \overrightarrow{MX}$

$= 3\overrightarrow{MX}$ — ½ mk

$\overrightarrow{MX} \parallel \overrightarrow{EM}$ as the two vectors are scalar multiples of each other. M is a common point and they E, M and X are collinear points or lie on the same straight line.

— ½ mk

MARKER'S COMMENTS

Alternatively

Some students showed that
$\vec{EM} \parallel \vec{EX}$

$\vec{EM} = \frac{1}{2}(4\underset{\sim}{b} - \underset{\sim}{a})$ — from above — 1 mk

$\vec{EX} = \frac{8}{3}\underset{\sim}{b} - \frac{2}{3}\underset{\sim}{a} = \frac{2}{3}(4\underset{\sim}{b} - \underset{\sim}{a})$ — 1 mk

$\frac{4}{3}\vec{EM} = \vec{EX}$ — ½ mk

$\vec{EX} \parallel \vec{EM}$ as the two vectors — ½ mk
are scalar multiples of each other.
E is a common point and thus
E, M and X are collinear points
or lie on the same straight
line.

Not done as expected.
Students are encouraged to
show all the relevant working
and reasoning in "prove",
"show" and "justify" questions.

MARKER'S COMMENTS

Alternatively

Some students showed that

$\overrightarrow{XM} \parallel \overrightarrow{XE}$

$\overrightarrow{XM} = -\frac{1}{6}(4\underset{\sim}{b} - \underset{\sim}{a})$

$\overrightarrow{XE} = -\frac{2}{3}(4\underset{\sim}{b} - \underset{\sim}{a})$

$\overrightarrow{XM} = \frac{1}{4}\overrightarrow{XE}$

$\overrightarrow{XM} \parallel \overrightarrow{XE}$ as the 2 vectors are scalar multiples of each other

X is a common point and they E, M and X are collinear points or lie on the same straight line.

Not done as expected.
Students are encouraged to show all the relevant working and reasoning in "prove", "show" and "justify" questions.

MARKER'S COMMENTS

b(i) $\underset{\sim}{r}_A(t) = \underset{\sim}{r}_B(t)$

$\underset{\sim}{r}_A(t) = 10t\,\underset{\sim}{i} + (-4.9t^2 + 10t)\underset{\sim}{j}$

$\underset{\sim}{r}_B(t) = (-5t + 30)\underset{\sim}{i} + (-4.9t^2 + 2.5t + 15)\underset{\sim}{j}$

$10t = -5t + 30$ — ½ mk

$15t = 30$

$t = 2s$ — ½ mk

$\therefore$ the two projectiles collide when $t = 2s$

Generally, well done.

ii) Coordinates of the point of collision:

$x = 10t = 10(2) = 20m$

$y = -4.9t^2 + 10t$

$= -4.9(2)^2 + 10(2)$

$= 0.4m$

Coordinates of the point of collision are $(20, 0.4)$

½ mk ½ mk

Generally, well done.

MARKER'S COMMENTS

iii) $\underset{\sim}{r}_A(t) = 10t\,\underset{\sim}{i} + (-4.9t^2 + 10t)\,\underset{\sim}{j}$

$\dot{\underset{\sim}{r}}_A(t) = 10\,\underset{\sim}{i} + (-9.8t + 10)\,\underset{\sim}{j}$ – 1mk

At $t = 2s$

$\dot{\underset{\sim}{r}}_A(t) = \sqrt{10^2 + (-9.8(2) + 10)^2}$

$= \sqrt{100 + (-9.6)^2}$

$= \sqrt{192.16}$

$= 13.862\ m/s$ – 1mk

Some students did not differentiate the displacement expression to obtain the expression for velocity.