



SYDNEY BOYS HIGH SCHOOL
MOORE PARK, SURRY HILLS

2003
TRIAL HIGHER SCHOOL
CERTIFICATE

Mathematics Extension 1

General Instructions

- Reading Time – 5 Minutes
- Working time – 3 hours
- Write using black or blue pen. Pencil may be used for diagrams.
- Board approved calculators may be used.
- All necessary working should be shown in every question.

Total Marks – 84

- Attempt all questions.
- All questions are of equal value.
- Each section is to be answered in a separate bundle, labeled Section A (Questions 1, 2, 3), Section B (Questions 4, 5, 6) and Section C (Questions 7 and 8).

Examiner: *A.M. Gainford*

Note: This is an assessment task only and does not necessarily reflect the content or format of the Higher School Certificate.

Total marks - 84.

Attempt Questions 1-7.

All questions are of equal value.

Answer each Section in a SEPARATE writing booklet. Extra writing booklets are available

Section A Use a SEPARATE writing booklet

Question 1	(12 marks)	Marks
(a)	Differentiate	
	(i) $x \sin 3x$	1
	(ii) e^{1-x^2}	1
(b)	Find the acute angle between the lines $3y = 2x + 8$ and $5x - y - 9 = 0$.	2
(c)	Evaluate	
	(i) $\int_0^2 \frac{dx}{4+x^2}$	2
	(ii) $\int_0^1 \frac{x^2}{2+x^3} dx$	2
(d)	The letters of the word INTEGRAL are arranged in a row. If one of these arrangements is selected at random, what is the probability that the vowels are in the same position?	2
(e)	Solve the inequality $\frac{\theta - 4}{\theta} > 0$.	2

Section A continued.

Question 2. (12 marks)

Marks

- (a) If α, β and γ are the roots of the equation $2x^3 - 5x^2 - 3x + 1 = 0$, evaluate

(i) $\alpha + \beta + \gamma$ and $\alpha\beta\gamma$.

1

(ii) $\alpha^2 + \beta^2 + \gamma^2$.

2

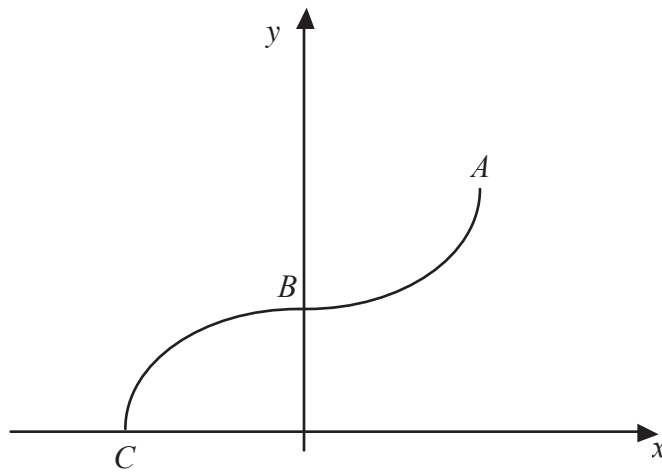
- (b) Use the substitution $u = x^2 + 4$ to find the exact value of $\int_0^{2\sqrt{3}} \frac{x}{\sqrt{x^2 + 4}} dx$.

3

- (c) Determine the exact value of $\cos\left(\tan^{-1}\left(\frac{8}{15}\right)\right)$.

2

- (d)



The diagram shows the graph of $y = \pi + 2 \sin^{-1} 3x$.

- (i) Find the coordinates of A and C .

2

- (ii) Find the gradient of the tangent at B .

2

Section A continued.

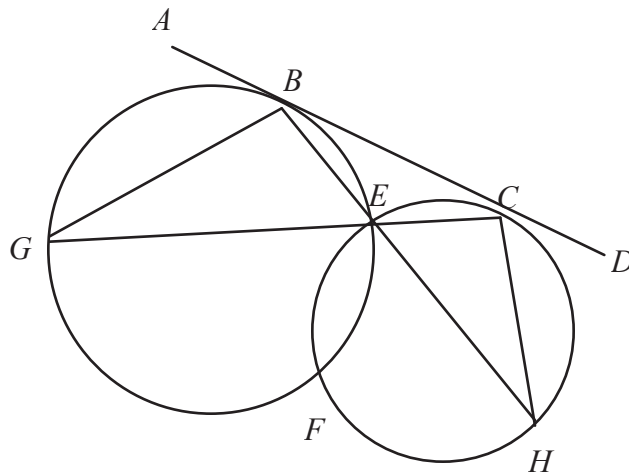
Question 3. (12 marks)

Marks

- (a) A function is defined as $f(x) = 1 + e^{2x}$. 2
Find the inverse function $f^{-1}(x)$ and state the domain and range.

- (b) Consider the quadratic expression $Q(x) = (5k - 4)x^2 - 6x + (6k + 3)$, 3
where k is a constant.
Find the values of k for which $Q(x) = 0$ has rational roots.

(c)



$ABCD$ is a common tangent to the two circles.

- (i) Prove that $\angle ABG = \angle DCH$. 2
- (ii) Prove that $\triangle BCG \parallel \triangle BCH$. 2
- (d) Consider the series $2^N + 2^{N-1} + 2^{N-2} + \dots + 2^{1-N} + 2^{-N}$,
where N is a positive integer.
- (i) Find an expression in terms of N for the number of terms in the series. 2
- (ii) Find an expression in terms of N for the sum of the series. 1

Section B Use a SEPARATE writing booklet.

Question 4. (12 marks)

Marks

(a) Consider the function $f(\theta) = \frac{\sin \theta + \sin \frac{\theta}{2}}{1 + \cos \theta + \cos \frac{\theta}{2}}$

- (i) Show that $f(\theta) = t$ where $t = \tan \frac{\theta}{2}$. 3
- (ii) Write down the general solution of $f(\theta) = 1$. 1

- (a) A certain particle moves along the straight line in accordance with the law: $t = 2x^2 - 5x + 3$, where x is measured in centimetres and t in seconds.

Initially, the particle is 1.5 centimetres to the right of the origin O , and moving away from O .

- (i) Show that the velocity, $v \text{ cms}^{-1}$, is given by 1

$$v = \frac{1}{4x - 5}$$

- (ii) Find an expression for the acceleration, $a \text{ cms}^{-2}$, of the particle, in terms of x . 2

- (iii) Find the velocity and acceleration of the particle when: 3

(α) $x = 2 \text{ cm}$

(β) $t = 6 \text{ seconds}$

- (iv) Describe carefully in words the motion of the particle. 2

Section B continued.

Question 5.

Marks

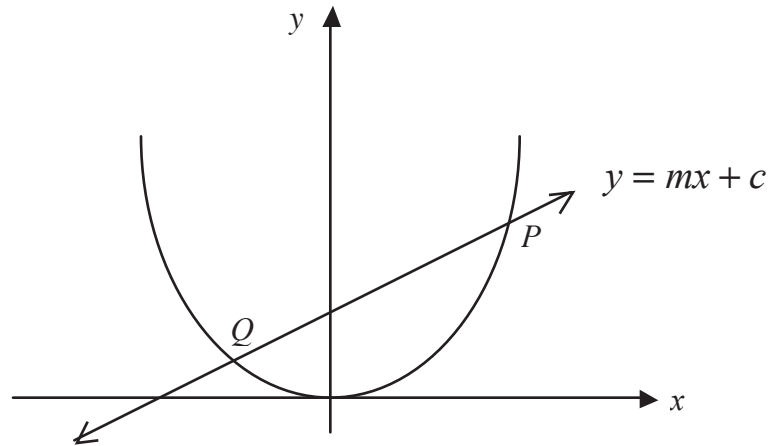
- | | | | |
|----|-------|--|---|
| a) | (i) | Prove the identity $\frac{\cos y - \cos(y + 2\alpha)}{2\sin\alpha} = \sin(y + \alpha)$ | 2 |
| | (ii) | Hence prove by mathematical induction that for positive integers n ,
$\sin\alpha + \sin 3\alpha + \sin 5\alpha + \dots + \sin(2n-1)\alpha = \frac{1 - \cos 2n\alpha}{2\sin\alpha}.$ | 4 |
| b) | (i) | Show that the curve $y = \frac{x^3 + 4}{x^2}$ has one stationary point and no points of inflexion. | 2 |
| | (ii) | Write down the equation(s) of any asymptotes. | 1 |
| | (iii) | Sketch the curve. | 1 |
| | (iv) | Hence, use the graph to find the values of k for which the equation $x^3 - kx^2 + 4 = 0$ has 3 real roots. | 2 |

Section C Use a SEPARATE writing booklet.

Question 6. (12 marks)

Marks

The straight line $y = mx + c$ meets the parabola $x = 2t, y = t^2$ in real distinct points P and Q which correspond respectively to the values $t = p$ and $t = q$.



- (i) Prove that $pq = -c$. 2
- (ii) Prove that $p^2 + q^2 = 4m + 2c$. 2
- (iii) Show that the equation of the normal to the parabola at P is $x + py = 2p + p^3$. 2
- (iv) The point N is the point of intersection of the normals to the parabola at P and Q . 2
Show that the coordinates at N are $(-pq(p + q), (2 + p^2 + pq + q^2))$
- (v) If the chord PQ is free to move while maintaining a fixed gradient.
 - (α) Show that the locus of N is a straight line. 2
 - (β) Hence, or otherwise, show that this straight line is a normal to the parabola. 2

Section C continued.

Question 7. (12 marks)

Marks

- (a) When the polynomial $P(x)$ is divided by $(x + 4)$ the remainder is 5 and when $P(x)$ is divided by $(x - 1)$ the remainder is 9. Find the remainder when $P(x)$ is divided by $(x - 1)(x + 4)$. 3

- (b) A projectile is fired from a point on horizontal ground with initial speed $V \text{ ms}^{-1}$ and angle of projection θ . The cartesian equation of the path is given by

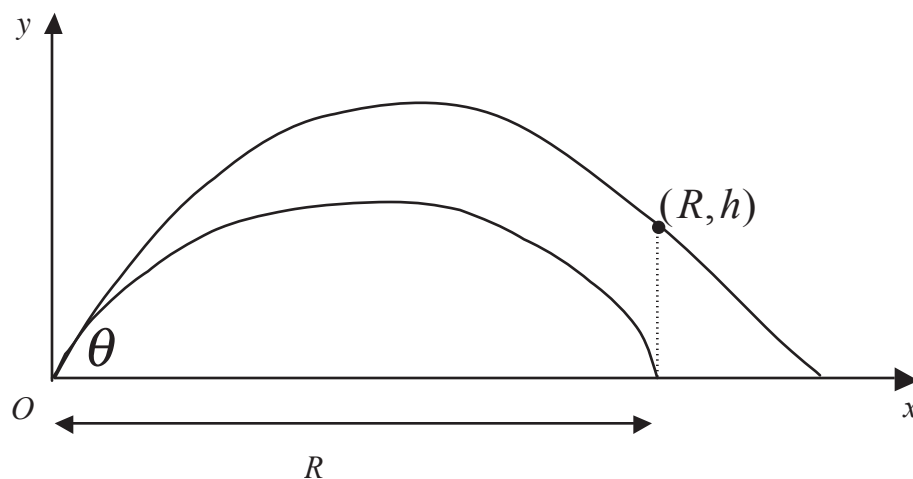
$$y = x \tan \theta - \frac{gx^2}{2V^2 \cos^2 \theta}$$

where x and y are the horizontal and vertical displacements of the particle from O , the point of projection.

The acceleration due to gravity is g and air resistance has been neglected.

- (i) Use the given equation to show that the maximum range R on the horizontal plane is given by $R = \frac{V^2}{g}$. 2

- (ii) Show that to hit a target h metres above the ground at the same horizontal distance R using the same angle of projection θ , the speed of projection must be increased to $\frac{V^2}{\sqrt{V^2 - gh}}$. 4

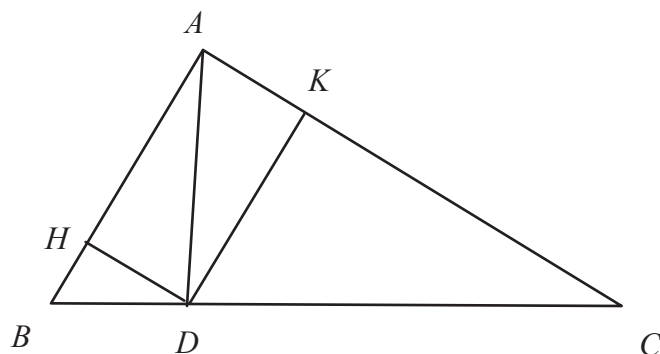


Section C continued.

Question 7.

Marks

(c)



In the triangle ABC , $\angle BAC = 90^\circ$. AD bisects $\angle BAC$.
 $DH \perp AB$ and $DK \perp AC$.

Copy the diagram.

- (i) Show that $\frac{AD}{DH} = \sqrt{2}$. 1
- (ii) By considering the areas of the triangles or otherwise, 2
show that $\frac{\sqrt{2}}{AD} = \frac{1}{AB} + \frac{1}{AC}$

THIS IS THE END OF THE PAPER



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Mathematics Extension 1

Sample Solutions

3 UNIT TRIAL HSC 2003.

QUESTION 1

(a) (i) $\sin 3x \times 1 + x \times 3 \cos 3x$
 $\sin 3x + 3x \cos 3x$

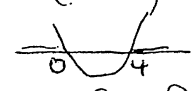
(ii) $e^{1-x^2} \times -2x = -2xe^{1-x^2}$

(b) $y = \frac{2}{3}x + \frac{8}{3}$ $y = 5x - 9$
 $m_1 = \frac{2}{3}$ $m_2 = 5$
 $\tan \theta = \left| \frac{\frac{2}{3} - 5}{1 + \frac{2}{3} \times 5} \right|$
 $\theta = 45^\circ$

(c) (i) $\left[\frac{1}{2} \tan^{-1} \frac{x}{2} \right]_0^2$
 $\frac{1}{2} \left[\frac{\pi}{4} - 0 \right] = \frac{\pi}{8}$

(ii) $\frac{1}{3} \int_0^1 \frac{3x^2}{x^3+2}$
 $= \frac{1}{3} \left[\log(x^3+2) \right]_0^1$
 $= \frac{1}{3} (\log 3 - \log 2)$
 ≈ 0.135 or $\frac{1}{3} \log \frac{3}{2}$

(d) $\frac{5!}{8!} = \frac{1}{336}$

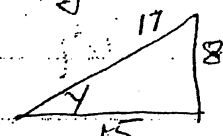
(e) $\theta < 0$ or $\theta > 4$
 $1 - \frac{4}{\theta} > 0$ or $\frac{\theta^2 - 4\theta}{\theta} > 0$
 $1 > \frac{4}{\theta}$
 $\theta < 0$ or $\theta > 4$
 $\theta^2 - 4\theta > 0$
 $\theta(\theta - 4) > 0$

 $\theta < 0$ or $\theta > 4$

QUESTION 2

(a) (i) $\alpha + \beta + \gamma = -\frac{b}{a} = \frac{5}{2}$
 $\alpha\beta\gamma = -\frac{1}{2}$

(ii) $(\alpha + \beta + \gamma)^2 - 2(\alpha\beta + \alpha\gamma + \beta\gamma)$
 $\frac{25}{4} - 2 \times \frac{-3}{2} = 9\frac{1}{4}$

(b) $\int_0^{2\sqrt{3}} \frac{x}{\sqrt{u}} \times \frac{du}{2x}$ $u = x^2 + 4$
 $\frac{1}{2} \int_0^{2\sqrt{3}} u^{-\frac{1}{2}} du$ $\frac{du}{dx} = 2x$
 $= \frac{1}{2} \left[\frac{u^{\frac{1}{2}}}{\frac{1}{2}} \right]_4^{16}$ $dx = \frac{du}{2x}$
 $= \left[\sqrt{4} - 2 \right] = 2$

(c) $y = \tan^{-1} \frac{8}{15}$
 $\tan y = \frac{8}{15}$


$\cos y = \frac{15}{17}$

(d) (i) $A\left(\frac{1}{3}, 2\pi\right)$ $C\left(-\frac{1}{3}, 0\right)$

(ii) $y = \pi + 2 \sin^{-1} 3x$
 $\frac{dy}{dx} = 2 \times \frac{3}{\sqrt{1-9x^2}}$

$x = 0$ grad of tangent = $\frac{6}{\sqrt{1-0}}$
 $= 6$

QUESTION 3

(a) $y = f(x) \quad y = 1 + e^{2x}$
 $f^{-1}(x) \quad x = 1 + e^{2y}$
 $e^{2y} = x - 1$
 $2y = \log(x - 1)$
 $y = \frac{1}{2} \log(x - 1)$

Domain $x > 1$ Range All real y

(b) Rational roots when $\Delta = b^2 - 4ac = 0$ or has rational square root

$$36 - 4(5k - 4)(6k + 3) = 0$$

$$36 - 120k^2 + 36k + 48 = 0$$

$$-120k^2 + 36k + 84 = 0$$

$$10k^2 - 3k - 7 = 0$$

$$(10k + 7)(k - 1) = 0$$

rational roots when $k = -\frac{7}{10}$ or 1

multiple solutions when $-120k^2 + 36k + 84$ has rational roots

(c) (i) $\angle ABG = \angle BEG$ (angle in alternate segment)

$\angle BEG = \angle CEH$ (vertically opposite)

$\angle CEH = \angle DCH$ (angle in alternate segment)

$\therefore \angle ABG = \angle DCH$ as required

(ii) $\angle CBH = \angle BGC$ (alternate segment)

$\angle BCE = \angle CHE$ "

$\therefore \angle GBC = \angle HCB$ (angle sum of Δ)

$\therefore \triangle BCG \cong \triangle BCH$ (equiangular)

(d) (i) $a = 2^N \quad r = 2^{-1}$

$$S_n = \frac{a}{1-r}$$

$T_n \quad ar^{n-1} = 2^{-N}$

$$2^N (2^{-1})^{n-1} = 2^{-N}$$

$$2^{-n+1} = 2^{-2N}$$

$$-n+1 = -2N$$

$$n = 2N+1$$

2

$$= \frac{2^N}{1 - \frac{1}{2}}$$

$$= 2 \cdot 2^N = 2^{N+1}$$

1

4) (a) $\sin 2\theta = 2 \sin \theta \cos \theta$
 (i) $\sin \theta = 2 \sin \frac{\theta}{2} \cos \frac{\theta}{2}$

$$\cos 2\theta = \frac{\cos^2 \theta - \sin^2 \theta}{1 - 2 \sin^2 \theta}$$

$$2 \cos^2 \theta - 1$$

$$\text{so } \cos \theta = \frac{\cos^2 \frac{\theta}{2} - \sin^2 \frac{\theta}{2}}{1 - 2 \sin^2 \frac{\theta}{2}}$$

$$2 \cos^2 \frac{\theta}{2} - 1$$

$$f(\theta) = \frac{2 \sin \frac{\theta}{2} \cos \frac{\theta}{2} + \sin \frac{\theta}{2}}{1 + 2 \cos^2 \frac{\theta}{2} - 1 + \cos \frac{\theta}{2}}$$

$$= \frac{\sin \frac{\theta}{2} [2 \cos \frac{\theta}{2} + 1]}{\cos \frac{\theta}{2} [2 \cos \frac{\theta}{2} + 1]}$$

$$= \tan \frac{\theta}{2} = t$$

(3)

(ii) $f(\theta) = \tan \frac{\theta}{2} = 1$ general soln.

$\therefore$ If $\tan \theta = a$, then $\theta = n\pi + \tan^{-1}(a)$

$\tan \frac{\theta}{2} = 1$, then $\frac{\theta}{2} = n\pi + \frac{\pi}{4}$
 $\theta = 2n\pi + \frac{\pi}{2}$

(1) n an integer

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 Section B

12

$$4 \text{ (b) (i) } t = 2x^2 - 5x + 3$$

$$\frac{dt}{dx} = 4x - 5$$

$$\frac{dx}{dt} = v = \frac{1}{4x-5} \quad (1)$$

$$\begin{aligned} \text{(ii) using } a &= \frac{d}{dx} \left(\frac{1}{2} v^2 \right) \\ &= \frac{d}{dx} \left(\frac{1}{2(4x-5)^2} \right) \\ &= \frac{d}{dx} \left[\frac{1}{2} (4x-5)^{-2} \right] \\ &= -(4x-5)^{-3} \times 4 \\ &= \frac{-4}{(4x-5)^3} \quad (2) \end{aligned}$$

$$\text{(iii) (a) when } x=2, \quad v = \frac{1}{3} \text{ cm/s} \quad \left(\frac{1}{2} \right)$$

$$a = -\frac{4}{27} \text{ cm/s}^2 \quad \left(\frac{1}{2} \right)$$

$$\begin{aligned} \text{(b) when } t=6, \quad 6 &= 2x^2 - 5x + 3 \\ (2x+1)(x-3) &= 0 \\ x &= -\frac{1}{2}, x=3 \quad (1) \end{aligned}$$

$$\begin{aligned} \text{take } x &= 3 \\ \text{At } x=3, \quad v &= \frac{1}{7} \text{ cm/s} \quad \left(\frac{1}{2} \right) \\ a &= \frac{4}{343} \quad \left(\frac{1}{2} \right) \end{aligned}$$

(iv) particle is travelling to the right but is slowing down. (2)

$$(5) (a) (i) \frac{\cos y - \cos(y+2\alpha)}{2\sin\alpha} = \sin(y+\alpha)$$

$$\text{LHS} = \frac{\cos y - (\cos y \cos 2\alpha - \sin y \sin 2\alpha)}{2\sin\alpha}$$

$$\frac{\cos y - (\cos y (1-2\sin^2\alpha) - \sin y 2\sin\alpha \cos\alpha)}{2\sin\alpha}$$

$$\frac{\cancel{\cos y} - \cancel{\cos y} + \cancel{2\sin^2\alpha} \cos y + \cancel{2\sin\alpha} \cos\alpha \sin y}{2\sin\alpha}$$

$$= \sin\alpha \cos y + \cos\alpha \sin y$$

$$= \sin(y+\alpha)$$

(2)

$$(ii) \sin\alpha + \sin 3\alpha + \sin 5\alpha + \dots + \sin(2n-1)\alpha = \frac{1 - \cos 2n\alpha}{2\sin\alpha}$$

step 1 Prove true for $n=1$

$$\text{LHS} = \sin\alpha$$

$$\text{RHS} = \frac{1 - \cos 2\alpha}{2\sin\alpha} = \frac{1 - (1 - 2\sin^2\alpha)}{2\sin\alpha} = \sin\alpha$$

$$= \text{LHS}$$

true for $n=1$

step 2 Assume true for $n=k$ (a positive integer) so

$$\sin\alpha + \sin 3\alpha + \sin 5\alpha + \dots + \sin(2k-1)\alpha = \frac{1 - \cos 2k\alpha}{2\sin\alpha}$$

and we must prove it true for $n=k+1$, so

$$\sin\alpha + \sin 3\alpha + \sin 5\alpha + \dots + \sin(2k-1)\alpha + \sin(2k+1)\alpha = \frac{1 - \cos 2(k+1)\alpha}{2\sin\alpha}$$

$$\text{LHS} = \frac{1 - \cos 2k\alpha}{2\sin\alpha} + \sin(2k+1)\alpha$$

=

$$\frac{1 - \cos 2k\alpha}{2\sin\alpha} + \sin(2k\alpha + \alpha).$$

now using (a)(i) $\sin(y + \alpha) = \frac{\cos y - \cos(y + 2\alpha)}{2\sin\alpha}$

then $\sin(2k\alpha + \alpha) = \frac{\cos 2k\alpha - \cos(2k\alpha + 2\alpha)}{2\sin\alpha}$

now, $\frac{1 - \cos 2k\alpha}{2\sin\alpha} + \frac{\cos 2k\alpha - \cos 2(k+1)\alpha}{2\sin\alpha}$

$$= \frac{1 - \cos 2(k+1)\alpha}{2\sin\alpha}$$

$\therefore$ RHS.

True for $n = k+1$.

step 3 If the statement is true for $n=k$, then it is also true for $n=k+1$. Since the statement is true for $n=1$, it follows that it must also be true for $n=2$ and so on. $\therefore$ the statement is true for all positive integers n .

(4)

(5) (b) (i) $y = \frac{x^3+4}{x^2} = \frac{x^3}{x^2} + \frac{4}{x^2} = x + 4x^{-2} = x + \frac{4}{x^2}$

$$y' = 1 - 8x^{-3} = 1 - \frac{8}{x^3}$$

$$y'' = 24x^{-4} = \frac{24}{x^4}$$

Stat points exist when $y' = 0$, $1 - \frac{8}{x^3} = 0$

$$\frac{8}{x^3} = 1 \Rightarrow x^3 = 8 \Rightarrow x = 2$$

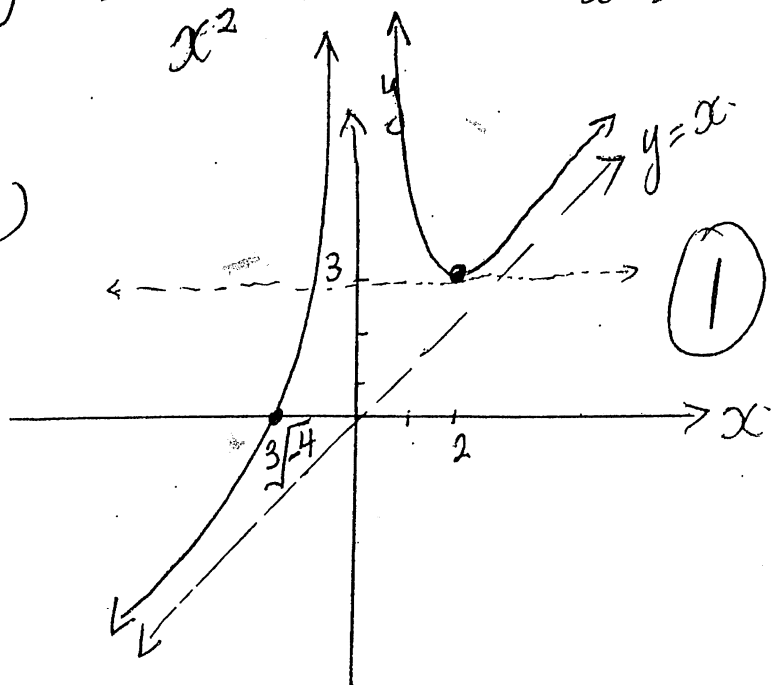
At $x=2$, $y = 2 + \frac{4}{2^2} = 3$ $(2, 3)$ (1) (min stat pt) $y'' > 0$

Inflexions occur when $y'' = 0$ and $\exists$ a sign change
 $\frac{24}{x^4} = 0 \Rightarrow 24 = 0x^4$ does not exist. (1)

(ii) $y = \frac{x^3+4}{x^2} \Rightarrow x \neq 0$ (y axis) (1/2) vertical asymptote

$y = \frac{x^3(1 + \frac{4}{x^3})}{x^2} = x(1 + \frac{4}{x^3})$ and as $x \rightarrow \infty$ $y \approx x$. (1/2) oblique asymptote

(iv)



when $y=0$,
 $0 = \frac{x^3+4}{x^2}$

so $x^3+4=0$
 $x^3 = -4$
 $x = \sqrt[3]{-4}$

$$5 \text{ (b) (iv)} \quad x^3 - kx^2 + 4 = 0$$

$$x^3 + 4 = kx^2$$

$$\text{So} \quad \frac{x^3 + 4}{x^2} = k$$

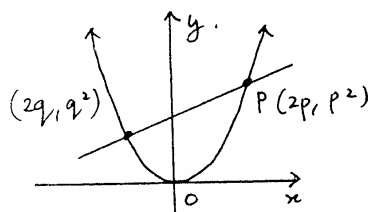
$$\Rightarrow y = \frac{x^3 + 4}{x^2} = k$$

3 intersections will occur between $y = k$ and $y = \frac{x^3 + 4}{x^2}$ if $k > 3$.

(2)

Solution — Section C
Question (6) [12]

$$x = 2t, y = t^2 \therefore y = \frac{x^2}{4}$$



Solve:

$$\begin{cases} y = \frac{x^2}{4} \\ y = mx + c \end{cases}$$

$$\therefore x^2 - 4mx - 4c = 0 \quad \text{--- (1)}$$

The roots to (1) are: $2p, 2q$.

$$\therefore \sum d_i : 2p + 2q = 4m$$

$$\text{i.e. } p + q = 2m \quad \text{--- (2)}$$

$$\text{Product of roots : } 4pq = -4c$$

$$(i) \therefore pq = -c \quad \text{--- (3)} \quad [2]$$

$$(ii) \text{ Now, } p^2 + q^2 = (p+q)^2 - 2pq \\ = 4m^2 - 2(-c)$$

$$\therefore p^2 + q^2 = 4m^2 + 2c \quad [2]$$

$$(iii) \text{ gradient of } \text{tgt.} = p$$

$$\therefore \text{gradient of normal} = -\frac{1}{p}$$

$$\therefore \text{equation of normal : } y - p^2 = -\frac{1}{p}(x - 2p)$$

$$\therefore x + py = p^3 + 2q$$

(iv) The equation of normal at Q

$$x + qy = q^3 + 2q \quad \text{--- (5)}$$

$\therefore$ (4) - (5) we have:

$$(p-q)y = (p^3 - q^3) + 2(p-q)$$

$$\therefore y = 2 + p^2 + pq + q^2 \quad \text{--- (6)}$$

Substitute (6) into (4) we have:

$$x + 2p + p^3 + p^2q + pq^2 = p^3 + 2p$$

$$\therefore x = -p^2q - pq^2 = -pq(p+q)$$

$$\therefore N(-pq(p+q), (2 + p^2 + p^2q + q^2))$$

[2]

(8)

Question (6).

(V). $\boxed{\begin{matrix} pq = -c, & p+q = 2m \\ p^2+q^2 = 4m^2+2c. \end{matrix}}$

The x-coord. of N becomes $c(2m)$

The y-coord. of N becomes $\sum 2+(4m^2+2c)-c^2$

$\therefore \boxed{N = (2mc, 4m^2+c+2)}$

(x) Chord PQ, whose equation is $y = mx + c$, is free to move whilst maintaining a fixed grad. i.e. $m_{PQ} = m$ (a constant), but c is a variable.

Now $x = 2mc, \Rightarrow c = \frac{x}{2m}$.

$y = 2 + 4m^2 + \frac{x}{2m}$ [2]

$\therefore \boxed{y = \frac{x}{2m} + 2(1+2m^2)}$

i.e. Equation of locus of N is a straight line with gradient $\frac{1}{2m}$ and y-intercept $2(1+m^2)$.

The points of intersection of the locus of N and $x^2 = 4y$ are found by solving

$$\begin{cases} y = \frac{x}{2m} + 2(1+2m^2) \\ x = 2t, y = t^2 \end{cases}$$

i.e. $t^2 = \frac{2t}{2m} + 2(1+2m^2)$. (4)

$mt^2 - t - 2m(1+2m^2) = 0$.

$\therefore t = \frac{1 \pm \sqrt{1+8m^2(1+2m^2)}}{2m}$
 $= \frac{1 \pm (1+4m^2)}{2m}$

$\therefore \boxed{t = \frac{1+2m^2}{m}, \text{ or } t = -2m}$

$\therefore$ locus of N cut parabola in 2 pt say U, V with parameters

$t = \begin{cases} \frac{1+2m^2}{m} \\ -2m \end{cases}$ [2]

From gradient of tgt, ($=t$) $\Rightarrow$

the gradients of tgts at U, V are

$\frac{1+2m^2}{m}$, and $-2m$. [In particular,

the tgt at V has gradient $-2m$ while the locus of N has gradient $\frac{1}{2m}$. Hence the locus of N is perp to tgt at V $\Rightarrow$ normal at V

Question (7). [12]

(a) $P(x) = (x+4)u(x) + 5$
 $= (x-1)u(x) + 9.$ [3]

$\therefore P(-4) = 5, P(1) = 9.$ — (1)

$P(x) = (x-1)(x+4)q(x) + (ax+b).$

From (1) $\begin{cases} -4a+b = 5 \\ a+b = 9 \end{cases}$ — (2)

$\therefore 5a = 4 \Rightarrow a = 4/5$

$\therefore b = 9 - 4/5 = 41/5$

i.e. $\boxed{\frac{4x}{5} + \frac{41}{5}}$

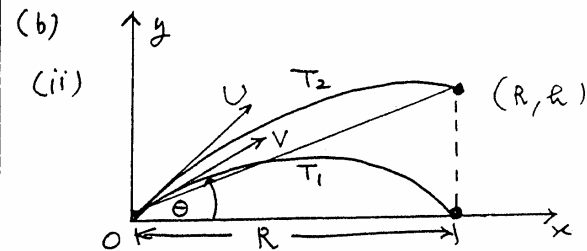
(b) To find the range, set $y=0$.

(i) i.e. $x \left(\tan \theta - \frac{gx}{2v^2 \cos^2 \theta} \right) = 0.$

i.e. $x = 0$, or $x = \frac{2v^2 \cos^2 \theta \times \tan \theta}{g}$

i.e. $\text{range} = \frac{v^2 (2 \sin \theta \cos \theta)}{g}$
 $= \frac{v^2 \sin 2\theta}{g}$ [2]

$\therefore$ Maximum range occurs when $\sin 2\theta = 1 \Rightarrow \boxed{R = \frac{v^2}{g}}$



Equation of higher trajectory (T_2) is

(1) $\boxed{h = R \tan \theta - \frac{gR^2}{2U^2 \cos^2 \theta}}$ (velocity)

When the speed of projectile was V , the range was:

$\boxed{R = \frac{V^2 \sin 2\theta}{g}}$ — (2)

Substitute (2) into (1) we have.

$h = \frac{V^2 \sin 2\theta \tan \theta}{g} - \frac{V^4 \sin^2 2\theta}{g U^2 \cos^2 \theta}.$

Note: $\sin 2\theta = 2 \sin \theta \cos \theta, \tan \theta = \sin \theta / \cos \theta$

$\therefore h = \frac{2V^2 \sin^2 \theta}{g} - \frac{2V^4 \sin^2 \theta}{g U^2}$

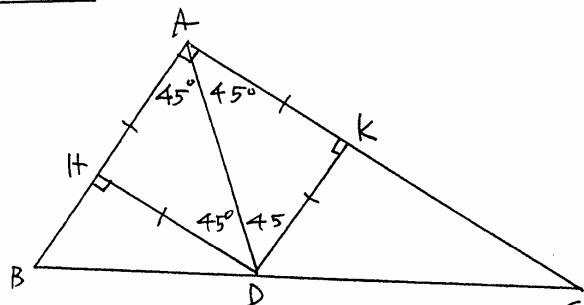
When $v = V$, Range is $R_{\max} \therefore \theta = 45^\circ$

$\therefore h = \frac{V^2}{g} - \frac{V^4}{g U^2}$ [4]

$\therefore U^2 g h = V^2 U^2 - V^4, U^2 (V^2 - gh) =$

$\therefore U^2 = \frac{V^4}{V^2 - gh} \therefore \boxed{U = \frac{V^2}{\sqrt{V^2 - gh}}}$

∴ Question
7(c)



∴ AD bisects $\angle BAC (=90^\circ)$

∴ $\angle BAD = \angle DAC = 45^\circ$

$\Rightarrow \angle HDA = \angle KDA = 45^\circ$

(Angle sum of a Δ).

i.e. ΔAHD is isos. $\Rightarrow AH = DH$.

In ΔAHD , $AD^2 = AH^2 + DH^2$

(Pythagoras) $= 2DH^2$

∴ $\left(\frac{AD}{DH}\right)^2 = 2$

(i) $\Rightarrow \frac{AD}{DH} = \sqrt{2}$ [1].

∴ $\boxed{\frac{1}{DH} = \frac{\sqrt{2}}{AD}}$ — (1)

(ii)

$\Delta AHD \equiv \Delta AKD$ (AAS).

∴ $DH = DK$. — (2)

Area of ΔABC

$$= \frac{1}{2} AB \cdot AC.$$

but area of ΔABC

$$= \text{area of } \Delta ABD + \text{area of } \Delta ACD.$$

$$\text{Area of } \Delta ABD = \frac{1}{2} AB \cdot DH$$

$$\text{Area of } \Delta ACD = \frac{1}{2} AC \cdot DK.$$

from (2) ∴ $DK = DH$

$$\therefore \text{area of } \Delta ACD = \frac{1}{2} AC \cdot DH.$$

$$\therefore \frac{1}{2} AB \cdot AC = \frac{1}{2} (AB \cdot DH + AC \cdot DH)$$

$$\therefore DH (AB + AC) = AB \cdot AC. [2]$$

$$\therefore \frac{AB + AC}{AB \cdot AC} = \frac{1}{DH}$$

$$\therefore \boxed{\frac{1}{AC} + \frac{1}{AB} = \frac{\sqrt{2}}{AD}} \text{ from (1)}$$