

Question One (12 marks)**Marks**

a) Find $\int \frac{dx}{9+x^2}$ [1]

b) Given $f(x) = \frac{1}{2} \sin^{-1} 2x$:

i) State the range of $f(x)$ [1]

ii) State the domain of $f(x)$ [1]

iii) Sketch $f(x)$ [1]

c) Solve $\frac{5}{x-4} < 1$ [2]

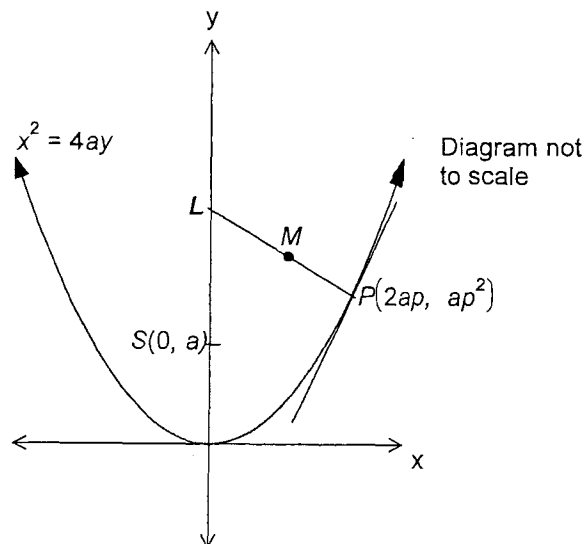
d) Evaluate $\int_0^1 x\sqrt{1-x^2} dx$ using the substitution $u = 1-x^2$ [3]

e) i) Show that there is a solution to $x^3 = x+1$ between $x=1$ and $x=2$ [1]

ii) Use one application of Newton's method and $x=1.5$ to find a further approximation correct to one decimal place. [2]

Question Two (12 marks)**Marks**

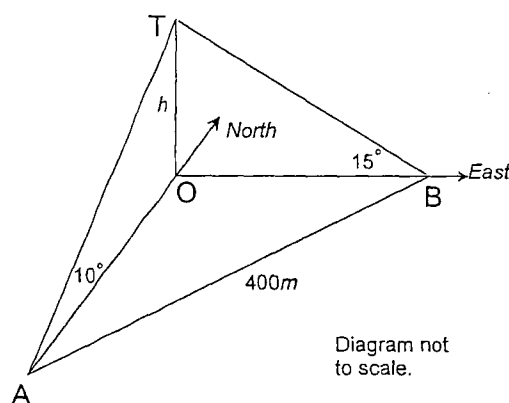
- a) The point $P(2, 3)$ divides the interval AB internally in the ratio $2:3$.
If A has coordinates $(-1, 6)$ find the coordinates of B . [2]
- b) A function is defined $f(x) = \frac{3}{x} - 4$.
- i) Find f^{-1} [1]
- ii) Evaluate $f^{-1}(4)$ [1]
- c) Given $P(x) = 2x^3 - 17x^2 + 7x + 8$:
- i) Show that $(x-1)$ is a factor of $P(x)$ [1]
- ii) Hence fully factorise $P(x)$ [2]
- d) The diagram shows the parabola $x^2 = 4ay$. The point $P(2ap, ap^2)$ where $p \neq 0$ lies on the parabola. The normal at P cuts the y -axis at L . M is the midpoint of LP .



- i) Show that the equation of the normal to the parabola at P is $x + py = ap^3 + 2ap$. [2]
- ii) Find the coordinates of L , the point where the normal cuts the y -axis. [1]
- iii) Show that SM is parallel to the tangent at P . [2]

Question Three (12 marks)**Marks**

- a) Find the size of the acute angle between the lines whose equations are $x - 2y - 1 = 0$ and $x + 3y + 2 = 0$ [3]
- b) A tower TO is due north of an observer at A. The angle of elevation from A to the top of the tower T is 10° . From a point B due east of the tower, the angle of elevation to the top of the tower is 15° . The distance from A to B is 400m



- i) Find an expression for AO in terms of h . [1]
- ii) Calculate the height h of the tower. [3]
- iii) Find the bearing of A from B [2]
- c) The polynomial $P(x)$ is defined as $P(x) = x^3 + ax^2 + 2ax + b$ [3]
where a and b are constants. The zeros of $P(x)$ are 2, -3 and γ .
Find the values a, b and γ

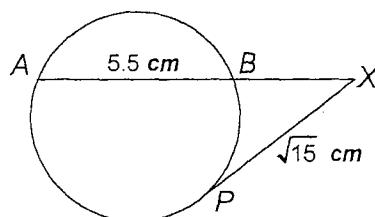
Question Four (12 marks)**Marks**

- a) Find $\int 2\cos^2 4x dx$ [2]
- b) The velocity of a particle is given by $\dot{x} = 2 - 3e^{-t}$ where x is the displacement in metres and t is the time in seconds. Initially the particle is at the origin.
- i) Find an expression for the acceleration $\ddot{x}$ of the particle at any time t . [1]
 - ii) Find an expression for the displacement x of the particle at any time t . [2]
 - iii) Find the time when the particle is next at rest (give exact answer). [2]
 - iv) Explain what happens to the acceleration and hence the velocity as t becomes very large. [2]
- c) Prove by mathematical induction that $2 \times 5^{n-1} + 12^n$ is divisible by 7 for all integers $n \geq 1$ [3]

Question Five (12 marks)

Marks

- a) In the diagram below the tangent at P meets AB at X .



If $AB = 5.5\text{ cm}$ and $PX = \sqrt{15}\text{ cm}$ find the length of BX .

[2]

b) Evaluate $\lim_{x \rightarrow \infty} \frac{2x^2 - 3x + 1}{5x^2 + x - 4}$

[1]

c) i) Write $\sqrt{12} \sin x + 2 \cos x$ in the form $r \sin(x + \alpha)$

[2]

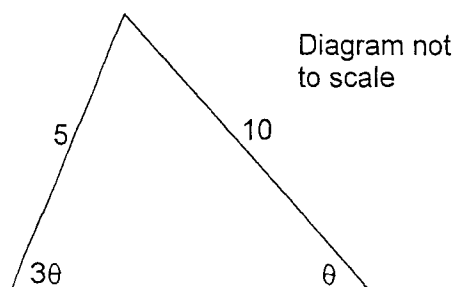
ii) Hence or otherwise solve $\sqrt{12} \sin x + 2 \cos x = -2\sqrt{2}$ for $0 \leq x \leq 2\pi$

[3]

d) i) Prove $\sin 3\theta = 3 \sin \theta - 4 \sin^3 \theta$

[2]

ii)

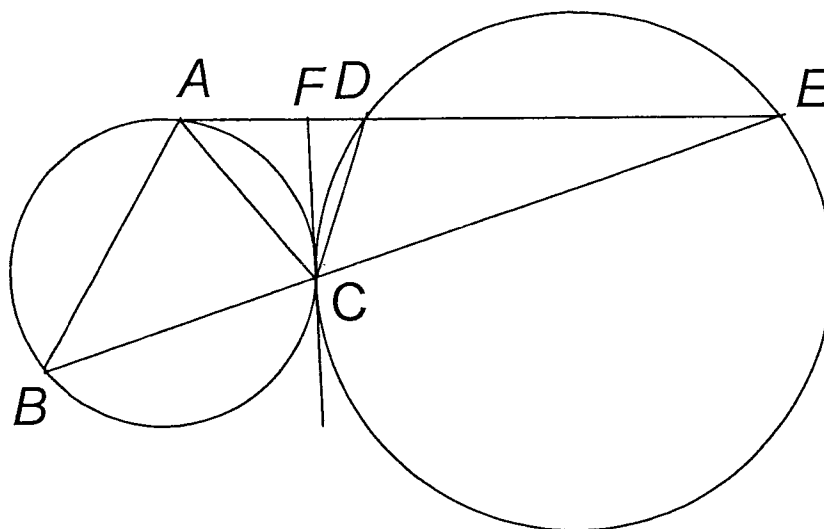


Hence find the value of θ in the triangle

[2]

Question Six (12 marks)**Marks**

- a) If the displacement of a particle is given by $x = 2 \sin 2t + 3 \cos 2t$, show that the motion of the particle is simple harmonic. [2]
- b) Jane is inflating balloons for the Year 12 Formal. Each empty balloon is being inflated so that its volume increases at the rate of $8 \text{ cm}^3 / \text{s}$.
- i) Show that the radius at any time t is $r = \sqrt[3]{\frac{6t}{\pi}}$ [2]
- ii) Find the rate of increase of the surface area after 4 seconds [2]
- iii) The balloon will burst when the surface area reaches 3000 cm^2 . [3]
After how many seconds should Jane cease inflation?
- c) Two circles touch each other externally at C . The tangent to the smaller circle at A meets the larger circle at D and E . EC meets the smaller circle at B . FC is the common tangent to both circles. Copy or trace the diagram.



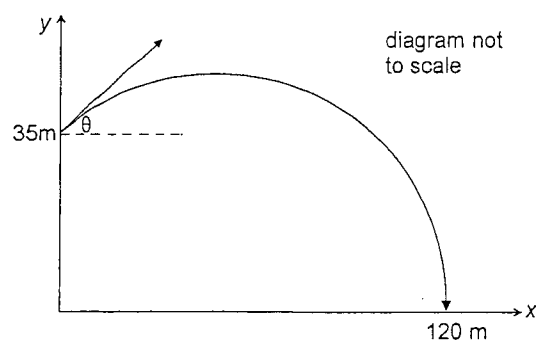
- i) Prove $\angle FAC = \angle FCA$ [2]
- ii) Prove $\angle ACD = \angle ACB$ [2]

Question Seven (12 Marks)

Marks

- a) Find the gradient of the tangent to $y = \sin^{-1}(\tan x)$ at $x = 0$. [2]

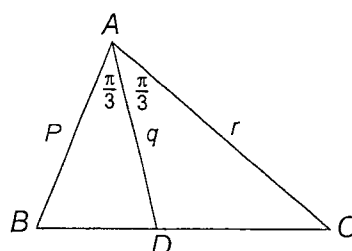
b)



A particle is projected from the top of a tower with a velocity of 30ms^{-1} to hit an object that is 120 metres away in the horizontal direction and 35 metres below in the vertical direction (as shown above). The components of its displacement after t seconds are:

$$\left. \begin{aligned} x &= 30t \cos \theta \\ y &= 30t \sin \theta - 5t^2 + 35 \end{aligned} \right\} \text{Do not prove these.}$$

- i) If the particle hits the object prove $80 \sec^2 \theta - 120 \tan \theta - 35 = 0$ [3]
 ii) Find the angle of projection to the nearest minute [3]
 iii) Find the time taken for the particle to reach the object [2]
- c) In triangle ABC below $AB = p$, $AD = q$, $AC = r$, $\angle BAD = \frac{\pi}{3} = \angle DAC$ [2]



Show that $\frac{1}{p} + \frac{1}{r} = \frac{1}{q}$

End of paper

Extension One Mathematics TRIAL HSC 2011 SOLUTIONS

S.H.S.

Question One:

a) $\frac{1}{3} \tan^{-1} \frac{x}{3} + C$ ✓

b) Let $y = \frac{1}{2} \sin^{-1} 2x$
 $\therefore 2y = \sin^{-1} 2x$

i) $-\frac{\pi}{2} \leq 2y \leq \frac{\pi}{2}$

$\therefore -\frac{\pi}{4} \leq y \leq \frac{\pi}{4}$ ✓

ii) $-1 \leq 2x \leq 1$

$-\frac{1}{2} \leq x \leq \frac{1}{2}$ ✓

c) $\frac{5}{x-4} < 1$

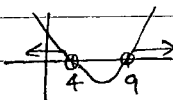
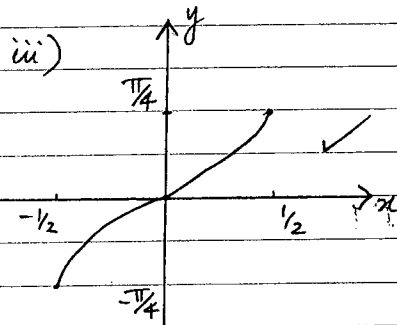
$5(x-4) < (x-4)^2$

$5x - 20 < x^2 - 8x + 16$

$0 < x^2 - 13x + 36$

$0 < (x-4)(x-9)$

$x < 4$ or $x > 9$ ✓✓



d) $\int_0^1 x \sqrt{1-x^2} dx$
 $= \int_0^1 x \cdot \sqrt{u} \cdot \frac{du}{-2x}$ ✓

$= -\frac{1}{2} \int_1^0 u^{1/2} du$

$= -\frac{1}{2} \left[\frac{2u^{3/2}}{3} \right]_1^0$ ✓

$= -\frac{1}{2} \left[0 - \frac{2}{3} \right]$

$= \frac{1}{3}$ ✓

$u = 1 - x^2$
 $\frac{du}{dx} = -2x$

$\frac{du}{-2x} = dx$

when $x=0$, $u=1$
 when $x=1$, $u=0$

e) i) Let $y = x^3 - x - 1$

when $x=1$, $y=-1 < 0$

when $x=2$, $y=5 > 0$

Since there is a change of sign ✓
 then there is a solution between $x=1$ and $x=2$

ii) $x_1 = x - \frac{f(x)}{f'(x)}$

$= 1.5 - \frac{f(1.5)}{f'(1.5)}$ ✓

$= 1.5 - \frac{0.875}{3.5}$

$= 1.25$

$\therefore x_1 = 1.3$ (1 dec. pl.) ✓

Question Two (12 marks).

a) $P(2,3)$ $m:n$ x, y $A(-1,6)$ $B(x, y)$
 xy $2:3$

$$x = \frac{nx_1 + mx_2}{m+n} \quad y = \frac{ny_1 + my_2}{m+n}$$

$$2 = \frac{3(-1) + 2(x_2)}{2+3} \quad 3 = \frac{3(6) + 2(y_2)}{2+3}$$

$$10 = -3 + 2x_2 \quad 15 = 18 + 2y_2$$

$$13 = 2x_2$$

$$\therefore x_2 = 6\frac{1}{2}$$

$$-3 = 2y_2$$

$$y_2 = -\frac{3}{2}$$

$$\therefore B(6\frac{1}{2}, -\frac{3}{2}).$$

(2)

b) $f(x) = \frac{3}{x} - 4$

i) let $y = \frac{3}{x} - 4$

ii) Evaluate

$$f^{-1}(4) = \frac{3}{x+4} = \frac{3}{4+4}$$

$$\therefore f^{-1}: x = \frac{3}{y} - 4$$

$$x+4 = \frac{3}{y}$$

$$y = \frac{3}{x+4}$$

(1)

$$f^{-1}(4) = \frac{3}{8} \quad (1)$$

c) i) $P(x) = 2x^3 - 17x^2 + 7x + 8$ $(x-1)$ is a factor

$$\therefore P(1) = 0$$

$$= 2(1)^3 - 17(1)^2 + 7(1) + 8$$

$$= 2 - 17 + 7 + 8$$

$$P(1) = 0$$

$$\therefore (x-1) \text{ is a factor} \quad (1)$$

c) ii) $(x-1) \overline{2x^3 - 17x^2 + 7x + 8}$
 $\underline{2x^3 - 2x^2}$

$$-15x^2 + 7x + 8$$

$$\underline{-15x^2 + 15x}$$

$$-8x + 8$$

$$\underline{-8x + 8}$$

$$0$$

$$\therefore P(x) = (x-1)(2x^2 - 15x - 8)$$

$$= (x-1)(2x+1)(x-8)$$

(2)

d) i) Gradient of tangent at $P = p$ $P(2ap, ap^2)$

$$\text{Gradient of normal at } P = -\frac{1}{p}$$

$$\therefore \text{Equation of normal at } P: y - y_1 = m(x - x_1)$$

$$y - ap^2 = -\frac{1}{p}(x - 2ap)$$

$$py - ap^3 = -x + 2ap$$

$$\therefore x + py = ap^3 + 2ap \quad (2)$$

ii) Coordinates of L at $x=0$

$$x + py = ap^3 + 2ap$$

$$py = ap^3 + 2ap$$

$$py = p(ap^2 + 2a)$$

$$y = ap^2 + 2a$$

$$\therefore L(0, ap^2 + 2a)$$

(1)

iii) Midpoint $M(\frac{0+2ap}{2}, \frac{ap^2+2a+ap^2}{2})$

$$L(0, ap^2+2a)$$

$$2(2ap, ap^2) \quad M(ap, \frac{2ap^2+2a}{2})$$

$$M(ap, ap^2+a) \quad S(0, a)$$

$$\text{Gradient of } SM = \frac{y_2 - y_1}{x_2 - x_1} = \frac{ap^2}{ap}$$

$$= \frac{ap^2 + a - a}{ap - 0} = p$$

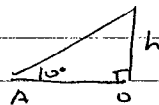
$$\therefore \text{Gradient of } SM = \text{Gradient of tangent at } P = p$$

$$\therefore SM \parallel \text{to tangent at } P.$$

(2)

Question 3

a) $x - 2y - 1 = 0 \Rightarrow y = \frac{1}{2}x - \frac{1}{2} \quad m_1 = \frac{1}{2}$
 $x + 3y + 2 = 0 \Rightarrow y = -\frac{1}{3}x - \frac{2}{3} \quad m_2 = -\frac{1}{3}$
 $\tan \theta = \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right|$
 $= \left| \left(\frac{1}{2} + \frac{1}{3} \right) \div \left(1 - \frac{1}{6} \right) \right|$
 $= 1$
 $\theta = 45^\circ \quad \text{or } \frac{\pi}{4}$ (3)

b) i)  $AO = \frac{h}{\tan 10^\circ} \quad \text{or } AO = h \cot 10^\circ$ (1)

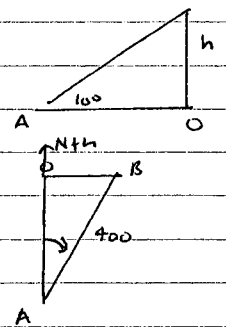
ii) $OB = \frac{h}{\tan 15^\circ} \quad \text{or } OB = h \cot 15^\circ$
 Now $OA^2 + OB^2 = 400^2$

$$\frac{h^2}{\tan^2 10^\circ} + \frac{h^2}{\tan^2 15^\circ} = 400^2$$

$$h^2 \left(\frac{1}{\tan^2 10^\circ} + \frac{1}{\tan^2 15^\circ} \right) = 400^2$$

$$h = 400 \div \sqrt{\frac{1}{\tan^2 10^\circ} + \frac{1}{\tan^2 15^\circ}} \quad (3)$$

$$= 58.9 \text{ m}$$

iii)  $\frac{h}{AO} = \tan 10^\circ$
 $AO = \frac{h}{\tan 10^\circ} = 334.03$
 $\cos \angle OAB = \frac{OA}{400} = \frac{334.03}{400}$
 $\angle OAB = 33^\circ \quad \text{or } \angle OBA = 2$ (2) ✓
 $\therefore \text{Bearing} = 180^\circ + 33^\circ = 213^\circ$

c) $P(x) = x^3 + ax^2 + 2ax + b$
 when $x = 2$, $8 + 4a + 4a + b = 0$
 $8a + b = -8$ (1)
 when $x = -3$, $-27 + 9a - 6a + b = 0$
 $3a + b = 27$ (2)
 (1) - (2) $5a = -35$
 $a = -7$ ✓
 $b = 48$ ✓ (the constant term)
 Now $(x-2)(x+3)(x-d) = 0$
 $\therefore 6d = 48$
 $d = 8$ (3)

2011 Ext 1 Trial – Solution to Question 4

(a) $\cos 2x = 2 \cos^2 x - 1 \Rightarrow 2 \cos^2 4x = \cos 8x + 1$

$$\int 2 \cos^2 4x \, dx = \int (\cos 8x + 1) \, dx$$

$$= \frac{\sin 8x}{8} + x + C$$

(b)(i) $\dot{x} = 2 - 3e^{-t} \quad \ddot{x} = 3e^{-t}$

(b)(ii) $\dot{x} = 2 - 3e^{-t} \quad x = \int (2 - 3e^{-t}) \, dt = 2t + 3e^{-t} + C$
 $x = 0$ when $t = 0 \quad 0 = 2(0) + 3e^0 + C \Rightarrow C = -3$
 $\therefore x = 2t + 3e^{-t} - 3$

(b)(iii) $\dot{x} = 2 - 3e^{-t} \quad \dot{x} = 0 \Rightarrow 0 = 2 - 3e^{-t}$
 $3e^{-t} = 2 \quad e^{-t} = \frac{2}{3} \Rightarrow -t = \ln \frac{2}{3}$
 $\therefore t = -\ln \frac{2}{3} = \ln \frac{3}{2} \text{ s}$

(b)(iv) as $t \rightarrow \infty \quad \ddot{x} \rightarrow 0^+$ and $\dot{x} \rightarrow 2^-$ i.e. acceleration approaches 0 and velocity approaches 2 m/s

(c) Step 1 : Prove true for $n = 1$.

$$2 \times 5^{1-1} + 12^1 = 2 + 12 = 14 = 2 \times 7$$

$\therefore$ Divisible by 7 for $n = 1$

Step 2 : Assume true for $n = k$.

$$2 \times 5^{k-1} + 12^k = 7A \quad \text{where } A \text{ is some integer.}$$

Step 3 : Prove true for $n = k + 1$.

i.e. prove $2 \times 5^{k+1-1} + 12^{k+1} = 7B$

$$\begin{aligned} LHS &= 2 \times 5^{k+1-1} + 12^{k+1} \\ &= 5^1 \times 2 \times 5^{k-1} + 12 \times 12^k \\ &= 5 \times (7A - 12^k) + 12 \times 12^k \\ &= 35A - 5 \times 12^k + 12 \times 12^k \\ &= 35A + 7 \times 12^k \\ &= 7(5A + 12^k) \\ &= 7B \quad \text{where } B = 5A + 12^k \\ LHS &= RHS \end{aligned}$$

Step 4 : If true for $n = k$, then proven true for $n = k + 1$. Since proven true for $n = 1$, must be true for $n = 2$. Since true for $n = 2$, must be true for $n = 3$, etc. Hence, proven true by mathematical induction for all positive integers.

Extension 1 Solutions Question 5.

1) let $BX = x$

$$x(x + 5 \cdot 5) = (15)^2$$

$$x^2 + 5 \cdot 5x - 15 = 0$$

$$2x^2 + 11x - 30 = 0$$

$$(2x + 15)(x - 2) = 0$$

$$\therefore x = -7.5, 2$$

but $x > 0$

$$\therefore x = 2$$

$$\begin{aligned} \lim_{x \rightarrow \infty} \frac{2x^2 - 3x + 1}{5x^2 + x - 4} &= \lim_{x \rightarrow \infty} \frac{2 - \frac{3}{x} + \frac{1}{x^2}}{5 + \frac{1}{x} - \frac{4}{x^2}} \\ &= \frac{2}{5} \end{aligned}$$

$$1) \text{ i) } \frac{\sqrt{12}}{r} \sin x + \frac{2 \cos x}{r} = \sin x \cos \alpha + \cos x \sin \alpha$$

$$\cos \alpha = \frac{\sqrt{12}}{r}$$

$$\sin \alpha = \frac{2}{r}$$

$$\cos^2 \alpha = \frac{12}{r^2}$$

$$\sin^2 \alpha = \frac{4}{r^2}$$

$$r^2 = 16$$

$$r = 4$$

$$\sin \alpha = \frac{2}{4}$$

$$\alpha = \frac{\pi}{6}$$

$$\therefore \sqrt{12} \sin x + 2 \cos x = 4 \sin(x + \frac{\pi}{6})$$

$$ii) 4 \sin(x + \frac{\pi}{6}) = -2\sqrt{2}$$

$$\sin(x + \frac{\pi}{6}) = -\frac{\sqrt{2}}{2}$$

$$x + \frac{\pi}{6} = \frac{5\pi}{4}, \frac{7\pi}{4}$$

$$x = \frac{13\pi}{12}, \frac{19\pi}{12}$$

$$\begin{aligned} d) \text{ i) } LHS &= \sin 3\theta \\ &= \sin(2\theta + \theta) \\ &= \sin 2\theta \cos \theta + \sin \theta \cos 2\theta \\ &= 2 \sin \theta \cos^2 \theta + \sin \theta (1 - 2 \sin^2 \theta) \\ &= 2 \sin \theta (1 - \sin^2 \theta) + \sin \theta (1 - 2 \sin^2 \theta) \\ &= 3 \sin \theta - 4 \sin^3 \theta \\ &= RHS \end{aligned}$$

$$ii) \frac{\sin 3\theta}{10} = \frac{\sin \theta}{5}$$

$$\sin 3\theta = 2 \sin \theta$$

$$3 \sin \theta - 4 \sin^3 \theta = 2 \sin \theta$$

$$\sin \theta - 4 \sin^3 \theta = 0$$

$$\sin \theta (1 - 4 \sin^2 \theta) = 0$$

$$\sin \theta = 0$$

not a solution
to this problem

$$\sin^2 \theta = \frac{1}{4}$$

$$\sin \theta = \frac{1}{2}$$

$$\theta = \frac{\pi}{6}, \frac{5\pi}{6}$$

however only $\theta = \frac{\pi}{6}$ satisfies the problem.

$$6 a) \ddot{x} = 4 \cos 2t - 6 \sin 2t$$

$$\ddot{x} = -8 \sin 2t - 12 \cos 2t$$

$$= -4x \quad \checkmark$$

$$b) i) V = \frac{4\pi r^3}{3}$$

$$\frac{dV}{dt} = \frac{dV}{dr} \times \frac{dr}{dt}$$

$$8 = 4\pi r^2 \times \frac{dr}{dt}$$

$$\frac{dt}{dr} = \frac{\pi r^2}{2}$$

$$t = \frac{\pi r^3}{6}$$

$$\frac{6t}{\pi} = r^3$$

$$r = \sqrt[3]{\frac{6t}{\pi}} \quad \checkmark \checkmark$$

$$ii) \text{ when } t=4$$

$$r = \sqrt[3]{\frac{24}{\pi}}$$

$$S = 4\pi r^2$$

$$\frac{dS}{dt} = \frac{dS}{dr} \times \frac{dr}{dt}$$

$$= 8\pi r \times \frac{2}{\pi r^2}$$

$$= \frac{16}{r}$$

$$= \frac{16}{\sqrt[3]{\frac{24}{\pi}}} \quad \checkmark \checkmark$$

$$iii) 4\pi r^2 = 3000$$

$$r^2 = \frac{750}{\pi}$$

$$r = \sqrt{\frac{750}{\pi}}$$

$$t = \frac{\pi \times \left(\sqrt{\frac{750}{\pi}}\right)^3}{6} \quad \checkmark \checkmark$$

$$c) i) \hat{F}\hat{A}\hat{C} = \hat{B} (\angle \text{ in alternate segments})$$

$$\hat{F}\hat{C}\hat{A} = \hat{B} (\text{ " " " " " " }) \quad \checkmark \checkmark$$

$$ii) \hat{A}\hat{C}\hat{B} = \hat{F}\hat{A}\hat{C} + \hat{E} (\text{exterior } \angle \text{ of } \triangle)$$

$$\hat{F}\hat{C}\hat{B} = \hat{E} (\angle \text{ in alternate segments})$$

$$\therefore \hat{A}\hat{C}\hat{B} = \hat{F}\hat{A}\hat{C} + \hat{F}\hat{C}\hat{B} \\ = \hat{A}\hat{C}\hat{D} \quad \checkmark \checkmark$$

Question 7:

a) let $u = \tan x$

$$\frac{dy}{dx} = \sec^2 x$$

$$y = \sin^{-1} u$$

$$\frac{dy}{du} = \frac{1}{\sqrt{1-u^2}}$$

$$\frac{dy}{dx} = \sec^2 x \cdot \frac{1}{\sqrt{1-u^2}}$$

$$= \frac{\sec^2 x}{\sqrt{1-\tan^2 x}}$$

when $x=0$ $m_1 = \frac{\sec^2 0}{\sqrt{1-0^2}} = 1$

b) When particle hits object, $x = 120$:

$$120 = 30t \cos \theta$$

$$t = \frac{4}{\cos \theta}$$

When particle hits object, $y = 0$:

$$0 = 30t \sin \theta - 5t^2 + 35 \quad (2)$$

Sub ① into ②:

$$0 = 30 \times \frac{4}{\cos \theta} \sin \theta - 5 \left(\frac{4}{\cos \theta} \right)^2 + 35 \quad (1)$$

$$= 120 \tan \theta - 80 \sec^2 \theta + 35$$

$$80 \sec^2 \theta - 120 \tan \theta - 35 = 0 \quad (1)$$

b) $80(1 + \tan^2 \theta) - 120 \tan \theta - 35 = 0 \quad (1)$

$$80 \tan^2 \theta - 120 \tan \theta + 45 = 0$$

$$16 \tan^2 \theta - 24 \tan \theta + 9 = 0$$

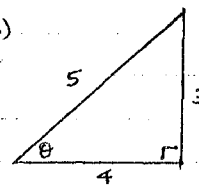
$$(4 \tan \theta - 3)^2 = 0 \quad (1)$$

$$4 \tan \theta = 3$$

$$\tan \theta = \frac{3}{4}$$

$$\therefore \theta = 36^\circ 52' \text{ (nearest minute)} \quad (P)$$

c)



$$x = 30t \cos \theta$$

$$120 = 30 \times t \times \frac{4}{5}$$

$$t = 5s.$$

(2)

c) In $\triangle ABD$:

$$\text{Area } \triangle ABD = \frac{1}{2} pq \sin \frac{\pi}{3}$$

In $\triangle ADC$:

$$\text{Area } \triangle ADC = \frac{1}{2} qr \sin \frac{\pi}{3}$$

In $\triangle ABC$:

$$\text{Area } \triangle ABC = \frac{1}{2} pr \sin \frac{2\pi}{3}$$

$$\text{Area } \triangle ABC = \text{Area } \triangle ABD + \text{Area } \triangle ADC$$

$$\frac{1}{2} pr \sin \frac{2\pi}{3} = \frac{1}{2} pq \sin \frac{\pi}{3} + \frac{1}{2} qr \sin \frac{\pi}{3}$$

$$p r \sin \frac{\pi}{3} = p q \sin \frac{\pi}{3} + q r \sin \frac{\pi}{3} \quad (1)$$

$$pr = pq + qr$$

$$\therefore \frac{1}{q} = \frac{1}{r} + \frac{1}{p}$$