Sydney Girls High School
2025
Trial Higher School Certificate Examination

Mathematics Extension 1

General Instructions

- Reading time – 10 minutes
- Working time – 2 hours
- Write using a black pen
- Calculators approved by NESA may be used
- A reference sheet is provided
- In Questions 11-14, show relevant mathematical reasoning and/or calculations

Total marks: 70

Section I – 10 marks

- Attempt Questions 1-10
- Allow about 15 minutes for this section

Section II – 60 marks

- Attempt Questions 11-14
- Allow about 1 hour and 45 minutes for this section

Name: ……………………………………………………… Teacher: ………………………………………………………	**THIS IS A TRIAL PAPER ONLY** It does not necessarily reflect the format or the content of the 2025 HSC Examination Paper in this subject.

Section I

10 marks
Attempt Questions 1–10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1–10.

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1 Find the length of the vector $\begin{pmatrix} -7 \\ 8 \end{pmatrix}$.

A. 13

B. $\sqrt{15}$

C. $\sqrt{113}$

D. $\sqrt{117}$

2 A circle with centre $(3, -4)$ passes through the origin.
Which of the following is the equation of the circle?

A. $x = 3 - 5\cos\theta, \quad y = 4 + 5\sin\theta$

B. $x = 3 - 5\cos\theta, \quad y = 4 - 5\sin\theta$

C. $x = -3 + 5\cos\theta, \quad y = 4 + 5\sin\theta$

D. $x = 3 + 5\cos\theta, \quad y = -4 + 5\sin\theta$

3 The vectors $\underset{\sim}{a}$ and $\underset{\sim}{b}$ satisfy $\hat{\underset{\sim}{a}} \cdot \hat{\underset{\sim}{b}} = -\dfrac{1}{2}$.
Which of the following is the angle between $\underset{\sim}{a}$ and $\underset{\sim}{b}$?

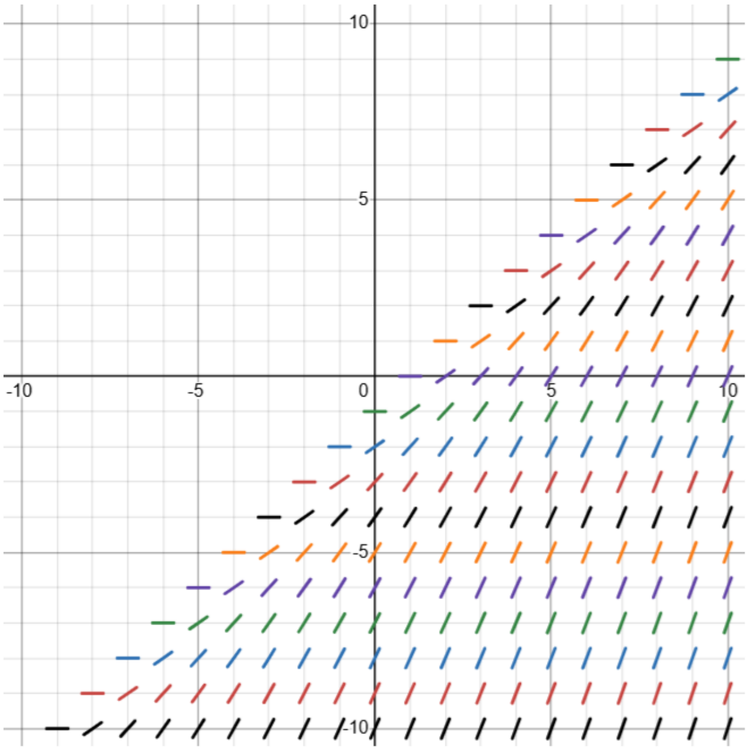
A. $\dfrac{\pi}{3}$

B. $\dfrac{2\pi}{3}$

C. $\dfrac{5\pi}{6}$

D. $\dfrac{3\pi}{4}$

4 Consider the slope field shown below.



Which of the following DEs could generate this slope field?

A. $y' = \ln(x + y)$

B. $y' = \ln(x - y)$

C. $y' = \ln(y - x)$

D. $y' = \ln(-y - x)$

5 It is given that $mx^4 + 3x^3 + 1 = 0$ has a double root.
Which of the following is a correct expression for this double root?

A. $-\dfrac{9m}{4}$

B. $\dfrac{9m}{4}$

C. $-\dfrac{9}{4m}$

D. $\dfrac{9}{4m}$

6 Using a graphical approach, how many solutions are there to $\ln|x| = \dfrac{1}{\ln x}$?

A. 0

B. 1

C. 2

D. 3

7 Which of the following is a particular solution to $\dfrac{dy}{dx} = -\dfrac{y}{3y^2 + x}$?

A. $y^3 - y = 1$

B. $y^3 + y = 1$

C. $y^3 - xy = 1$

D. $y^3 + xy = 1$

8 Which of the following is a correct expression for $\arccos(1 - 2x)$?

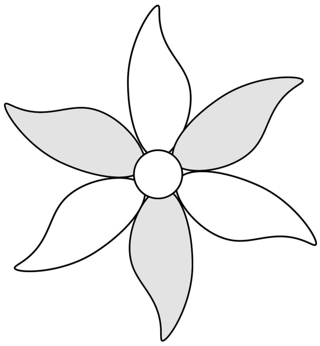
A. $\dfrac{\pi}{2} - \arccos(2x - 1)$

B. $\dfrac{\pi}{2} + \arccos(2x - 1)$

C. $\dfrac{\pi}{2} - \arcsin(2x - 1)$

D. $\dfrac{\pi}{2} + \arcsin(2x - 1)$

9 Flowers are made with exactly six petals.
Each petal is coloured either white or grey.
A flower with three white and three grey petals is shown below.



Any two rotations of the same flower are considered to be identical.
Which of the following is equal to the number of possible flowers?

A. 12

B. 14

C. 16

D. 18

10 The function $f(x+a)-b$ is odd, where a and b are positive.

When the domain of $f(x)$ is restricted to $x \geqslant 2a$, $f(x)$ becomes one-to-one.
For $f(x)$ subject to this restriction, let $f^{-1}(x)$ be the inverse function.

Which of the following is a correct expression for $f^{-1}\big(2b - f(-1)\big)$?

A. $a+1$

B. $a-1$

C. $2a+1$

D. $2a-1$

Section II

60 marks
Attempt Questions 11–14
Allow about 1 hours and 45 minutes for this section

Start each question on a new sheet of paper.

Question 11 (15 marks) Start a new page.

(a) Find the remainder when $x^4 - 3x^3 + 7$ is divided by $x + 3$. **2**

(b) Evaluate $\displaystyle\int_0^2 \frac{8}{4+x^2}\,dx$. **2**

(c) Find the coefficient of x^6 in the expansion of $(4 + 3x^2)^7$. **3**

(d) A block of ice is melting in such a way that it always remains in the shape of a 500mm tall cylinder. **2**
The radius of the cylinder decreases at a rate of 3mm/min.
Find how fast the volume is decreasing when the radius is 30mm.

(e) Determine whether A, B and C are collinear if: **2**

$$\overrightarrow{OA} = -3\underset{\sim}{i} + 15\underset{\sim}{j}$$
$$\overrightarrow{OB} = 5\underset{\sim}{i} - 9\underset{\sim}{j}$$
$$\overrightarrow{OC} = \underset{\sim}{i} + 3\underset{\sim}{j}.$$

(f) If $t = \tan\left(\dfrac{x}{2}\right)$, show that $\dfrac{1}{\cot x + \mathrm{cosec} x} = t$. **2**

(g) Find the general solution to the differential equation $\dfrac{dy}{dx} = 2x\sec(y)$. **2**

End of Question 11

Question 12 (15 marks) Start a new page.

(a) Find the equation of the tangent to the curve $y = \arctan(\cos x)$ at the point $\left(\frac{\pi}{2}, \frac{\pi}{4}\right)$. **2**

(b) Suppose that $y = Ae^{rx}$ is a solution to the differential equation **3**

$$y'' - 6y' + 8y = 0.$$

Find the possible values of A and r if $y = e^3$ when $x = 1$.

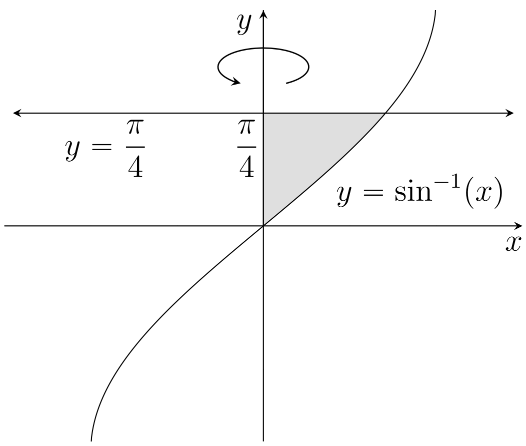
(c) Sketch the graph of $y = 5\cos^{-1}(2x)$, indicating the domain and range. **2**

(d) Let $\underset{\sim}{a} = \begin{pmatrix} k \\ 4 \end{pmatrix}$ and $\underset{\sim}{b} = \begin{pmatrix} 12 \\ -5 \end{pmatrix}$.

(i) Find $\text{proj}_{\underset{\sim}{b}}\, \underset{\sim}{a}$ in terms of k. **2**

(ii) Find the possible values of k if $\left|\text{proj}_{\underset{\sim}{b}}\, \underset{\sim}{a}\right| = 8$. **3**

(e) Find the volume of revolution when the region bounded by the $y-$axis, the curve $y = \sin^{-1} x$, and the line $y = \frac{\pi}{4}$ is rotated about the $y-$axis. **3**



End of Question 12

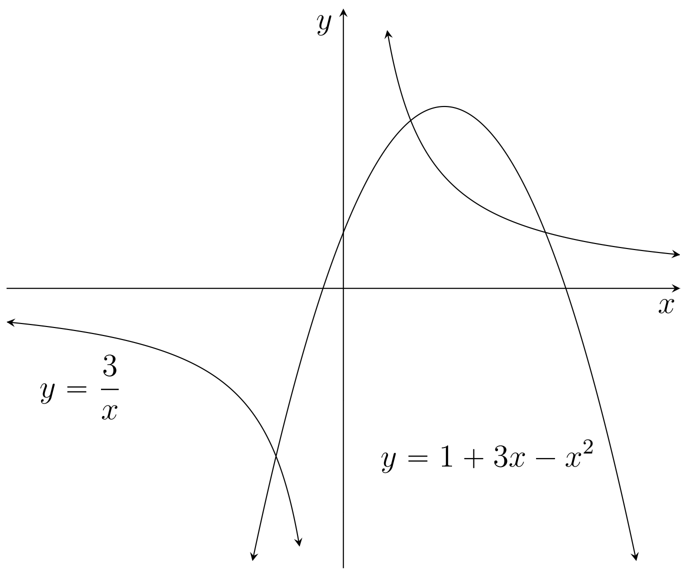
Question 13 (14 marks) Start a new page.

(a) Use mathematical induction to prove that **3**

$$1\times 3 + 2\times 4 + 3\times 5 + \cdots + n\times(n+2) = \frac{n(n+1)(2n+7)}{6}$$

for all integers $n \geqslant 1$.

(b) The curves $y = 1 + 3x - x^2$ and $y = \dfrac{3}{x}$ are sketched below. **3**



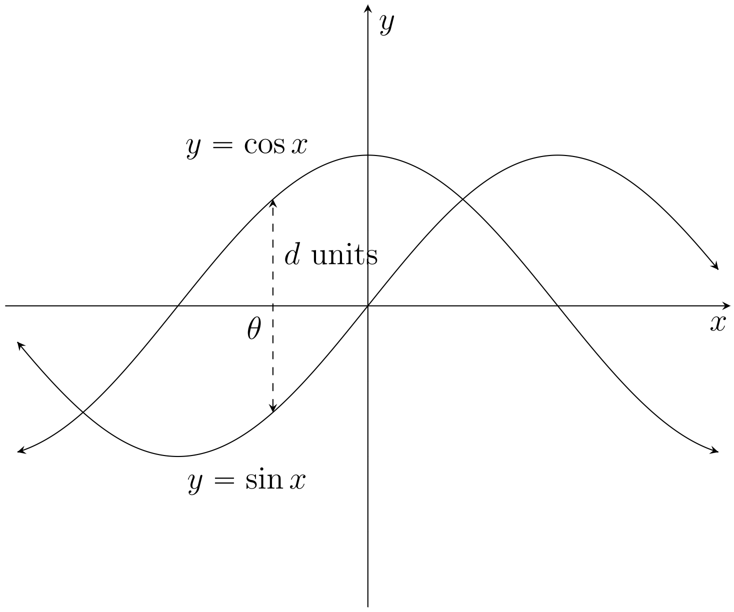
Using this diagram, or otherwise, solve $1 + 3x - x^2 \leqslant \dfrac{3}{x}$.

(c) Using the substitution $u = 2e^x + e^{-x}$, and by considering u^2, find **3**

$$\int \frac{2e^x - e^{-x}}{\sqrt{5 - 4e^{2x} - e^{-2x}}}\, dx.$$

Question 13 continues on the following page

(d) The graphs of $y = \cos x$ and $y = \sin x$ are sketched below.



The dashed line indicates the greatest vertical distance, d units, between the two curves.

The $x-$coordinate of the dashed line is θ.

(i) Find d, giving reasons for your answer. **1**

(ii) Find θ, giving reasons for your answer. **1**

(e) A fair coin is tossed repeatedly until either 100 heads or 100 tails have been recorded (not necessarily consecutive), at which point the experiment ends.

(i) Explain, using the pigeonhole principle, why the experiment must end after at most 199 tosses. **1**

(ii) Find the number of outcomes of the experiment which have 199 tosses. **2**

End of Question 13

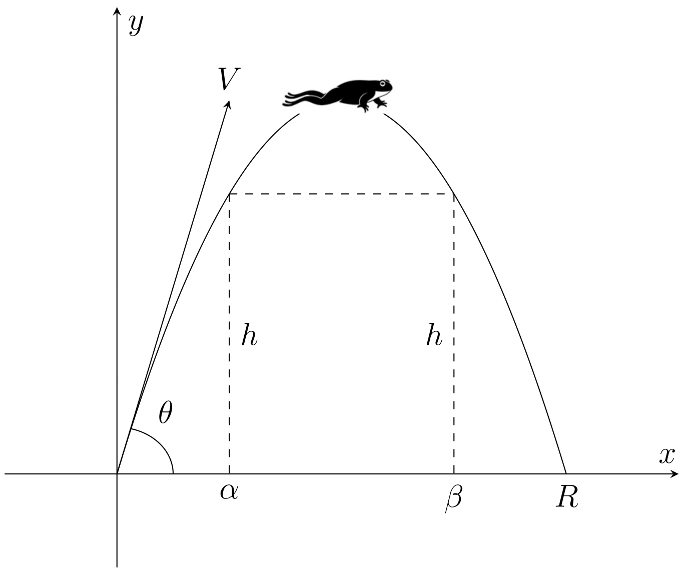
Question 14 (16 marks) Start a new page.

(a) A frog leaps with an initial speed of V ms^{-1}.

The angle of projection is θ, as shown below.

The horizontal range of the frog is R.

Let α and β be the $x-$coordinates of the frog when $y = h$.



The vector equation of motion is given by

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} Vt\cos\theta \\ Vt\sin\theta - \frac{1}{2}gt^2 \end{pmatrix}. \quad \text{[Do not prove this.]}$$

(i) Show that $R = \dfrac{2V^2 \sin\theta\cos\theta}{g}$. **1**

(ii) Hence, show that $y = x\left(1 - \dfrac{x}{R}\right)\tan\theta$. **2**

(iii) Hence, show that α and β are the roots of **1**

$$x^2\tan\theta - xR\tan\theta + Rh = 0.$$

(iv) Deduce that $(\beta - \alpha)^2 = R\,(R - 4h\cot\theta)$. **2**

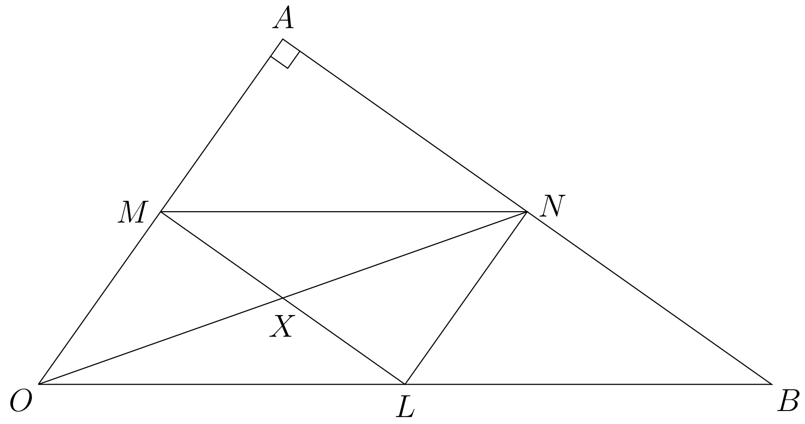
Question 14 continues on the following page

Question 14 (continued)

(b) In the diagram below, ΔAOB is right-angled with $\angle OAB = 90°$.

The midpoints of OA, AB and OB are M, N and L, respectively.

The intersection of ML and ON is X.



Let $\underset{\sim}{a} = \overrightarrow{OA}$ and $\underset{\sim}{b} = \overrightarrow{OB}$.

(i) Use vectors to show that $OMNL$ is a parallelogram. **2**

(ii) Suppose that AX is perpendicular to MN. **3**

Use vectors to show that $|\underset{\sim}{b}| = \sqrt{3}|\underset{\sim}{a}|$.

(You may assume that the diagonals of a parallelogram bisect each other.)

(c) By considering $\dfrac{d}{dx}\left(\dfrac{x}{y}\right)$, or otherwise, find the general solution to **2**

$$y' = \frac{y}{y^2 + x},$$

expressing your answer with y as the subject.

(d) Solve for $0 \leqslant x \leqslant 2\pi$: **3**

$$\cos^2 x + \cos^2 2x + \cos^2 3x = 1.$$

End of paper

2025 Extension 1 Trial

Multiple choice answers

Question	Answer
Q1	C
Q2	D
Q3	B
Q4	B
Q5	C
Q6	C
Q7	D
Q8	D
Q9	B
Q10	C

Section I

10 marks
Attempt Questions 1–10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1–10.

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1 Find the length of the vector $\begin{pmatrix} -7 \\ 8 \end{pmatrix}$.

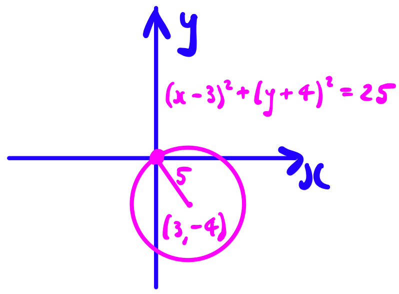
A. 13

B. $\sqrt{15}$

C. $\sqrt{113}$

D. $\sqrt{117}$

$$\left|\begin{pmatrix} -7 \\ 8 \end{pmatrix}\right| = \sqrt{(-7)^2 + 8^2}$$
$$= \sqrt{113}$$



2 A circle with centre $(3, -4)$ passes through the origin.
Which of the following is the equation of the circle?

A. $x = 3 - 5\cos\theta, \quad y = 4 + 5\sin\theta$

B. $x = 3 - 5\cos\theta, \quad y = 4 - 5\sin\theta$

C. $x = -3 + 5\cos\theta, \quad y = 4 + 5\sin\theta$

D. $x = 3 + 5\cos\theta, \quad y = -4 + 5\sin\theta$

$x - 3 = 5\cos\theta$ ①

$y + 4 = 5\sin\theta$ ②

①^2 + ②^2:

$$(x-3)^2 + (y+4)^2 = 5^2\cos^2\theta + 5^2\sin^2\theta$$
$$= 25(\sin^2\theta + \cos^2\theta)$$
$$= 25$$

3 The vectors $\underset{\sim}{a}$ and $\underset{\sim}{b}$ satisfy $\hat{\underset{\sim}{a}} \cdot \hat{\underset{\sim}{b}} = -\frac{1}{2}$.
Which of the following is the angle between $\underset{\sim}{a}$ and $\underset{\sim}{b}$?

A. $\frac{\pi}{3}$

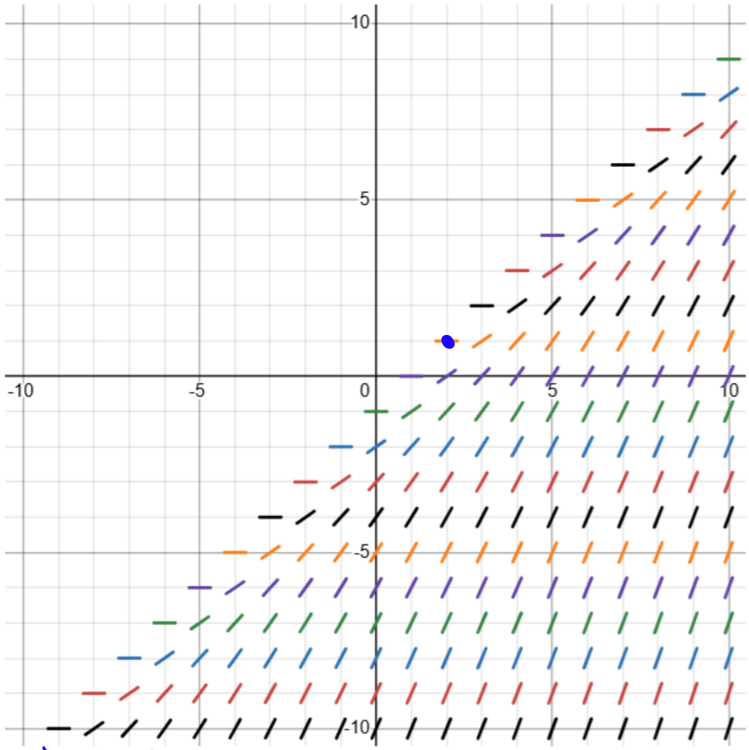
B. $\frac{2\pi}{3}$

C. $\frac{5\pi}{6}$

D. $\frac{3\pi}{4}$

$$\hat{\underset{\sim}{a}} \cdot \hat{\underset{\sim}{b}} = |\hat{\underset{\sim}{a}}||\hat{\underset{\sim}{b}}|\cos\theta$$
$$-\frac{1}{2} = 1 \times 1 \times \cos\theta$$
$$\theta = \cos^{-1}\left(-\frac{1}{2}\right)$$
$$= \frac{2\pi}{3}$$

4 Consider the slope field shown below.



At $(2,1)$, $y' = 0$.

Which of the following DEs could generate this slope field?

Sub $x = 2$, $y = 1$:

A. $y' = \ln(x+y) = \ln 3$ ✗

B. $y' = \ln(x-y) = \ln 1 = 0$ ✓

C. $y' = \ln(y-x) = \ln(-1)$ ✗ undefined

D. $y' = \ln(-y-x) = \ln(-3)$ ✗ undefined

Let $P(x) =$

5 It is given that $mx^4 + 3x^3 + 1 = 0$ has a double root.
Which of the following is a correct expression for this double root?

A. $-\dfrac{9m}{4}$

B. $\dfrac{9m}{4}$

C. $-\dfrac{9}{4m}$

D. $\dfrac{9}{4m}$

Double roots satisfy $P(x) = P'(x) = 0$.

$P'(x) = 4mx^3 + 9x^2 = 0$

$x^2(4mx + 9) = 0$

$4mx + 9 = 0$ OR $x = 0$

$\therefore x = -\dfrac{9}{4m}$

But $P(0) = 1$

$\therefore$ not a root

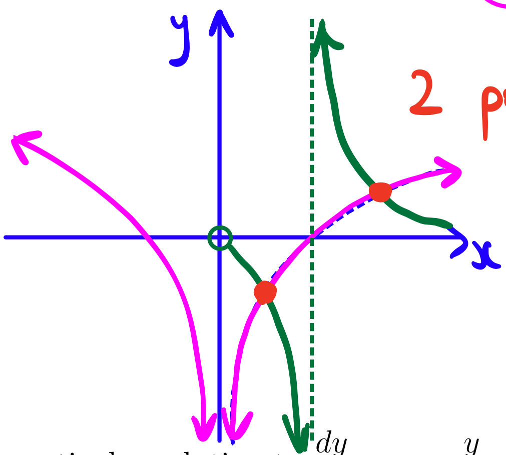
6 Using a graphical approach, how many solutions are there to $\ln|x| = \dfrac{1}{\ln x}$?

A. 0

B. 1

C. 2

D. 3



2 points of intersection

7 Which of the following is a particular solution to $\dfrac{dy}{dx} = -\dfrac{y}{3y^2 + x}$?

Differentiate each option:

A. $y^3 - y = 1$ $\Rightarrow 3y^2y' - y' = 0 \Rightarrow y'(3y^2 - 1) = 0$ ✗

B. $y^3 + y = 1$ $\Rightarrow 3y^2y' + y' = 0 \Rightarrow y'(3y^2+1) = 0$ ✗

C. $y^3 - xy = 1$ $\Rightarrow 3y^2y' - y - xy' = 0 \Rightarrow y'(3y^2 - x) = y$ ✗

D. $y^3 + xy = 1$ $\Rightarrow 3y^2y' + y + xy' = 0 \Rightarrow y'(3y^2 + x) = -y$ ✓

8 Which of the following is a correct expression for $\arccos(1 - 2x)$?

Use $\sin^{-1}x + \cos^{-1}x = \dfrac{\pi}{2}$

i.e. $\cos^{-1}x = \dfrac{\pi}{2} - \sin^{-1}x$

A. $\dfrac{\pi}{2} - \arccos(2x - 1)$

B. $\dfrac{\pi}{2} + \arccos(2x - 1)$

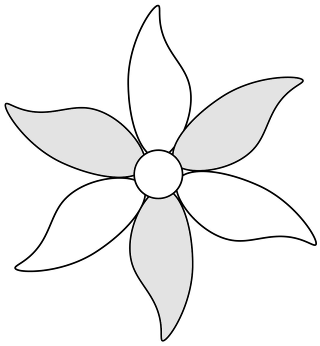
C. $\dfrac{\pi}{2} - \arcsin(2x - 1)$

D. $\dfrac{\pi}{2} + \arcsin(2x - 1)$ $\therefore \arccos(1-2x) = \dfrac{\pi}{2} - \arcsin(1-2x)$

$= \dfrac{\pi}{2} + \arcsin(2x-1)$,

since $\sin^{-1}x$ is an odd function.

9 Flowers are made with exactly six petals.
Each petal is coloured either white or grey.
A flower with three white and three grey petals is shown below.



Any two rotations of the same flower are considered to be identical.
Which of the following is equal to the number of possible flowers?

A. 12

B. 14

C. 16

D. 18

6W	W W W W W W	1
5W 1G	G W W W W W	1
4W 2G	0 gap, 1 gap, 2 gap	3
3W 3G		4
2W 4G	same as 4W 2G	3
1W 5G	same as 5W 1G	1
6G	same as 6W	1

$\therefore 1+1+3+4+3+1+1 = \underline{\underline{14}}$

10 The function $f(x+a) - b$ is odd, where a and b are positive.

When the domain of $f(x)$ is restricted to $x \geqslant 2a$, $f(x)$ becomes one-to-one.
For $f(x)$ subject to this restriction, let $f^{-1}(x)$ be the inverse function.

Which of the following is a correct expression for $f^{-1}(2b - f(-1))$? $= f^{-1}(f(2a+1))$

$= \underline{2a+1}$

A. $a+1$

B. $a-1$

C. $2a+1$

D. $2a-1$

$g(x) = f(x+a) - b$ is odd

$g(-x) = f(-x+a) - b$

$-g(x) = -f(x+a) + b$

sub $x+a = -1$

i.e. $x = -a-1$

$f(a+1+a) - b = -f(-1) + b$

$f(2a+1) = 2b - f(-1)$

Question 11 a) 2 marks total

$P(-3) = (-3)^4 - 3(-3)^3 + 7$ ①

$= 169$ ①

OR polynomial division

$$
\begin{array}{r l}
 & x^3 - 6x^2 + 18x - 54 \\
x+3 \,\big) & x^4 - 3x^3 + 0x^2 + 0x + 7 \\
 & \underline{x^4 + 3x^3} \\
 & -6x^3 + 0x^2 \\
 & \underline{-6x^3 - 18x^2} \\
 & 18x^2 + 0x \\
 & \underline{18x^2 + 54x} \\
 & -54x + 7 \\
 & \underline{-54x - 162} \\
 & 169
\end{array}
$$

① correct method

① correct answer

Marker feedback:

- Most students did this well
- Some students forgot $0x^2 + 0x$ section which led to errors in the division

Question 11 b) 2 marks total

$$\int_0^2 \frac{8}{4+x^2}\,dx = 8\int_0^2 \frac{1}{2^2+x^2}\,dx$$

$$= 8\left[\frac{1}{2}\tan\left(\frac{x}{2}\right)\right]_0^2 \quad ①$$

$$= \frac{8}{2}\left(\tan^{-1}\left(\frac{2}{2}\right) - \tan^{-1}\left(\frac{0}{2}\right)\right)$$

$$= 4\left(\frac{\pi}{4} - 0\right)$$

$$= \pi \quad ①$$

Marker feedback:

- Most students did this well
- Some students made errors with factoring the 8

Question 11c) 3 marks total

$(4+3x^2)^7$

$= \binom{7}{0}(4)^7(3x^2)^0 + \binom{7}{1}(4)^6(3x^2)^1 + \binom{7}{2}(4)^5(3x^2)^2 + \binom{7}{3}(4)^4(3x^2)^3 + \ldots\ldots$ 1

$\uparrow x^6$

$\therefore$ Coefficent of $x^6 = \binom{7}{3}(4)^4(3)^3$ 1

$= 35 \times 256 \times 27$

$= 241920$ 1

OR

$(4+3x^2)^7 = \sum_{k=0}^{7} {}^7C_k (4)^{7-k}(3x^2)^k$ 1

Term of x^6: let $(x^2)^k = x^6$

$\therefore 2k = 6$

$\therefore k = 3$ 1

$\therefore {}^7C_3 (4)^{7-3}(3x^2)^3$

$\therefore$ Coefficient of $x^6 = \binom{7}{3}(4)^4(3)^3$

$= 241920$ 1

OR

by reversing the terms: $\binom{7}{4}(4)^4(3)^{7-4}$

$= 241920$

Marker feedback:

- Student gained full marks for correct answer
- Students should practice showing where their answer came from ie stating general term formula
- Some students forgot to include $\binom{7}{4}$

Question 11d) 2 marks total

$V = 500\pi r^2$

$\therefore \frac{dV}{dr} = 1000\pi r$ 1

$\frac{dV}{dt} = \frac{dV}{dr} \times \frac{dr}{dt}$

$= 1000\pi r \times -3$, when $r = 30$ mm

$= -90000\pi$ mm³/min 1

Markers feedback

- Most students did this well
- Students should note negative means decreasing
- Some students used the cone formula instead of cylinder

Question 11e) 2 marks total

Collinear if: $\overrightarrow{AB} = k_1\overrightarrow{BC}$, OR $\overrightarrow{AB} = k_2\overrightarrow{AC}$, OR $\overrightarrow{BC} = k_3\overrightarrow{AC}$

$\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB} = 3\underset{\sim}{i} - 15\underset{\sim}{j} + 5\underset{\sim}{i} - 9\underset{\sim}{j} = 8\underset{\sim}{i} - 24\underset{\sim}{j} = 8(\underset{\sim}{i} - 3\underset{\sim}{j})$

$\overrightarrow{BC} = \overrightarrow{BO} + \overrightarrow{OC} = -5\underset{\sim}{i} + 9\underset{\sim}{j} + \underset{\sim}{i} + 3\underset{\sim}{j} = -4\underset{\sim}{i} + 12\underset{\sim}{j} = -4(\underset{\sim}{i} - 3\underset{\sim}{j})$

$\overrightarrow{AC} = \overrightarrow{AO} + \overrightarrow{OC} = 3\underset{\sim}{i} - 15\underset{\sim}{j} + \underset{\sim}{i} + 3\underset{\sim}{j} = 4\underset{\sim}{i} - 12\underset{\sim}{j} = 4(\underset{\sim}{i} - 3\underset{\sim}{j})$

1 calculated scalar multiple or same gradient

$\therefore \overrightarrow{AB} = -\frac{1}{2}\overrightarrow{BC}$ OR $\overrightarrow{AB} = \frac{1}{2}\overrightarrow{AC}$ OR $\overrightarrow{BC} = -1\overrightarrow{AC}$ 1

$\therefore$ A, B, C are collinear (reason is // by gradient or scalar multiple

Marker feedback

- Most students showed algebraic understanding of proving the scalar multiple
- Some students tried to prove $\overrightarrow{OA} = k\overrightarrow{OB} = k_2\overrightarrow{OC}$ which led to incorrect solutions.

Question 11f) 2 marks total

$$\frac{1}{\cot x + \operatorname{cosec} x} = t$$

$$\text{LHS} = \frac{1}{\frac{\cos x}{\sin x} + \frac{1}{\sin x}}$$

$$= \frac{\sin x}{\cos x + 1}$$

$$= \frac{\frac{2t}{1+t^2}}{\frac{1-t^2}{1+t^2} + \frac{1+t^2}{1+t^2}} \quad (1)$$

$$= \frac{2t}{1-t^2+1+t^2}$$

$$= \frac{2t}{2}$$

$$= t \quad (1)$$

$$= \text{RHS}$$

OR $$\frac{1}{\frac{1-t^2}{2t} + \frac{1+t^2}{2t}} \quad (1)$$

$$= \frac{2t}{1-t^2+1+t^2}$$

$$= \frac{2t}{2}$$

$$= t \quad (1)$$

Marker feedback

- Most students did this well

Question 11g) 2 marks total

$$\frac{dy}{dx} = 2x\sec(y)$$

$$\frac{1}{\sec(y)}\,dy = 2x\,dx$$

$$\therefore \int \cos(y)\,dy = \int 2x\,dx \quad (1)$$

$$\therefore \sin y = x^2 + c \quad (1)$$

Marker feedback

- Most students did this well

Question 12 (a)

Criteria	Marks
• Provides correct derivative and finds correct gradient.	1
• Finds correct equation.	1

Solution

$$y = \tan^{-1}(\cos x)$$

$$y' = \frac{f'(x)}{1+\left[f(x)\right]^2}$$

$$y' = \frac{-\sin x}{1+\cos^2 x}$$

Let $x = \frac{\pi}{2}$

$$y' = \frac{-\sin\frac{\pi}{2}}{1+(\cos\frac{\pi}{2})^2}$$

$$y' = -1 \quad ✓$$

Equation of the tangent:

$$y - \frac{\pi}{4} = -1(x - \frac{\pi}{2})$$

$$y = \frac{3\pi}{4} - x \quad ✓$$

Comments

- This question was well done. Some students made small errors.
- A common issue was to overcomplicate the working.

Question 12 (b)

Criteria	Marks
• Find first and second derivative and substitute into the differential equation. • Find the correct values of r and A	1 2

Solution

$y = Ae^{rx}$
$y' = rAe^{rx}$
$y'' = r^2 Ae^{rx}$,

$y'' - 6y' + 8y = 0$

$r^2 Ae^{rx} - 6rAe^{rx} + 8Ae^{rx} = 0$ ✓
$Ae^{rx}(r^2 - 6r + 8) = 0$
$Ae^{rx}(r-2)(r-4) = 0$

Ae^{rx}, has no solution.
$r = 2$, or $r = 4$

Solve for A, when $x = 1$ and $y = e^3$
When $r = 2$
$e^3 = Ae^r$
$A = e^{3-r}$
$A = e^{3-2}$
$A = e$ ✓

When $r = 4$
$e^3 = Ae^4$
$A = e^{-1}$
$A = \frac{1}{e}$ ✓

Comments

- Very well done by most students.

Question 12 (c)

Criteria	Marks
• Determine correct domain and range • Draw correct graph labelling intercepts and endpoints.	1 1

Solution

$y = 5\cos^{-1}(2x)$

Domain

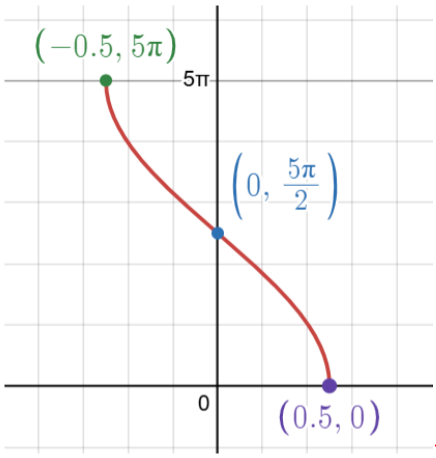
$-1 \le 2x \le 1$

$-\frac{1}{2} \le x \le \frac{1}{2}$

Range

$0 \le \frac{y}{5} \le \pi$

$0 \le y \le 5\pi$ ✓



✓

Comments

- One mark was allocated for the correct domain and correct range.
- One mark was allocated for correct shape and labelling the y-intercept.
- A common error was not labelling the intercept.

Question 12 (d)

Criteria	Marks
• Set up solution correctly. • Found the correct values of k.	1 2

Solution

(i)

$$\text{proj}_{\underset{\sim}{b}}\underset{\sim}{a} = \frac{\underset{\sim}{a} \cdot \underset{\sim}{b}}{|\underset{\sim}{b}|^2}\underset{\sim}{b}$$

$$= \frac{12k-20}{12^2+5^2} \cdot \begin{pmatrix} 12 \\ -5 \end{pmatrix}$$

$$= \frac{12k-20}{169} \cdot \begin{pmatrix} 12 \\ -5 \end{pmatrix} ✓✓$$

(ii)

$$|\lambda \underset{\sim}{v}| = |\lambda||\underset{\sim}{v}|$$

$$\frac{|12k-20|}{169} \times \sqrt{169} = 8 ✓$$

$$|12k-20| = 104$$

Positive case

$$12k - 20 = 104$$

$$12k = 124$$

$$k = \frac{31}{3} ✓$$

Negative case

$$12k - 20 = -104$$

$$12k = -84$$

$$k = -7 ✓$$

Comments

- A common issue for many students was to overcomplicate the question. This resulted in numerous errors and an inability in some cases to find solutions.

Question 12(e)

Criteria	Marks
• Set up correct integral and convert using double angle formula.	1
• Integrates correctly.	1
• Provides correct solution	1

Solution

$$y = \sin^{-1}(x)$$
$$x = \sin y$$

$$\begin{aligned} V &= \pi \int_0^{\frac{\pi}{4}} \sin^2 y \ dy \\ &= \frac{\pi}{2} \int_0^{\frac{\pi}{4}} (1 - \cos 2y) \ dy \\ &= \frac{\pi}{2} \left[y - \frac{1}{2} \sin 2y \right] \quad ✓✓ \\ &= \frac{\pi}{2} \left[\frac{\pi}{4} - \frac{1}{2} \sin \frac{\pi}{2} - (0 - \frac{1}{2} \sin 0) \right] \\ &= (\frac{\pi^2}{8} - \frac{\pi}{4}) \ u^3 \end{aligned}$$

✓

Comments

- This question was well done.
- Some students were not sure how to use the double angle rule to integrate.

Question 13 a) 3 marks total

$1\times3 + 2\times4 + 3\times5 + \cdots + n\times(n+2) = \dfrac{n(n+1)(2n+7)}{6}$

Let $n=1$

$1\times(1+2) = \dfrac{1(1+1)(2(1)+7)}{6}$

$3 = 3$ ①

Assume $n=k$

$1\times3 + 2\times4 + 3\times5 + \cdots + k\times(k+2) = \dfrac{k(k+1)(2k+7)}{6}$ *

Prove for $n=k+1$

$1\times3 + 2\times4 + 3\times5 + \cdots + k\times(k+2) + (k+1)((k+1)+2) = \dfrac{(k+1)(k+1+1)(2(k+1)+7)}{6}$

$= \dfrac{(k+1)(k+2)(2k+9)}{6}$

LHS using assumption *

$= \dfrac{k(k+1)(2k+7)}{6} + (k+1)(k+3)$ ①

$= \dfrac{k(k+1)(2k+7)+6(k+1)(k+3)}{6}$

$= \dfrac{(k+1)(k(2k+7)+6(k+3))}{6}$

$= \dfrac{(k+1)(2k^2+7k+6k+18)}{6}$

$= \dfrac{(k+1)(2k^2+4k+9k+18)}{6}$

$= \dfrac{(k+1)(2k+9)(k+2)}{6}$

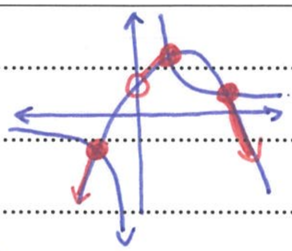
$= \text{RHS}$

① must show working between assumption and final answer

Markers feedback:

- Most students did this well
- some students showed insufficent working out for third mark
- some students working used expansion instead of factoring $(k+1)$, this is the faster method

Question 13 b) 3 marks total



$$1 + 3x - x^2 \leq \frac{3}{x}$$

Solve for intercepts:

$$1 + 3x - x^2 = \frac{3}{x} \quad , \; x \neq 0$$

$$x + 3x^2 - x^3 + 3 = 0$$

$$f(-1) = (-1) + 3(-1)^2 - (-1)^3 + 3 = 0$$

$$f(1) = (1) + 3(1)^2 - (1)^3 + 3 = 0$$

$$\begin{array}{r} x^2 - 4x + 3 \\ x+1 \overline{\smash{)}\, x^3 - 3x^2 - x + 3} \\ \underline{x^3 + x^2 } \\ -4x^2 - x \\ \underline{-4x^2 - 4x } \\ 3x + 3 \\ \underline{3x + 3} \\ 0 \end{array} \rightarrow (x-3)(x-1)$$

Many ways to solve for roots, including trial and error substitution

(1) 2 correct roots

$\therefore$ The three factors are $-1, 1, 3$ (1)

From the graph: $x \leq -1$, $0 < x \leq 1$, $x \geq 3$ (1)

important for full marks

Marker feedback

- Most students did this well
- Some students forgot $x \neq 0$
- Some students used trial and error to solve for roots, if you do this at least write one line of working as shown above

Question 13c) 3 marks total

$u = 2e^{x} + e^{-x}$

$du = 2e^{x} - e^{-x}\,dx$

$\therefore dx = \dfrac{du}{2e^{x} - e^{-x}}$

$u^2 = (2e^{x} + e^{-x})^2$

$= 4e^{2x} + 4 + e^{-2x}$

$\left.\begin{array}{l} \therefore 4e^{2x} + e^{-2x} = u^2 - 4 \\ \text{OR } -4e^{2x} - e^{-2x} = 4 - u^2 \end{array}\right\}$ 1

$$\int \frac{2e^{x} - e^{-x}}{\sqrt{5 - 4e^{2x} - e^{-2x}}}\,dx$$

$$= \int \frac{\cancel{2e^{x} - e^{-x}}}{\sqrt{5 - (u^2 - 4)}} \times \frac{du}{\cancel{2e^{x} - e^{-x}}}$$

$$= \int \frac{1}{\sqrt{9 - u^2}}\,du$$ 1

$$= \sin^{-1}\frac{(u)}{3} + c$$

$$= \sin^{-1}\frac{(2e^{x} + e^{-x})}{3} + c$$ 1

(second method)

$$\text{OR} = \int \frac{\cancel{2e^{x} - e^{-x}}}{\sqrt{5 + 4 - u^2}} \times \frac{du}{\cancel{2e^{x} - e^{-x}}}$$

$$= \int \frac{1}{\sqrt{9 - u^2}}\,du$$ 1

$$= \sin^{-1}\frac{(u)}{3} + c$$

$$= \sin^{-1}\frac{(2e^{x} + e^{-x})}{3} + c$$ 1

Marker Feedback

- Most students did this well
- Some students forgot the final line of substitution, which is the best form of the answer.
- some students didn't take the hint of u^2 and/or forgot the $+4$ in the u^2 expansion

Question 13d) i) ii) 2 marks total

$d = \cos x - \sin x$

$\dfrac{d}{dx} = -\sin x - \cos x$, T.P: $\dfrac{d}{dx} = 0$

$\dfrac{d^2}{dx^2} = -\cos x + \sin x$, Max T.P: $\dfrac{d^2}{dx^2} > 0$

$\therefore x = -\dfrac{\pi}{4} + k\pi$ 1, $d\left(-\dfrac{\pi}{4}\right) = \dfrac{1}{\sqrt{2}} + \dfrac{1}{\sqrt{2}} = \sqrt{2}$ 1

OR

let $\cos x - \sin x = R\cos(x + \alpha)$

$\alpha = \dfrac{\pi}{4}$, $R = \sqrt{2}$

$\therefore \cos x - \sin x = \sqrt{2}\cos\left(x + \dfrac{\pi}{4}\right)$

$x = -\dfrac{\pi}{4}$ 1, $d = \sqrt{2}$ 1

Marker feedback

- Many alternate solutions to this question
- one mark per correct answer was awarded

Question 13 e) i) 1 mark total

i) Pigeons = 199 , Pigeon holes = 2

$\frac{n}{2} > 99 \quad \therefore n > 198$ ①

Therefore by PHP there must have been 199 tosses

OR

Worse case scenario 99H + 99T + 1 = 199 tosses ①

Marker feedback

- some student were able to use one of the methods above
- some students used incorrect pigeonholes number
- 100 + 100 − 1 was not accepted, as it shows no understanding of PHP

Question 13 e) ii) 2 marks total

Either ends with a H or a T

Case 1: _ _ _ _ _ _ _ $\underset{\text{198 spots}}{\text{\_}}$ . . . . . . _ _ _ H

Case 2: _ _ _ _ _ _ _ $\underset{\text{198 spots}}{\text{\_}}$ . . . . . . _ _ _ T

①

$\therefore$ Total $= 2\binom{198}{99}$ OR $2\left(\frac{198!}{99!\,99!}\right)$ ①

Markers feedback

- one mark was given for forming a valid case scenario, or calculating only one case
- one mark was given for the small error in $2\binom{199}{99}$
- most students either didn't attempt this question, or struggled to see and execute the two cases successfully.

Question 14(a)(i)	Marks
• Provides correct solution.	1 mark

Solution

Range achieved when $y = 0$, that is:

$$\begin{aligned} Vt\sin\theta - \frac{1}{2}gt^2 &= 0 \\ t\left(V\sin\theta - \frac{1}{2}gt\right) &= 0 \\ t &= \frac{2V\sin\theta}{g} \end{aligned}$$

Substitute this into $x = Vt\cos\theta$:

$$\begin{aligned} R &= V\left(\frac{2V\sin\theta}{g}\right)\cos\theta \\ &= \frac{2V^2\sin\theta\cos\theta}{g} \checkmark \end{aligned}$$

Question 14(a)(ii)	Marks
• Provides correct solution.	2 marks
• Substitutes correctly and makes progress simplifying.	1 mark

Solution

Substitute $x = Vt\cos\theta$ into RHS of $y = x\left(1 - \frac{x}{R}\right)\tan\theta$:

$$\begin{aligned} \text{RHS} &= Vt\cos\theta\left(1 - \frac{Vt\cancel{\cos\theta}}{\frac{2V^2}{g}\sin\theta\cancel{\cos\theta}}\right)\frac{\sin\theta}{\cos\theta} \\ &= \frac{Vt\cancel{\cos\theta}\sin\theta}{\cancel{\cos\theta}}\left(1 - \frac{g\cancel{V}t}{2V^{\cancel{2}}\sin\theta}\right) \checkmark \\ &= Vt\sin\theta\left(1 - \frac{gt}{2V\sin\theta}\right) \\ &= Vt\sin\theta - \frac{\cancel{V}gt^2\cancel{\sin\theta}}{2\cancel{V}\cancel{\sin\theta}} \\ &= Vt\sin\theta - \frac{1}{2}gt^2 \\ &= y \\ &= \text{LHS} \checkmark \end{aligned}$$

Question 14(a)(iii)	Marks
• Provides correct solution.	1 mark

Solution
Substitute $y = h$ into $y = x\left(1 - \frac{x}{R}\right)\tan\theta$, then rearrange:

$$h = x\left(1 - \frac{x}{R}\right)\tan\theta$$

$$Rh = x\,(R - x)\tan\theta$$

$$Rh = Rx\tan\theta - x^2\tan\theta$$

$$x^2\tan\theta - xR\tan\theta + Rh = 0 \checkmark$$

Hence, α and β are the roots of $x^2\tan\theta - xR\tan\theta + Rh = 0$, since they are the $x-$coordinates when $y = h$.

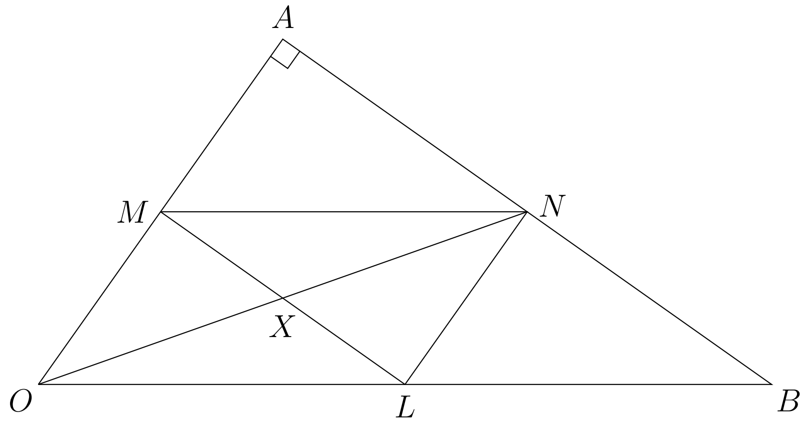
Question 14(a)(iv)	Marks
• Provides correct solution.	2 marks
• Uses the identity $(\alpha-\beta)^2 = (\alpha+\beta)^2 - 4\alpha\beta$ and sum and product of roots.	1 mark

$$\begin{aligned}(\beta - \alpha)^2 &= (\alpha + \beta)^2 - 4\alpha\beta \\ &= \left(-\frac{b}{a}\right)^2 - 4\left(\frac{c}{a}\right) \checkmark \\ &= \left(-\frac{-R\cancel{\tan\theta}}{\cancel{\tan\theta}}\right)^2 - 4\left(\frac{Rh}{\tan\theta}\right) \\ &= R^2 - 4Rh\cot\theta \\ &= R\,(R - 4h\cot\theta) \checkmark\end{aligned}$$

Comments
• Quite well done overall.
• Part (ii) could also be done by subbing $x = Vt\cos\theta$ into $y = Vt\sin\theta - \frac{1}{2}gt^2$
• Part (iv) could also be done using the quadratic formula, though this was not as elegant.

Question 14(b)(i)	Marks
• Provides correct solution.	2 marks
• Finds $\overrightarrow{ON} = \frac{1}{2}\underset{\sim}{a} + \frac{1}{2}\underset{\sim}{b}$ or $\overrightarrow{AN} = \frac{1}{2}\underset{\sim}{b} - \frac{1}{2}\underset{\sim}{a}$.	1 mark

Solution



$$\begin{aligned}\overrightarrow{MN} &= \overrightarrow{MA} + \overrightarrow{AN}\\ &= \frac{1}{2}\overrightarrow{OA} + \frac{1}{2}\overrightarrow{AB} \quad (M,\ N \text{ midpoints of } OA, AB)\\ &= \frac{1}{2}\underset{\sim}{a} + \frac{1}{2}(\underset{\sim}{b} - \underset{\sim}{a}) \checkmark\\ &= \frac{1}{2}\underset{\sim}{b}\\ &= \overrightarrow{OL} \quad (L \text{ midpoint of } OB)\end{aligned}$$

Therefore $OMNL$ is a parallelogram (one pair of equal and parallel sides). ✓

Comments

• Some students assumed that X was the midpoint of ON and ML, but this is circular reasoning. The diagonals of a parellelogram do bisect each other, but we cannot assume that $OMNL$ is a parallelogram, it is our job to prove it.

• It was sufficient to prove $\overrightarrow{MN} = \overrightarrow{OL}$. This implies one pair of equal and parallel sides.

• There was an interesting approach that used the converse of the parallelogram rule for adding vectors. Explicitly, students who managed to prove $\overrightarrow{OM} + \overrightarrow{OL} = \overrightarrow{ON}$ without circular reasoning were awarded full marks.

Question 14(b)(ii)	Marks
• Provides correct solution.	3 marks
• Finds $\underset{\sim}{b} \cdot \underset{\sim}{b} = 3\underset{\sim}{a} \cdot \underset{\sim}{b}$.	2 marks
• Finds $\overrightarrow{AX} = \frac{1}{4}\underset{\sim}{b} - \frac{3}{4}\underset{\sim}{a}$.	1 mark

Solution

Since AX is perpendicular to MN, we have

$$\begin{aligned}
\overrightarrow{AX} \cdot \overrightarrow{MN} &= 0 \\
(\overrightarrow{OX} - \overrightarrow{OA}) \cdot (\overrightarrow{ON} - \overrightarrow{OM}) &= 0 \\
\left(\frac{1}{2}(\frac{1}{2}\underset{\sim}{a} + \frac{1}{2}\underset{\sim}{b}) - \underset{\sim}{a}\right) \cdot \left(\frac{1}{2}\underset{\sim}{a} + \frac{1}{2}\underset{\sim}{b} - \frac{1}{2}\underset{\sim}{a}\right) &= 0 \\
\left(\frac{1}{4}\underset{\sim}{b} - \frac{3}{4}\underset{\sim}{a}\right) \cdot \left(\frac{1}{2}\underset{\sim}{b}\right) &= 0 \\
(\underset{\sim}{b} - 3\underset{\sim}{a}) \cdot \underset{\sim}{b} &= 0 \\
\underset{\sim}{b} \cdot \underset{\sim}{b} - 3\underset{\sim}{a} \cdot \underset{\sim}{b} &= 0 \\
\underset{\sim}{b} \cdot \underset{\sim}{b} &= 3\underset{\sim}{a} \cdot \underset{\sim}{b} \qquad (*)
\end{aligned}$$

But OA is perpendicular to AB.

$$\begin{aligned}
\overrightarrow{OA} \cdot \overrightarrow{AB} &= 0 \\
\underset{\sim}{a} \cdot (\underset{\sim}{b} - \underset{\sim}{a}) &= 0 \\
\underset{\sim}{a} \cdot \underset{\sim}{b} - \underset{\sim}{a} \cdot \underset{\sim}{a} &= 0 \\
\underset{\sim}{a} \cdot \underset{\sim}{b} &= \underset{\sim}{a} \cdot \underset{\sim}{a}
\end{aligned}$$

Substitute this into $(*)$:

$$\begin{aligned}
\underset{\sim}{b} \cdot \underset{\sim}{b} &= 3\underset{\sim}{a} \cdot \underset{\sim}{a} \\
|\underset{\sim}{b}|^2 &= 3|\underset{\sim}{a}|^2 \\
|\underset{\sim}{b}| &= \sqrt{3}|\underset{\sim}{a}|
\end{aligned}$$

Comments

• One mark was also awarded for $\overrightarrow{OX} = \frac{1}{4}\underset{\sim}{a} + \frac{1}{4}\underset{\sim}{b}$.

• There was a cool approach which used the geometrical definition of the dot product. Namely, $\underset{\sim}{a} \cdot \underset{\sim}{b} = |\underset{\sim}{a}||\underset{\sim}{b}|\cos\theta = |\underset{\sim}{a}||\underset{\sim}{b}| \times \frac{|\underset{\sim}{a}|}{|\underset{\sim}{b}|} = |\underset{\sim}{a}|^2$, which works because $\triangle OAB$ is right angled (cos is adjacent over hypotenuse).

Question 14(c)	Marks
• Provides correct solution.	2 marks
• Finds $\frac{d}{dx}\left(\frac{x}{y}\right) = y'$	1 mark

Solution

By the quotient rule, $\dfrac{d}{dx}\left(\dfrac{x}{y}\right) = \dfrac{y - xy'}{y^2}(*)$.

The given DE can be rearranged as follows:

$$\begin{aligned} y' &= \frac{y}{y^2 + x} \\ y'y^2 + y'x &= y \\ y'y^2 &= y - xy' \\ y' &= \frac{y - xy'}{y^2} \checkmark \end{aligned}$$

Now, substitute $(*)$:

$$\begin{aligned} y' &= \frac{d}{dx}\left(\frac{x}{y}\right) \quad \text{(Integrate both sides with respect to } x\text{)} \\ y + C &= \frac{x}{y} \\ y^2 + Cy &= x \\ y^2 + Cy - x &= 0 \\ y &= \frac{-C \pm \sqrt{C^2 + 4x}}{2} \checkmark \end{aligned}$$

Comments

- This was the hardest part of Question 14. The median mark was 0 out of 2.
- The difficulty was in making a connection between the hint and the given DE.

Question 14(d)	Marks
• Provides correct solution.	3 marks
• Finds $2\cos 3x(\cos x + \cos 3x) = 0$.	2 marks
• Finds $\cos 2x + \cos 4x + 2\cos^2 3x = 0$.	1 mark

Solution

$$\cos^2 x + \cos^2 2x + \cos^2 3x = 1$$

$$2\cos^2 x + 2\cos^2 2x + 2\cos^2 3x = 2 \qquad \text{(Multiply both sides by 2)}$$

$$2\cos^2 x - 1 + 2\cos^2 2x - 1 + 2\cos^2 3x = 0 \qquad \text{(Subtract 1 twice from both sides)}$$

$$\cos 2x + \cos 4x + 2\cos^2 3x = 0 \checkmark \qquad (\text{Use } \cos 2\theta = 2\cos^2\theta - 1)$$

$$2\cos 3x \cos x + 2\cos^2 3x = 0 \qquad \left(\cos P + \cos Q = 2\cos\frac{P+Q}{2}\cos\frac{P-Q}{2}\right)$$

$$2\cos 3x\,(\cos x + \cos 3x) = 0 \checkmark \qquad \text{(Take out highest common factor)}$$

$$2\cos 3x\,(2\cos x \cos 2x) = 0 \qquad \text{(Use sums to products identity again)}$$

$$4\cos x \cos 2x \cos 3x = 0$$

If $\cos x = 0$, then $x = \frac{\pi}{2}, \frac{3\pi}{2}$.

If $\cos 2x = 0$, then $x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

If $\cos 3x = 0$, then $x = \frac{\pi}{6}, \frac{3\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{9\pi}{6}, \frac{11\pi}{6}$

Therefore, accounting for duplicates, there are 10 unique solutions, namely:

$$x = \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{5\pi}{4}, \frac{3\pi}{2}, \frac{7\pi}{4}, \frac{11\pi}{6} \checkmark$$

Comments

• Guessing answers was not awarded marks, as it did not demonstrate understanding of the ideas involved in a complete solution.

• One alternative approach was to make the subsitution $y = \cos x$. If applied correctly, this resulted in $16u^6 - 20u^4 + 6u^2 = 0$, which factorises as $2u^2(2u^2 - 1)(4u^2 - 3) = 0$.

• Another approach was to make the substitution $v = \cos 2x$ which if applied correctly becomes $2v^3 + v^2 - v = 0$, which factorises as $v(2v - 1)(v + 1) = 0$

• One mark was awarded for finding $\cos 2x + \cos 4x + \cos 6x = -1$, since this constitutes a simplification and good progress.