Name: ______________________________

Teacher: ______________________________

Sydney Technical High School

2025 | HIGHER SCHOOL CERTIFICATE **TRIAL EXAMINATION**

Mathematics Extension 1

General Instructions	• Working Time – 2 hours • Write using black pen
Total marks: 70	**Section I – MULTIPLE CHOICE – 10 marks** (pages 2-6) • Attempt Questions 1-10 • Use the multiple-choice answer sheet provided. • Allow about 15 minutes for this section. **Section II – 60 marks** (page 7-11) • Attempt Questions 11-14 • Allow about 1 hour 45 minutes for this section. • Answer each question in the writing booklet and begin each question on a new page.

SECTION I MULTIPLE CHOICE
10 Marks
Attempt questions 1-10
Allow approximately 15 minutes for this section.
Use the multiple-choice answer sheet for questions 1 – 10

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1. Determine which inverse trigonometric function below could be described as an odd function with a domain of $[-3\,,3]$?

A. $y = 2\cos^{-1}\left(\frac{x}{3}\right)$

B. $y = 2\sin^{-1}\left(\frac{x}{3}\right)$

C. $y = 3\sin^{-1}\left(\frac{x}{2}\right)$

D. $y = 2\tan^{-1}\left(\frac{x}{3}\right)$

2. Consider the three statements below:

#1: In a group of thirteen people, at least two must have their birthday in the same month.

#2: A room which has five windows in its four walls must have at least one wall which has at least two windows.

#3: A drawer contains ten socks which are identical except that they are a mixture of seven different colours. If six socks are drawn from the drawer they must include at least one pair of socks of matching colours.

Which statements confirm the Pigeonhole Principle?

A. Statements 1 and 2 only

B. Statements 1 and 3 only

C. Statements 2 and 3 only

D. All three statements

3. How many distinct arrangements are possible from the letters of the word NECESSITIES ?

A. 50 400

B. 554 400

C. 1 663 200

D. 39 916 800

4. The vector $\overrightarrow{AB}$ joins the points $A(-5,4)$ and $B(3,10)$.
Three other vectors are given:

$\underset{\sim}{u} = \underset{\sim}{i} + 2\underset{\sim}{j}$

$\underset{\sim}{v} = 2\underset{\sim}{i} + \underset{\sim}{j}$

$\underset{\sim}{w} = 3\underset{\sim}{i} + \underset{\sim}{j}$

Which of the following vectors is parallel to $\overrightarrow{AB}$?

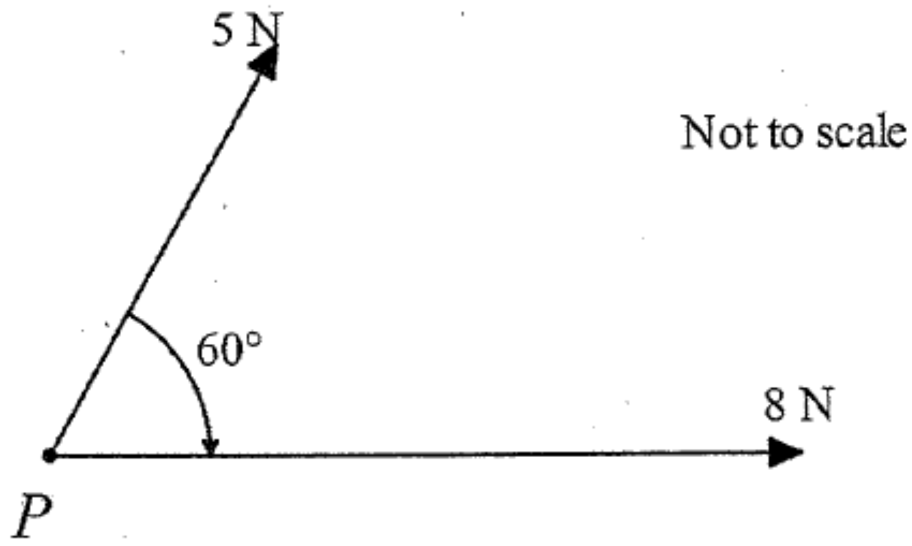
A. $\underset{\sim}{u} - \underset{\sim}{v}$

B. $\underset{\sim}{u} + \underset{\sim}{v}$

C. $\underset{\sim}{w} + \underset{\sim}{v}$

D. $\underset{\sim}{w} + \underset{\sim}{u}$

5. Forces of magnitude $8N$ and $5N$ act on a particle P. The angle between the directions of the two forces is 60° as shown in the diagram.



Which of the following is the correct magnitude and direction of the resultant force acting on P?

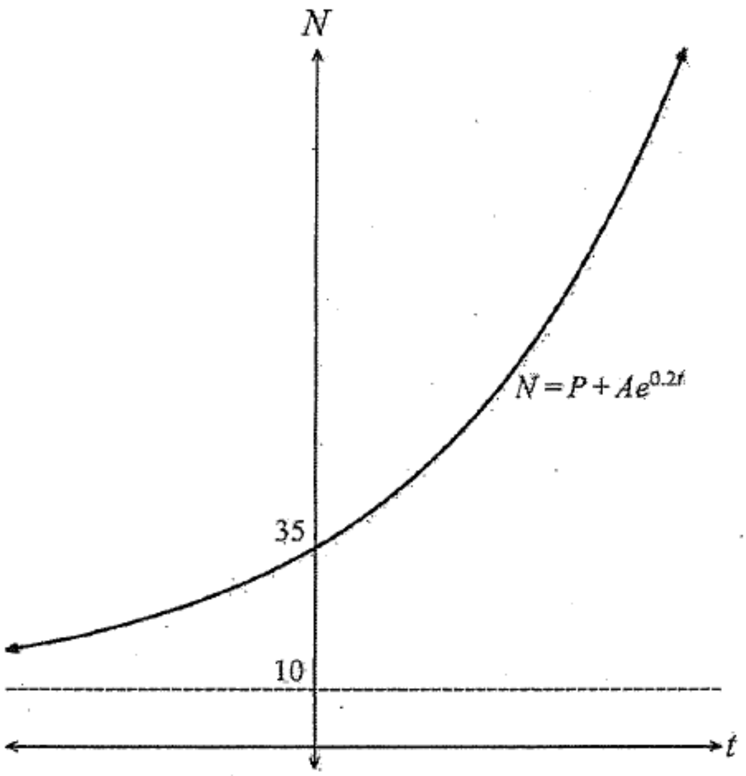
A. 11.36N, 22°25’ to the horizontal

B. 11.36N, 67°35’ to the horizontal

C. 12.58N, 22°25’ to the horizontal

D. 12.58N, 67°35’ to the horizontal

6. The quantity N increases exponentially over time according to the equation $N = P + Ae^{0.2t}$
The graph below shows the change in N over time (t).



Which equation describes the rate of change of N with respect to t?

A. $\frac{dN}{dt} = 0.2(N - 10)$

B. $\frac{dN}{dt} = 0.2(N - 25)$

C. $\frac{dN}{dt} = 10(N - 0.2)$

D. $\frac{dN}{dt} = 25(N - 10)$

7. Consider the polynomial $P(x) = x^3 + x^2 + cx - 10$. It is known that two of its zeros are equal in magnitude but opposite in sign.
What is the value of c?

A. $\sqrt{10}$

B. 10

C. -10

D. $-\sqrt{10}$

8. A spherical balloon is being inflated at a constant rate of $200\pi\ cm^3s^{-1}$.
At what rate is the radius of the balloon increasing when the radius is 10cm?

A. $0.25\ cms^{-1}$

B. $0.5\ cms^{-1}$

C. $1.0\ cms^{-1}$

D. $2.0\ cms^{-1}$

9. What is the value of k such that the function $f(x) = \frac{k}{1+x^2}$ $, -1 \leq x \leq 1$ is a probability density function?

A. $k = \frac{\pi}{4}$

B. $k = \frac{\pi}{2}$

C. $k = \frac{2}{\pi}$

D. $k = \frac{4}{\pi}$

10. The graph of the function $y = \tan^{-1}\frac{1}{2}(x-2)$ is to be transformed by a translation left by 1 unit, then a horizontal dilation with a scale factor of 2. The equation of the transformed graph is:

A. $y = \tan^{-1}\left(\frac{x-3}{4}\right)$

B. $y = \tan^{-1}\left(\frac{x-2}{4}\right)$

C. $y = \tan^{-1}(x-2)$

D. $y = \tan^{-1}\left(x-\frac{1}{2}\right)$

End of Section I

Section II

60 marks
Attempt Questions 11 - 14
Allow approximately 1 hour 45 minutes for this section.

Answer all questions in the writing booklet and begin each question on a new page.
Your responses should include relevant mathematical reasons and/or calculations.

QUESTION 11 (15 Marks) **Marks**

a) Solve $\frac{1}{x+1} \leq -1$ **2**

b) Find $\int_0^{\ln 2} \frac{e^{2x}}{1+e^{4x}}\,dx$ by using the substitution $u = e^{2x}$, correct to 2 decimal places **4**

c) Find the term independent of x in the expansion of $\left(3x - \frac{1}{2x^2}\right)^9$ (answer can be left in index form) **3**

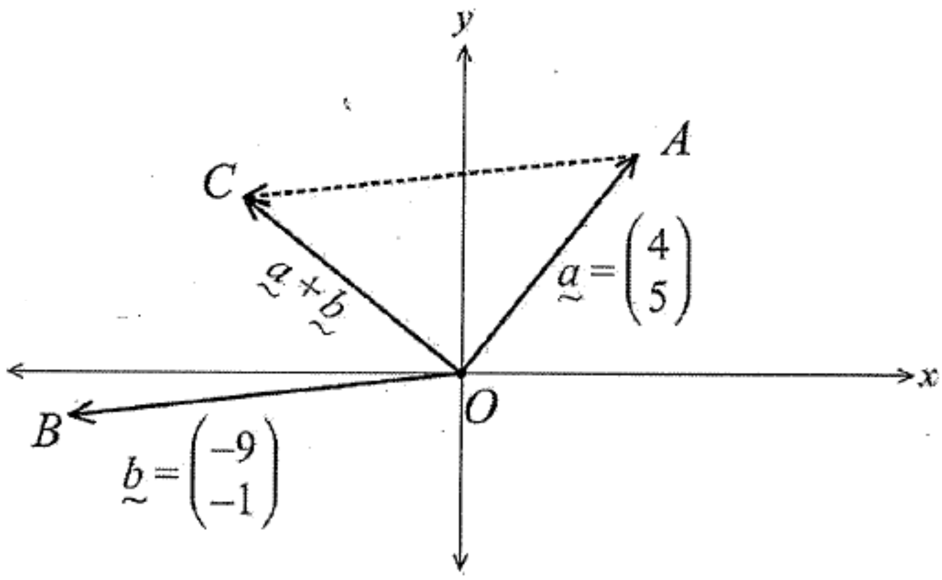
d) Given that $P(x) = x^4 - 9x^3 + 25x^2 - 27x + 10$ has a double root, factorise $P(x)$ into linear factors. **2**

e) Find the exact value of $\tan\left(2\sin^{-1}\frac{2}{3}\right)$ **2**

f) Given the function $f(x) = \sqrt{(2x+1)}$ and that $f^{-1}(x)$ is the inverse function of $f(x)$, find $f^{-1}(5)$. **2**

QUESTION 12 (15 Marks) **Marks**

a) The diagram shows the vectors $\underset{\sim}{a} = \begin{pmatrix} 4 \\ 5 \end{pmatrix}$, $\underset{\sim}{b} = \begin{pmatrix} -9 \\ -1 \end{pmatrix}$ and the resultant vector $\underset{\sim}{a} + \underset{\sim}{b}$. **3**



Show that $\triangle\, OAC$, formed by the vectors, is an isosceles triangle and find the size of $\angle AOC$.

b) Express $\sqrt{6}\sin x + \sqrt{2}\cos x$ in the form $R\cos(x - \alpha)$ and hence or otherwise solve: $\sqrt{6}\sin x + \sqrt{2}\cos x = 2$ over $[0\,, 2\pi]$ **3**

c) Given that ${}^{n}C_{3} = p$ and ${}^{n}C_{4} = q$, use the Pascal Triangle relationships between coefficients to find an expression, in terms of p and q, for ${}^{n+1}C_{n-3} + {}^{n}C_{n-3}$ **2** d)

(i) Find the unit vector that is in the direction of vector $\overrightarrow{AB}$ if A is the point (-1,-1) and B is the point (2,-3). **2**

(ii) If $\underset{\sim}{F}$ is the vector $\begin{pmatrix} 2 \\ 5 \end{pmatrix}$ find the projection of $\underset{\sim}{F}$ in the direction of the vector $\overrightarrow{AB}$ **2**

e) Use the t-formulae to solve the equation $\cos x - \sin x + 1 = 0$ where $0 \leq x \leq 360°$ **3**

QUESTION 13 (15 Marks) **Marks**

a) Use the method of separation of variables to solve the differential equation: $\frac{dy}{dx} = \frac{3y}{x}$, given that $\frac{dy}{dx} = 6$ when $x = 2$ **4**

b) (i) Prove the trigonometric identity **3**

$$\sin 3A = 3\sin A - 4\sin^3 A$$

(ii) Using the identity in part (i) and the substitution $x = \sin A$, show that the equation $6x - 8x^3 = 1$ has the roots **3**

$\sin\frac{\pi}{18}$, $\sin\frac{5\pi}{18}$ and $\sin\frac{25\pi}{18}$.

(iii) Show that $\sin\frac{\pi}{18} \times \sin\frac{5\pi}{18} \times \sin\frac{25\pi}{18} = \frac{-1}{8}$ **1**

c) ACE Electrical is a wholesale business which sells electrical components to manufacturers. From many years of experience, the probability of making a sale when calling a regular customer is 0.8.
In a week, the sales representative at ACE made 900 calls to customers.

(i) This is a binomial probability distribution where X is the variable for the number of calls which result in sales.
Find the expected number of sales in the week $E(X)$ and the variance $Var(X)$. **2**

(ii) Use a normal approximation to the binomial distribution to estimate the probability that there are more than 744 sales in the week.
(You do not need to first test if a normal approximation is appropriate for this distribution). **2**

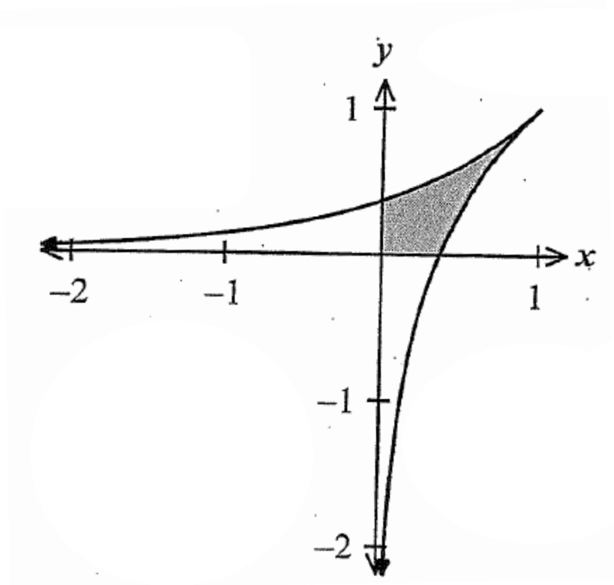
QUESTION 14 (15 Marks) **Marks**

a) Use mathematical induction to prove that $3^{2n+4} - 2^{2n}$ is divisible by 5 for all integers $n \geq 1$. **3**

b) The function $f(x) = 1 + \ln x$ is defined in the domain (0,1] .

(i) Show that $\frac{d}{dx}(x \ln x) = 1 + \ln x$ **1**

(ii) The diagram shows the graphs of the function $y = f(x)$ and the inverse function $y = f^{-1}(x)$. **3**



Find in simplest exact form the area of the region in the first quadrant bounded by the curves $y = f(x)$, $y = f^{-1}(x)$ and the coordinate axes.

c) By using the fact that $(1+x)^{11} = (1+x)^3(1+x)^8$, show that **2**

$$\binom{11}{5} = \binom{8}{5} + \binom{3}{1}\binom{8}{4} + \binom{3}{2}\binom{8}{3} + \binom{8}{2}$$

d) Find $\int \frac{3}{\sqrt{4-9x^2}}\ dx$ **2**

e) At a football club a junior team of 13 players is to be chosen from a pool of 25 players consisting of 20 female and 5 male players. **2**
What is the probability that the team will consist of at least 2 male players?

f) RSV is an infectious respiratory virus. The probability of contracting an RSV infection after being exposed to the virus is 0.74. A group of ten friends are all exposed to RSV on one occasion in a classroom. **2**
Find the P(eight or more of the group contract an RSV infection).

END OF EXAM

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2025 STHS Ext. 1 Trial HSC Sol'ns

1. B 2. A 3. $\frac{11!}{3!2!3!} = \frac{11\times10\times9\times8\times7\times\cancel{6}\times5\times\cancel{4}^{2}\times\cancel{3}\times\cancel{2}}{\cancel{6}\times\cancel{2}\times\cancel{6}}$ B

4. D 5. A 6. $\frac{dN}{dt} = 0.2Ae^{0.2t}$

$= 0.2(N-P)$

As $t \to -\infty$, $N \to 10 \downarrow \therefore P = 10$

$= 0.2(N-10)$

A

7. $\sum: \alpha + -\alpha + \beta = -1$ Also $-\alpha^2 \times -1 = 10$

$\beta = -1$ $\alpha^2 = 10$

$\cancel{\alpha\times-1} + \cancel{-\alpha\times-1} - \alpha^2 = c$

$-\alpha^2 = c$

$\therefore c = -10$

C

8. $\frac{dV}{dt} = 200\pi$ $\frac{dV}{dt} = \frac{dV}{dr} \times \frac{dr}{dt}$

$V = \frac{4}{3}\pi r^3$ $200\pi = 4\times\pi\times r^2 \times \frac{dr}{dt}$

$\frac{dV}{dr} = 4\pi r^2$ When $r = 10$

$\frac{dr}{dt} = \frac{200\pi}{400\pi} = \frac{1}{2}$ cm/s.

B

9. $\int_{-1}^{1} \frac{k}{1+x^2}\,dx = 1$

$k\left[\tan^{-1}x\right]_{-1}^{1} = 1$

$k\left(\frac{\pi}{4} - -\frac{\pi}{4}\right) = 1$

$k \times \frac{\pi}{2} = 1$

$k = \frac{2}{\pi}$

C

10. $y = \tan^{-1}\frac{1}{2}(x-1)$ is left by 1

Dilation $\frac{x}{2}$

$y = \tan^{-1}\frac{1}{2}\left(\frac{x}{2} - 1\right)$

$y = \tan^{-1}\left(\frac{x}{4} - \frac{1}{2}\right)$ B

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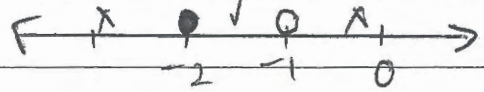
11a. $\frac{1}{x+1} \leq -1$

$x = -1$ a critical pt.

$\frac{1}{x+1} = -1$

$1 = -x-1$

$x = -2$ a critical pt.

Test 

$-2 \leq x < -1$

b) $\int_0^{\ln 2} \frac{e^{2x}}{1+(e^{2x})^2}\,dx$

Let $u = e^{2x}$

$du = 2e^{2x}\,dx$

$\frac{1}{2}du = e^{2x}\,dx$

Now $x=0$ gives $u=1$

$x = \ln 2$ $u = 4$

$\frac{1}{2}\int_1^4 \frac{du}{1+u^2}$

$= \frac{1}{2}\left[\tan^{-1} u\right]_1^4$

$= \frac{1}{2}\left[\tan^{-1}4 - \tan^{-1}1\right]$

$= 0 \cdot 27$

c) $\left(3x - \frac{1}{2x^2}\right)^9$

General term:

$T_{k+1} = \binom{9}{k}(3x)^{9-k}\left(\frac{-1}{2x^2}\right)^k$

$= \binom{9}{k} 3^{9-k} x^{9-k} \left(\frac{-1}{2}\right)^k (x)^{-2k}$

$= \binom{9}{k} 3^{9-k} \left(\frac{-1}{2}\right)^k x^{9-3k}$

$9-3k = 0 \therefore k = 3$ eliminates x

$\therefore$ Constant term is $\binom{9}{3} 3^6 \left(\frac{-1}{2}\right)^3$

d) $P'(x) = 4x^3 - 27x^2 + 50x - 27$

$P'(1) = 0$ and $P(1) = 0$ so double root is $x = 1$

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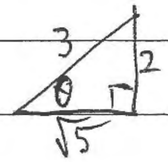
Now by inspection

$x^4 - 9x^3 + 25x^2 - 27x + 10 = (x^2 - 2x + 1)(x^2 - 7x + 10)$

$= (x-1)^2(x-2)(x-5)$

e) $\tan\left(2\sin^{-1}\frac{2}{3}\right)$

$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$



$\therefore \tan\theta = \frac{2}{\sqrt{5}}$

$\tan 2\theta = \frac{2 \times \frac{2}{\sqrt{5}}}{1 - \left(\frac{2}{\sqrt{5}}\right)^2}$

$= \frac{\frac{4}{\sqrt{5}}}{1 - \frac{4}{5}}$

$= \frac{4}{\sqrt{5}} \times \frac{5}{1}$

$= 4\sqrt{5}$

f) $f^{-1}(5) \Rightarrow (5, y)$

Curve must pass through $(y, 5)$

$\therefore 5 = \sqrt{2x+1}$

$25 = 2x + 1$

$x = 12$

i.e: $(12, 5)$ $\therefore$ inverse passes through $(5, 12)$

$\therefore f^{-1}(5) = 12$

12. a) $\underset{\sim}{a} + \underset{\sim}{b}$

$= \begin{pmatrix} 4 \\ 5 \end{pmatrix} + \begin{pmatrix} -9 \\ -1 \end{pmatrix}$

$= \begin{pmatrix} -5 \\ 4 \end{pmatrix}$

Now ΔOAC is isosceles if $|\underset{\sim}{a}| = |\underset{\sim}{a} + \underset{\sim}{b}|$

$|\underset{\sim}{a}| = \sqrt{4^2 + 5^2} = \sqrt{41}$

$|\underset{\sim}{a} + \underset{\sim}{b}| = \sqrt{(-5)^2 + 4^2} = \sqrt{41}$

$\therefore$ isosceles.

$\underset{\sim}{a} \cdot (\underset{\sim}{a} + \underset{\sim}{b}) = |\underset{\sim}{a}||\underset{\sim}{a} + \underset{\sim}{b}| \cos\theta$

$4 \times -5 + 5 \times -4 = \sqrt{41}\sqrt{41} \times \cos\theta$

$0 = 41 \cos\theta$

$\theta = 90°$

$\therefore \angle AOC = 90°$

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b) $\sqrt{6}\sin x + \sqrt{2}\cos x = R\cos(x-\alpha)$

$\sqrt{2}\cos x + \sqrt{6}\sin x = R\cos(x-\alpha)$

$R = \sqrt{(\sqrt{2})^2+(\sqrt{6})^2}$ $\quad \tan\alpha = \frac{\sqrt{6}}{\sqrt{2}}$

$R = \sqrt{8}$ $\quad \tan\alpha = \sqrt{3}$

$\alpha = \frac{\pi}{3}$

$\therefore \sqrt{6}\sin x + \sqrt{2}\cos x = \sqrt{8}\cos\left(x-\frac{\pi}{3}\right)$

Now solve $\quad 0 \le x \le 2\pi$

$\sqrt{8}\cos\left(x-\frac{\pi}{3}\right) = 2 \quad -\frac{\pi}{3} \le x-\frac{\pi}{3} \le \frac{5\pi}{3}$

$\cos\left(x-\frac{\pi}{3}\right) = \frac{\cancel{2}}{\cancel{2}\sqrt{2}}$

$\therefore x-\frac{\pi}{3} = \frac{-\pi}{4}, \frac{7\pi}{4}, \frac{\pi}{4}$

$\therefore x = \frac{\pi}{12}, \cancel{\frac{25\pi}{12}}, \frac{7\pi}{12}$

$x = \frac{\pi}{12}, \frac{7\pi}{12}$

c) ${}^nC_3 = p$ and ${}^nC_4 = q$

$\therefore {}^nC_{n-3} = p$ (symmetry of Pascal's Triangle)

${}^nC_3 + {}^nC_4 = {}^{n+1}C_4 = p+q$ (Sum of terms below)

${}^{n+1}C_4 = {}^{n+1}C_{n+1-4} = {}^{n+1}C_{n-3} = p+q$ (symmetry)

$\therefore {}^{n+1}C_{n-3} + {}^nC_{n-3} = p+q+p$

$= 2p+q$

d) (i) $\overrightarrow{AB} = \begin{pmatrix} 2--1 \\ -3--1 \end{pmatrix}$

$= \begin{pmatrix} 3 \\ -2 \end{pmatrix}$

$\hat{\overrightarrow{AB}} = \frac{1}{\sqrt{3^2+(-2)^2}}$

$= \frac{1}{\sqrt{13}}\begin{pmatrix} 3 \\ -2 \end{pmatrix}$

(ii) $\text{proj}_{\underset{\sim}{F}}\overrightarrow{AB} = \frac{\underset{\sim}{F}\cdot\overrightarrow{AB}}{|\overrightarrow{AB}|^2}\overrightarrow{AB}$

$= \frac{\begin{pmatrix} 2 \\ 5 \end{pmatrix}\cdot\begin{pmatrix} 3 \\ -2 \end{pmatrix}}{13}\begin{pmatrix} 3 \\ -2 \end{pmatrix}$

$= \frac{6-10}{13}\begin{pmatrix} 3 \\ -2 \end{pmatrix}$

$= -\frac{4}{13}\begin{pmatrix} 3 \\ -2 \end{pmatrix}$

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e) $\cos x - \sin x + 1 = 0$ Let $t = \tan\frac{x}{2}$

$\frac{1-t^2}{1+t^2} - \frac{2t}{1+t^2} + 1 = 0$ $\therefore\ 0 \le \frac{x}{2} \le 180$

$1 - \cancel{t^2} - 2t + 1 + \cancel{t^2} = 0$

$2 - 2t = 0$

$t = 1$

$\tan\frac{x}{2} = 1$

$\therefore \frac{x}{2} = 45$

$x = 90^\circ.$

Also must test $x = 180$

$\cos 180 - \sin 180 + 1 = 0$

$-1 - 0 + 1 = 0$ ✓

$\therefore$ Sol'ns are $90^\circ, 180^\circ.$

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13 a) $\frac{dy}{dx} = 6,\ x = 2$

$\frac{dy}{dx} = \frac{3y}{x}$

$\frac{dy}{3y} = \frac{dx}{x}$

$\frac{1}{3}\ln y = \ln x + c$

When $x = 2,\ \frac{dy}{dx} = 6$

$6 = \frac{3y}{2}$

$y = 4 \therefore (2,4)$ satisfies equation

$\frac{1}{3}\ln 4 = \ln 2 + c$

$\ln 2^{2/3} = \ln 2 + c$

$\therefore c = \ln 2^{-\frac{1}{3}}$

so $\frac{1}{3}\ln y = \ln x + \ln 2^{-\frac{1}{3}}$

$\ln y = 3\ln(x \times 2^{-\frac{1}{3}})$

$\ln y = \ln\left(\frac{x}{2^{\frac{1}{3}}}\right)^3$

$\therefore y = \frac{x^3}{2}$

b) (i) $\sin 3A = 3\sin A - 4\sin^3 A$

$LHS = \sin(2A + A)$

$= \sin 2A\cos A + \cos 2A \sin A$

$= 2\sin A\cos^2 A + (1 - 2\sin^2 A)\sin A$

$= 2\sin A(1 - \sin^2 A) + \sin A - 2\sin^3 A$

$= 2\sin A - 2\sin^3 A + \sin A - 2\sin^3 A$

$= 3\sin A - 4\sin^3 A$

$= RHS$

(ii) $6x - 8x^3 = 1$

Let $x = \sin A$

$6\sin A - 8\sin^3 A = 1$

$3\sin A - 4\sin^3 A = \frac{1}{2}$

$\sin 3A = \frac{1}{2}$ using (i)

$3A = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{13\pi}{6}, \frac{17\pi}{6}, \frac{25\pi}{6}, \frac{29\pi}{6}$

$\therefore A = \frac{\pi}{18}, \frac{5\pi}{18}, \cancel{\frac{13\pi}{18}}, \cancel{\frac{17\pi}{18}}, \frac{25\pi}{18}, \frac{29\pi}{18}$

There can only be 3 distinct solutions

$\sin\frac{\pi}{18} = \sin\frac{17\pi}{18}$ $\sin\frac{5\pi}{18} = \sin\frac{13\pi}{18}$ $\sin\frac{25\pi}{18} = \sin\frac{29\pi}{18}$

$\therefore$ 3 solutions are $\sin\frac{\pi}{18}, \sin\frac{5\pi}{18}, \sin\frac{25\pi}{18}$

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(iii) $6x - 8x^3 = 1$

$8x^3 - 6x + 1 = 0$

$8x^3 + 0x^2 - 6x + 1 = 0$

a b c d

$\sin\frac{\pi}{18} \times \sin\frac{5\pi}{18} \times \sin\frac{25\pi}{18}$

is the product of all 3 roots.

Product $= \frac{-d}{a}$

$= \frac{-1}{8}$ as required.

c) (i) $X \sim Bin(900, 0\cdot8)$

$n = 900 \quad p = 0\cdot8 \quad q = 0\cdot2$

$E(X) = np$

$= 900 \times 0\cdot8$

$= 720$

$Var(X) = np(1-p) = npq$

$= 900 \times 0\cdot8 \times 0\cdot2$

$= 144$

(ii) So for the Normal distribution

$\mu = 720$ and $\sigma^2 = 144 \therefore \sigma = 12$

$P(x > 744)$ is required.

This is 2 standard deviations above mean

From the normal distribution, the proportion having 2 standard deviations above the mean is $2\cdot5\%$

$\therefore P(x > 744) = 2\cdot5\%$ or $0\cdot025$

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14 a) Show true for $n=1$

$3^{2+4} - 2^{2} = 725$

which divides by 5

$\therefore$ true for $n=1$.

Assume result is true for $n=k$

ie: $3^{2k+4} - 2^{2k} = 5M$ where M is an integer

Now to show result is true for $n=k+1$

ie: that $3^{2(k+1)+4} - 2^{2(k+1)}$ divides by 5

$= 3^{2} \times 3^{2k+4} - 2^{2} \times 2^{2k}$

$= 9(5M + 2^{2k}) - 4 \times 2^{2k}$ using assumption

$= 45M + 9.2^{2k} - 4 \times 2^{2k}$

$= 45M + 5 \times 2^{2k}$

$= 5(9M + 2^{2k})$

Since M and k are integers, $9M + 2^{2k}$ is integral

so $N = 9M + 2^{2k}$, N is an integer

$= 5N$ ie: divisible by 5

Hence if true for $n=k$, then it is true for $n=k+1$

So by the principle of mathematical induction, result is true for all integers $n \geq 1$.

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b) (i) $\frac{d}{dx}(x\ln x) = x \times \frac{1}{x} + 1 \times \ln x$ (product rule)
$= 1 + \ln x$

(ii) $\text{Area} = \text{Square} - 2\int_{\frac{1}{e}}^{1} 1 + \ln x \, dx$

(by symmetry)

If $f(x) = 0 = 1 + \ln x$
$-1 = \ln x$
$e^{-1} = x = \frac{1}{e}$ is x intercept

Using part (i) $\frac{d}{dx}(x\ln x) = 1 + \ln x$
$x\ln x = \int 1 + \ln x \, dx$

$\therefore \text{Area} = 1 \times 1 - 2\left[x\ln x\right]_{\frac{1}{e}}^{1}$

$= 1 - 2\left[1 \times \ln 1 - \frac{1}{e}\ln\frac{1}{e}\right]$
$= 1 - 2\left(0 - \frac{1}{e}\ln e^{-1}\right)$
$= 1 - 2\left(+\frac{1}{e}\ln e\right)$
$= 1 - \frac{2}{e}$ or $\frac{e-2}{e}$

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c) $(1+x)^{11} = (1+x)^3(1+x)^8$

On LHS, $\binom{11}{5}$ is the coefficient of the x^5 term in its expansion. On the RHS x^5 terms occur when

1st exp.		2nd exp.
constant	$\times$	x^5
x	$\times$	x^4
x^2	$\times$	x^3
x^3	$\times$	x^2

ie: $1 \times \binom{8}{5} + \binom{3}{1}\binom{8}{4} + \binom{3}{2}\binom{8}{3} + \binom{3}{3}\binom{8}{2}$

So polynomial holds when x^5 terms on LHS = x^5 terms on RHS

ie: $\binom{11}{5} = \binom{8}{5} + \binom{3}{1}\binom{8}{4} + \binom{3}{2}\binom{8}{3} + \binom{8}{2}$ as required

d) $\int \frac{3}{\sqrt{4-9x^2}}\, dx$

$\frac{3}{3}\int \frac{1}{\sqrt{\frac{4}{9}-x^2}}\, dx$

$\int \frac{1}{\sqrt{(\frac{2}{3})^2-x^2}}\, dx$

$\sin^{-1} \frac{x}{2/3}$

$=\sin^{-1} \frac{3x}{2} + C$

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e) P(at least 2 males) = P(2 males) + P(3 males) + P(4 males) + P(5 males)

$= 1 - P(\text{no males}) - P(\text{one male})$

$= 1 - \dfrac{^{20}C_{13}}{^{25}C_{13}} - \dfrac{^{5}C_{1} \times {}^{20}C_{12}}{^{25}C_{13}}$

$= 0.86$ to 2 d.p.'s

f) Binomial Probability:

P(eight or more contract) = P(8) + P(9) + P(10)

$= {}^{10}C_8(0.74)^8 \times (0.26)^2 +$
${}^{10}C_9(0.74)^9 \times 0.26 +$
${}^{10}C_{10}(0.74)^{10}$

$= 0.50$

THE END