Trinity Grammar School

Mathematics Department

2016

TRIAL EXAMINATION

HSC ASSESSMENT TASK 4

Year 12

Mathematics Extension 1

Board of Studies Student Number
(Year 12 only)

Class Teacher:

………………………………………………………………

Name:
(Year 11 only)

………………………………………………………………

Do NOT write solutions on this question paper. Any working on this question paper will NOT be marked.

Date of examination:

Wednesday, 17 August 2016

General Instructions

- Reading time – 5 minutes
- Working time – 2 hours
- Write using black or blue pen
- Only approved calculators for this course are allowed in this task
- A Reference Sheet is supplied
- Show all necessary working
- Write your Board of Studies Student Number and your Class teacher on the question paper and on any answer sheets or writing booklets used to write your responses to the questions submitted
- If you do not attempt a question you must submit an answer sheet or writing booklet for that question clearly indicating **N/A** and your Student Number or Name.

- **HSC Assessment Weighting:** 40%

Total marks – 70

Section I

10 marks

- Attempt Questions 1-10
- Allow about **15** minutes for this section

Section II

60 marks

- Attempt Questions 11-14
- Allow about **1** hour **45** minutes for this section

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Section I 10 marks

- Attempt Questions 1 – 10
- Allow about 15 minutes for this section
- Shade the correct response on the multiple-choice answer sheet for Questions 1 – 10
- Each question is worth 1 mark

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1 What is the acute angle between the lines $x-2y+1=0$ and $2x-y-1=0$?

(A) 37°

(B) 45°

(C) 90°

(D) 143°

2 What is the number of asymptotes on the graph of $y=\dfrac{4}{x^2-4}$?

(A) 1

(B) 2

(C) 3

(D) 4

3 Which of the following is the correct expression for $\displaystyle\int \frac{dx}{\sqrt{9-x^2}}$?

(A) $\cos^{-1}\dfrac{x}{3}+C$

(B) $\cos^{-1}3x+C$

(C) $\sin^{-1}\dfrac{x}{3}+C$

(D) $\sin^{-1}3x+C$

4 A curve has parametric equations $x=t-3$ and $y=t^2+2$. What is the Cartesian equation of this curve?

(A) $y=x^2-x-1$

(B) $y=x^2+x-1$

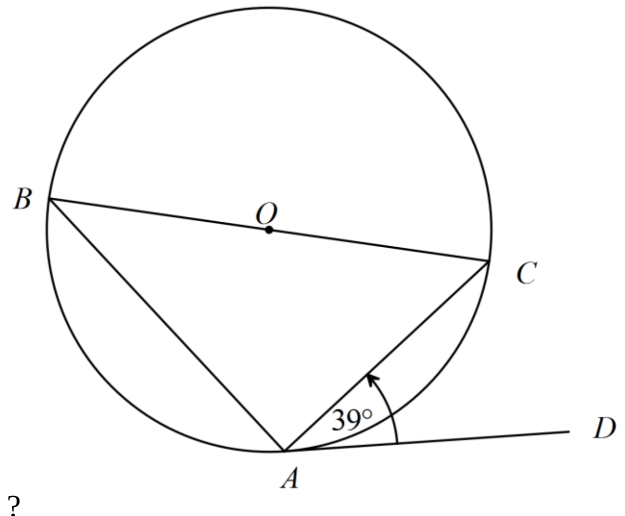
(C) $y=x^2-6x+11$

(D) $y=x^2+6x+11$

5 In the circle, centre O, BC is a diameter.

AD is a tangent to the circle with A being the point of contact. $\angle DAC = 39°$.

NOT TO SCALE



What is the size of $\angle BCA$?

(A) 39°

(B) 51°

(C) 78°

(D) 89°

6 What is the solution to the inequality $\dfrac{x^2-4}{x} \geq 0$?

(A) $-2 \leq x < 0$ or $x \geq 2$

(B) $-2 \geq x > 0$ or $x \leq 2$

(C) $-4 \leq x < 0$ or $x \geq 4$

(D) $-4 \geq x > 0$ or $x \leq 4$

7 What is the value of $\int_0^1 \dfrac{4x}{2x+1}dx$? Use the substitution $u = 2x+1$.

(A) $2-\log_e 2$

(B) $2-\log_e 3$

(C) $4-2\log_e 2$

(D) $4-2\log_e 3$

8 When the polynomial $P(x)$ is divided by $(x+1)(x-2)$ its remainder is $18x+17$.

What is the remainder when $P(x)$ is divided by $(x-2)$?

(A) $18x+15$

(B) -19

(C) 35

(D) 53

9 What is the term independent of x in the expansion of $\left(x^2-\frac{2}{x}\right)^9$?

(A) ${}^9C_3(-2)^3$

(B) ${}^9C_6(-2)^6$

(C) ${}^9C_3(2)^3$

(D) ${}^9C_6(2)^6$

10 A particle moves in a straight line with a displacement of x and velocity of v.

When $t=0$ the acceleration is $3x^2$, velocity $-\sqrt{2}$ and displacement is 1.

Which of the following is the correct equation for x as a function of t?

(A) $x=\dfrac{-2}{(t+\sqrt{2})^2}$

(B) $x=\dfrac{-2}{(t-\sqrt{2})^2}$

(C) $x=\dfrac{2}{(t+\sqrt{2})^2}$

(D) $x=\dfrac{2}{(t-\sqrt{2})^2}$

End of Section I

Section II commences on the next page

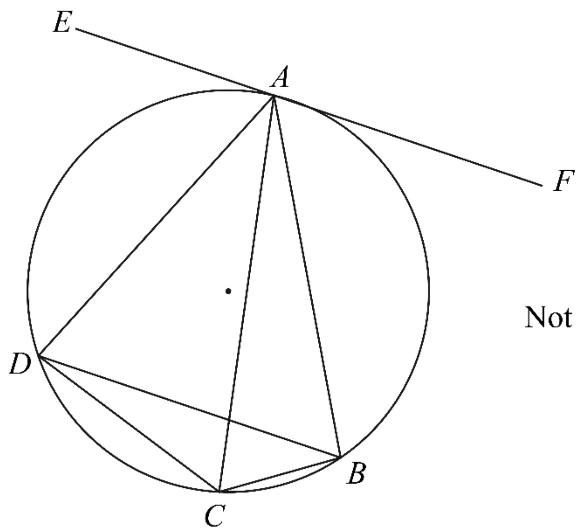
Section II 60 marks

- Attempt Questions 11 – 14
- Allow about 1 hour and 45 minutes for this section
- Answer each question in a SEPARATE writing booklet or answer sheet. Extra writing booklets are available.
- In Questions 11 – 14, your responses should include all relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Use a SEPARATE writing booklet **Marks**

(a) Use Newton's method to find a second approximation to the positive root of $x - 2\sin x = 0$. Take $x = 1.6$ as the first approximation, correct to 1 decimal places. **2**

(b) $ABCD$ is a cyclic quadrilateral. EAF is a tangent at A to the circle. CA bisects $\angle BCD$. **3**



Not to scale

Show that $EF \parallel DB$.

(c) What is the exact value of the definite integral $\int_0^{\frac{\pi}{3}} \sin^2 x \, dx$? **2**

(d) A particle moves in a straight line and its position at any time is given by:

$$x = 1 + \sqrt{3}\cos 4t + \sin 4t$$

(i) Prove the motion is simple harmonic. **2**

(ii) Find the amplitude of the motion. **1**

(iii) When does the particle first reach maximum speed after time $t = 0$? **2**

(e) The velocity V of a particle decreases according to the equation:

$$\frac{dV}{dt} = -k(V - P)$$

where t is the time in seconds and k is a positive constant. The initial velocity of the particle is 0 ms^{-1} and the terminal velocity or P is 60 ms^{-1}.

(i) Verify that $V = P + Ae^{-kt}$ is a solution of the above equation, where A is a constant. **1**

(ii) What is the value of k if the velocity of the particle after 10 seconds is 35 ms^{-1}? Answer correct to two significant places. **2**

End of Question 11

Question 12 **(15 marks)** Use a SEPARATE writing booklet **Marks**

(a) $P(2p, p^2)$ and $Q(2q, q^2)$ are two points on the parabola $x^2 = 4y$.

M is the midpoint of PQ.

(i) Show that $(p-q)^2 = 2(p^2+q^2)-(p+q)^2$. **1**

(ii) If P and Q move on the parabola so that $p-q=4$, show that the locus of M is the parabola $x^2 = 4y-16$. **2**

(iii) What is the focus of the locus of M? **1**

(b) The angle between the lines $y = mx$ and $y = \frac{1}{2}x$ is 45°. Find the value of m. **2**

(c) State the domain and range of $y = 4\cos^{-1}\left(\frac{3x}{2}\right)$. **2**

(d) (i) Show that $\cos(A+B)+\cos(A-B)=2\cos A\cos B$. **1**

(ii) Hence find $\int_0^1 \cos 2x \cos x \, dx$ correct to 3 significant figures. **3**

(e) By considering the expansion of $(1+x)^{2n}$ show that $\sum_{r=1}^{2n} r\binom{2n}{r} = n \times 2^{2n}$. **3**

End of Question 12

Question 13 **(15 marks)** Use a SEPARATE writing booklet **Marks**

(a) Use the substitution $u = x+1$ to evaluate $\int_0^{15} \frac{x}{\sqrt{x+1}}\,dx$. **3**

(b) Use the principle of mathematical induction to prove for $n \geq 1$ that **3**

$$\frac{1}{2!}+\frac{2}{3!}+\frac{3}{4!}+...+\frac{n}{(n+1)!}=1-\frac{1}{(n+1)!}$$

(c) (i) Express $\cos x - \sqrt{3}\sin x$ in the form $R\cos(x+\alpha)$ where $R > 0$ and α is an acute angle. **2**

(ii) Hence, or otherwise, solve the equation $\cos x - \sqrt{3}\sin x = 2$ for $0 \leq x \leq 2\pi$. **1**

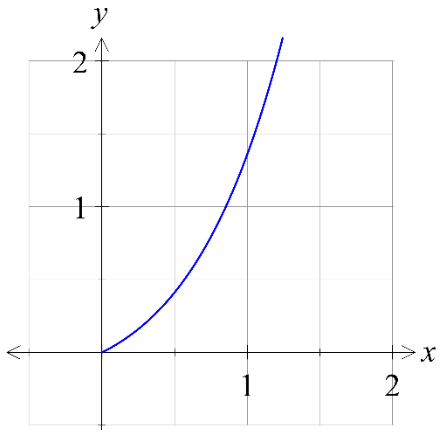
(d) What are the roots of the equation $x^3 + 6x^2 - x - 30 = 0$ given one root is the sum of the other two roots? **3**

(e) Consider the function $f(x) = \dfrac{xe^x}{2}$ for $x \geq 0$.

(i) Show that $f'(x) > 0$ for all x in the domain. **1**

(ii) Explain why $f(x)$ has an inverse function $f^{-1}(x)$. **1**

(iii) Copy the sketch of $y = f(x)$ below and insert a sketch of $y = f^{-1}(x)$. **1**



End of Question 13

Question 14 (15 marks) Use a SEPARATE writing booklet **Marks**

(a) A particle is moving in a straight line with simple harmonic motion.

At time t seconds it has displacement x metres from a fixed point 0 on the line, velocity v ms^{-1} and acceleration a ms^{-2} is given by $a = -4x + 4$.

Initially, the particle is 2 metres to the right of 0 and is moving away from 0 with a speed of $2\sqrt{3}$ ms^{-1}.

(i) Use integration to show that $v^2 = -4(x-3)(x+1)$. **2**

(ii) Hence, find the centre and amplitude of the motion. **1**

(b) A projectile is fired with velocity $V\ ms^{-1}$ at an angle θ above the horizontal.

The horizontal and vertical displacements (in metres) from the point of projection at time t seconds are given by $x = Vt\cos\theta$ and $y = Vt\sin\theta - \frac{1}{2}gt^2$ respectively (where g is the acceleration due to gravity).

The particle reaches a maximum height of H metres after T seconds.

(i) Show that when $t = \frac{1}{2}T$ the height of the particle is $\frac{3}{4}H$. **3**

(ii) Show that when $t = \frac{1}{2}T$ the particle is moving on a path **3**

inclined at an angle α to the horizontal such that $\tan\alpha = \frac{1}{2}\tan\theta$.

(c) (i) Expand $(1+x)^n$ using the binomial theorem. **1**

(ii) Show that $\sum_{r=1}^{n} {}^{n}C_r = 2^n - 1$ **1**

(iii) Show that $\frac{1}{n+1}\sum_{r=1}^{n+1} {}^{n+1}C_r = \sum_{r=0}^{n} \frac{1}{r+1} {}^{n}C_r$ **2**

Hint: Integrate the identity in part (i) between the limits of 0 and 1.

(d) Find all real x such that $|4x-1| > 2\sqrt{x(1-x)}$ **2**

End of Question 14

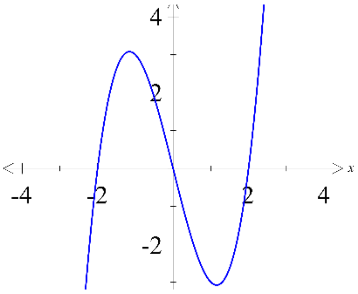
End of Section II

End of Examination

Trinity Grammar School Examination 2016

HSC Mathematics Extension 1 Yearly Examination

Worked solutions and marking guidelines

Section I		
	Solution	**Criteria**
1	$x-2y+1=0$ $y=\frac{1}{2}x+\frac{1}{2}$ $m_1=\frac{1}{2}$ $2x-y-1=0$ $y=2x-1$ $m_2=2$ $\tan\theta=\left\|\frac{m_1-m_2}{1+m_1m_2}\right\|=\left\|\frac{\frac{1}{2}-2}{1+\frac{1}{2}\times 2}\right\|=\frac{3}{4}$ $\theta=36.86989765...$ $\approx 37^\circ$	1 Mark: A
2	$x\neq 1$ or $x\neq -1$ $x\to\infty \quad y=\frac{4}{(x+2)(x-2)}=\frac{2}{(x+2)}\times\frac{2}{(x-2)}\to 0$ $\therefore x=2,\ x=-2$ and $y=0$ are asymptotes. Number of asymptotes is 3.	1 Mark: C
3	$\int\frac{dx}{\sqrt{9-x^2}}=\sin^{-1}\frac{x}{3}+C$	1 Mark: C
4	$x=t-3$ or $t=x+3$ Substitute $x+3$ for t into $y=t^2+2$ $y=(x+3)^2+2=x^2+6x+11$	1 Mark: D
5	$\angle CAB=90°$ (angle in a semicircle) $\angle ABC=\angle DAC=39°$ (angle between a tangent and a chord at point of contact is equal to the angle in the alternate segment) $\angle BCA=51°$ (angle sum of ΔABC)	1 Mark: B
6	$x^2\times\frac{x^2-4}{x}\geq 0\times x^2 \quad x\neq 0$ $x(x^2-4)\geq 0$ $x(x-2)(x+2)\geq 0$ $-2\leq x<0$ or $x\geq 2$  	1 Mark: A

7	$u = 2x+1 \qquad x=1$ then $u=3$ $du = 2dx \qquad x=0$ then $u=1$ $\int_0^1 \frac{4x}{2x+1}dx = \int_1^3 \frac{2(u-1)}{u} \times \frac{1}{2}du$ $= \int_1^3 1 - \frac{1}{u}du$ $= \left[u - \log_e u\right]_1^3$ $= \left[(3-\log_e 3) - (1-\log_e 1)\right] = 2 - \log_e 3$	1 Mark: B
8	$P(x) = (x+1)(x-2)Q(x) + 18x + 17$ When $P(x)$ is divided by $(x-2)$ the remainder is $P(2)$. $P(2) = (2+1)(2-2)Q(2) + 18(2) + 17$ $= 3(0)Q(2) + 36 + 17$ $= 0 + 53$ $= 53$	1 Mark: D
9	$T_{r+1} = {}^9C_r (x^2)^{9-r}\left(-\frac{2}{x}\right)^r$ $= {}^9C_r x^{18-2r}(-2)^r x^{-r} = {}^9C_r(-2)^r x^{18-3r}$ Term independent of x $18-3r=0$ $r=6$ $T_7 = {}^9C_6(-2)^6 x^{18-3\times6} = {}^9C_6(-2)^6$	1 Mark: B
10	$a = 3x^2$ $v^2 = 2\int\left(3x^2\right)dx = 2x^3 + C$ When $x=1$, $v=-\sqrt{2}$ then $C=0$ $v = -\sqrt{2x^3}$ ($v<0$ when $x=1$) $\frac{dx}{dt} = -\sqrt{2x^3}$ $\frac{dt}{dx} = -\frac{1}{\sqrt{2}}x^{-\frac{3}{2}}$ $t = \frac{2}{\sqrt{2}}x^{-\frac{1}{2}} + C$ When $t=0$, $x=1$ then $C=-\sqrt{2}$	1 Mark: C

	$t=\sqrt{2}x^{-\frac{1}{2}}-\sqrt{2}$ $x^{-\frac{1}{2}}=\dfrac{t+\sqrt{2}}{\sqrt{2}}$ $x=\dfrac{2}{(t+\sqrt{2})^2}$	

Section II		
11(a)	$f(x)=x-2\sin x$ $f'(x)=1-2\cos x$ $x_1=x_0-\dfrac{f(x_0)}{f'(x_0)}$ $=1.6-\dfrac{1.6-2\sin 1.6}{1-2\cos 1.6}$ $=1.977123551...$ ≈ 1.98	2 Marks: Correct answer. 1 Mark: Finds $f(1.6)$, $f'(1.6)$ or shows some understanding of Newton's method.
11(b)	$\angle BAF=\angle BCA$ (Angle between a tangent and a chord is equal to the angle in the alternate segment) $\angle BCA=\angle DCA$ (CA bisects $\angle BCD$) $\angle DCA=\angle DBA$ (Angles in the same segment of a circle are equal) $\angle BAF=\angle DBA$ (from above) $\therefore EAF \parallel DB$ (Alternate angles $\angle BAF=\angle DBA$ are only equal when two lines are parallel)	3 Marks: Correct answer. 2 Marks: Makes some progress towards the solution. 1 Mark: States one relevant statement and circle theorem.
11(c)	$\int_0^{\frac{\pi}{3}}\sin^2 x\,dx=\int_0^{\frac{\pi}{3}}\frac{1}{2}(1-\cos 2x)dx$ $=\left[\frac{1}{2}x-\frac{1}{4}\sin 2x\right]_0^{\frac{\pi}{3}}$ $=\left[\left(\frac{\pi}{6}-\frac{1}{4}\sin\frac{2\pi}{3}\right)-\left(0-\frac{1}{4}\sin 0\right)\right]$ $=\frac{\pi}{6}-\frac{\sqrt{3}}{8}$ $=\dfrac{4\pi-3\sqrt{3}}{24}$	2 Marks: Correct answer. 1 Mark: Applies the double angle trig identity.
11(d) (i)	Simple harmonic motion occurs when $\ddot{x}=-n^2x$	2 Marks: Correct answer.

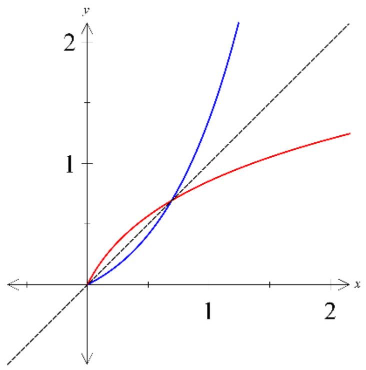
	Now $x = 1 + \sqrt{3}\cos 4t + \sin 4t$ $\dot{x} = -\sqrt{3} \times 4\sin 4t + 4\cos 4t$ $\ddot{x} = -\sqrt{3} \times 4^2 \cos 4t - 4^2 \sin 4t$ $= -4^2\left(\sqrt{3}\cos 4t + \sin 4t\right)$ $\ddot{x} = -4^2(x-1)$	1 Mark: Recognises the condition for SHM.

11(d) (ii)	$x = 1+\sqrt{3}\cos 4t + \sin 4t$ $= 1 + 2\sin\frac{\pi}{3}\cos 4t + 2\cos\frac{\pi}{3}\sin 4t$ $= 1 + 2\left[\sin 4t\cos\frac{\pi}{3} + \cos 4t\sin\frac{\pi}{3}\right]$ $= 1 + 2\sin(4t + \frac{\pi}{3})$ (In the form $x = b + a\sin(nt+\alpha)$) Amplitude is 2.	1 Mark: Correct answer.
11(d) (iii)	Maximum speed at $\ddot{x} = 0$ or $x = 0$ (centre of motion) $\ddot{x} = -4^2\left(\sqrt{3}\cos 4t + \sin 4t\right) = 0$ $\sin 4t + \sqrt{3}\cos 4t = 0$ $\frac{\sin 4t}{\cos 4t} = -\sqrt{3}$ $\tan 4t = -\sqrt{3}$ $4t = \frac{2\pi}{3}, \frac{5\pi}{3}, \ldots$ $t = \frac{\pi}{6}, \frac{5\pi}{12}, \ldots$ Particle first reaches maximum speed at $t = \frac{\pi}{6}$	2 Marks: Correct answer. 1 Mark: Makes some progress towards the solution.
11(e) (i)	$V = P + Ae^{-kt}$ or $Ae^{-kt} = V - P$ $\frac{dV}{dt} = -kAe^{-kt}$ $= -k(V-P)$	1 Mark: Correct answer.
11(e) (ii)	Initially $t = 0$ and $V = 0$, $P = 60$ $V = P + Ae^{-kt}$ $0 = 60 + Ae^{-k\times 0}$ $A = -60$ Also $t = 10$ and $V = 35$ $35 = 60 - 60e^{-k\times 10}$ $e^{-10k} = \frac{25}{60}$ $-10k = \log_e\frac{25}{60}$ $k = -\frac{1}{10}\log_e\frac{25}{60} = \frac{1}{10}\log_e\frac{60}{25}$ $= 0.087546873\ldots \approx 0.088$	2 Marks: Correct answer. 1 Mark: Finds the value of *A*.

12(a) (i)	$\text{RHS} = 2(p^2+q^2)-(p+q)^2$ $= 2p^2+2q^2-p^2-2pq-q^2$ $= p^2-2pq+q^2$ $=(p-q)^2$ $=$ LHS	1 Mark: Correct answer.
12(a) (ii)	Coordinates of $M\left(\dfrac{2p+2q}{2},\dfrac{p^2+q^2}{2}\right)$ or $\left(p+q,\dfrac{p^2+q^2}{2}\right)$ Using the result in (i) and $p-q=4$ (given) $(p-q)^2 = 2(p^2+q^2)-(p+q)^2$ $4^2 = 2(2y)-(x)^2$ $x^2 = 4y-16$ Therefore the locus of M is the parabola $x^2 = 4y-16$	2 Marks: Correct answer. 1 Mark: Determines the coordinates of M or makes some progress.
12(a) (iii)	Now $x^2 = 4y-16$ $=4(y-4)$ Focal length is 1, vertex $(0,4)$ and parabola is concave up. Focus is $(0,5)$	1 Mark: Correct answer.
12(b)	For $y=mx$ then $m_1=m$. For $y=\frac{1}{2}x$ then $m_2=\frac{1}{2}$ $\tan 45^\circ = \left\lvert\dfrac{m-\frac{1}{2}}{1+m\times\frac{1}{2}}\right\rvert$ $1=\left\lvert\dfrac{2m-1}{2}\times\dfrac{2}{2+m}\right\rvert$ or $\left\lvert\dfrac{2m-1}{2+m}\right\rvert=1$ $\dfrac{2m-1}{2+m}=1$ or $\dfrac{2m-1}{2+m}=-1$ $2m-1=2+m$ $2m-1=-2-m$ $m=3$ $m=-\frac{1}{3}$	2 Marks: Correct answer. 1 Mark: Uses the formula with one correct gradient.
12(c)	Domain: $-1\le(\dfrac{3x}{2})\le 1$. Range: $0\le\cos^{-1}\left(\dfrac{3x}{2}\right)\le\pi$ $-\dfrac{2}{3}\le x\le\dfrac{2}{3}$ $0\le\dfrac{y}{4}\le\pi$ $0\le y\le 4\pi$	2 Marks: Correct answer. 1 Mark: Finds the domain or the range.
12 (d) (i)	$\cos(A+B)+\cos(A-B)=(\cos A\cos B-\sin A\sin B)+(cosA\cos B+\sin A\sin B)$ $=2\cos A\cos B$	1 Mark: correct answer

12 (d) (ii)	From (i) $2\cos 2x\cos x=\cos(2x+x)+\cos(2x-x)$ $\cos 2x\cos x=\frac{1}{2}(\cos 3x+\cos x)$ $\int_0^1 \cos 2x\cos x\,dx=\frac{1}{2}\int_0^1(\cos 3x+\cos x)\,dx$ $=\frac{1}{2}\left[\frac{1}{3}\sin 3x+\sin x\right]_0^1$ $=\frac{1}{2}\left(\frac{1}{3}\sin 3+\sin 1\right)$ $=0.444255....$ $=0.444$ (3 sig fig.)	3 Marks: Correct answer (no marks lost for not writing to 3 sig. fig.) 2 Marks: Correct integration. 1 Mark: Some attempt to rewrite integral using result from (i)
12 (e)	$(1+x)^{2n}=\binom{2n}{0}+\binom{2n}{1}x+\binom{2n}{2}x^2+......+\binom{2n}{2n-1}x^{2n-1}+\binom{2n}{2n}x^{2n}$ differentiate both sides $2n(1+x)^{2n-1}=\binom{2n}{1}+2\binom{2n}{2}x+......+(2n-1)\binom{2n}{2n-1}x^{2n-2}+2n\binom{2n}{2n}x^{2n-1}$ substitute $x=1$ $2n(2)^{2n-1}=\binom{2n}{1}+2\binom{2n}{2}+......+(2n-1)\binom{2n}{2n-1}+2n\binom{2n}{2n}$ $n(2\times 2^{2n-1})=\sum_{r=1}^{2n} r\binom{2n}{r}$ $\sum_{r=1}^{2n} r\binom{2n}{r}=n\times 2^{2n}$	3 Marks: Correctly establishes the result 2 Marks: Differentiates both sides of the expression 1 Mark: Correct expansion of $(1+x)^{2n}$
13(a)	$u=x+1$ when $x=0$ then $u=1$ $\frac{du}{dx}=1$ or $du=dx$ $x=15$ then $u=16$ $\int_0^{15}\frac{x}{\sqrt{x+1}}\,dx=\int_1^{16}\frac{u-1}{\sqrt{u}}\,du=\int_1^{16}\left(u^{\frac{1}{2}}-u^{-\frac{1}{2}}\right)du$ $=\left[\frac{2}{3}u^{\frac{3}{2}}-2u^{\frac{1}{2}}\right]_1^{16}$ $=\left(\frac{2}{3}\times 16^{\frac{3}{2}}-2\times 16^{\frac{1}{2}}\right)-\left(\frac{2}{3}\times 1^{\frac{3}{2}}-2\times 1^{\frac{1}{2}}\right)=36$	3 Marks: Correct answer. 2 Marks: Finds the primitive as a function of u. 1 Mark: Sets up the integration using the substitution

13(b)	Step 1: To prove the statement true for $n=1$ $\text{LHS}=\frac{1}{2!}=\frac{1}{2}\qquad \text{RHS}=1-\frac{1}{2!}=1-\frac{1}{2}=\frac{1}{2}$ Result is true for $n=1$ Step 2: Assume the result true for $n=k$ $\frac{1}{2!}+\frac{2}{3!}+\frac{3}{4!}+...+\frac{k}{(k+1)!}=1-\frac{1}{(k+1)!}$ To prove the result is true for $n=k+1$ $\frac{1}{2!}+\frac{2}{3!}+\frac{3}{4!}+...+\frac{k}{(k+1)!}+\frac{(k+1)}{(k+2)!}=1-\frac{1}{(k+2)!}$ $\text{LHS}=\frac{1}{2!}+\frac{2}{3!}+\frac{3}{4!}+...+\frac{k}{(k+1)!}+\frac{(k+1)}{(k+2)!}$ $=1-\frac{1}{(k+1)!}+\frac{(k+1)}{(k+2)!}$ $=1-\frac{(k+2)}{(k+2)(k+1)!}+\frac{(k+1)}{(k+2)!}$ $\text{LHS}=1-\frac{(k+2)-(k+1)}{(k+2)!}$ $=1-\frac{1}{(k+2)!}=$ RHS Result is true for $n=k+1$ if true for $n=k$ Step 3: Result true by principle of mathematical induction.	3 Marks: Correct answer. 2 Marks: Proves the result true for $n=1$ and attempts to use the result of $n=k$ to prove the result for $n=k+1$. 1 Mark: Proves the result true for $n=1$.
13(c) (i)	$R\cos(x+\alpha)=\cos x-\sqrt{3}\sin x$ $R\cos(x+\alpha)=R\cos x\cos\alpha-R\sin x\sin\alpha$ Hence $R\cos\alpha=1$ and $R\sin\alpha=\sqrt{3}$ Dividing these equations $\tan\alpha=\frac{\sqrt{3}}{1}$ or $\alpha=\tan^{-1}\sqrt{3}$ or $\frac{\pi}{3}$ Squaring and adding the equations $R^2=1^2+\sqrt{3}^2$ or $R=2$ $\cos x-\sqrt{3}\sin x=2\cos(x+\frac{\pi}{3})$	2 Marks: Correct answer. 1 Mark: Determines the value of α or R
13(c) (ii)	$2\cos(x+\frac{\pi}{3})=2$ from (i) $\cos(x+\frac{\pi}{3})=1$ $x+\frac{\pi}{3}=0,2\pi,4\pi,...$ $x=\frac{5\pi}{3}$	1 Mark: Correct answer.

13(d)	Let the roots be α, β and γ with $\alpha = \beta + \gamma$ $x^3 + 6x^2 - x - 30 = 0$ $\alpha + (\beta + \gamma) = -\frac{b}{a} = -\frac{6}{1} = -6$ $\alpha + \alpha = -6$ $\alpha = -3$ $\alpha\beta\gamma = -\frac{d}{a} = -\frac{-30}{1} = 30$ $-3 \times \beta\gamma = 30$ $\beta\gamma = -10 \quad (1)$ Also $\beta + \gamma = -3 \quad (2)$ By inspection (or solving the equations 1 & 2 simultaneously) $\beta = -5$ and $\gamma = 2$ Roots are $x = -5,\ x = -3$ and $x = 2$	3 Marks: Correct answer. 2 Marks: Makes significant progress towards the solution. 1 Mark: Finds the sum or product of the roots.
13(e) (i)	$f'(x) = \frac{x}{2}e^x + e^x\frac{1}{2}$ $= \frac{e^x(x+1)}{2}$ $> 0 \quad$ when $x \geq 0$ then $e^x \geq 1$	1 Mark: Correct answer.
13(e) (ii)	The function $y = f(x)$ is an increasing function defined for $x \geq 0$. It has no turning points in this domain. Hence an inverse function $y = f^{-1}(x)$ exists as the function $y = f(x)$ is a one-to-one increasing function (it satisfies the horizontal line test).	1 Mark: Correct answer.
13(e) (iii)	$y = f^{-1}(x)$ 	1 Mark: Correct answer.

14(a) (i)	$\frac{d}{dx}(\frac{1}{2}v^2) = -4x+4$ $\frac{1}{2}v^2 = -2x^2+4x+C$ When $t=0$, $x=2$, $v=2\sqrt{3}$ $\frac{1}{2}\times\left(2\sqrt{3}\right)^2 = -2\times 2^2+4\times 2+C$ $C=6$ $\frac{1}{2}v^2 = -2x^2+4x+6$ $v^2 = -4x^2+8x+12$ $= -4(x^2-2x-3)$ $v^2 = -4(x-3)(x+1)$	2 Marks: Correct answer. 1 Mark: Finds an expression for $\frac{1}{2}v^2$ in terms of *x*.
14(a) (ii)	When $v=0$ indicates the boundaries for displacement. $\{x:-1\le x\le 3\}$ Therefore motion is centred at 1 m to the right of *O* with an amplitude of 2 m.	1 Mark: Correct answer.

14(b) (i)	Maximum height occurs when $\dot{y}=0$ $V\sin\theta - gt = 0$ $T=\dfrac{V\sin\theta}{g}$ $y_{\max}=H=V\left(\dfrac{V\sin\theta}{g}\right)\sin\theta-\dfrac{1}{2}g\left(\dfrac{V\sin\theta}{g}\right)^2$ $=\dfrac{V^2\sin^2\theta}{g}-\dfrac{V^2\sin^2\theta}{2g}$ $H=\dfrac{V^2\sin^2\theta}{2g}$ when $t=\dfrac{1}{2}T=\dfrac{V\sin\theta}{2g}$ $y=V\left(\dfrac{V\sin\theta}{2g}\right)\sin\theta-\dfrac{1}{2}g\left(\dfrac{V\sin\theta}{2g}\right)^2$ $=\dfrac{V^2\sin^2\theta}{2g}-\dfrac{V^2\sin^2\theta}{8g}$ $=\dfrac{3V^2\sin^2\theta}{8g}$ $=\dfrac{3}{4}\left(\dfrac{V^2\sin^2\theta}{2g}\right)$ $=\dfrac{3}{4}H$	3 Marks: Correctly shows result. 2 Maarks: Correctly finds the maximum height of the particle in terms of θ. 1 Mark: Makes some progress in finding maximum height (e.g. makes some attempt to solve $\dot{y}=0$.
14(b) (ii)	When $t=\dfrac{1}{2}T=\dfrac{V\sin\theta}{2g}$ $\dot{x}=V\cos\theta$ $\dot{y}=V\sin\theta-g\left(\dfrac{V\sin\theta}{2g}\right)$ $=\dfrac{V\sin\theta}{2}$ horizontal component of velocity $=V\cos\theta$ vertical component of velocity $=\dfrac{V\sin\theta}{2}$ $\tan\alpha=\dfrac{\frac{V\sin\theta}{2}}{V\cos\theta}$ $=\dfrac{1}{2}\tan\theta$	3 Marks: Correctly shows result. 2 Marks: Finds horizontal and vertical component of velocity and makes some attempt to find $\tan\alpha$. 1 Mark: Finds horizontal or vertical component of velocity.

14(c) (i)	$(1+x)^n = {}^nC_0 1^n + {}^nC_1 1^{n-1}x^1 + {}^nC_2 1^{n-2}x^2 + \ldots + {}^nC_n 1^1 x^n$ $= {}^nC_0 + {}^nC_1 x^1 + {}^nC_2 x^2 + \ldots + {}^nC_n x^n$	1 Mark: Correct answer.
14(c) (ii)	Substitute $x=1$ into the above identity. $(1+1)^n = {}^nC_0 + {}^nC_1 \times 1^1 + {}^nC_2 \times 1^2 + \ldots + {}^nC_n \times 1^n$ $2^n = 1 + {}^nC_1 + {}^nC_2 + \ldots + {}^nC_n \qquad ({}^nC_0 = 1)$ $\sum_{r=1}^{n} {}^nC_r = 2^n - 1$	1 Marks: Correct answer. 0 Mark: Shows some understanding.
14 (c) (iii)	$\int_0^1 (1+x)^n\,dx = \int_0^1 {}^nC_0 + {}^nC_1 x^1 + {}^nC_2 x^2 + \ldots + {}^nC_n x^n dx$ $\left[\frac{(1+x)^{n+1}}{n+1}\right]_0^1 = \left[{}^nC_0 x + \frac{1}{2}\,{}^nC_1 x^2 + \frac{1}{3}\,{}^nC_2 x^3 + \ldots + \frac{1}{n+1}\,{}^nC_n x^{n+1}\right]_0^1$ $\frac{1}{n+1}\left[2^{n+1}-1\right] = {}^nC_0 + \frac{1}{2}\,{}^nC_1 + \frac{1}{3}\,{}^nC_2 + \ldots + \frac{1}{n+1}\,{}^nC_n$ $\frac{1}{n+1}\sum_{r=1}^{n+1} {}^{n+1}C_r = \sum_{r=0}^{n} \frac{1}{r+1}\,{}^nC_r$	2 Marks: Correct answer. 1 Mark: Makes some progress towards the solution.
14(d)	Inequality is only defined for $x(1-x) \ge 0$ (Cannot find the square root of a negative number) $\therefore 0 \le x \le 1 \quad (1)$ Using the result $\|x\| = \sqrt{x^2}$ or $\|4x-1\| = \sqrt{(4x-1)^2}$ $\sqrt{(4x-1)^2} > 2\sqrt{x(1-x)}$ $(4x-1)^2 > 4x(1-x)$ $16x^2 - 8x + 1 > 4x - 4x^2$ $20x^2 - 12x + 1 > 0$ $(10x-1)(2x-1) > 0$ $\therefore x < \frac{1}{10}$ or $x > \frac{1}{2} \quad (2)$ Combining results (1) and (2) $\therefore 0 \le x < \frac{1}{10}$ and $\frac{1}{2} < x \le 1$	2 Marks: Correct answer. 1 Marks: Finds one correct region or makes significant progress. 0 Mark: Finds $0 \le x \le 1$ or uses $\|x\| = \sqrt{x^2}$ or shows some understanding.