TRINITY GRAMMAR SCHOOL

Mathematics Department

(NESA Student Number | Year 12 only)

2020

Year 12

Mathematics Extension 1

TRIAL EXAMINATION

HSC ASSESSMENT TASK 4

Date of Assessment Task: Monday, 24 August 2020

General Instructions

- Reading time – 10 minutes
- Working time – 2 hours
- Write using black pen
- NESA approved calculators may be used
- A reference sheet is provided
- In Questions 11 – 14, show relevant mathematical reasoning and/or calculations
- Write your NESA Student Number (Year 12 HSC) and your Class teacher on the question paper and on any answer sheets or writing booklets used to write your responses to the questions submitted
- If you do not attempt a question you must submit an answer sheet or writing booklet for that question clearly indicating N/A and your NESA Student Number or Name

Total marks: 70

Section I – 10 marks (pages 2 – 5)

- Attempt Questions 1 – 10
- Allow about 15 minutes for this section

Section II – 60 marks (pages 6 – 12)

- Attempt Questions 11 – 14
- Allow about 1 hour and 45 minutes for this section

- **HSC Assessment Weighting:** 30%

Section I

10 marks
Attempt Questions 1 – 10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1 – 10.

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1 What is the derivative of $5\cos^{-1}\left(\dfrac{x}{9}\right)$?

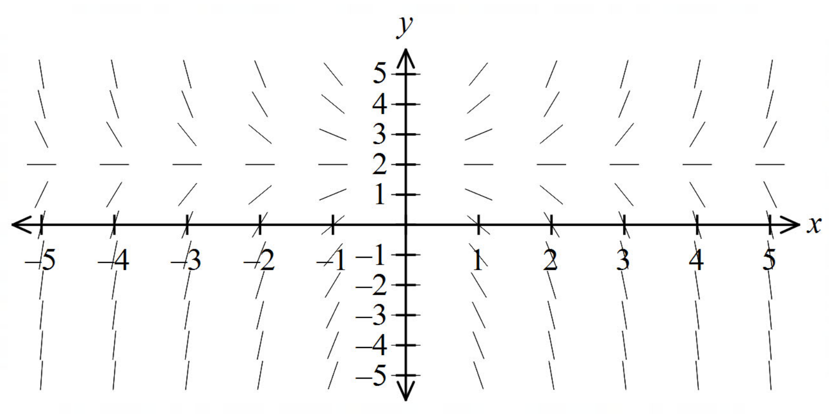
A. $-\dfrac{1}{\sqrt{81-x^2}}$

B. $-\dfrac{5}{\sqrt{81-x^2}}$

C. $-\dfrac{1}{\sqrt{9-x^2}}$

D. $-\dfrac{5}{\sqrt{9-x^2}}$

2 Which of the following differential equations could be represented by the slope field diagram below?



A. $y' = x(y-2)$

B. $y' = y(x-2)$

C. $y' = x(y+2)$

D. $y' = y(x+2)$

3 Over a long period of time, an archer finds that she is successful on 90% of her attempts. If X represents the number of successful attempts, which of the following correctly describes the skewness of the binomial distribution for $P(X = x)$?

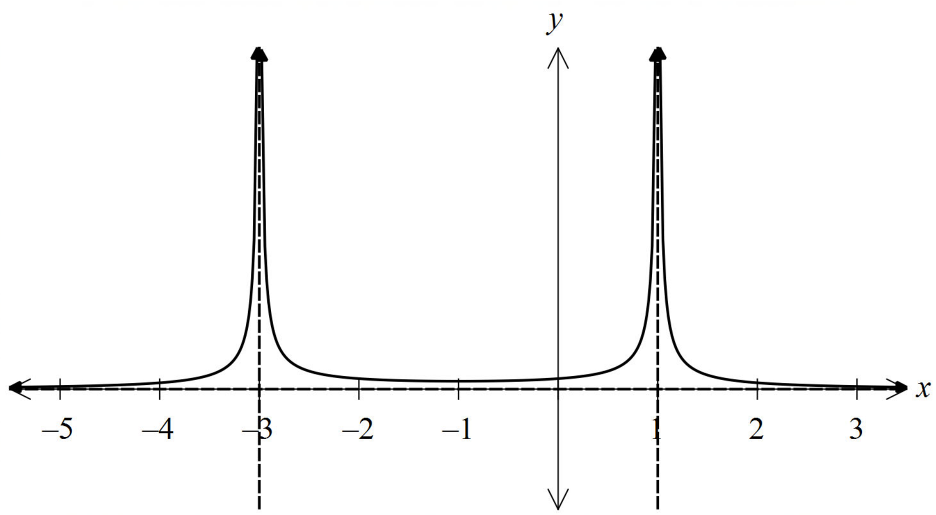
A. The distribution is negatively skewed

B. The distribution is normal

C. The distribution is positively skewed

D. The distribution is symmetrical

4



The graph above shows $y = \dfrac{1}{\sqrt{f(x)}}$. Note that $y = 0$ is a horizontal asymptote.

Which of the equations below best represents $y = f(x)$?

A. $f(x) = (x-1)(x+3)$

B. $f(x) = (x-1)(x+3)^2$

C. $f(x) = (x+3)^2(x-1)^2$

D. $f(x) = (x-3)^2(x+1)^2$

5 A group of 75 Trinity Mathematics Extension 1 students were going into 8 different venues to complete their Trial exam due to physical distancing. Which of the following statements are true if all the students were allocated into various venues? Note that all venues have more than enough space to hold everyone.

A. There will be at least one venue with at most 8 students.

B. There will be at least one venue with at least 10 students.

C. There will be at most three venues with at least 12 students.

D. There will always be 1 venue that has no student.

6 If $(x-2)$ is a common factor of $x^2+(a-3)x-(a+b)$ **and** $2x^2-(a-1)x+b$, then which of the following is the value of *a* and of *b*.

A. $a=-8,\ b=6.$

B. $a=-6,\ b=8.$

C. $a=6,\ b=8.$

D. $a=8,\ b=6.$

7 From 8 green and 8 white cards, 4 are chosen at random. If each outcome is equally likely, how many combinations are possible that have at most 1 green card?

A. 518

B. 930

C. 1820

D. 3920

8 If $a < 0$ and $b < 0$, then $|a - b|$ is equal to

A. $|a| - |b|$.

B. $|a| + |b|$.

C. $-|b - a|$.

D. Neither A, nor B, nor C.

9 A curve H has parametric equation $x = \sin^2 t$ and $y = 9\cos^2 t$ for $t \in \mathbb{R}$.

What is the Cartesian equation of H?

A. $y = 3x - 3$ for $0 \le x \le 1$

B. $y = 3x - 3$ for $x \in \mathbb{R}$

C. $y = 9 - 9x$ for $0 \le x \le 1$

D. $y = 9 - 9x$ for $x \in \mathbb{R}$

10 $\underset{\sim}{\mathbf{a}}$ and $\underset{\sim}{\mathbf{b}}$ are vectors with angle θ between them such that $0 < \theta < 90^\circ$, which of the following is true?

A. $|\underset{\sim}{\mathbf{a}} + \underset{\sim}{\mathbf{b}}| < |\underset{\sim}{\mathbf{a}}| + |\underset{\sim}{\mathbf{b}}|$

B. $|\underset{\sim}{\mathbf{a}} + \underset{\sim}{\mathbf{b}}| > |\underset{\sim}{\mathbf{a}}| + |\underset{\sim}{\mathbf{b}}|$

C. $|\underset{\sim}{\mathbf{a}} + \underset{\sim}{\mathbf{b}}| < \sqrt{|\underset{\sim}{\mathbf{a}}|^2 + |\underset{\sim}{\mathbf{b}}|^2}$

D. $|\underset{\sim}{\mathbf{a}} + \underset{\sim}{\mathbf{b}}| = \sqrt{|\underset{\sim}{\mathbf{a}}|^2 + |\underset{\sim}{\mathbf{b}}|^2}$

Section II

60 marks
Attempt Questions 11 – 14
Allow about 1 hour and 45 minutes for this section

Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.

In Questions 11 – 14, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Use a SEPARATE writing booklet.

(a) Solve $|4x-5| \geq 2$. **2**

(b) Evaluate $\int_{-4}^{0} \frac{x}{\sqrt{2-x}}\,dx$ using the substitution $u = 2-x$. **3**

(c) Consider the vectors $\underset{\sim}{a} = 2\underset{\sim}{i} + 5\underset{\sim}{j}$ and $\underset{\sim}{b} = -3\underset{\sim}{i} + 8\underset{\sim}{j}$.

(i) Determine the unit vector in the direction of $\underset{\sim}{b}$. **2**

(ii) Find the vector projection of $\underset{\sim}{a}$ onto $\underset{\sim}{b}$. **2**

(d) Find the general solution of the differential equation $y\frac{dy}{dx}(1+x^2) = 2x(y^2+1)$, **3**

expressing y in terms of x.

(e) Find the coefficient of x^4 in the expansion $(1+x+x^2)(1-x)^9$. **3**

End of Question 11

Question 12 (15 marks) Use a SEPARATE writing booklet.

(a) According to an election poll, 20% of Australians voted for Greens Party.

(i) A random sample of 40 voters is to be taken to determine the proportion of those who have voted for Green Party. For the distribution of sample proportion, Find: **2**

(α) the mean;

(β) the standard deviation (correct to 2 decimal places).

(ii) Part of a table of $P(Z \leq z)$ values, where Z is a standard normal variable shown. **2**

Use the table below to estimate the probability that at most 4 will have voted for Greens Party from a random sample of 40 voters. Give your answer correct to 2 significant figures.

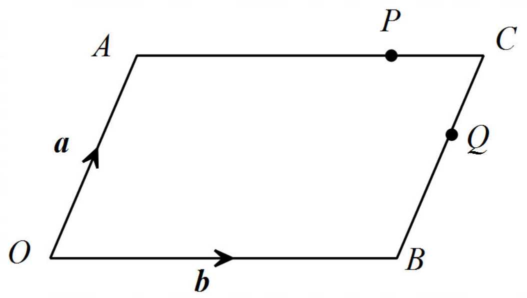
z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
-2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
-1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
-1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
-1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
-1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455

(iii) Find the probability that the sample proportion is equal to the population proportion, if a random sample of 40 voters is taken. Give your answer correct to four decimal places. **2**

Question 12 continues on page 8

Question 12 (continued)

(b) In the diagram, $OACB$ is a parallelogram. $\overrightarrow{OA} = \underset{\sim}{a}$ and $\overrightarrow{OB} = \underset{\sim}{b}$. P and Q are points on AC and BC respectively such that AP: PC = 3: 1 and BQ : QC = 3 : 1. **3**



NOT TO SCALE

(i) Prove that $AB \parallel PQ$.

(ii) Hence, show that $AB = 4PQ$.

(c) Find $\int_0^{\pi} \sin 5x \cos 8x \, dx$. **3**

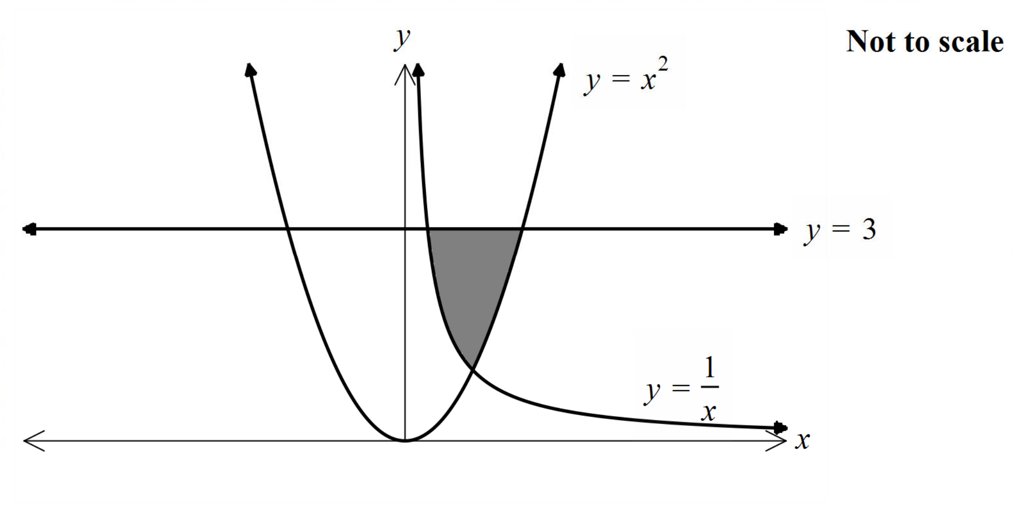
(d) Given that $\cos 4\theta = 8\cos^4\theta - 8\cos^2\theta + 1$. **3**

Find all solutions to the equation $8x^4 - 8x^2 + 1 = 0$.

End of Question 12

Question 13 (14 marks) Use a SEPARATE writing booklet.

(a) A region is bounded by the curve $y = \dfrac{1}{x}$, $y = x^2$ and $y = 3$ as shown in the diagram below. **3**



Find the exact volume of the solid generated when the region is rotated $360°$ about the y-axis.

(b) The functions f and g are defined as:

$$f(x) = \sqrt{x-2} + 5$$

$$g(x) = \frac{1}{x+1}$$

(i) Find $h(x)$ where $h(x) = (f \circ g)(x)$. **1**

(ii) Find $h^{-1}(x)$. **2**

(iii) Find the range of $h^{-1}(x)$. **4**

Question 13 continues on page 10

(c) Newton's Law of Cooling states that the rate at which the temperature of body falls is proportional to the amount by which its temperature exceeds that of its surroundings, $B°$C. At time t minutes after cooling commences, the temperature of the body is $\theta°$C. Assuming that the room temperature remains constant at $25°$C and the body has an initial unknown temperature of θ_0 .The body has a temperature of $85°$C and $65°$C when $t = 5$ mins and 10 mins respectively. **4**

(i) Explain why $\dfrac{d\theta}{dt} = k\left(\theta - B\right)$.

(ii) By using separation of variables, show that $\theta = 25 + Ae^{kt}$.

(iii) Another body of initial temperature $125°$C is brought into the room. Find the time taken for the body to cool down to a temperature of $35°$C.

End of Question 13

Question 14 (16 marks) Use a SEPARATE writing booklet.

(a) Prove by mathematical induction that, for all integers $n \geq 1$, **3**

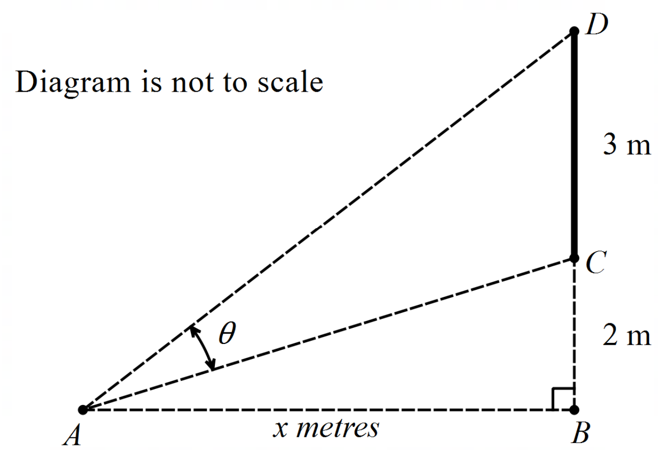
$$\frac{5}{1\times2\times3}+\frac{6}{2\times3\times4}+\frac{7}{3\times4\times5}+\ldots+\frac{n+4}{n(n+1)(n+2)}=\frac{n(3n+7)}{2(n+1)(n+2)}$$

(b) Let A, B, C be the interior angles of a triangle. **3**

(i) Explain why $\tan(A+B)=\tan(\pi-C)$

(ii) Hence, or otherwise, show that $\tan A+\tan B+\tan C=\tan A\tan B\tan C$.

(c)



Ms. Dunbar has decided to go to a cinema to watch a recent blockbuster movie. She is initially seated x metres at the point A, as shown in the side view diagram of the cinema above, and she has a movie screen viewing angle of θ radians (that is $\angle DAC=\theta$). The base of the movie screen at point C is placed 2 metres above the level ground (that is vertically above point B) and the viewing height of the movie screen, DC, is 3 metres. You may assume x and θ are functions of time t (in seconds).

(i) Show that $\theta=\arctan\left(\dfrac{5}{x}\right)-\arctan\left(\dfrac{2}{x}\right)$. **2**

(ii) If x is increasing at the rate of 3 m/s, at what rate is the viewing angle θ changing when $x=6$ metres. **3**

Question 14 continues on page 12

(d) A particle is projected in a vertical plane at right angles to a wall of height h metres standing on horizontal ground at a distance c metres from the point of projection, O. The particle just clears the wall at the highest point of its path. **5**

Prove that the speed, v, of projection is given by $v^2 = \dfrac{g}{2h}(c^2 + 4h^2)$.

You may assume the following equations (DO NOT PROVE THESE):

$$\ddot{x} = 0$$
$$\dot{x} = v\cos\theta\,\underset{\sim}{\mathbf{i}}$$
$$x = vt\cos\theta\,\underset{\sim}{\mathbf{i}}$$

$$\ddot{y} = -g\underset{\sim}{\mathbf{j}}$$
$$\dot{y} = (v\sin\theta - gt)\underset{\sim}{\mathbf{j}}$$
$$y = \left(vt\sin\theta - \frac{1}{2}gt^2\right)\underset{\sim}{\mathbf{j}}$$

End of Task

2020 Ext 1 Trial Solutions

2020 EXT 1 SOLUTIONS

Section I

Q1. $\frac{d}{dx} 5\cos^{-1}\left(\frac{x}{9}\right) = -\frac{5}{\sqrt{81-x^2}}$

B

Q2. A

Q3. $p = 0.9$ A

Since $p > 0.5$, the distribution is negatively skewed

Q4. vertical asymptote: $(x+3)(x-1)$

C

Q5. B

Q6. $(2)^2 + (a-3)(2) - (a+b) = 0$

$a - b - 2 = 0 \ldots (1)$

$2(2)^2 - (a-1)(2) + b = 0$

$8 - 2a + 2 + b = 0$

$b - 2a + 10 = 0 \ldots (2)$

$(1) + (2)$

$-a + 8 = 0$

$a = 8$

D

Q7. At most 1 Green = 0 Green & 1 Green

$^8C_0 \, ^8C_4 + ^8C_1 \, ^8C_3 = 518$

A

Q8. D

Q9. $x = \sin^2 t$ $\quad \frac{y}{9} = \cos^2 t$ 1

$\sin^2 t + \cos^2 t = x + \frac{y}{9}$

$1 =$ $\quad 0 \leq \sin^2 t \leq 1$

$9 = 9x + y$

$y = 9 - 9x \quad 0 \leq x \leq 1$

C

Q10. $(\bar{A}+\bar{B})(\bar{A}+\bar{B}) = (\bar{A}+\bar{B})(\bar{A}+\bar{B})$

using scalar product,

$|\bar{A}+\bar{B}|^2 = |\bar{A}|^2 + |\bar{B}|^2 + 2|\bar{A}||\bar{B}|$

$= |\bar{A}|^2 + |\bar{B}|^2 + 2|\bar{A}||\bar{B}|\cos\theta$

since $0 < \cos\theta < 1$,

$0 < 2|\bar{A}||\bar{B}|\cos\theta < 2|\bar{A}||\bar{B}|$ adding $|\bar{A}|^2 + |\bar{B}|^2$

$|\bar{A}|^2 + |\bar{B}|^2 < |\bar{A}|^2 + |\bar{B}|^2 + 2|\bar{A}||\bar{B}|\cos\theta < |\bar{A}|^2 + |\bar{B}|^2 + 2|\bar{A}||\bar{B}|$

$|\bar{A}|^2 + |\bar{B}|^2 < |\bar{A}+\bar{B}|^2 < (|\bar{A}| + |\bar{B}|)^2$

Since all terms are positive,

$\sqrt{|\bar{A}|^2 + |\bar{B}|^2} < |\bar{A}+\bar{B}| < (|\bar{A}| + |\bar{B}|)$

A

Section II

Q11.

a) $|4x-5| \geq 2$

① makes some correct progress

$4x-5 \geq 2$ $\qquad$ $4x-5 \leq -2$

$4x \geq 7$ $\qquad$ $4x \leq 3$

$\therefore\ x \geq 7/4$ $\qquad$ $x \leq \frac{3}{4}$ #.

① Correct answer.

b) let $u = 2-x \rightarrow x = 2-u$

① some relevant work towards using the given substitution

$\frac{du}{dx} = -1$ $\qquad$ $x=0$ $\qquad$ $x=-4$

$-du = dx$ $\qquad$ $u=2$ $\qquad$ $u=6$

$$\int_6^2 -\frac{2-u}{\sqrt{u}}\,du = \int_6^2 -2u^{-1/2} + u^{1/2}\,du$$

① correct integral in terms of u including upper and lower bounds or equivalent merit

$$= \left[-4u^{1/2} + \frac{2u^{3/2}}{3}\right]_6^2$$

$$= \left(-4\sqrt{2} + \tfrac{4}{3}\sqrt{2}\right) - \left(-4\sqrt{6} + 4\sqrt{6}\right)$$

$$= -\frac{8\sqrt{2}}{3}$$

① correct answer

c) i) $|\underset{\sim}{b}| = \sqrt{(-3)^2 + 8^2}$

$= \sqrt{73}$

① correct magnitude or equivalent merit

unit vector $= \dfrac{\underset{\sim}{b}}{|\underset{\sim}{b}|} = \dfrac{1}{\sqrt{73}}(-3\underset{\sim}{i} + 8\underset{\sim}{j})$ ① correct answer

ii) $\underset{\sim}{a} \cdot \underset{\sim}{b} = (2)(-3) + (5)(8)$

$= 34$

① makes some correct progress

$$\text{proj}_{\underset{\sim}{b}}\underset{\sim}{a} = \frac{34}{(\sqrt{73})^2}(-3\underset{\sim}{i} + 8\underset{\sim}{j})$$

$$= -\frac{102}{73}\underset{\sim}{i} + \frac{272}{73}\underset{\sim}{j}$$ ① Correct answer

d) $y\frac{dy}{dx}(1+x^2)=2x(y^2+1)$

① makes some correct progress

$\frac{1}{2}\cdot\frac{2y}{y^2+1}\,dy = \frac{2x}{1+x^2}\,dx$

$\frac{1}{2}\ln|y^2+1| = \ln|1+x^2|+C$

① correct integration or equivalent merit

$\ln|y^2+1| = \ln(1+x^2)^2 + C_1$

$y^2+1 = (1+x^2)^2 \times e^{C_1}$

$y^2 = A(1+x^2)^2-1$ (where $e^{C_1}=A$)

① correct expression of y in terms of x.

e) $(1+x+x^2)$

① makes some correct progress

For 1, $\binom{9}{4}(-x)^4$

$= 126x^4$

For x, $\binom{9}{3}(-x)^3$

$= -84x^3$

The coefficient of x^4

$\Rightarrow$ $126-84+36 = 78$ $\#$.

① correct answer

For x^2, $\binom{9}{2}(-x)^2$

$= 36x^2$

① at least one correct coefficient of x^4

Q12.

a) i) $\mu = 0.2$ ① correct mean

$$\sigma = \sqrt{\frac{0.2(1-0.2)}{40}}$$

$= 0.06$ ① correct standard deviation

ii) $$z = \frac{\frac{4}{40} - 0.2}{0.06}$$

① correct z-score

$= -1.67$

$P(Z \leq -1.67)$ Using the table provided,

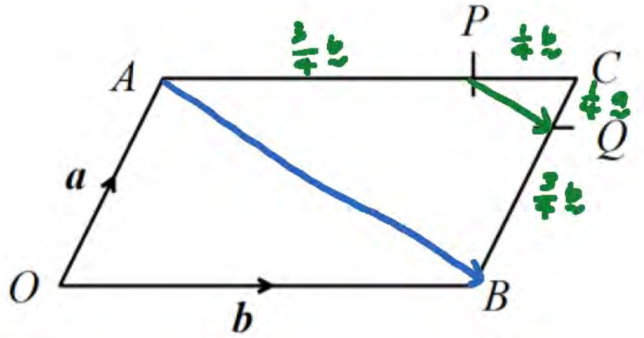
$= 0.0475$ ① correct answer

iii) $X \sim Bin(40, 0.2)$ ① makes correct progress

$= \binom{40}{8}(0.2)^8(0.8)^{32}$

$= 0.15598$ ① correct answer

b)



① correct expression for $\overrightarrow{AB}$ or $\overrightarrow{PQ}$

i) $\overrightarrow{AB} = \underset{\sim}{b} - \underset{\sim}{a}$ $\quad \overrightarrow{PQ} = \frac{1}{4}\underset{\sim}{b} - \frac{1}{4}\underset{\sim}{a}$

$= -\frac{1}{4}(\underset{\sim}{b} - \underset{\sim}{a})$

① Proves that $AB \parallel PQ$

$\therefore AB \parallel PQ$.

ii) $|\overrightarrow{AB}| = |4(-\frac{1}{4}(\underset{\sim}{b} - \underset{\sim}{a}))|$

$= 4PQ$ ① correctly proves that $AB = 4PQ$

$\therefore |AB| = 4PQ$

c) $\int_0^{\pi} \sin 5x \cos 8x \, dx$

$= \frac{1}{2}\int_0^{\pi}(\sin 13x + \sin(-3x))\, dx$ ① makes some correct progress

$= \frac{1}{2}\int_0^{\pi}(\sin 13x - \sin 3x)\, dx$

$= \frac{1}{2}\left[-\frac{\cos 13x}{13} + \frac{\cos 3x}{3}\right]_0^{\pi}$ ① correct integration or equivalent merit

$= \frac{1}{2}\left[\left(-\frac{\cos 13\pi}{13} + \frac{\cos 3\pi}{3}\right) - \left(-\frac{\cos 0}{13} + \frac{\cos 0}{3}\right)\right]$

$= \frac{1}{2}\left[\left(\frac{1}{13} - \frac{1}{3}\right) - \left(-\frac{1}{13} + \frac{1}{3}\right)\right]$

$= -\frac{10}{39}$ ※. ① correct answer

d) let $\cos\theta = x$

$0 \le \theta \le \pi$

$8x^4 - 8x^2 + 1 = \cos 4\theta$

$0 \le 4\theta \le 4\pi$

$\cos 4\theta = 0$ ① deduces that $\cos 4\theta = 0$

$4\theta = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2} \dots$ ① makes some significant correct progress

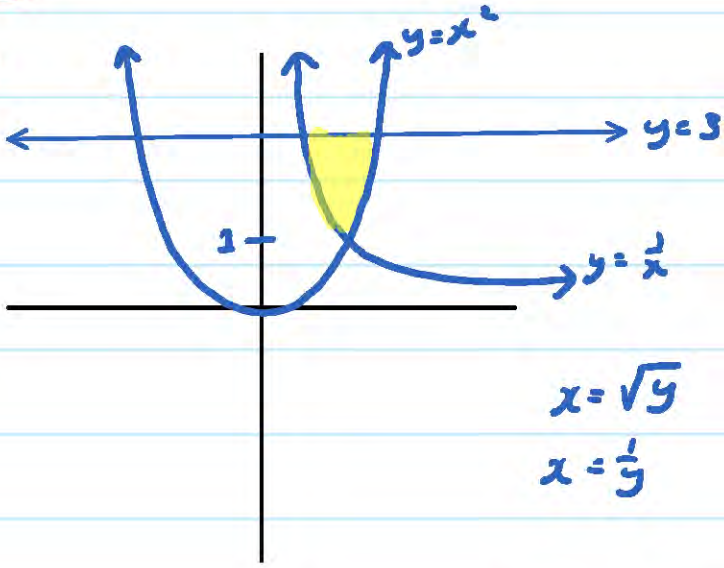
$\theta = \frac{\pi}{8}, \frac{3\pi}{8}, \frac{5\pi}{8}, \frac{7\pi}{8}, \frac{(2n-1)\pi}{8}$

$\therefore x = \cos\frac{\pi}{8}, \cos\frac{3\pi}{8}, \cos\frac{5\pi}{8}, \cos\frac{7\pi}{8}, \cos\frac{(2n-1)\pi}{8}$ (where $n \ge 1$) $n \in \mathbb{Z}$

① correct answer

Q13.

a)



$x = \sqrt{y}$

$x = \frac{1}{y}$

$\pi \int_1^3 (\sqrt{y})^2 - \left(\frac{1}{y}\right)^2 \, dy$

① convert all functions in terms of y correctly or equivalent merit

$= \pi \int_1^3 (y - y^{-2}) \, dy$

① makes some correct progress

$= \pi \left[\frac{y^2}{2} - \frac{y^{-1}}{-1} \right]_1^3$

$= \pi \left[\left(\frac{9}{2} + \frac{1}{3}\right) - \left(\frac{1}{2} + 1\right) \right]$

$= \frac{10\pi}{3} u^3$ *

① correct answer

b) i) $f(g(x)) = f\left(\frac{1}{x+1}\right)$

$= \sqrt{\frac{1}{x+1} - 2} + 5$ ① correct answer

ii) $x = \sqrt{\frac{1}{y+1} - 2} + 5$ ① interchanging x and y

$\frac{1}{y+1} = (x-5)^2 + 2$

$y = \frac{1}{(x-5)^2 + 2} - 1$ ① correct answer

iii) The range of $h^{-1}(x)$ = the domain of $h(x)$

$$h(x) = \sqrt{\frac{1}{x+1} - 2} + 5$$

① makes some correct progress

$$\frac{1}{x+1} - 2 \geq 0 \qquad x \neq -1$$

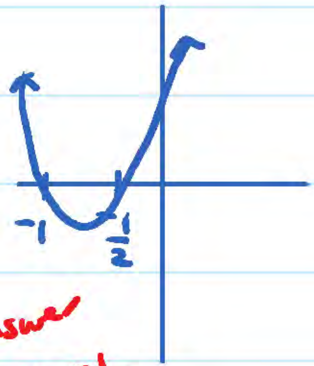
$$\frac{1}{x+1} \geq 2$$

①

$$x+1 \geq 2(x+1)^2$$

$$(x+1)(2(x+1) - 1) \leq 0$$

$$(x+1)(2x+1) \leq 0$$



D: $-1 < x \leq -\frac{1}{2}$

① final answer including −1.

$\therefore$ The range of $h^{-1}(x)$ $-1 < y \leq -\frac{1}{2}$ ① correct answer

c) i) $\frac{d\theta}{dt} \propto k(\theta - B)$ 1

ii) $\frac{d\theta}{dt} = k(\theta - 25)$

$\int \frac{d\theta}{\theta - 25} = \int k \, dt$

$\ln|\theta - 25| = kt + c$ 1

$\theta - 25 = e^{kt+c}$

$= Ae^{kt}$ (where $A = e^c$)

$\theta = 25 + Ae^{kt}$

iii) @ $t = 5$ $\theta = 85°$

$85 = 25 + Ae^{5k}$

$Ae^{5k} = 60$

$A = \frac{60}{e^{5k}}$

$= \frac{60}{e^{5(\frac{\ln\frac{2}{3}}{5})}}$

$= \frac{60}{\frac{2}{3}}$

$= 90°C$ 1

@ $t = 10$ $\theta = 65°$

$65 = 25 + Ae^{10k}$

$Ae^{10k} = 40$

$\frac{60}{e^{5k}} e^{10k} =$

$e^{5k} = \frac{2}{3}$

$k = \frac{1}{5}\ln\left(\frac{2}{3}\right)$

$\therefore$ The initial temperature is $90 + 25 = 115°C$

Another body of initial temperature of $125°C$

$125 = 25 + Ae^{\frac{1}{5}\ln(\frac{2}{3})(0)}$

$A = 100°C$

$$35 = 25 + 100e^{\frac{1}{5}\ln\left(\frac{2}{3}\right)t}$$

$$e^{\frac{1}{5}\ln\left(\frac{2}{3}\right)t} = \frac{1}{10}$$

$$\frac{1}{5}\ln\left(\frac{2}{3}\right)t = \ln\left(\frac{1}{10}\right)$$

$$t = \frac{\ln\frac{1}{10}}{\frac{1}{5}\ln\left(\frac{2}{3}\right)}$$

$$= 28.394\ldots \quad 1$$

$$\doteq 28.4 \text{ mins}$$

Q14

a) Step 1: Let $n=1$

$$LHS = \frac{5}{1\times2\times3} = \frac{5}{6}$$

$$RHS = \frac{1(3+7)}{2(2)(3)} = \frac{5}{6} = LHS$$

① establishes the $n=1$ case

$\therefore$ True for $n=1$

Step 2: Assume true for $n=k$ $\quad k \in \mathbb{Z}^+, \ 1 \le k \le n$

$$\frac{5}{1\times2\times3} + \dots + \frac{k+4}{k(k+1)(k+2)} = \frac{k(3k+7)}{2(k+1)(k+2)}$$

Step 3: Prove true for $n=k+1$ $\quad k+1 \le n$

$$\frac{5}{1\times2\times3} + \dots + \frac{k+5}{(k+1)(k+2)(k+3)} = \frac{(k+1)(3k+10)}{2(k+2)(k+3)}$$

① stating for $n=k+1$

$$LHS = \underbrace{\frac{5}{1\times2\times3} + \dots + \frac{k+4}{k(k+1)(k+2)}}_{\text{from assumption}} + \frac{k+5}{(k+1)(k+2)(k+3)}$$

$$= \frac{k(3k+7)}{2(k+1)(k+2)} \times \frac{(k+3)}{(k+3)} + \frac{k+5}{(k+1)(k+2)(k+3)} \times \frac{2}{2}$$

$$= \frac{k(3k+7)(k+3) + 2(k+5)}{2(k+1)(k+2)(k+3)}$$

$$= \frac{3k^3 + 16k^2 + 21k + 2k + 10}{2(k+1)(k+2)(k+3)}$$

$$= \frac{3k^3 + 16k^2 + 23k + 10}{2(k+1)(k+2)(k+3)}$$

$\rightarrow k=-1$ is a factor

$(k+1)(3k^2+13k+10)$

$(k+1)(3k+10)(k+1)$

$$= \frac{\cancel{(k+1)^2}(3k+10)}{2\cancel{(k+1)}(k+2)(k+3)}$$

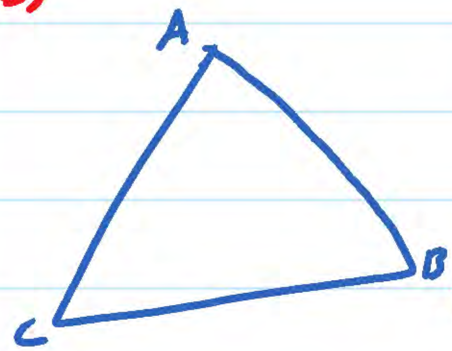
① Correctly deriving the LHS = RHS with full working

$$= \frac{(k+1)(3k+10)}{2(k+2)(k+3)}$$

$\therefore$ true for $n = k+1$.

Since the statement is true for $n = 1, 2, 3$ and so on, by mathematical induction, it is true for all integers $n \geq 1$.

b)



i) $A + B + C = \pi$ ① recognises that $A+B+C=\pi$ or equivalent merit

$A + B = \pi - C$

$\tan(A+B) = \tan(\pi - C)$

ii) $\dfrac{\tan A + \tan B}{1 - \tan A \tan B} = -\tan C$ ① Uses sums & diff identities or equivalent merit

① Correct proof with full reasoning

$\tan A + \tan B = -\tan C + \tan A \tan B \tan C$

$\tan A + \tan B + \tan C = \tan A \tan B \tan C$ #.

c) i) let $\angle CAB = \alpha$

$\therefore \tan\alpha = \frac{2}{x}$

$\alpha = \tan^{-1}\left(\frac{2}{x}\right)$ ①

$\tan(\theta+\alpha) = \frac{5}{x}$

$\theta+\alpha = \tan^{-1}\left(\frac{5}{x}\right)$ ①

$\theta = (\theta+\alpha)-\alpha$

$= \tan^{-1}\left(\frac{5}{x}\right) - \tan^{-1}\left(\frac{2}{x}\right)$ #.

ii) $\frac{dx}{dt} = 3$ m/s $\quad \frac{d\theta}{dt} = ?$ when $x=6$

$$\frac{d\theta}{dt} = \frac{dx}{dt} \times \frac{d\theta}{dx}$$

$$\frac{d\theta}{dx} = \frac{\frac{d}{dx}\left(\frac{5}{x}\right)}{1+\left(\frac{5}{x}\right)^2} - \frac{\frac{d}{dx}\left(\frac{2}{x}\right)}{1+\left(\frac{2}{x}\right)^2}$$ ① attempt

$$= \frac{-\frac{5}{x^2}}{1+\frac{25}{x^2}} + \frac{\frac{2}{x^2}}{1+\frac{4}{x^2}}$$

$$= \frac{-5}{x^2+25} + \frac{2}{x^2+4}$$ ①

$$\frac{d\theta}{dt} = 3 \times \left(\frac{-5}{(6)^2+25} + \frac{2}{(6)^2+4}\right)$$

$$= 3\left(\frac{-5}{61} + \frac{2}{40}\right)$$

$$= -\frac{117}{1220}$$

$\doteqdot -0.0959$ rad/s #. ①

d)

$$\dot{y}=0, \quad 0=(v\sin\theta-gt)\underset{\sim}{j}$$

$$t=\frac{v\sin\theta}{g} \quad 1$$

$$@\ t=\frac{v\sin\theta}{g}, \quad y=h$$

$$h=v\left(\frac{v\sin\theta}{g}\right)\sin\theta-\frac{1}{2}g\left(\frac{v\sin\theta}{g}\right)^2 \quad 1$$

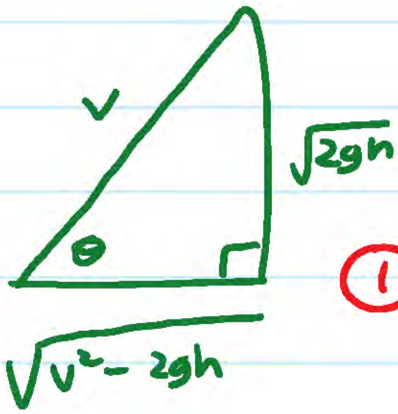
$$=\frac{2\times v^2\sin\theta}{2\times g}\sin\theta-\frac{v^2\sin^2\theta}{2g}$$

$$=\frac{2v^2\sin^2\theta-v^2\sin^2\theta}{2g}$$

$$=\frac{v^2\sin^2\theta}{2g}$$

$$\sin^2\theta=\frac{2gh}{v^2}$$

$$\sin\theta=\frac{\sqrt{2gh}}{v}$$



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$$@\ t=\frac{v\sin\theta}{g}, \quad x=c \quad 1$$

$$c=v\left(\frac{v\sin\theta}{g}\right)\cos\theta$$

$$=\frac{v^2\sin\theta\cos\theta}{g}$$

$$v^2=\frac{cg}{\sin\theta\cos\theta}$$

$$v^2 = \frac{cg}{\frac{\sqrt{2gh}}{v} \times \frac{\sqrt{v^2-2gh}}{v}} \quad 1$$

$$\cancel{v^2} = \frac{\cancel{v^2}cg}{\sqrt{2gh(v^2-2gh)}}$$

$$\sqrt{2gh(v^2-2gh)} = cg$$

$$2gh(v^2-2gh) = c^2g^2$$

$$v^2 = \frac{c^2g^{\cancel{2}}}{2\cancel{g}h} + 2gh$$

$$= \frac{g}{2h}(c^2+4h^2) \ \#.$$