BAULKHAM HILLS HS PERSEVERE

2022 **YEAR 11 ASSESSMENT TASK 2**

Mathematics Extension

General Instructions	• Reading time – 5 minutes • Working time – 1 hour 30 minutes • Write using black pen. • Calculators approved by NESA may be used. • In Questions 11 – 14, show relevant mathematical reasoning and/or calculations. • Answer all questions in the booklet provided. • A reference sheet is provided at the back of this paper
Total marks: 59	**Section I – 10 marks** (pages 2 – 4) • Attempt Questions 1 – 10 • Allow about 15 minutes for this section. **Section II – 49 marks** (pages 5 – 8) • Attempt Questions 11 – 14 • Allow about 1 hour and 15 minutes for this section.

Section I
10 marks
Attempt questions 1 - 10
Allow about 15 minutes for this section

Use the multiple choice answer page in your booklet to answer Questions 1 – 10.

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1) What is the solution of $|x-5| \le 2$?

A) $-7 \le x \le -3$ B) $x \le -7,\quad x \ge 3$

C) $3 \le x \le 7$ D) $x \le 3,\quad x \ge 7$

2) The function $f(x)$ is defined as $f(x) = x^3 - 4$. Which expression is equal to $f^{-1}(x)$?

A) $\dfrac{1}{x^3} - 4$ B) $\dfrac{1}{x^3 - 4}$ C) $\sqrt[3]{x+4}$ D) $\sqrt[3]{x} + 4$

3) If there are 5 boxes to store 22 pens, what does the pigeonhole principle state?

A) At least 4 boxes will contain more than 4 pens.
B) At least 1 box will contain more than 4 pens.
C) At most 4 boxes will contain more than 4 pens.
D) At least 1 box will contain less than 4 pens.

4) How many three-digit PIN codes can be formed from the digits 0, 1, 2, 3, 4 and 5 if repetition is not allowed?

A) 720 B) 24 C) 120 D) 60

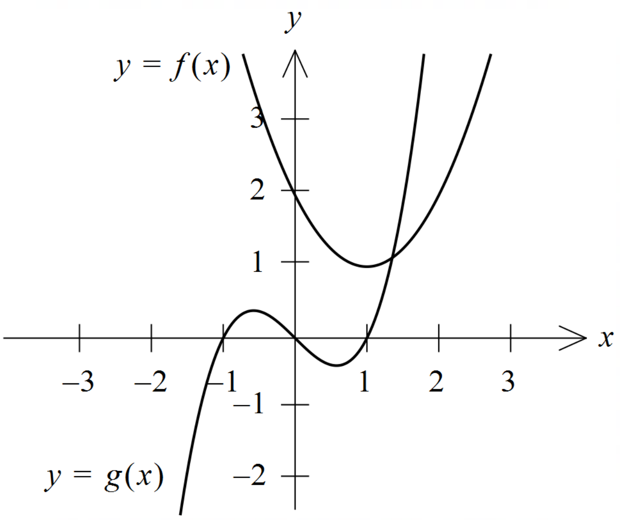
5) The polynomial $P(x) = 2x^3 + ax - 3$ has a remainder of -1 when it is divided by $(x+1)$. What will be the remainder if it is divided by $(x-1)$?

A) -6 B) -5 C) 1 D) 5

6) The polynomial $P(x) = x^3 - x^2 - 5x - 3$ has a double root at $x = \alpha$. What is the value of α ?

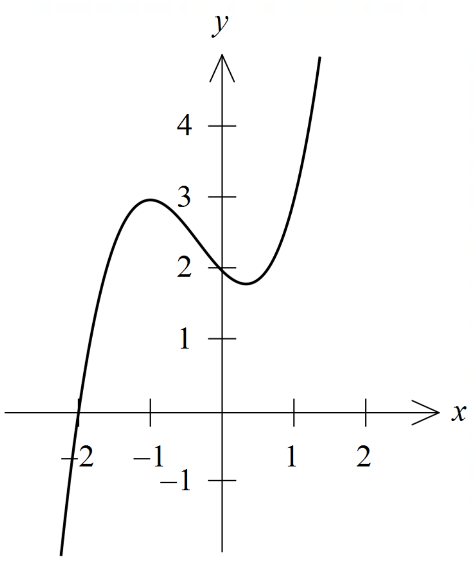
A) $-\dfrac{5}{3}$ B) -1 C) 1 D) $\dfrac{5}{3}$

7) The graphs of a parabolic function $y = f(x)$ and a cubic function $y = g(x)$ are shown below:

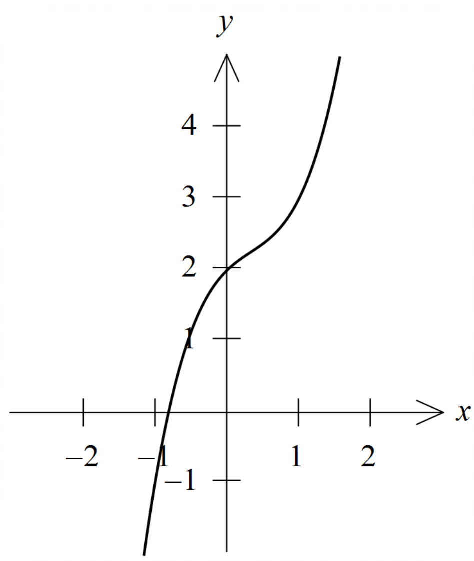


Which of the following represents the graph of $y = f(x) + g(x)$?

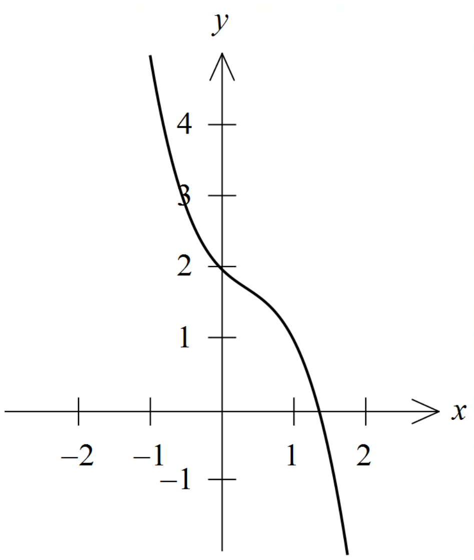
A)



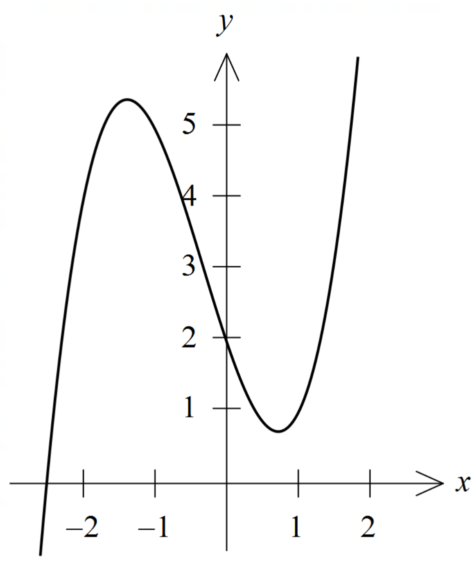
B)



C)



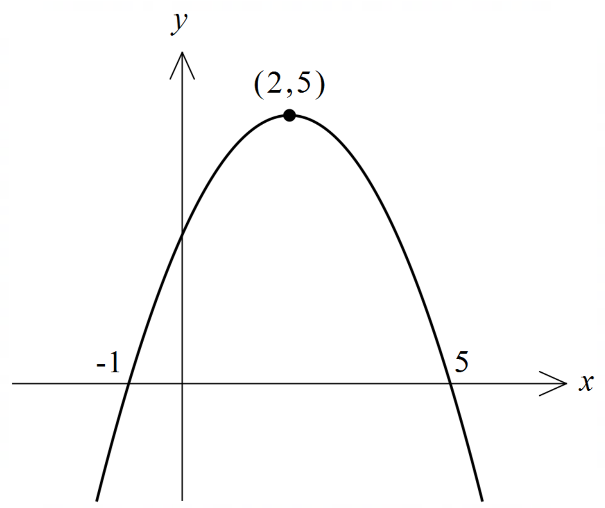
D)



8) In how many ways can the letters of the word **INDEPENDENCE** be arranged in a row, if all the letters 'E' being together?

A) $\dfrac{12!}{4!3!2!}$ B) $\dfrac{9!}{4!3!2!}$ C) $\dfrac{9!}{3!2!}$ D) $\dfrac{1}{55}$

9) The graph of $y = f(x)$ is given below:



What is the range of $y = \dfrac{1}{f(x)}$?

A) $(-\infty,0) \cup \left[\dfrac{1}{5},\infty\right)$ B) $\left(0,\dfrac{1}{5}\right)$

C) $(-\infty,0) \cup \left(\dfrac{1}{5},\infty\right)$ D) $\left(0,\dfrac{1}{5}\right]$

10) Given $f(x)$ is a quadratic function whose graph is symmetrical about $x = c$, what is the value of $f^{-1}(f(a))$ if $a > c$ and a is NOT in the restricted domain of $f(x)$ such that $f^{-1}(x)$ exists?

A) $a-2c$ B) $2c-a$ C) $2a-c$ D) a

End of Section I

Section II

49 marks
Attempt Questions 11 - 14
Allow about 1 hour 15 minutes for this section

Answer each question on the appropriate pages of your answer booklet. Extra paper is available.
Your responses should include relevant calculations and/or mathematical reasoning.

Question 11 (14 marks)

a) Solve for x, and graph the solution on a number line: **3**

$$\frac{2}{x-1} > 3$$

b) A team of five is to be chosen at random from 7 women and 5 men.

i) In how many ways can this team be chosen? **1**

ii) What is the probability that the team will consist of at most 2 women? **2**

c) i) Find the inverse function of $f(x) = \frac{3}{x-2} + 1$. **2**

ii) Hence show that $f^{-1}\left(f\left(x\right)\right) = x$. **1**

d) The polynomial $P\left(x\right) = 2x^3 - 10x^2 - 40x + 175$ has roots $\alpha,\ \beta,$ and γ. Find the value of:

i) $\alpha + \beta + \gamma$ **1**

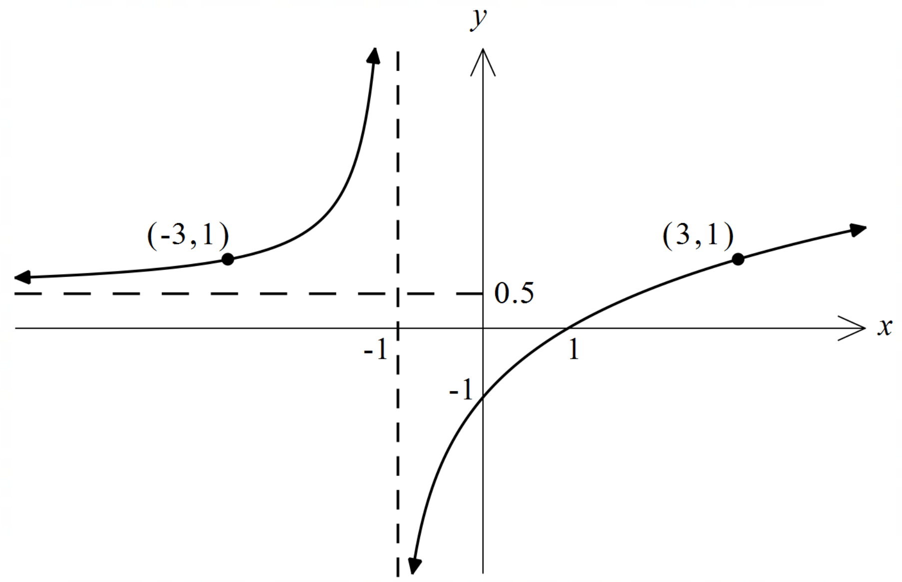
ii) $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma}$ **2**

iii) $\alpha^2 + \beta^2 + \gamma^2$ **2**

End of Question 11

Question 12 (12 marks)

a) The diagram shows the graph of $y = f(x)$.



On separate axes provided in the answer booklet, sketch the graphs of each of the following functions, clearly labelling all important features on your graph.

i) $y = f\left(\left|x\right|\right)$ **1**

ii) $y = \sqrt{f(x)}$ **2**

iii) $y = \dfrac{1}{f(x)}$ **2**

b) A bowl contains 10 red cubes and 10 blue cubes, all of the same size. A woman reaches into the bowl and selects cubes at random without looking at them.

i) What is the minimum number of cubes she must select to be sure of having at least three cubes of the same colour? **1**

ii) What is the minimum number of cubes she must select to be sure of having at least three blue cubes? **1**

c) Solve $\dfrac{1}{x(x+2)} \leq -1$. **3**

d) When $P(x)$ is divided by $(x-3)$, the remainder is 4, and when it is divided by $(x+5)$, the remainder is 12. What would the remainder be if $P(x)$ is divided by $(x-3)(x+5)$? **2**

End of Question 12

Question 13 (12 marks)

a) The letters of the word **BEGINNING** are to be arranged in a row.

i) In how many arrangements are the letter G's together? **1**

ii) How many arrangements will **NOT** contain adjacent N's? **2**

(For example, **NGENINBIG** is one such arrangement but the arrangements **NNNBEGIIG** and **GNNEGBIIN** are NOT.)

b) Consider the function $f\left(x\right)=x^{2}+4x+1$.

i) Briefly explain why the domain has to be restricted for an inverse function to exist. **1**

ii) What is the largest domain containing the value $x=0$, for which the function has an inverse function $f^{-1}\left(x\right)$? **1**

iii) Show that the inverse function in its restricted domain is $f^{-1}\left(x\right)=-2+\sqrt{x+3}$. **2**

iv) Sketch the graphs of $y=f(x)$, $y=f^{-1}(x)$ and $y=x$ on the same set of axes within the restricted domain. (DO NOT find any points of intersection) **2**

c) Two roots of the equation $x^{3}+px^{2}-qx+3=0$ are reciprocals of each other.

i) Evaluate the third root. **1**

ii) Hence find the relation between p and q, expressing p in terms of q. **2**

End of Question 13

Question 14 (11 marks)

a) It is given that the polynomial $P(x) = 4x^3 - 13x^2 + 4x + 12$ has a double root at $x = \alpha$.

i) By differentiating $P(x)$, find the value of α. **2**

ii) Hence or otherwise factorise $P(x)$. **1**

b) Find the Cartesian equation of the curve given by the parametric equations: **2**

$$x = \cos\theta + \sin\theta$$
$$y = \cos\theta - \sin\theta$$

c) Nine people are partaking in a competition. In a particular round, the group of nine needs to split themselves into two teams of four and a score keeper.

i) In how many ways can this be done? **2**

ii) These nine competitors are to be seated around a circular table for the competition. In how many ways can they be seated if the teammates of each team must sit together? **2**

d) How many five-digit numbers can be formed using digits 0, 2, 4, 5, 7 if none of the digits are repeated and the numbers formed are even? **2**

End of Examination

Solutions (Yr-11 Ext-1) Assessment Task-2, 2022

Section-I Multiple Choice

1) $|x-5| \le 2$

$-2 \le x-5 \le 2$

$3 \le x \le 7$ (C)

2) $f(x) = x^3 - 4$

$y = x^3 - 4$

Swap x with y

$x = y^3 - 4$

$y^3 = x + 4$

$y = \sqrt[3]{x+4}$

$f^{-1}(x) = \sqrt[3]{x+4}$ (C)

3) $\left\lceil \frac{22}{5} \right\rceil = 5$

or $5 \times 4 + 2 = 22$

$\therefore$ At least 1 box contain more than 4. (B)

4) $6 \times 5 \times 4 = 120$ (C)

5) $P(x) = 2x^3 + ax - 3$

$P(-1) = -1 \quad P(1) = ?$

$-2 - a - 3 = -1$

$a = -4$ (B)

$P(1) = 2 - 4 - 3 = -5$

6) $P(x) = x^3 - x^2 - 5x - 3$

$P'(x) = 3x^2 - 2x - 5$

Solve $P'(x) = 0$

$3x^2 - 2x - 5 = 0$

$x = -1, 5/3$ (B)

Since $P(-1) = 0$

$\therefore \alpha = -1$

7) (D)

8) $\dfrac{9!}{3!\ 2!}$ (C)

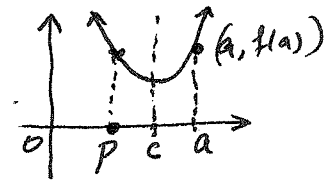
9) $(-\infty, 0) \cup \left[\frac{1}{5}, \infty\right)$ (A)

10) Since $x = c$ is the axis of symmetry of the parabola.

$\therefore f(a) = f(2c-a)$

$f^{-1}(f(a)) = f^{-1}(f(2c-a))$ (B)

$= 2c - a$



At $P = c - (a-c)$

$= 2c - a$

Question-11a

$\dfrac{2}{x-1} > 3 \quad , \quad x \neq 1$

Multiply both sides by $(x-1)^2$

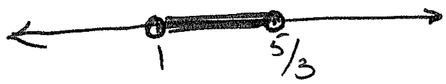
$2(x-1) > 3(x-1)^2$

$3(x-1)^2 - 2(x-1) < 0$

$(x-1)(3x-3-2) < 0$

$(x-1)(3x-5) < 0$

$1 < x < \frac{5}{3}$



* Can be done by very different methods.

(3 marks correct solution & graph
2 marks for correct solution
1 mark for significant progress towards the sol.

11(b)

i) $^{12}C_5$ or 792 (1 mark)

ii) 2 women $= {}^7C_2\,{}^5C_3 = 210$

1 women $= {}^7C_1\,{}^5C_4 = 35$

0 women $= {}^5C_5 = 1$

$P(\text{at most 2 women}) = \dfrac{210+35+1}{792}$

$= \dfrac{246}{792}$ or $\dfrac{41}{132}$

(2 marks correct answer.
1 mark for significant progress towards the answer.

11(c)

i) $y = \dfrac{3}{x-2} + 1$

Swap x & y.

$x = \dfrac{3}{y-2} + 1$

$\dfrac{3}{y-2} = x - 1$

$y - 2 = \dfrac{3}{x-1}$

$y = 2 + \dfrac{3}{x-1}$ or $f^{-1}(x) = 2 + \dfrac{3}{x-1}$

(2 marks for correct inverse function.
1 mark for correct process to find inverse function

11(c)

ii) $f^{-1}(f(x)) = f^{-1}\left(\frac{3}{x-2}+1\right)$ (1 mark correct Solution or correct process)

$= \frac{3}{\frac{3}{x-2}+\cancel{1}-\cancel{1}} + 2$

$= x-2+2$

$= x$

Hence shown.

11(d)

$P(x) = 2x^3-10x^2-40x+175$

i) $\alpha+\beta+\gamma = -\frac{b}{a} = \frac{10}{2} = 5$ (1 mark)

ii) $\frac{1}{\alpha}+\frac{1}{\beta}+\frac{1}{\gamma} = \frac{\alpha\beta+\beta\gamma+\gamma\alpha}{\alpha\beta\gamma}$

(2 marks correct answer
1 mark to simplify $\frac{1}{\alpha}+\frac{1}{\beta}+\frac{1}{\gamma}$)

$= \frac{-40/2}{-175/2}$

$= \frac{8}{35}$

iii) $\alpha^2+\beta^2+\gamma^2 = (\alpha+\beta+\gamma)^2-2(\alpha\beta+\beta\gamma+\gamma\alpha)$

$=(5)^2-2(-20)$

$=25+40$

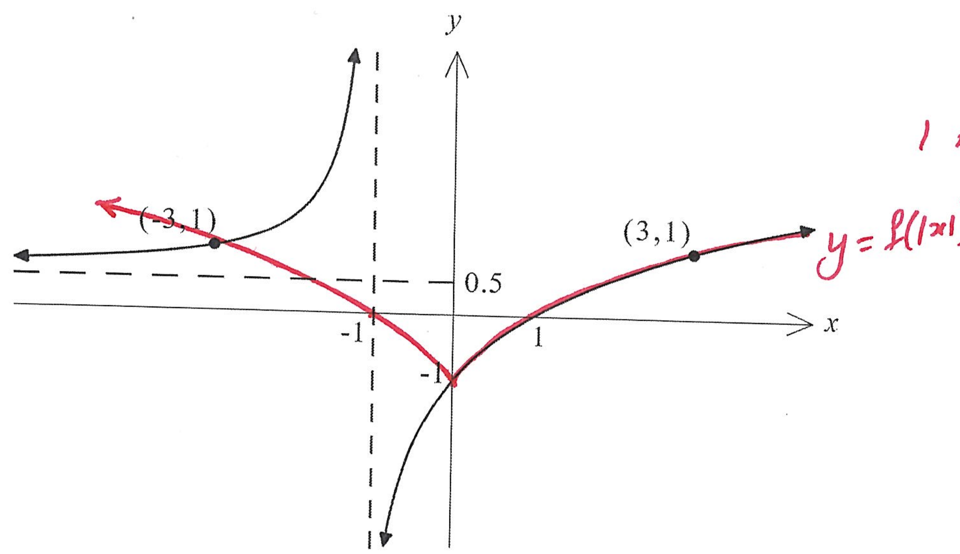
$=65$

(2 marks correct answer
1 mark correct process towards the answer.
OR correct Expansion of $\alpha^2+\beta^2+\alpha^2$)

Question 12

Name: ....

Diagram for Question 12 a), part i), $y = f\left(\left|x\right|\right)$:



1 mark.

Diagram for Question 12 a), part ii), $y = \sqrt{f(x)}$:

2 marks correct graph and considering graph needs to be above the original graph from $x=1$ to $x=3$.
1 mark for correct shape only or correct graph either $x>0$ or $x<0$

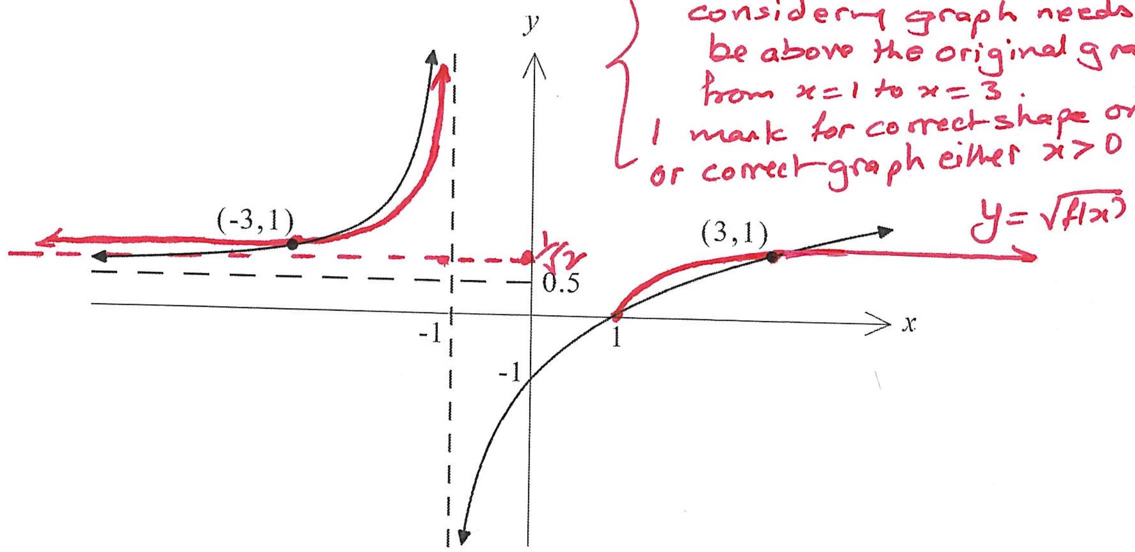
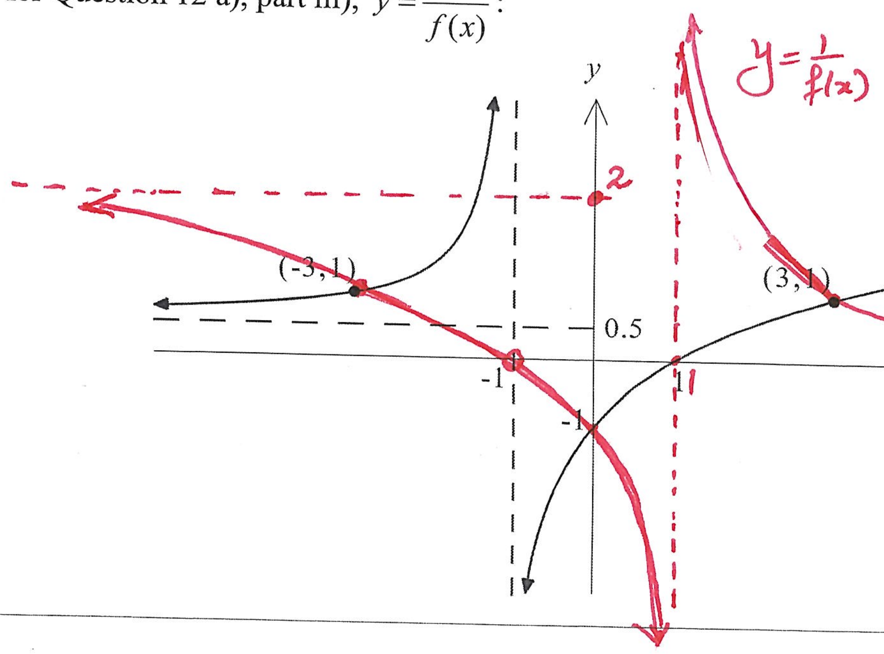


Diagram for Question 12 a), part iii), $y = \dfrac{1}{f(x)}$:



2 marks correct graph including asym. & $y=1$.
1 mark for either showing correct asym.
OR One correct branch $(x>1)$ taking the point $(3,1)$ into consideration

Question 12(b)

12b i) 5 cubes (1 mark)

Worst case is $\underline{r}\ \underline{b}\ \underline{r}\ \underline{b}\ \underline{r}$

$r = $ red, $b = $ blue.

ii) 13 cubes. (1 mark)

Worst case is $\underbrace{r, r, \ldots, r}_{10\text{ red}}, \underline{b}\ \underline{b}\ \underline{b}$

12c) $\dfrac{1}{x(x+2)} \le -1, \quad x \ne 0, -2$

$$\frac{1}{x(x+2)} + 1 \le 0$$

$$\frac{1 + x(x+2)}{x(x+2)} \le 0$$

$$\frac{x^2+2x+1}{x(x+2)} \le 0$$

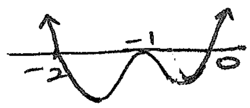
$$\frac{(x+1)^2}{x(x+2)} \le 0$$

3 marks for correct solution
2 marks for significant progress (sketches polynomial OR identifies $-2 \le x \le 0$
1 mark to obtain $x(x+2)(x+1)^2 \le 0$ or identifies $0, -1, -2$ as critical values

Multiply both sides by $x^2(x+2)^2$

$$x(x+2)(x+1)^2 \le 0$$

$\therefore$ Solution is $-2 < x < 0$



* It can be done by using different methods

12d) $P(3) = 4, \quad P(-5) = 12$

$P(x) = (x-3)(x+5)Q(x) + (ax+b)$

Since $P(3) = 4$

$\therefore 4 = 3a + b$ —— ①

& $P(-5) = 12$

$12 = -5a + b$ —— ②

Solve ① & ② simultaneously

$a = -1, \quad b = 7$

$\therefore$ Remainder $= -x + 7$

2 marks correct solution
1 mark for evaluating $P(3)$ or $P(-5)$
OR Getting two equation correctly

Question-13

13a) i) BEGINNING

3N's 2G's 2I's 1B, 1E

(1 mark for correct answer)

G's together

No. of ways. $\dfrac{8!}{2!\,3!} = 3360$

2 identical I's ; 3 identical N's

ii) Total Arrangements $= \dfrac{9!}{2!\,2!\,3!} = 15120$

2G's 2I's 3N's.

2 marks for correct solution.
1 mark for taking 3N's case but not 2N's

3 N's together $= \dfrac{7!}{2!\,2!} = 1260$

2 N's together. N N _ x x x x x x Third N can be placed at 6 places

Illy If two N's are on other end. " " " " 6 places.

_ N N _ x x x x x Third N can be placed at 5 places

x _ N N _ x x x x " " " " 5 "

_ _ _ N N _ _ _ _

N N _. Total ways are 6

$\therefore$ Total $= 6 + 6 + 5\times 6 = 42$

No. of ways $= 42 \times \dfrac{6!}{2!\times 2!} = 7560$

Using Complementary N's are NOT adjacent is

Total − 3 N's together − 2 N's together

15120 − 1260 − 7560 = 6300.

Method-2.

x B x E x G x I x I x G x

N can be placed at 7 different places so that they are not adjacent.

$\dfrac{{}^7P_3}{3!} \times \dfrac{6!}{2!\times 2!} = 6300$

3 Identical N's ; 2G's ; 2I's

2 marks for correct sol.
1 mark if they just write answer without explanation

Question 13

13b) $f(x) = x^2 + 4x + 1$

i) The domain has to be restricted as the given function is not one to one or does not pass the horizontal line test.

(1 mark).

ii) Largest domain containing $x = 0$ is $x \geq -2$ (1 mark)

iii) $y = x^2 + 4x + 1, \quad x \geq -2$

Swap x and y

$$x = y^2 + 4y + 1$$

$$x = y^2 + 4y + 4 - 3$$

$$x + 3 = (y+2)^2$$

$$y + 2 = \pm\sqrt{x+3}$$

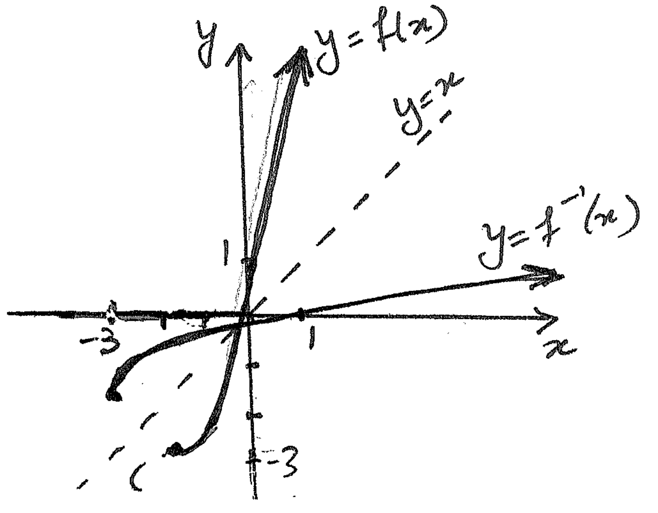
$$y = -2 + \sqrt{x+3} \quad (\because y \geq -2)$$

$$f^{-1}(x) = -2 + \sqrt{x+3}$$

Hence shown.

2 marks for correct process
1 mark if $y \geq -2$ is not taken into consideration

iv)



2 marks for each graph draw correctly with all the import points clearly mentioned.
1 mark for either $f(x)$ & $f^{-1}(x)$ drawns correctly labels points as well.

Question-13

13(c). $x^3 + px^2 - qx + 3 = 0$ (1 mark corret solution)

Let roots are $\alpha, \frac{1}{\alpha}, \beta$

Product of roots $= -\frac{d}{a} = -3$

$\therefore \alpha \times \frac{1}{\alpha} \times \beta = -3$

$\beta = -3$

$\therefore$ Third root is -3.

ii) Since -3 is a root

$\therefore (-3)^3 + p(-3)^2 - q(-3) + 3 = 0$

$-27 + 9p + 3q + 3 = 0$

$9p + 3q = 24$

$3p + q = 8$

$p = \frac{8-q}{3}$

2 marks correct solution

1 mark for significant progress towards answer

OR Correct sum of roots & sum of two at a time including $\beta = -3$

OR. Sum of roots $= -\frac{b}{a} = -p$

$\alpha + \frac{1}{\alpha} - 3 = -p$

$\alpha + \frac{1}{\alpha} = 3 - p$ —— (1)

& Sum of product of two at a time $= \frac{c}{a} = -q$

$1 - 3\alpha - \frac{3}{\alpha} = -q$

$1 - 3(\alpha + \frac{1}{\alpha}) = -q$

$1 - 3(3-p) = -q$ (from (1))

$1 - 9 + 3p = -q$

$-8 + 3p = -q$

$p = \frac{8-q}{3}$

Question-14

14a $P(x) = 4x^3 - 13x^2 + 4x + 12$

i) $P'(x) = 12x^2 - 26x + 4$

Solve $P'(x) = 0$

$12x^2 - 26x + 4 = 0$

$(6x-1)(x-2) = 0$

$x = \frac{1}{6}$ or 2

{ 2 marks for correct solution.
1 mark for solving $P'(x) = 0$

Test for $x = \frac{1}{6}$, $P(\frac{1}{6}) \neq 0$

Test for $x = 2$, $P(2) = 0$

$\because 4(2)^3 - 13(2)^2 + 4(2) + 12 = 0$

$\therefore x = 2$ is double root of $P(x)$ as $P(2) = 0$ & $P'(2) = 0$

$\alpha = 2$

ii) $P(x) = (x-2)^2(ax+b)$ (1 mark)

By comparing coeff. of x^3 & constant

$a = 4$, $b = 3$

$\therefore P(x) = (x-2)^2(4x+3)$

OR Can be done by using long division.

b) $x = \cos\theta + \sin\theta$ —— ①

$y = \cos\theta - \sin\theta$ —— ②.

{ 2 marks finding correct Cartesian equation
1 mark towards significant progress to eliminate θ.

①^2 + ②^2

$x^2 + y^2 = \cos^2\theta + \sin^2\theta + \cancel{2\sin\theta\cos\theta} + \cos^2\theta + \sin^2\theta - \cancel{2\sin\theta\cos\theta}$

$= 1 + 1$

$\therefore x^2 + y^2 = 2.$

Question - 14

14(c) i) $$\frac{{}^9C_4 \times {}^5C_4 \times 1}{2} = 315$$

2 ← for two identical groups.

(2 marks correct sol. 1 mark for ${}^9C_4 \times {}^5C_4$)

ii) $$\underbrace{2!}_{\text{arranging the groups.}} \times \underbrace{4! \times 4!}_{\text{arranging teammates within the groups.}} = 1152$$

(2 marks for correct sol. 1 mark $4! \times 4!$ OR $2 \times 5! \times 5!$)

14(d). Five-digit Number $\{0, 2, 4, 5, 7\}$

Number starting with even numbers 2, 4, (0 can't take that place)

$$\underset{\text{Two options}}{\underline{2}} \times \underbrace{\underline{3} \times \underline{2} \times \underline{1}}_{3!} \times \underset{\text{Two options.}}{\underline{2}} = 24$$

Number starting with odd numbers 5, 7

$$\underset{\text{Two options}}{\underline{2}} \times \underbrace{\underline{3} \times \underline{2} \times \underline{1}}_{3!} \quad \underset{\text{Three options.}}{\underline{3}} = 36$$

(2 marks correct Solution
1 mark for significant progress/clear explanation towards the answer.)

Total no. of ways $= 24 + 36$
$= 60$

The End.