Mrs Cooper	Mr Feng	**Name:** ………………………………..
Mrs Greenberg	Mrs Kerr	
Ms Lau	Mrs Squires	**Teacher:**………………………………

Pymble Ladies' College

MATHEMATICS EXTENSION 1

Year 11 ASSESSMENT TASK 2

Term 2 2023

General Instructions	• Working time – 55 minutes • Write using black or blue pen • Erasable pens are not to be used • Calculators approved by NESA may be used • Reference sheets are provided
Total marks 33	**SECTION 1 – 5 marks** • Attempt Questions 1-5 **SECTION II – 28 marks** • Attempt all Questions • Answer each question in the appropriate space in the Answer Booklet.

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SECTION I

Multiple Choice (5 marks)

Answer using the Multiple-Choice Answer Sheet.

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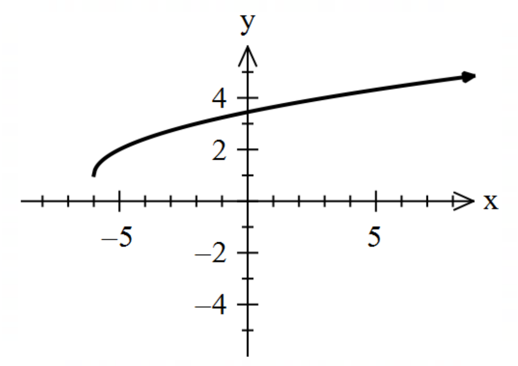
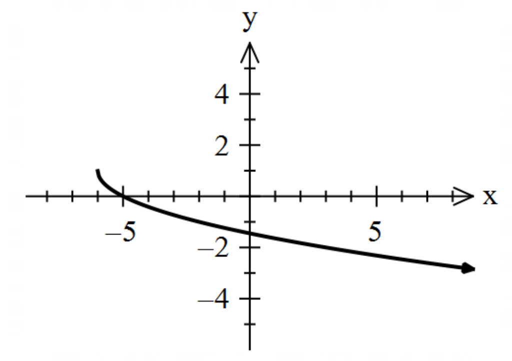
1 A function is defined as $f(x) = x^2 - 2x - 5$ over the domain $x \leq 1$.

Which of the following is the graph of $f^{-1}(x)$?

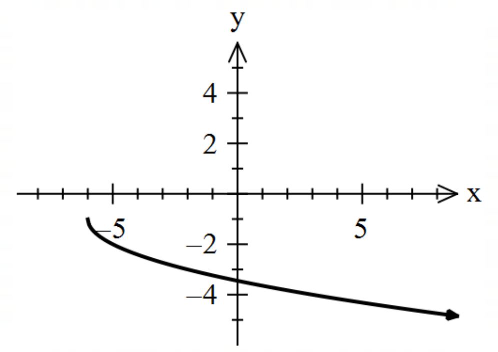
(A)



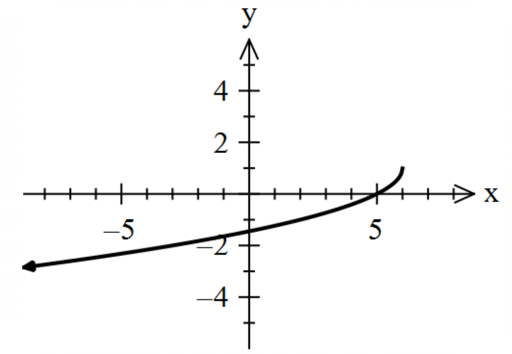
(B)



(C)



(D)



2 The function $f(x) = 96\cos(3x)\sin(3x)$ can be written as $f(x) = a\sin(bx)$.

What are the values of a and b?

(A) $a = 48,\ b = 3$

(B) $a = 48,\ b = 6$

(C) $a = 192,\ b = 3$

(D) $a = 192,\ b = 6$

3 What is the exact value of $\sin\left(2\cos^{-1}\left(\frac{-\sqrt{15}}{8}\right)\right)$?

(A) $\frac{7}{8}$

(B) $\frac{-7}{8}$

(C) $\frac{7\sqrt{15}}{32}$

(D) $\frac{-7\sqrt{15}}{32}$

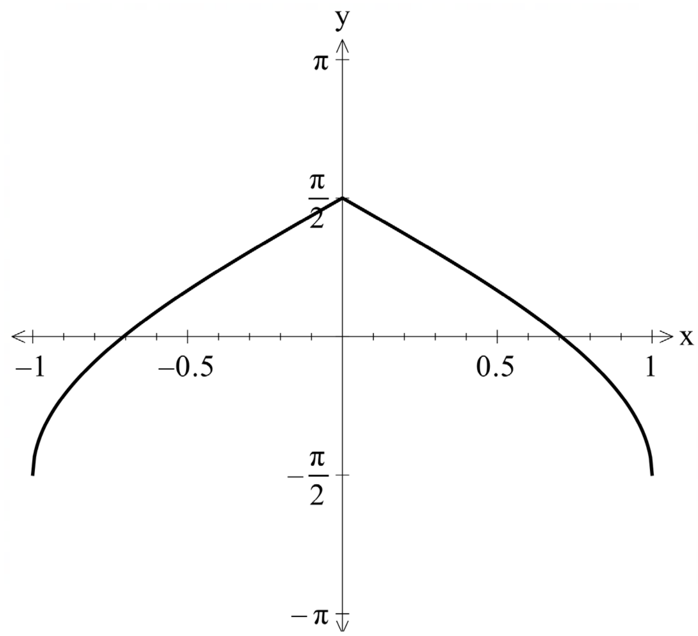
4 What is the equation of the function shown in the graph below?

(A) $y = \left|2\sin^{-1}(x)\right|$

(B) $y = \left|2\cos^{-1}(x) - \frac{\pi}{2}\right|$

(C) $y = 2\cos^{-1}(|x|) - \frac{\pi}{2}$

(D) $y = 2\sin^{-1}(|x|)$



5 A curve is defined by the parametric equations $x = \cos 2\theta$ and $y = \sin\theta\cos\theta$.
Which of the following represents the Cartesian equation?

(A) $x^2 + y^2 = 1$

(B) $x^2 + 4y^2 = 1$

(C) $x^2 + 2y^2 = 1$

(D) $x^2 + y^2 = 4$

SECTION II
25 Marks

Answer these questions in the Answer Book provided

Your responses should include relevant mathematical reasoning and/or calculations.

Question 6 (10 marks) **Marks**

(a) Find the exact value of the following, showing all necessary working.

(i) $\arccos\left(-\frac{\sqrt{3}}{2}\right)$ **1**

(ii) $\cos\left(\tan^{-1}(-1)\right)$ **2**

(iii) $\sin^{-1}\left(\sin\frac{9\pi}{4}\right)$ **2**

(b) Sketch the graph expressed parametrically by $\left(p^2, 3\right)$. **1**

(c) If $t = \tan\theta$, express $\frac{\cot 2\theta}{1-\operatorname{cosec}2\theta}$ in terms of t in simplest form. **2**

(d) Find the exact value of $\sin\frac{5\pi}{24}\sin\frac{\pi}{24}$, showing all necessary working. **2**

Question 7 (11 marks) **Marks**

(a) Sketch $y = 3\sin^{-1} 2x$. **2**

(b) Find the exact value of $\cos 255°$. **2**

(c) (i) Prove that $\dfrac{1-\tan^2\theta}{1+\tan^2\theta} = \cos 2\theta$. **2**

(ii) Hence, or otherwise, show that $\tan^2\dfrac{\pi}{8} = 3 - 2\sqrt{2}.$ **3**

Question 8 (9 marks) **Marks**

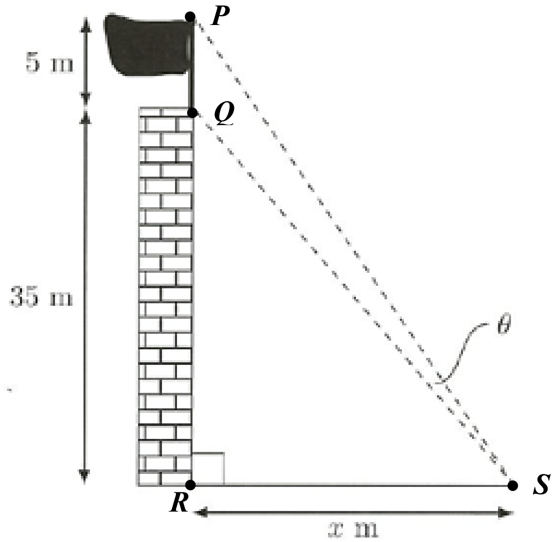
(a) A curve C is given by the parametric equations **3**

$$x = 2t^2 - 1 \text{ and } y = 3(t+1)$$

Determine the points of intersection between C and the straight line with equation $3x - 4y = 3$.

(b) Prove $\dfrac{\sin 3\theta + \sin \theta}{\cos 3\theta + \cos \theta} = \tan(2\theta)$. **3**

(c) A 5-metre flag stands on top of a 35-metre tall building as shown in the diagram below. **3**



The flag subtends an angle of θ with the eye of an observer on the ground x metres away from the building. Show that

$$\theta = \arctan\left(\frac{5x}{x^2 + 1400}\right).$$

END OF ASSESSMENT TASK

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NSW Education Standards Authority

2022 **HIGHER SCHOOL CERTIFICATE EXAMINATION**

Mathematics Advanced
Mathematics Extension 1
Mathematics Extension 2

REFERENCE SHEET

Measurement

Length

$$l = \frac{\theta}{360} \times 2\pi r$$

Area

$$A = \frac{\theta}{360} \times \pi r^2$$

$$A = \frac{h}{2}(a + b)$$

Surface area

$$A = 2\pi r^2 + 2\pi rh$$

$$A = 4\pi r^2$$

Volume

$$V = \frac{1}{3}Ah$$

$$V = \frac{4}{3}\pi r^3$$

Functions

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

For $ax^3 + bx^2 + cx + d = 0$:

$$\alpha + \beta + \gamma = -\frac{b}{a}$$

$$\alpha\beta + \alpha\gamma + \beta\gamma = \frac{c}{a}$$

and $\alpha\beta\gamma = -\frac{d}{a}$

Relations

$$(x - h)^2 + (y - k)^2 = r^2$$

Financial Mathematics

$$A = P(1 + r)^n$$

Sequences and series

$$T_n = a + (n - 1)d$$

$$S_n = \frac{n}{2}[2a + (n - 1)d] = \frac{n}{2}(a + l)$$

$$T_n = ar^{n-1}$$

$$S_n = \frac{a(1 - r^n)}{1 - r} = \frac{a(r^n - 1)}{r - 1},\ r \neq 1$$

$$S = \frac{a}{1 - r},\ |r| < 1$$

Logarithmic and Exponential Functions

$$\log_a a^x = x = a^{\log_a x}$$

$$\log_a x = \frac{\log_b x}{\log_b a}$$

$$a^x = e^{x \ln a}$$

Trigonometric Functions

$$\sin A = \frac{\text{opp}}{\text{hyp}}, \quad \cos A = \frac{\text{adj}}{\text{hyp}}, \quad \tan A = \frac{\text{opp}}{\text{adj}}$$

$$A = \frac{1}{2}ab\sin C$$

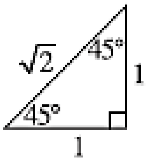
$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

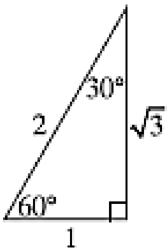
$$c^2 = a^2 + b^2 - 2ab\cos C$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

$$l = r\theta$$

$$A = \frac{1}{2}r^2\theta$$





Trigonometric identities

$$\sec A = \frac{1}{\cos A}, \ \cos A \neq 0$$

$$\operatorname{cosec} A = \frac{1}{\sin A}, \ \sin A \neq 0$$

$$\cot A = \frac{\cos A}{\sin A}, \ \sin A \neq 0$$

$$\cos^2 x + \sin^2 x = 1$$

Compound angles

$$\sin(A + B) = \sin A\cos B + \cos A\sin B$$

$$\cos(A + B) = \cos A\cos B - \sin A\sin B$$

$$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}$$

If $t = \tan\frac{A}{2}$ then $\sin A = \frac{2t}{1+t^2}$

$$\cos A = \frac{1-t^2}{1+t^2}$$

$$\tan A = \frac{2t}{1-t^2}$$

$$\cos A\cos B = \frac{1}{2}\left[\cos(A - B) + \cos(A + B)\right]$$

$$\sin A\sin B = \frac{1}{2}\left[\cos(A - B) - \cos(A + B)\right]$$

$$\sin A\cos B = \frac{1}{2}\left[\sin(A + B) + \sin(A - B)\right]$$

$$\cos A\sin B = \frac{1}{2}\left[\sin(A + B) - \sin(A - B)\right]$$

$$\sin^2 nx = \frac{1}{2}(1 - \cos 2nx)$$

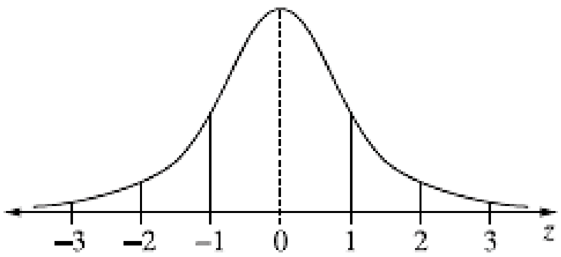
$$\cos^2 nx = \frac{1}{2}(1 + \cos 2nx)$$

Statistical Analysis

$$z = \frac{x - \mu}{\sigma}$$

An outlier is a score
less than $Q_1 - 1.5 \times IQR$
or
more than $Q_3 + 1.5 \times IQR$

Normal distribution



- approximately 68% of scores have z-scores between –1 and 1
- approximately 95% of scores have z-scores between –2 and 2
- approximately 99.7% of scores have z-scores between –3 and 3

$$E(X) = \mu$$

$$\operatorname{Var}(X) = E\left[(X - \mu)^2\right] = E\left(X^2\right) - \mu^2$$

Probability

$$P(A \cap B) = P(A)P(B)$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}, \ P(B) \neq 0$$

Continuous random variables

$$P(X \le r) = \int_a^r f(x)\,dx$$

$$P(a < X < b) = \int_a^b f(x)\,dx$$

Binomial distribution

$$P(X = r) = {}^nC_r p^r(1 - p)^{n-r}$$

$$X \sim \operatorname{Bin}(n, p)$$

$$\Rightarrow \ P(X = x)$$

$$= \binom{n}{x}p^x(1 - p)^{n-x}, \ x = 0, 1, \ldots, n$$

$$E(X) = np$$

$$\operatorname{Var}(X) = np(1 - p)$$

Differential Calculus

Function	Derivative
$y = f(x)^n$	$\dfrac{dy}{dx} = nf'(x)[f(x)]^{n-1}$
$y = uv$	$\dfrac{dy}{dx} = u\dfrac{dv}{dx} + v\dfrac{du}{dx}$
$y = g(u)$ where $u = f(x)$	$\dfrac{dy}{dx} = \dfrac{dy}{du} \times \dfrac{du}{dx}$
$y = \dfrac{u}{v}$	$\dfrac{dy}{dx} = \dfrac{v\dfrac{du}{dx} - u\dfrac{dv}{dx}}{v^2}$
$y = \sin f(x)$	$\dfrac{dy}{dx} = f'(x)\cos f(x)$
$y = \cos f(x)$	$\dfrac{dy}{dx} = -f'(x)\sin f(x)$
$y = \tan f(x)$	$\dfrac{dy}{dx} = f'(x)\sec^2 f(x)$
$y = e^{f(x)}$	$\dfrac{dy}{dx} = f'(x)e^{f(x)}$
$y = \ln f(x)$	$\dfrac{dy}{dx} = \dfrac{f'(x)}{f(x)}$
$y = a^{f(x)}$	$\dfrac{dy}{dx} = (\ln a)f'(x)a^{f(x)}$
$y = \log_a f(x)$	$\dfrac{dy}{dx} = \dfrac{f'(x)}{(\ln a)f(x)}$
$y = \sin^{-1} f(x)$	$\dfrac{dy}{dx} = \dfrac{f'(x)}{\sqrt{1-[f(x)]^2}}$
$y = \cos^{-1} f(x)$	$\dfrac{dy}{dx} = -\dfrac{f'(x)}{\sqrt{1-[f(x)]^2}}$
$y = \tan^{-1} f(x)$	$\dfrac{dy}{dx} = \dfrac{f'(x)}{1+[f(x)]^2}$

Integral Calculus

$$\int f'(x)[f(x)]^n dx = \frac{1}{n+1}[f(x)]^{n+1} + c$$

where $n \neq -1$

$$\int f'(x)\sin f(x)dx = -\cos f(x) + c$$

$$\int f'(x)\cos f(x)dx = \sin f(x) + c$$

$$\int f'(x)\sec^2 f(x)dx = \tan f(x) + c$$

$$\int f'(x)e^{f(x)}dx = e^{f(x)} + c$$

$$\int \frac{f'(x)}{f(x)}dx = \ln|f(x)| + c$$

$$\int f'(x)a^{f(x)}dx = \frac{a^{f(x)}}{\ln a} + c$$

$$\int \frac{f'(x)}{\sqrt{a^2 - [f(x)]^2}}dx = \sin^{-1}\frac{f(x)}{a} + c$$

$$\int \frac{f'(x)}{a^2 + [f(x)]^2}dx = \frac{1}{a}\tan^{-1}\frac{f(x)}{a} + c$$

$$\int u\frac{dv}{dx}dx = uv - \int v\frac{du}{dx}dx$$

$$\int_a^b f(x)\,dx$$

$$\approx \frac{b-a}{2n}\left\{f(a) + f(b) + 2\left[f(x_1) + \cdots + f(x_{n-1})\right]\right\}$$

where $a = x_0$ and $b = x_n$

Combinatorics

$$^{n}P_{r} = \frac{n!}{(n-r)!}$$

$$\binom{n}{r} = {}^{n}C_{r} = \frac{n!}{r!\,(n-r)!}$$

$$(x+a)^{n} = x^{n} + \binom{n}{1}x^{n-1}a + \cdots + \binom{n}{r}x^{n-r}a^{r} + \cdots + a^{n}$$

Vectors

$$\left|\underset{\sim}{u}\right| = \left|x\underset{\sim}{i} + y\underset{\sim}{j}\right| = \sqrt{x^{2}+y^{2}}$$

$$\underset{\sim}{u}\cdot\underset{\sim}{v} = \left|\underset{\sim}{u}\right|\left|\underset{\sim}{v}\right|\cos\theta = x_{1}x_{2} + y_{1}y_{2},$$

where $\underset{\sim}{u} = x_{1}\underset{\sim}{i} + y_{1}\underset{\sim}{j}$

and $\underset{\sim}{v} = x_{2}\underset{\sim}{i} + y_{2}\underset{\sim}{j}$

$$\underset{\sim}{r} = \underset{\sim}{a} + \lambda\underset{\sim}{b}$$

Complex Numbers

$$\begin{aligned} z = a + ib &= r(\cos\theta + i\sin\theta) \\ &= re^{i\theta} \end{aligned}$$

$$\begin{aligned} \left[r(\cos\theta + i\sin\theta)\right]^{n} &= r^{n}(\cos n\theta + i\sin n\theta) \\ &= r^{n}e^{in\theta} \end{aligned}$$

Mechanics

$$\frac{d^{2}x}{dt^{2}} = \frac{dv}{dt} = v\frac{dv}{dx} = \frac{d}{dx}\left(\frac{1}{2}v^{2}\right)$$

$$x = a\cos(nt+\alpha) + c$$

$$x = a\sin(nt+\alpha) + c$$

$$\ddot{x} = -n^{2}(x-c)$$

Mrs Cooper
Mrs Greenberg
Ms Lau

Mr Feng
Mrs Kerr
Mrs Squires

Name: Solutions

Teacher:

Pymble Ladies' College

MATHEMATICS EXTENSION 1

Year 11 ASSESSMENT TASK 1

Term 2 2023

Answer Booklet

Time allowed: 55 minutes

Total marks: 33

Section I **5 marks**

Section II **28 marks**

Instructions

- **Put your Name and Teacher's name on this Booklet.**
- For Questions 1-5 use the Multiple-Choice Answer Sheet.
- Answer Questions 6, 7 and 8 in the spaces allocated for each question as these spaces provide guidance for the expected length of your response.
- Write using black or blue pen. Black is preferred.
- NESA approved calculators may be used.
- Reference sheets have been provided.

Mark	/33
Rank	/
Highest Mark	/33

Mathematics Extension I
Year 11

Section I – Multiple Choice Answer Sheet

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample: $2 + 4 =$ (A) 2 (B) 6 (C) 8 (D) 9

A ○ B ● C ○ D ○

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ● B ~~●~~ C ○ D ○

If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word ***correct*** and drawing an arrow as follows.

correct

A ~~●~~ B ~~●~~ C ○ D ○

Assessment Task 2

1	A ~~●~~	B ●	C ○	D ○
2	A ○	B ●	C ○	D ○
3	A ○	B ○	C ○	D ●
4	A ○	B ○	C ●	D ○
5	A ○	B ●	C ○	D ○

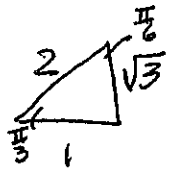
SECTION II
28 Marks

Answer all questions in the spaces provided.

Extra writing space is provided at the end of this booklet.

Your responses should include relevant mathematical reasoning and/or calculations.

Question 6 (10 marks) **Marks**



(a)

(i) 1

$\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right) = A$

$\cos A = -\frac{\sqrt{3}}{2}$

$A = \pi - \frac{\pi}{6}$ ✓

$A = \boxed{\frac{5\pi}{6}}$

$\boxed{1}$ $A = \pi - \frac{\pi}{6}$

(ii) 2

$\cos(\underbrace{\tan^{-1}(-1)}_{A}) = \cos A$

$A = \tan^{-1}(-1)$ $\quad = \cos\left(-\frac{\pi}{4}\right)$

$\tan A = -1$

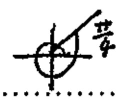
$A = -\frac{\pi}{4}$ $\quad = \cos\frac{\pi}{4}$

$= \boxed{\frac{1}{\sqrt{2}}}$

$\boxed{1}$ reaches $\tan A = -1$ or equivalent

$\boxed{2}$ correct soln

(iii) 2



$\sin^{-1}\left(\sin\frac{9\pi}{4}\right)$

$= \sin^{-1}\left(\sin\frac{\pi}{4}\right)$

$= \boxed{\frac{\pi}{4}}$

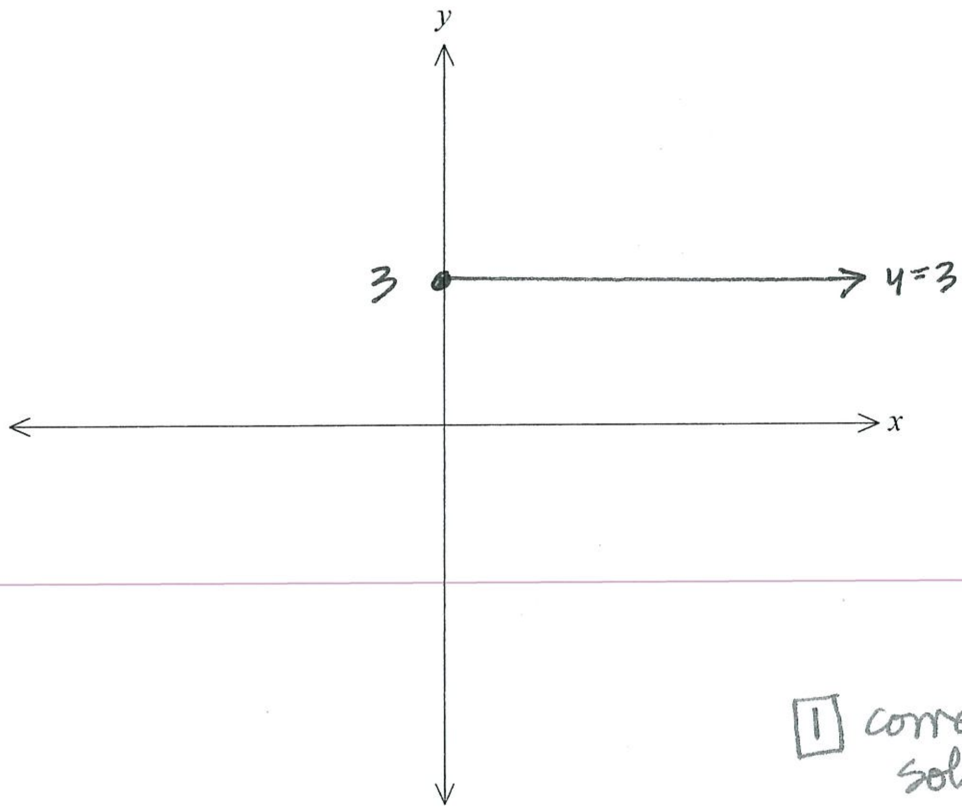
$\boxed{1}$ equates $\frac{9\pi}{4}$ with $\frac{\pi}{4}$

$\boxed{2}$ correct soln

Question 6 continues next page

Question 6 continued. **Marks**

(b) **1**



$\boxed{1}$ correct solution

$x = p^2$

$\boxed{y = 3}$

eqn is $y=3$

but $p^2 \geq 0$

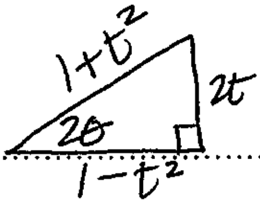
$\therefore x \geq 0$

Domain: $x \geq 0$

Question 6 continues next page

Marks

(c) $t = \tan\theta$



2

$$\frac{\cot 2\theta}{1-\operatorname{cosec} 2\theta} = \frac{\frac{1-t^2}{2t}}{1-\frac{1+t^2}{2t}} \times \frac{2t}{2t}$$

$$= \frac{1-t^2}{2t-1-t^2}$$

$$= \frac{1-t^2}{-(t^2-2t+1)}$$

$$= \frac{(1+t)(1-t)}{-(t-1)^2}$$

$$= \frac{(t+1)(t-1)}{(t-1)^2}$$

$$= \frac{t+1}{t-1}$$

correct subst

$\boxed{1}$ $\dfrac{\frac{1-t^2}{2t}}{1-\frac{1+t^2}{2t}}$

$\boxed{2}$ correct soln

Question 6 continues next page

Question 6 continued. **Marks**

(d) $\sin\frac{5\pi}{24}\cdot\sin\frac{\pi}{24} = \frac{1}{2}\left[\cos\left(\frac{5\pi}{24}-\frac{\pi}{24}\right)-\cos\left(\frac{5\pi}{24}+\frac{\pi}{24}\right)\right]$ **2**

A B

$= \frac{1}{2}\left[\cos\frac{\pi}{6} - \cos\frac{\pi}{4}\right]$

$= \frac{1}{2}\left(\frac{1}{2}-\frac{1}{\sqrt{2}}\right)$

$= \frac{1}{4} - \frac{1}{2\sqrt{2}}$

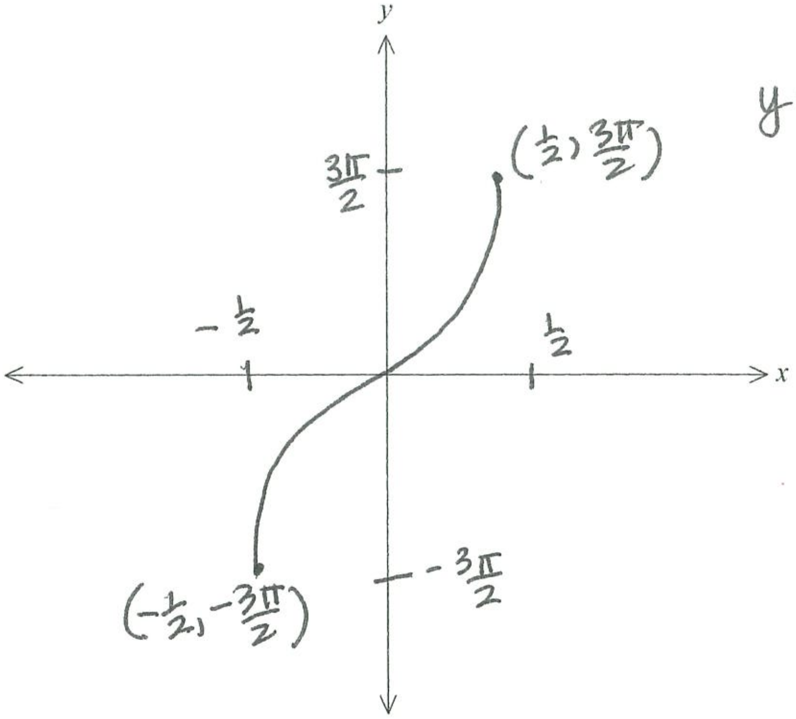
$\boxed{1}$ attempts to use product to sum

$\boxed{2}$ $\frac{1}{2}\left(\frac{1}{2}-\frac{1}{\sqrt{2}}\right)$ full marks (rationalisation not required)

End of Question 6

Question 7 (9 marks) **Marks**

(a) **2**



$y = 3\sin^{-1}2x$

$\boxed{1}$ correct range or domain

$\boxed{2}$ correct graph.

$y = 3\sin^{-1}2x$

$\frac{y}{3} = \sin^{-1}2x$

D: $-1 \le 2x \le 1$

$-\frac{1}{2} \le x \le \frac{1}{2}$

R: $-\frac{\pi}{2} \le \frac{y}{3} \le \frac{\pi}{2}$

$-\frac{3\pi}{2} \le y \le \frac{3\pi}{2}$

(b) **2**

$\cos 255^\circ = \cos(210^\circ + 45^\circ)$

$= \cos 210 . \cos 45 - \sin 210^\circ \sin 45^\circ$

$= -\cos 30^\circ . \cos 45 - (-\sin 30^\circ) . \sin 45^\circ$

$= -\frac{\sqrt{3}}{2} . \frac{1}{\sqrt{2}} + \frac{1}{2} . \frac{1}{\sqrt{2}}$

$= \frac{1 - \sqrt{3}}{2\sqrt{2}}$

$\boxed{1}$ uses $\cos(A+B)$ expands correctly

$\boxed{2}$ correct soln (rationalisation not required)

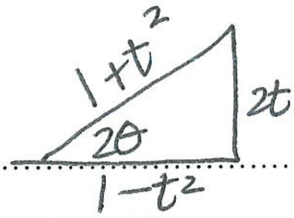
$-\frac{1}{2\sqrt{2}} + \frac{\sqrt{3}}{2\sqrt{2}}$ 2 marks

Question 7 continues next page

Question 7 continued. **Marks**

(c)

(i) let $t = \tan\theta$ **2**



$$LHS = \frac{1-t^2}{1+t^2}$$

$$RHS = \frac{1-t^2}{1+t^2} \quad (\text{from } \Delta)$$

$$= LHS \text{ as required}$$

[1] correct ~~set~~. set up for t-formula and applies t-formula ~~to~~ to either LHS or RHS correctly

[2] correct soln

or

$$LHS = \frac{1 - \frac{\sin^2\theta}{\cos^2\theta}}{1 + \frac{\sin^2\theta}{\cos^2\theta}} \times \frac{\cos^2\theta}{\cos^2\theta}$$

$$= \frac{\cos^2\theta - \sin^2\theta}{\cos^2\theta + \sin^2\theta}$$

$$= \frac{\cos 2\theta}{1}$$

$$= RHS \text{ as required}$$

OR

[1] converts to $\sin\theta$ and $\cos\theta$

or

correct use of one identity

$\cos^2\theta - \sin^2\theta = \cos^2\theta$

$\cos^2\theta + \sin^2\theta = 1$

[2] correct soln

Question 7 continues next page

 Marks

(ii) 3

let $\theta = \frac{\pi}{8}$ $\quad \therefore 2\theta = \frac{\pi}{4}$

by (i) $\dfrac{1-\tan^2\frac{\pi}{8}}{1+\tan^2\frac{\pi}{8}} = \cos\frac{\pi}{4}$

let $\tan\theta = t$

$\therefore \dfrac{1-t^2}{1+t^2} = \dfrac{1}{\sqrt{2}}$

$\sqrt{2}-\sqrt{2}t^2 = 1+t^2$

$\sqrt{2}-1 = t^2+\sqrt{2}t^2$

$\sqrt{2}-1 = t^2(1+\sqrt{2})$

$\therefore t^2 = \dfrac{\sqrt{2}-1}{\sqrt{2}+1} \times \dfrac{\sqrt{2}-1}{\sqrt{2}-1}$

$t^2 = \dfrac{2-2\sqrt{2}+1}{2-1}$

$t^2 = 3-2\sqrt{2}$ as required

$\boxed{1}$ reaches $\dfrac{1-\tan^2\frac{\pi}{8}}{1+\tan^2\frac{\pi}{8}} = \cos\frac{\pi}{4}$

gets $\boxed{2}$ $t^2 = \dfrac{\sqrt{2}-1}{\sqrt{2}+1}$

$\boxed{3}$ rationalise correctly to $t^2 = 3-2\sqrt{2}$

End of Question 7

Question 8 (9 marks) **Marks**

(a) 3

$x = 2t^2 - 1$ — $\boxed{1}$ $\quad y = 3(t+1)$

$t + 1 = \frac{y}{3}$

$t = \frac{y}{3} - 1$ — $\boxed{2}$

$\boxed{2} \to \boxed{1}$ $\quad x = 2\left(\frac{y}{3} - 1\right)^2 - 1$

$x = 2\left(\frac{y^2}{9} - \frac{2y}{3} + 1\right) - 1$

$x = \frac{2y^2}{9} - \frac{4y}{3} + 2 - 1$

$x = \frac{2y^2}{9} - \frac{4y}{3} + 1$ — $\boxed{3}$ is C

now $3x - 4y = 3$

$3x = 3 + 4y$

$x = 1 + \frac{4y}{3}$ — $\boxed{4}$

for pt of intersection of $\boxed{3}$ + $\boxed{4}$

$\boxed{4} \to \boxed{3}$

$\cancel{1+}\frac{4y}{3} = \frac{2y^2}{9} - \frac{4y}{3} \cancel{+1}$

$\boxed{\times 9}$ $\quad 12y = 2y^2 - 12y$

$0 = 2y^2 - 24y$

$0 = y^2 - 12y$

$0 = y(y - 12)$

$\therefore y = 0$ or $y = 12$

$\to \boxed{4}$ $\quad x = 1 + \frac{4}{3}(0)$ $\quad x = 1 + \frac{4}{3}(12)$

$x = 0$ $\quad x = 17$

$$\boxed{\therefore (0,0) \quad \text{and} \quad (12,17)}$$

$\boxed{1}$ subst t to $x = 2\left(\frac{y}{3} - 1\right)^2 - 1$

$\boxed{2}$ isolates variable from $3x - 4y = 3$ and subs into c eqn

$\boxed{3}$ correct solns

Question 8 continues next page

Question 8 continued. **Marks**

(b) 3

$$LHS = \frac{\sin 3\theta + \sin\theta}{\cos 3\theta + \cos\theta}$$

$A+B=3\theta$
$A-B=\theta$
$2A = 4\theta$
$\therefore A=2\theta,\ B=\theta$

$$= \frac{\sin(2\theta+\theta)+\sin(2\theta-\theta)}{\cos(2\theta+\theta)+\cos(2\theta-\theta)}$$

$$= \frac{\cancel{2}\sin 2\theta . \cancel{\cos\theta}}{\cancel{2}\cos 2\theta . \cancel{\cos\theta}}$$

$= \tan 2\theta$

$=$ RHS as required

[1] attempts to use sums to products

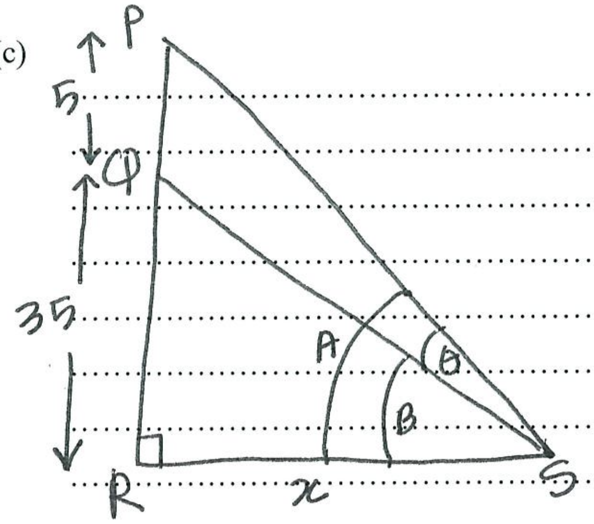
[2] reaches either $2\sin 2\theta . \cos\theta$ or $2\cos 2\theta . \cos\theta$

[3] correct soln

Question 8 continues next page

Question 8 continued. **Marks**

(c) **3**



let $A = \angle PSR$

$B = \angle QSR$

$\therefore \theta = A - B$

$\tan A = \dfrac{40}{x} \qquad \tan B = \dfrac{35}{x}$

$\tan\theta = \tan(A-B)$

$= \dfrac{\tan A - \tan B}{1 + \tan A . \tan B}$

$= \dfrac{\frac{40}{x} - \frac{35}{x}}{1 + \frac{40}{x} \cdot \frac{35}{x}} \times \dfrac{x^2}{x^2}$

$= \dfrac{40x - 35x}{x^2 + 1400}$

$\tan\theta = \dfrac{5x}{x^2+1400}$

$\therefore \theta = \tan^{-1}\left(\dfrac{5x}{x^2+1400}\right)$

or $\theta = \arctan\left(\dfrac{5x}{x^2+1400}\right)$

$\boxed{1}$ sets up $\theta = A - B$ or equivalent

or

$\tan A = \frac{40}{x}$

or

$\tan B = \frac{35}{x}$

or

attempts to use $\tan(A-B)$ expansion

$\boxed{2}$ uses $\tan(A-B)$ expansion and subst. correctly

$\boxed{3}$ reaches $\theta = \tan^{-1}\left(\frac{5x}{x^2+1400}\right)$

or

$\theta = \arctan\left(\frac{5x}{x^2+1400}\right)$

END OF ASSESSMENT TASK