AT KS GB DW KL

Name:	____________
Class:	11MTX________
Teacher:	____________

CHERRYBROOK TECHNOLOGY HIGH SCHOOL

2016
YEAR 11

AP2 EXAMINATION

MATHEMATICS EXTENSION 1

Time allowed - $1\frac{1}{2}$ HOURS
(Plus 5 minutes reading time)

Directions to candidates:

- Attempt all questions.
- Board approved calculators may be used.
- Write using black or blue pen
- Write your name at the top of every page

Section I (Multiple Choice) -Total Marks 10

- Attempt all questions.
- Write your answers on the multiple choice answer sheet provided.
- Allow about 15 minutes for this section.

Section II - Total Marks 48

- Attempt Questions 11-14.
- All questions are to be answered on the writing paper provided.
- Start each question on a new page.
- All necessary working should be shown in every question. Full marks may not be awarded for careless or badly arranged work.
- Allow about 1 hour and 15 minutes for this section.

YOUR ANSWERS WILL BE COLLECTED IN ONE BUNDLE STAPLED TOGETHER IN THE TOP LEFT HAND CORNER (MULTIPLE CHOICE SECTION I, THEN WRITTEN QUESTIONS SECTION II.

Section I: 10 Marks

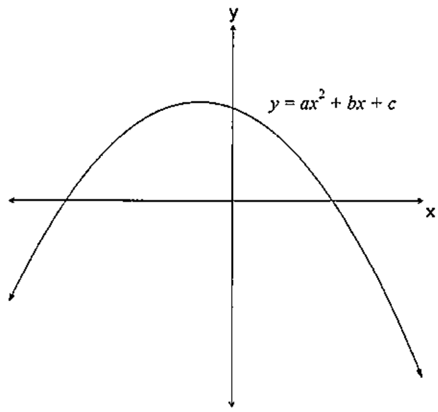
Attempt Questions 1 — 10.
Allow about 15 minutes for this section.
Use the multiple-choice answer sheet for Questions 1—10.

1. The point which divides the line segment joining A (2, $-$1) and B (8, 5) externally in the ratio 2 : 1 is

(A) (-4, -7) (B) (4, 1)

(C) (6, 3) (D) (14, 11)

2. The diagram shows the graph of $y = ax^2 + bx + c$. Which of the following is true?



(A) b > 0, c > 0 (B) b > 0, c < 0

(C) b < 0, c > 0 (D) b < 0, c < 0

3. The acute angle between the lines

$2x - y + 7 = 0$ *and* $3x + 4y - 5 = 0$

to the nearest degree is

(A) 22° (B) 47°

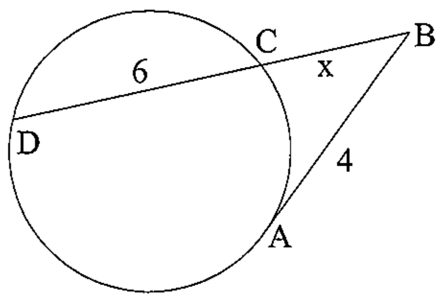
(C) 68° (D) 80°

4. The 4th term in the expansion of $\left(3x - \frac{2}{x}\right)^6$ is

(A) -4320 (B) 4320

(C) $\frac{2160}{x^2}$ (D) $2160\,x^2$

5. AB is a tangent to the circle at A. The value of x is



(A) 2 (B) 4

(C) 6 (D) 8

6. α and β are the roots of the equation $2x^2 - 3x + 5 = 0$. The value of $\frac{\alpha}{\beta} + \frac{\beta}{\alpha}$ is

(A) $\frac{9}{10}$ (B) $-\frac{11}{10}$

(C) $-\frac{1}{10}$ (D) $\frac{29}{10}$

7. Evaluate $\lim\limits_{x \to 2} \frac{x^3 - 8}{x - 2}$

(A) 0 (B) 4

(C) 12 (D) indeterminate

8. The derivative of $x(2x+1)^4$ is

(A) $(2x+1)^4$ (B) $8x(2x+1)^3$

(C) $(6x+1)(2x+1)^3$ (D) $(10x+1)(2x+1)^3$

9. Which of the following expressions is the same as $n\{n! + (n-1)!\}$?

(A) $n!$ (B) $(n+1)!$

(C) $n^2(n-1)!$ (D) $n(n+1)!$

10. P, Q are the end points of a horizontal focal chord of the parabola $x^2 + 7y - 6 = 0$. The length of PQ is

(A) $\frac{7}{4}$ (B) $\frac{7}{2}$

(C) 7 (D) 6

Section II: Short answer 48 Marks

Attempt Questions 11—14.
Allow about 1 hour 15 minutes for this section.

Answer each question on a SEPARATE page. Extra writing paper is available.
In Questions 11 — 14, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (12 Marks) **Start a NEW page** Marks

a) Evaluate $\lim_{x \to \infty} \frac{3x^2 - 5x - 7}{4x^2 + 6x + 8}$. 2

b) (i) Sketch the graph of $y = \frac{\sqrt{x^2}}{x}$. 2

(ii) Find the domain and range of $y = \frac{\sqrt{x^2}}{x}$ 2

c) Given $f(x) = = \begin{cases} (x+2)^2 + 5 & x \leq 0 \\ \frac{1}{x} - 16 & x > 0 \end{cases}$

(i) Show that $3a^2 - 2a + 8 > 0$ for all real values of a. 2

(ii) Find an expression for $f\left(\frac{1}{3a^2 - 2a + 8}\right)$, fully factorised if possible. 2

d) Given $x - \frac{1}{x} = 5$, find the value of $x^2 + \frac{1}{x^2}$ 2

Question 12 on next page...

Question 12 (12 Marks) **Start a NEW page** Marks

a) The roots of the equation $kx^2 - (k^2 + k - 1)x + 2k - 6 = 0$ are reciprocal of each other but opposite in sign.

(i) Find the value of k. 1

(ii) Hence, find the sum of the two roots of the equation. 1

b) (i) Find the value of λ if the line $12x + y - 6 = 0$ is a tangent to the parabola $y = x^2 - 2x + \lambda$. 2

(ii) Find the (α) vertex and 2

(β) focus 1

of the parabola in (i).

c) Find the equation of the parabola with vertex at (5, 2) and latus rectum of length 2. 2

d) A variable point $P(x, y)$ moves such that the square of its distance from the point A (-3, 5) is equal to 10 times its perpendicular distance from the line $L: 8x - 6y + 45 = 0$.

(i) Show that the loci of P are circles. 2

(ii) Find the centre of the circle that contains the origin. 1

Question 13 (12 Marks) **Start a NEW page**

a) (i) Rewrite $5\sin x + 12\cos x$ in the format of $A\sin(x + \alpha)$ where A > 0 and $0^\circ < \alpha < 90^\circ$. 2

(ii) Hence find the maximum value of $5\sin x + 12\cos x + 14$. 1

b) Given $3\sin 2A + 4\cos 2A + 5 = 0$. Find the value of $\tan A$. 2

c) Find the equation of the tangent with gradient -5 to the curve $y = x^3 - 6x^2 + 7x + 3$. 3

d) Given $x^2 - 3xy + 4x + 5y - 7 = 0$.

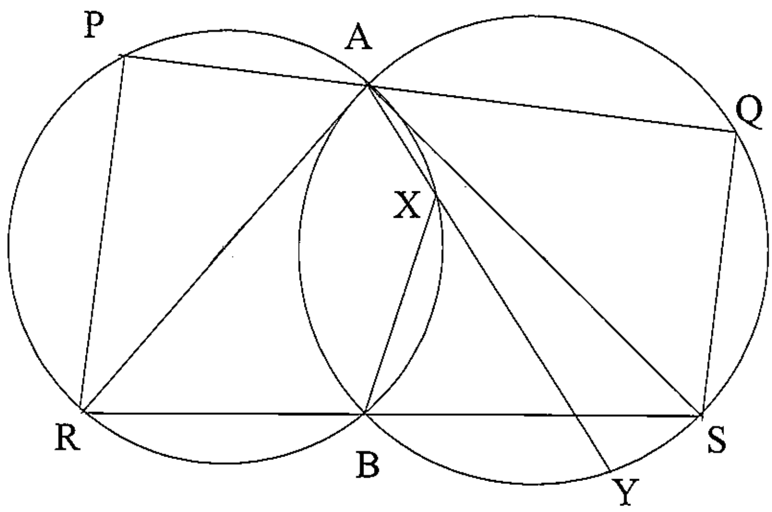
(i) Show that $y = \dfrac{x^2 + 4x - 7}{3x - 5}$. 1

(ii) Find the equation of the normal to the curve at (2, 5). 3

Question 14 on next page...

Question 14 (12 Marks) **Start a NEW page** Marks

a)



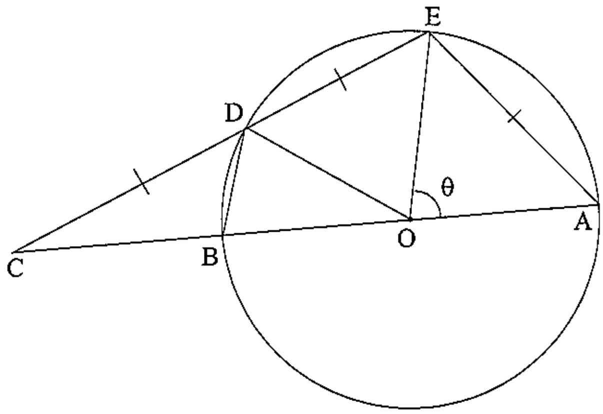
PAQ and RBS are straight lines.

(i) Prove that PR // QS. 2

(ii) If AP = BX, prove that AB = XP. 2

(iii) AR is a tangent to circle ABSQ while AS is a tangent to circle ABRP, prove that AR is a diameter of circle ABRP. 2

c) O is the centre of the circle. AE = CD = DE.



(i) Prove that $\angle OBD = \theta$. 2

(ii) Prove that $BD = \frac{1}{2} OE$. 2

(iii) Hence deduce that $\cos\theta = \frac{1}{4}$. 2

End of Paper

CTHS Yr 11 AP2 Ent 1 Sol^n

1. D   2. C   3. D   4. A   5. A

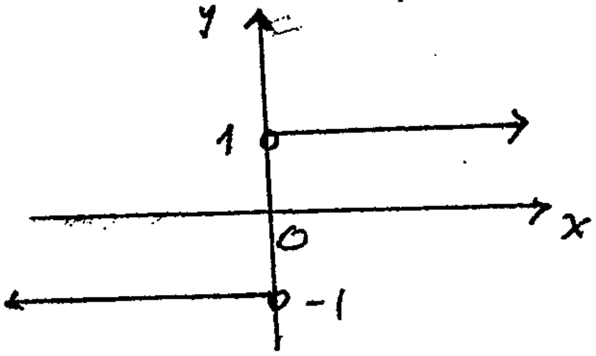
6. B   7. C   8. D   9. B   10. C

Question 11

a) $\lim_{x \to \infty} \dfrac{3x^2-5x-7}{4x^2+6x+8} = \lim_{x \to \infty} \dfrac{3-\frac{5}{x}-\frac{7}{x^2}}{4+\frac{6}{x}+\frac{8}{x^2}}$ $\boxed{1}$

$= \dfrac{3}{4}$ $\boxed{1}$

b) (i)



1 mark for eac branch. End-(must be correc to earn the mark.

(ii) Domain $= \{x \in \mathbb{R} : x \neq 0\}$ $\boxed{1}$

Range $= \{y \in \mathbb{R} : y = 1 \text{ or } y = -1\}$ $\boxed{1}$

c) (i) For $3a^2-2a+8$, $\Delta = (-2)^2 - 4(3)(8)$

$= -92 < 0$ $\boxed{1}$

$\therefore y = 3a^2-2a+8$ is positive definite $\boxed{1}$

hence $3a^2-2a+8 > 0 \quad \forall a \in \mathbb{R}$.

(ii) $f\left(\dfrac{1}{3a^2-2a+8}\right) = 3a^2-2a+8-16$

$= 3a^2-2a-8$ $\boxed{1}$

$= (a-2)(3a+4)$ $\boxed{1}$

d) $x^2 + \dfrac{1}{x^2} = \left(x - \dfrac{1}{x}\right)^2 + 2$ $\boxed{1}$

$= 5^2 + 2$

$= 27$ $\boxed{1}$

Question 12

a) Let the roots be $\alpha, -\frac{1}{\alpha}$

(i) $\therefore \frac{2k-6}{k} = -1$

$$2k-6 = -k$$
$$3k = 6$$
$$k = 2 \qquad \boxed{1}$$

(ii) Sum of roots $= \frac{k^2+k-1}{k}$

$$= \frac{4+2-1}{2}$$
$$= \frac{5}{2} \qquad \boxed{1}$$

b)(i) When the line $12x+y-6=0$ (1)

meets the parabola $y = x^2-2x+\lambda$ (2)

$$x^2-2x+\lambda = 6-12x$$
$$x^2+10x+(\lambda-6)=0 \qquad (3)$$

Since (1) is a tangent to (2),

(3) must have a double root, $\Delta=0$ $\boxed{1}$

$$10^2-4(\lambda-6)=0$$
$$\lambda-6=25$$
$$\lambda=31 \qquad \boxed{1}$$

(ii) (2) becomes $y = x^2-2x+31$

$$= (x-1)^2+30$$

or $(x-1)^2 = y-30$ $\boxed{1}$

(α) $\therefore$ vertex is $(1, 30)$ $\boxed{1}$

(β) Focal length is $\frac{1}{4}$

$\therefore$ Focus is $(1, 30\frac{1}{4})$ or $(1, \frac{121}{4})$ $\boxed{1}$

4 solutions, so 1 mark for a-value and 1 mark for any equation

c) $4a = 2$

$\therefore$ Equation of the parabolas are

$$(x-5)^2 = \pm 2(y-2)$$

ie $x^2 - 10x - 2y + 29 = 0$, $x^2 - 10x + 2y + 21 = 0$

d) (i) $$(x+3)^2 + (y-5)^2 = 10\left|\frac{8x-6y+45}{\sqrt{8^2+6^2}}\right|$$

$$x^2+6x+9+y^2-10y+25 = |8x-6y+45|$$

$$x^2+y^2-2x-4y-11=0 \;;\; x^2+y^2+14x-16y+79=0$$

$$(x-1)^2+(y-2)^2=4^2 \;;\; (x+7)^2+(y-8)^2=34 \quad \boxed{1}+\boxed{1}$$

$\therefore$ The possible loci of P are both circles.

(ii) The centre of $(x-1)^2+(y-2)^2=4^2$ is $C(1,2)$

radius $= 4$

$OC = \sqrt{1^2+2^2}$

$= \sqrt{5} < 4$

$\therefore$ The origin is inside this circle $(x-1)^2+(y-2)^2=4^2$.

For the circle $(x+7)^2+(y-8)^2=34$,

centre is $D(-7,8)$, $r=\sqrt{34}$

$OD = \sqrt{7^2+8^2}$

$= \sqrt{113} > \sqrt{34}$

$\therefore$ The origin lies outside of the circle.

$\therefore$ The circle is $(x-1)^2+(y-2)^2=4^2$

$\therefore$ centre is $(1,2)$ $\boxed{1}$

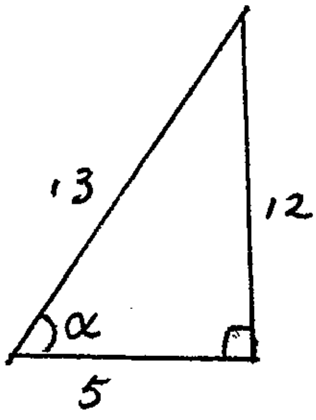
uestion 13

(i) $5\sin x + 12\cos x$

$= 13\left(\frac{5}{13}\sin x + \frac{12}{13}\cos x\right)$

$= 13(\cos\alpha \sin x + \sin\alpha \cos x)$

$= 13\sin(x+\alpha)$ where $\alpha = \tan^{-1}\frac{12}{5}$



$\boxed{1}$ mark for 13
$\boxed{1}$ mark for α

) $5\sin x + 12\cos x + 14$

$= 13\sin(x+\alpha) + 14$

$\leq 13 + 14$ since $-1 \leq \sin\theta \leq 1 \quad \forall \theta \in \mathbb{R}$

$= 27$

$\therefore$ max value $= 27$ $\boxed{1}$

' Let $t = \tan A$

$3\sin 2A + 4\cos 2A + 5 = 0$ becomes

$$\frac{3(2t)}{1+t^2} + \frac{4(1-t^2)}{1+t^2} + 5 = 0 \quad \boxed{1}$$

$$6t + 4 - 4t^2 + 5 + 5t^2 = 0$$

$$t^2 + 6t + 9 = 0$$

$$(t+3)^2 = 0$$

$$t = -3$$

$$\tan A = -3 \quad \boxed{1}$$

) $y = x^3 - 6x^2 + 7x + 3$

$y' = 3x^2 - 12x + 7$

gradient $= -5$

$\therefore 3x^2 - 12x + 7 = -5$ $\boxed{1}$

$3x^2 - 12x + 12 = 0$

$x^2 - 4x + 4 = 0$

$(x-2)^2 = 0$

Q13.(cont'd)

$$x = 2$$

At $x=2$, $\quad y = 1$ $\qquad \boxed{1}$

$\therefore$ Equation of the tangent is

$$y - 1 = -5(x-2)$$
$$= -5x + 10$$

$$5x + y - 11 = 0 \qquad \boxed{1}$$

d) (i)

$$x^2 - 3xy + 4x + 5y - 7 = 0$$
$$x^2 + 4x - 7 - (3x-5)y = 0$$
$$\therefore \ y = \frac{x^2+4x-7}{3x-5} \qquad \boxed{1}$$

(ii)

$$\frac{dy}{dx} = \frac{(2x+4)(3x-5) - 3(x^2+4x-7)}{(3x-5)^2}$$
$$= \frac{6x^2+2x-20-3x^2-12x+21}{(3x-5)^2}$$
$$= \frac{3x^2-10x+1}{(3x-5)^2} \qquad \boxed{1}$$

at $(2,5)$ $\dfrac{dy}{dx} = -7$

$\therefore$ gradient of normal $= \frac{1}{7}$ $\qquad \boxed{1}$

Equation of normal is

$$y - 5 = \frac{1}{7}(x-2)$$
$$7y - 35 = x - 2$$
$$x - 7y + 33 = 0 \qquad \boxed{1}$$

Question 14

a) (i) Join AB

$\angle APR = \angle ABS$ (ext $\angle$ of cyclic quad equals int. opp $\angle$)

but $\angle ABS + \angle AQS = 180°$ (opp $\angle$s of cyclic quad are supplementary) $\boxed{1}$

$\therefore \angle APR + \angle AQS = 180°$

$PR \parallel QS$ (co-int $\angle$s between // lines are supplementary) $\boxed{1}$

(ii) $AP = BX$ (given)

$\therefore \angle AXP = \angle BAX$ ($\angle$s subtended by equal chords are equal)

$\angle BXP = \angle BAP$ ($\angle$s in same seg. are equal)

$\angle AXP + \angle BXP = \angle BAX + \angle BAP$

$\angle AXB = \angle PAX$ $\boxed{1}$

In ΔPAX, ΔBXA

$AP = BX$ (given)

$\angle PAX = \angle AXB$ (proved)

$AX = AX$ (common)

$\therefore \Delta PAX \equiv \Delta BXA$ (SAS) $\boxed{1}$

$\therefore AB = XP$ (corr. sides of congruent Δs are equal)

ii) AR is tangent to circle ABSQ (given)

$\therefore \angle RAS = \angle AQS$ ($\angle s$ in alt segments are equal) $\boxed{1}$

Similarly $\angle SAR = \angle APR$ ($\angle s$ in alt segments are equal)

Hence $\angle APR = \angle AQS$.

but $\angle APR + \angle AQS = 180°$ (proved above)

$2\angle APR = 180°$

$\angle APR = 90°$ $\boxed{1}$

$\therefore$ AR is a diameter of circle ABRP ($\angle s$ in semicircle is rt$\angle$)

)(i) In $\triangle OAE$, OED

$OA = OE$ (radii)

$OE = OD$ (radii)

$AE = ED$ (given)

$\therefore \triangle OAE \equiv \triangle OED$ (SSS)

Hence $\angle EOD = \angle AOE$ (corr. $\angle s$ of congruent $\triangle s$ are equal) $\boxed{1}$

$= \theta$

In $\triangle OBD$

$OB = OD$ (radii)

$\angle OBD = \angle ODB$ (base $\angle s$ of isos. $\triangle$)

$\angle OBD + \angle ODB + \angle BOD = 180°$ ($\angle$ sum of $\triangle$ is $180°$)

$2\angle OBD = 180° - \angle BOD$

$= 180° - (180° - 2\theta)$ $\boxed{1}$

$= 2\theta$

$\therefore \angle OBD = \theta$

(ii) Hence $\angle OBD = \angle AOE$

$BD // OE$ (corr $\angle$s on // lines are equal)

In $\triangle CBD$, $\triangle COE$

$\angle BCD = \angle OCE$ (common)

$\angle CBD = \angle COE$ (corr $\angle$s, $BD//OE$)

$\triangle CBD \sim \triangle COE$ (equiangular $\triangle$s) $\boxed{1}$

$\frac{BD}{OE} = \frac{CD}{CE}$ (corr sides of similar $\triangle$s are proportional)

$= \frac{1}{2}$ ($CD = DE$, given) $\boxed{1}$

$\therefore BD = \frac{1}{2} OE$

(iii) In $\triangle BOD$, let $OB = OD = r$

$\therefore BD = \frac{r}{2}$

$OD^2 = BD^2 + BO^2 - 2BD \cdot BO \cos\theta$ (cosine rule)

$r^2 = \left(\frac{r}{2}\right)^2 + r^2 - 2 \cdot \frac{r}{2} r \cos\theta$ $\boxed{1}$

$r^2 \cos\theta = \frac{r^2}{4}$

$\therefore \cos\theta = \frac{1}{4}$ $\boxed{1}$