MK
KP
EW
AB
GB

NAME_______________

CLASS 11MTX____

CHERRYBROOK TECHNOLOGY HIGH SCHOOL

2017

YEAR 11

AP2

MATHEMATICS EXTENSION 1

Time allowed – 1½ HOURS

Plus 5 minutes reading time

DIRECTIONS TO CANDIDATES:

- Attempt all questions.
- Write your name on the question paper.
- Multiple choice questions should be answered on the separate multiple choice answer sheet.
- Each question in Section II is to be commenced in a new booklet clearly marked with your name, class and the relevant question number.
- All necessary working should be shown in every question. Full marks may not be awarded for careless or badly arranged work.
- Approved calculators may be used and the Reference Sheet is provided.

Section I (10 marks)

Attempt Questions 1–10

Allow about 15 minutes for this section.

Use the multiple choice answer sheet for Questions 1 - 10.

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1. If $\cos\alpha = \frac{1}{3}$, where α is an acute angle, find the value of $\cos 2\alpha$.

A. $-\frac{7}{9}$

B. $-\frac{1}{3}$

C. $\frac{2}{3}$

D. $\frac{2}{9}$

2. Simplify $\dfrac{(n+1)!+(n+2)!}{(n+1)!}$

A. $1+(n+2)!$

B. $n+2$

C. $n+3$

D. $(n+2)!$

3. The solution of $\dfrac{1}{4x} > x$ is:

A. $x < 0$ or $x > \frac{1}{2}$

B. $-\frac{1}{2} < x < \frac{1}{2}$

C. $x < -\frac{1}{2}$ or $x > \frac{1}{2}$

D. $x < -\frac{1}{2}$ or $0 < x < \frac{1}{2}$

4. Point P internally divides the interval between $(2,4)$ and $(4,8)$ in the ratio $2:1$. What are the coordinates of P?

A. $\left(2\frac{2}{3}, 5\frac{1}{3}\right)$

B. $(3,12)$

C. $\left(3\frac{1}{3}, 6\frac{2}{3}\right)$

D. $(4,6)$

5. Correct to the nearest degree, what is the size of the acute angle between the lines $3x - 2y = 0$ and $x + 2y = 0$?

A. 30°

B. 48°

C. 65°

D. 83°

6. If $5^x = 17$, then x is closest in value to:

A. 1.760

B. 1.761

C. 0.567

D. 0.568

7. When compared to the graph of $y = f(x)$, the graph of $y = f(2 - x)$ can be created by:

A. Reflecting $f(x)$ vertically, then shifting it 2 units to the right

B. Reflecting $f(x)$ vertically, then shifting it 2 units to the left

C. Reflecting $f(x)$ horizontally, then shifting it 2 units to the right

D. Reflecting $f(x)$ horizontally, then shifting it 2 units to the left

8. The coefficient of x^2 in the expansion of $\left(2x + \frac{3}{x}\right)^6$ is:

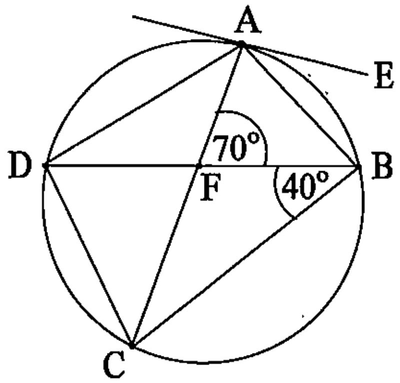
A. 135

B. 576

C. 2160

D. 4860

9. In the circle shown, the chords AC and BD meet at F. AE is a tangent to the circle, $\angle CBD = 40°$ and $\angle AFB = 70°$.



What is the size of $\angle EAB$?

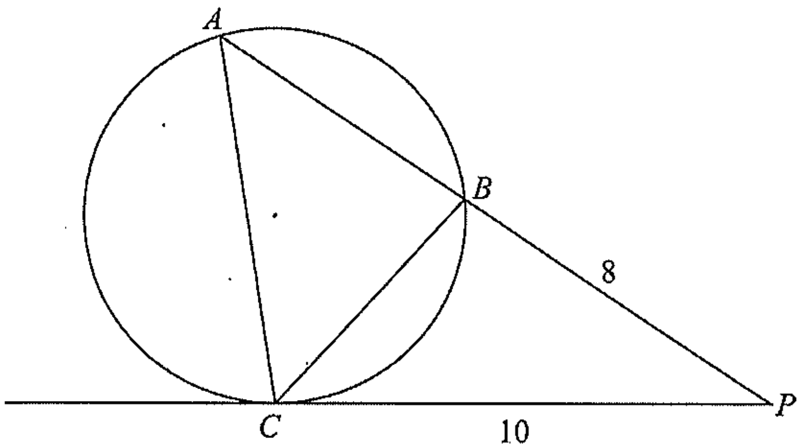
A. 30°

B. 35°

C. 40°

D. 70°

10. In the diagram below, ΔABC is inscribed in a circle. ABP is a straight line and the tangent to the circle at C intersects AP at P.



What is the length of AB?

A. 3.5

B. 4.5

C. 8.0

D. 12.5

Section II (48 marks)

Attempt Questions 11–14
Allow about 1 hour 15 minutes for this section.

Answer each question in the writing booklets provided. Begin each question in a new booklet.

Question 11 (12 marks) – **START A NEW WRITING BOOKLET** **Marks**

(a) α and β are the roots of $4x^2 - 2x - 5 = 0$.
Without evaluating α and β individually, find the exact values of:

(i) $\alpha + \beta$ **1**

(ii) $\alpha\beta$ **1**

(iii) $|\alpha - \beta|$ **2**

(iv) $\alpha - 2\alpha^2$ **1**

(b) Consider the function $y = x^2 - 2(k+1)x - k + 5$.

(i) Show that the discriminant is $4(k^2 + 3k - 4)$. **1**

(ii) Hence or otherwise find the values of k such that the function is indefinite. **2**

(c) A piece of wire 12cm long is cut into two pieces. The first length is bent to form a square, while the second length is bent to form a rectangle whose length is 3 times its breadth.

(i) Let the side length of the square be x cm and the breadth of the rectangle be y cm. Show that $x + 2y = 3$. **1**

(ii) Find the dimensions of the square such that the sum of the areas, A cm^2, is the smallest possible value. **3**

Question 12 (12 marks) – **START A NEW WRITING BOOKLET** **Marks**

(a) (i) State the domain of $y = \dfrac{x-1}{x(x+2)}$. **1**

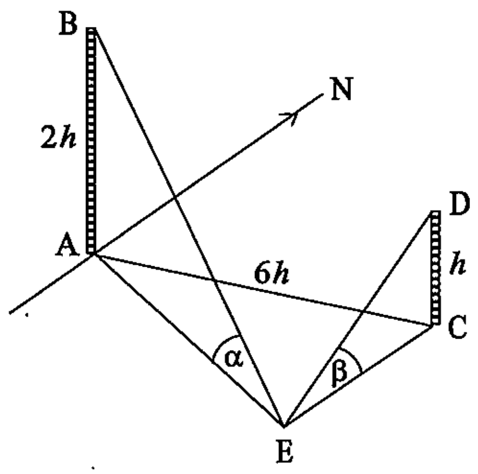
(ii) Hence, graph $y = \dfrac{x-1}{x(x+2)}$. **3**

Clearly indicate all intercepts & asymptotes with their equations.

(b) A and C are the bases of two buildings AB and CD with heights $2h$ metres and h metres respectively. **3**

CD is due east of AB and at a distance of $6h$ metres from AB.

From a point E due south of CD, the angles of elevations of AB and CD are α and β respectively.



Show that $4\cot^2\alpha - \cot^2\beta = 36$.

(c) Consider the curve $y = x^4 - 2x^2 - x$.

(i) Find the equation of line l, the tangent at $(1, -2)$. **2**

(ii) Show that line l is also tangent to the curve at another point, and find the coordinates of this point. **3**

Question 13 (12 marks) – **START A NEW WRITING BOOKLET** **Marks**

(a) M is the midpoint of hypotenuse AC in a right-angled triangle, ΔABC. Draw a neat diagram showing this information and hence prove that $BM = CM$. **3**

(b) (i) If $f(x) = \log_3(x^2)$, evaluate $f(-1)$. **1**

(ii) Neatly sketch the graph of $y = f(x)$, showing all features. **3**

(c) Show that $\dfrac{\tan^3\theta + 1}{\tan\theta + 1} = \dfrac{1}{2}\sec^2\theta\,(2 - \sin 2\theta)$ **3**

(d) x and y are rational numbers such that: **2**

- $\sqrt{x+y}$ is irrational
- $x - 2y + \sqrt{x+y} = \frac{1}{2}\sqrt{6}$

Find the values of x and y.

Question 14 (12 marks) – **START A NEW WRITING BOOKLET**

(a) Consider the function $E(x) = \displaystyle\sum_{n=0}^{\infty} \frac{x^n}{n!}$.

(i) Write out, in simplified form, the first 5 terms of $E(x)$. **1**

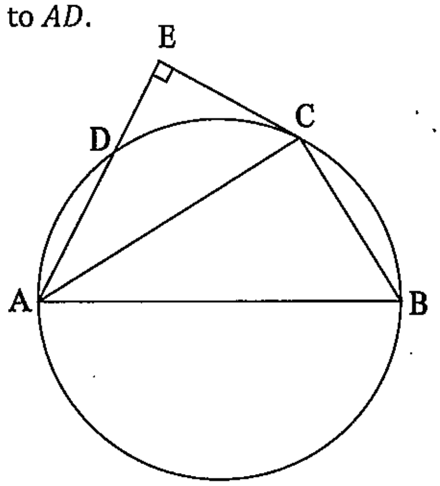
(ii) Hence prove that $E'(x) = E(x)$. **1**

(b) By using the auxiliary angle method, solve $\cos 2x + \sqrt{3}\sin 2x = \sqrt{2}$ in the domain $0 \leq x \leq 360°$. **3**

Question 14 continues on the next page

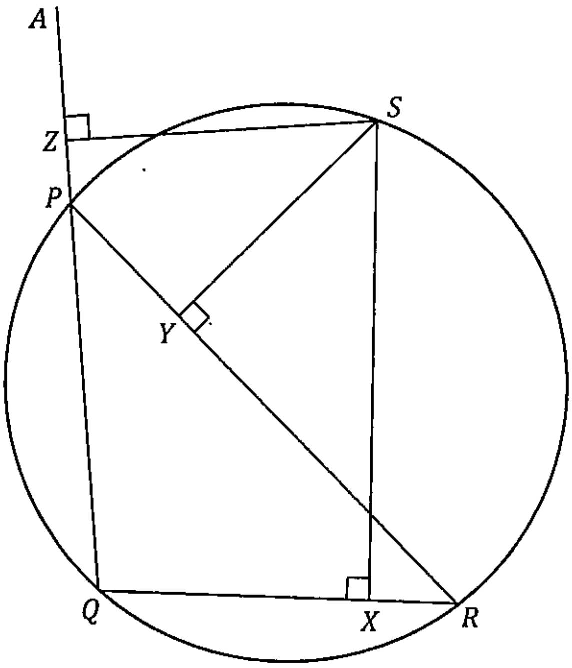
Marks

(c) A, B, C and D are points on a circle with diameter AB. **3**
The tangent at C meets the chord AD produced to E such that CE is perpendicular to AD.



Show that AC bisects $\angle EAB$.

(d) P, Q, R and S are points on the circumference of a circle. SX, SY and SZ have been constructed perpendicular to QR, PR and PQ as shown in the diagram below.



Neatly copy this diagram into your writing booklet.

(i) Prove that P, S, Y and Z are concyclic. **2**

(ii) Prove that R, S, X and Y are concyclic. **2**

End of Examination

2017 Year 11 Mathematics Extension 1 AP2 Solutions

Section I

1. $\cos 2\alpha = 2\cos^2\alpha - 1$

$= 2\left(\frac{1}{3}\right)^2 - 1$

$= \frac{2}{9} - 1$

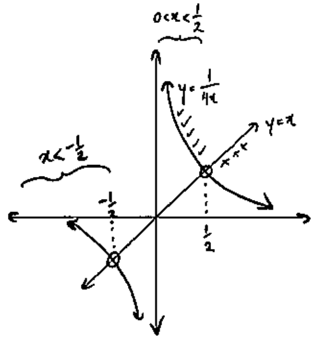
$= \frac{-7}{9} \quad \therefore$ (A)

2. $\frac{(n+1)! + (n+2)!}{(n+1)!} = \frac{(n+1)! + (n+2)(n+1)!}{(n+1)!}$

$= 1 + (n+2)$

$= n + 3 \quad \therefore$ (C)

3.



$\therefore$ (D)

4. $P = \left(\frac{lx_1 + kx_2}{k+l}, \frac{ly_1 + ky_2}{k+l}\right)$

$= \left(\frac{1(2) + 2(4)}{3}, \frac{1(4) + 2(8)}{3}\right)$

$= \left(\frac{11}{3}, \frac{20}{3}\right) \quad \therefore$ (C)

5. $3x - 2y = 0 \rightarrow y = \frac{3}{2}x$

$x + 2y = 0 \rightarrow y = \frac{-1}{2}x$

$\tan\theta = \left|\frac{\frac{3}{2} - \left(\frac{-1}{2}\right)}{1 + \left(\frac{3}{2} \times \frac{-1}{2}\right)}\right|$

$= \frac{2}{1 - \frac{3}{4}}$

$\tan\theta = 8$

$\theta = 82.87\ldots \quad \therefore$ (D)

6. $5^x = 17$

$x = \log_5 17$

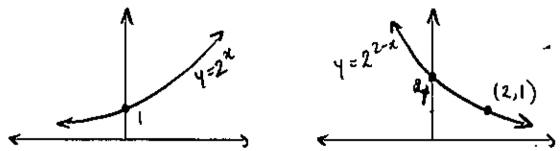
$= \frac{\log 17}{\log 5}$

$= 1.76037\ldots \quad \therefore$ (A)

7. $f(2-x) = f(-x+2)$

$= f[-(x-2)] \quad \therefore$ (C)

eg. Compare $y = 2^x$ with $y = 2^{2-x}$



8. For $\left(2x + \frac{3}{x}\right)^6$, the general term $T_n = \binom{6}{n}(2x)^n\left(\frac{3}{x}\right)^{6-n}$

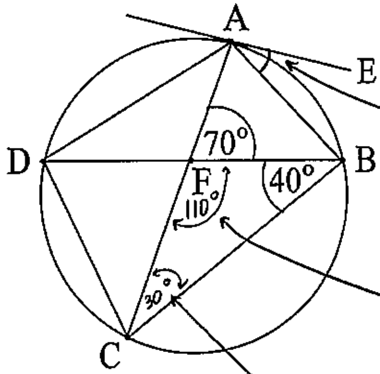
$= \binom{6}{n} 2^n x^n 3^{6-n} x^{n-6}$

$= \binom{6}{n} 2^n 3^{6-n} x^{2n-6}$

$\therefore x^2$ term is $T_4 = \binom{6}{4} 2^4 3^2 x^2$

$= 2160x^2 \quad \therefore$ (C)

9.



3. angle in the alternate segment
$\angle EAB = \angle ACB$
$= 30°$ $\therefore$ (A)

1. angle sum of $\angle AFC$

2. angle sum of $\triangle BCF$

10. $AP \times BP = PC^2$ (square of intercept on tangent equals product of intercepts on secant)

$(AB+8) \times 8 = 10^2$

$AB + 8 = 12.5$

$AB = 4.5$ $\therefore$ (B)

Section II

QUESTION 11

(a) $4x^2 - 2x - 5 = 0$

(i) $\alpha + \beta = \frac{-(-2)}{4}$
$= \frac{1}{2}$

(ii) $\alpha\beta = \frac{-5}{4}$

(iii) $|\alpha - \beta| = \sqrt{(\alpha-\beta)^2}$
$= \sqrt{(\alpha+\beta)^2 - 4\alpha\beta}$
$= \sqrt{\frac{1}{4} + 5}$
$= \sqrt{\frac{21}{4}}$
$= \frac{\sqrt{21}}{2}$

① manipulation of sum/product results

① final value

(iv) METHOD 1: USING SUM & PRODUCT RESULTS

From (i), $\beta = \frac{1}{2} - \alpha$

From (ii), $\beta = \frac{-5}{4\alpha}$

Equating gives $\frac{1}{2} - \alpha = \frac{-5}{4\alpha}$

$\frac{4\alpha}{2} - 4\alpha^2 = -5$

$2\alpha - 4\alpha^2 = -5$

$\alpha - 2\alpha^2 = \frac{-5}{2}$

METHOD 2: USING ORIGINAL EQUATION

α is a root of $4x^2 - 2x - 5 = 0$.

$\therefore 4\alpha^2 - 2\alpha - 5 = 0$

$4\alpha^2 - 2\alpha = 5$

$2\alpha^2 - \alpha = \frac{5}{2}$

$\alpha - 2\alpha^2 = \frac{-5}{2}$

(b) $y = x^2 - 2(k+1)x - k + 5$

(i) $\Delta = [-2(k+1)]^2 - 4(1)(-k+5)$

$= 4(k^2 + 2k + 1) + 4(k-5)$

$= 4(k^2 + 2k + k + 1 - 5)$

$= 4(k^2 + 3k - 4)$ as required.

(ii) Function is indefinite when $\Delta \geq 0$.

ie. $4(k^2 + 3k - 4) \geq 0$

$k^2 + 3k - 4 \geq 0$

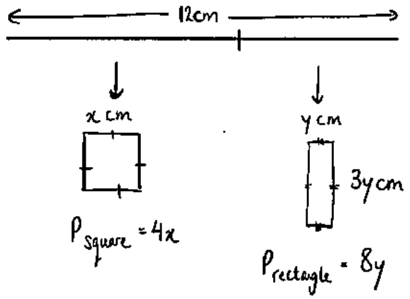
$(k+4)(k-1) \geq 0$

$\therefore \; k \leq -4$ or $k \geq 1$.

① Student makes progress in solving $\Delta \geq 0$

① correct values for k

(c) (i)



$\therefore \; 4x + 8y = 12$

$x + 2y = 3$ as required.

(ii) $x = 3 - 2y$

$\therefore A = \text{Square} + \text{Rectangle}$

$= (3-2y)^2 + (y \times 3y)$

$= 9 - 12y + 4y^2 + 3y^2$

$= 7y^2 - 12y + 9$ ① expression for unsimplified total area

This is a concave-up parabola, so its minimum value occurs along the axis of symmetry: ① reasoning to locate minimum

$y = \frac{-(-12)}{2(7)}$

$y = \frac{6}{7}$

$\therefore$ Side length of square is $3 - 2\left(\frac{6}{7}\right) = \frac{9}{7}$ cm.

① side length

(ii) Function is indefinite when $\Delta \geq 0$.

ie. $4(k^2+3k-4) \geq 0$

$k^2+3k-4 \geq 0$

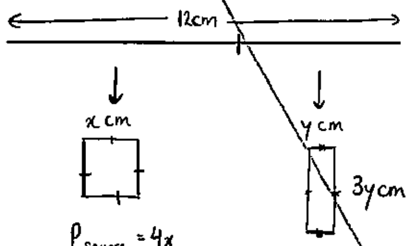
$(k+4)(k-1) \geq 0$

$\therefore\ k \leq -4$ or $k \geq 1$.

① Student makes progress in solving $\Delta \geq 0$

① correct values for k

(c) (i)



$P_{square} = 4x$

$P_{rectangle} = 8y$

$\therefore\ 4x + 8y = 12$

$x + 2y = 3$ as required.

(ii) $x = 3-2y$

$\therefore\ A =$ Square + Rectangle

$= (3-2y)^2 + (y \times 3y)$

$= 9 - 12y + 4y^2 + 3y^2$

$= 7y^2 - 12y + 9$ ① expression for total area

This is a concave-up parabola, so its minimum value occurs along the axis of symmetry: ① reasoning to locate minimum

$y = \dfrac{-(-12)}{2(7)}$

$y = \dfrac{6}{7}$ $\therefore$ Side length of square is $3-2\left(\frac{6}{7}\right) = \frac{9}{7}$ cm.

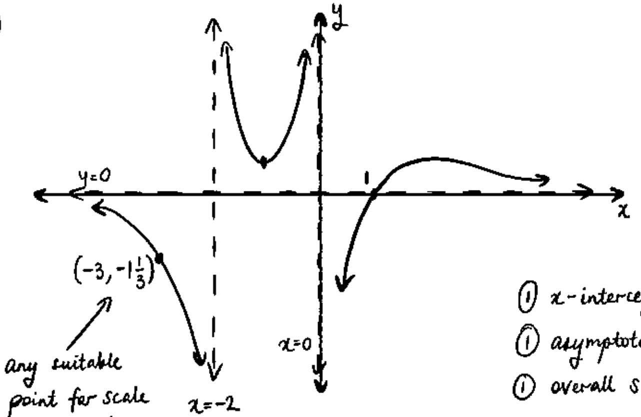
① side length

QUESTION 12

(a) $y = \dfrac{x-1}{x(x+2)}$

(i) Domain is $x<-2$, $-2<x<0$ or $x>0$.

(ii)



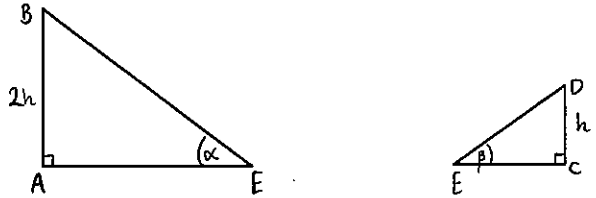
any suitable point for scale is acceptable

① x-intercept

① asymptotes

① overall shape

(b)



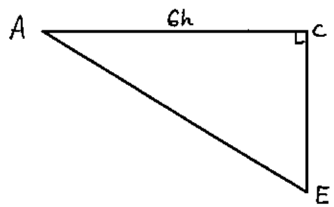
① expressions for AE & CE

In $\triangle ABE$, $\tan\alpha = \dfrac{2h}{AE}$

$AE = 2h\cot\alpha$

In $\triangle CDE$, $\tan\beta = \dfrac{h}{CE}$

$CE = h\cot\beta$



In $\triangle ACE$, $AC^2 + CE^2 = AE^2$ (Pythagoras' Theorem)

$(6h)^2 + (h\cot\beta)^2 = (2h\cot\alpha)^2$ ① substitution into Pythagoras

$36h^2 + h^2\cot^2\beta = 4h^2\cot^2\alpha$

$36 + \cot^2\beta = 4\cot^2\alpha$ ① algebraic manipulation

$4\cot^2\alpha - \cot^2\beta = 36$ as required.

(c) $y = x^4 - 2x^2 - x$

(i) $\frac{dy}{dx} = 4x^3 - 4x - 1$

When $x = 1$, $\frac{dy}{dx} = 4 - 4 - 1$

$= -1$ ① gradient at $(1,-2)$

$\therefore$ Equation of l is $y-(-2) = -1(x-1)$

$y + 2 = -x + 1$ ① equation of line

$y = -x - 1$ or $x + y + 1 = 0$

(ii) METHOD 1: TEST POINT OF INTERSECTION

The curve intersects with l when:

$x^4 - 2x^2 - x = -x - 1$

$x^4 - 2x^2 + 1 = 0$

$(x^2 - 1)^2 = 0$

$x = \pm 1$ ① locates points of intersection

Since $x=1$ gives the original intersection point, the other must be $x = -1$.

When $x = -1$, $\left.\begin{aligned}\frac{dy}{dx} &= -4+4-1\\ &= -1\end{aligned}\right\}$ ① shows that gradients are equal

$\therefore$ When $x=-1$, l intersects with the curve & their gradients are equal, so it is a tangent here as well.

When $x = -1$, $y = 1 - 2 + 1$

$= 0$ $\therefore$ Coordinates are $(-1, 0)$.

① full coordinates

METHOD 2: EQUATE GRADIENTS

If l is tangent to the curve elsewhere, there must be another point with the same gradient.

ie. $\frac{dy}{dx} = -1$

$4x^3 - 4x - 1 = -1$

$4x^3 - 4x = 0$

$x^3 - x = 0$

$x(x^2 - 1) = 0$

$x = 0$ or ± 1

$x=1$ gives the original intersection point.

When $x = 0$, $y = 0$.

When $x = -1$, $y = 1 - 2 + 1$

$= 0$

ie. The curve has gradient -1 at points $(0,0)$ and $(-1,0)$.

Test these coordinates with line l $(y = -x - 1)$:

$0 \neq -0 - 1 \rightarrow (0,0)$ is not on line l

$0 = -(-1) - 1 \rightarrow (-1,0)$ is on line l

$\therefore$ Line l is also tangent to the curve at $(-1,0)$.

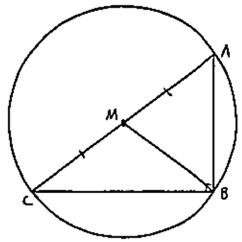
QUESTION 13

(a) METHOD 1: CIRCLE PROPERTIES

Since $\angle ABC$ is a right angle, it is the angle in a semicircle with diameter AC.

But M is the midpoint of AC, so it is the centre of the circle.

$\therefore BM = CM$ (radii)

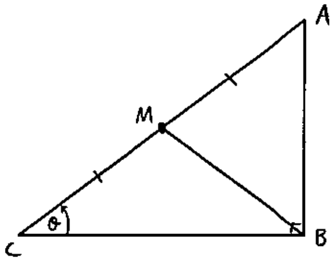


METHOD 2: TRIGONOMETRY

Let $\angle ACB = \theta$.

In $\triangle ABC$, $\cos\theta = \dfrac{BC}{AC}$

$$= \frac{BC}{2 \times CM} \quad \text{(since } M \text{ bisects } AC\text{)}$$



In $\triangle BCM$, $\cos\theta = \dfrac{CM^2 + BC^2 - BM^2}{2 \times CM \times BC}$

$$\frac{BC}{2 \times CM} = \frac{CM^2 + BC^2 - BM^2}{2 \times CM \times BC}$$

$$BC = \frac{CM^2 + BC^2 - BM^2}{BC}$$

$$BC^2 = CM^2 + BC^2 - BM^2$$

$$BM^2 = CM^2$$

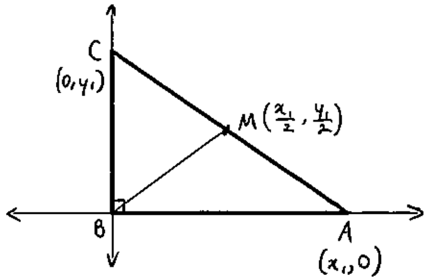
$\therefore BM = CM$ ($BM > 0$ & $CM > 0$ as they are lengths)

METHOD 3: COORDINATE GEOMETRY

Since $\triangle ABC$ is right-angled at B, position B at the origin with A & C on the x & y axes respectively.

Let coordinates of A be $(x_1, 0)$ and C be $(0, y_1)$.

$\therefore$ M is at $\left(\frac{x_1}{2}, \frac{y_1}{2}\right)$.



$$BM = \sqrt{\left(\tfrac{x_1}{2} - 0\right)^2 + \left(\tfrac{y_1}{2} - 0\right)^2}$$

$$= \sqrt{\frac{(x_1)^2}{4} + \frac{(y_1)^2}{4}}$$

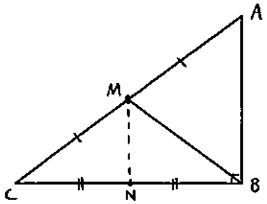
$$CM = \sqrt{\left(\tfrac{x_1}{2} - 0\right)^2 + \left(\tfrac{y_1}{2} - y_1\right)^2}$$

$$= \sqrt{\frac{(x_1)^2}{4} + \frac{(-y_1)^2}{4}}$$

$$= \sqrt{\frac{(x_1)^2}{4} + \frac{(y_1)^2}{4}}$$

$\therefore CM = BM$ as required.

METHOD 4 (triangle congruence)



Construct MN where N is the midpoint of BC.

$\therefore MN \parallel AB$ (interval joining midpoints of two sides in $\triangle ABC$ is parallel to the third side)

$\therefore \angle MNC = \angle ABC$ (corresponding angles on parallel lines are equal)

$= 90^\circ$

$\angle MNC + \angle MNB = 180^\circ$ (angle sum of a straight line is 180°)

$\angle MNB = 90^\circ$

In $\triangle MNC$ & $\triangle MNB$: MN is common

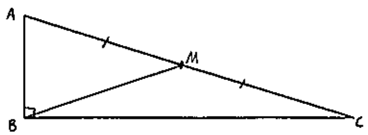
$\angle MNC = \angle MNB$ (proven above)

$CN = BN$ (N bisects BC)

$\therefore \triangle MNC \equiv \triangle MNB$ (SAS)

$\therefore BM = CM$ (corresponding sides in congruent triangles are equal), as required.

Note that the information in the question does not guarantee that BM bisects $\angle ABC$, nor that BM is perpendicular to AC. Consider the following possible $\triangle ABC$:



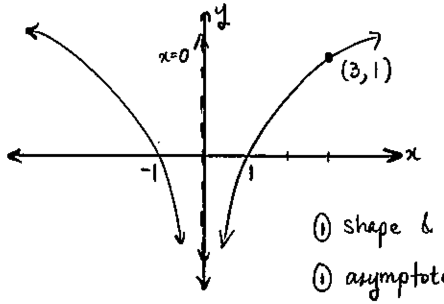
In this example, BM is clearly not perpendicular to AC, nor is $\angle ABM$ equal to $\angle CBM$ (though both of these were erroneously assumed by students).

(b) $f(x) = \log_3(x^2)$ Note that as $f(x) = f(-x)$, this is an even function.

(i) $f(-1) = \log_3 1$

$= 0$

(ii)



① shape & intercepts

① asymptote

① both branches

(c) RTP: $\dfrac{\tan^3\theta + 1}{\tan\theta + 1} = \frac{1}{2}\sec^2\theta\,(2 - \sin 2\theta)$

METHOD 1: STANDARD IDENTITIES

$$\text{LHS} = \frac{\tan^3\theta + 1}{\tan\theta + 1}$$

$$= \frac{(\tan\theta + 1)(\tan^2\theta - \tan\theta + 1)}{\tan\theta + 1}$$ ① sum of cubes factorisation

$$= \tan^2\theta - \tan\theta + 1$$

$$= \sec^2\theta - \tan\theta$$ ① Pythagorean identity

$$= \sec^2\theta - \frac{\sin\theta\cos\theta}{\cos^2\theta}$$

$$= \sec^2\theta - \frac{1}{2}\sin 2\theta \sec^2\theta$$ ① double angle identity

$$= \frac{1}{2}\sec^2\theta\,(2 - \sin 2\theta)$$ as required.

METHOD 2: t-RESULTS

Let $t = \tan\theta$.

$\therefore \sin 2\theta = \frac{2t}{1+t^2}$ and $\sec^2\theta = 1 + t^2$.

$$\text{LHS} = \frac{t^3+1}{t+1}$$

$$\text{RHS} = \frac{1}{2}(1+t^2)\left(2 - \frac{2t}{1+t^2}\right)$$

$$= (1+t^2) - t$$

$$= t^2 - t + 1$$

$$= \frac{(t+1)(t^2-t+1)}{t+1}$$

$$= \frac{t^3+1}{t+1} \text{ as required.}$$

(d) $x - 2y + \sqrt{x+y} = \frac{1}{2}\sqrt{6}$

$x - 2y + \sqrt{x+y} = \sqrt{\frac{6}{4}}$

$(x-2y) + \sqrt{x+y} = \sqrt{\frac{3}{2}}$

By comparing rational & irrational components, } ① working

$x - 2y = 0$ — ①

$x + y = \frac{3}{2}$ — ②

② − ① : $y - (-2y) = \frac{3}{2} - 0$

$3y = \frac{3}{2}$

$y = \frac{1}{2}$

$\therefore x = 1$ } ① final values

QUESTION 14

(a) $E(x) = \sum_{n=0}^{\infty} \frac{x^n}{n!}$

(i) First 5 terms $= \frac{x^0}{0!} + \frac{x^1}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!}$

$$= 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!}$$

(ii) $E'(x) = 0 + 1 + \frac{2x}{2!} + \frac{3x^2}{3!} + \frac{4x^3}{4!} + \cdots$

$$= 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$$

$= E(x)$ as required.

(b) $\cos 2x + \sqrt{3}\sin 2x = \sqrt{2}$ $\{0° \le x \le 360°\}$

Let $\cos 2x + \sqrt{3}\sin 2x = R\cos(2x - \alpha)$

$= R[\cos 2x \cos\alpha + \sin 2x \sin\alpha]$

$\cos 2x + \sqrt{3}\sin 2x = (R\cos\alpha)\cos 2x + (R\sin\alpha)\sin 2x$

By comparison of coefficients, } ① use of trigonometric expansion & formation of simultaneous equations

$R\cos\alpha = 1$ — ①

$R\sin\alpha = \sqrt{3}$ — ②

①^2 + ②^2 : $R^2\cos^2\alpha + R^2\sin^2\alpha = 1 + 3$

$R^2(1) = 4$

$R = 2$ $(R > 0)$

$\therefore \cos\alpha = \frac{1}{2}$ & $\sin\alpha = \frac{\sqrt{3}}{2}$ NOTE: do <u>not</u> accept responses that use tan without accounting for domain of α

$\rightarrow 0 < \alpha < 90^\circ$

$\rightarrow \alpha = 60^\circ$ (1) amplitude & phase shift (note that phase may differ based on choice of trigonometric expansion)

$\therefore \cos 2x + \sqrt{3}\sin 2x = 2\cos(2x - 60^\circ)$

$$\boxed{\begin{array}{c} 0 < x < 360 \\ 0 < 2x < 720 \\ -60 < 2x-60 < 660 \end{array}}$$

$2\cos(2x - 60^\circ) = \sqrt{2}$

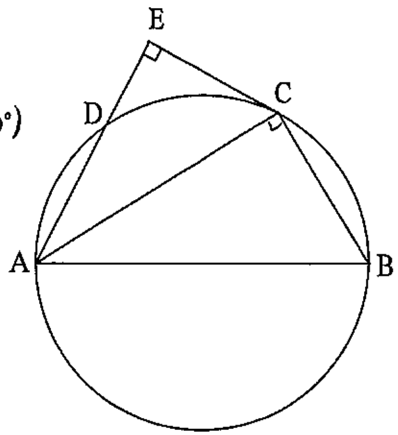
$\cos(2x - 60^\circ) = \frac{1}{\sqrt{2}}$

$2x - 60^\circ = -45^\circ, 45^\circ, 315^\circ, 405^\circ$

$2x = 15^\circ, 105^\circ, 375^\circ, 465^\circ$

$x = 7.5^\circ, 52.5^\circ, 187.5^\circ, 465^\circ$ (1) all solutions

(c) METHOD 1



$\angle AEC = 90^\circ$ ($CE \perp AD$)

$\angle ACB = 90^\circ$ (angle in a semicircle is 90°)

$\therefore \angle AEC = \angle ACB$

In ΔAEC & ΔACB:

$\angle AEC = \angle ACB$ (proven above)

$\angle ACE = \angle ABC$ (angle in the alternate segment)

$\therefore \Delta AEC \,|||\, \Delta ACB$ (equiangular)

$\therefore \angle CAE = \angle BAC$ (corresponding angles of similar triangles are equal)

$\therefore AC$ bisects $\angle EAB$, as required.

METHOD 2

$\angle ACE + \angle CAE + \angle AEC = 180^\circ$ (angle sum of ΔACE is 180°)

$\angle ACE + \angle CAE + 90^\circ = 180^\circ$

$\angle ACE = 90^\circ - \angle CAE$

But $\angle ABC = \angle ACE$ (angle in the alternate segment) (1)

$\angle ABC = 90^\circ - \angle CAE$

$\angle ACB = 90^\circ$ (angle in a semicircle is 90°) (1)

$\angle BAC + \angle ACB + \angle ABC = 180^\circ$ (angle sum of ΔABC is 180°) (1)

$\angle BAC + 90^\circ + 90^\circ - \angle CAE = 180^\circ$

$\angle BAC = \angle CAE$

$\therefore AC$ bisects $\angle EAB$, as required.

(d)(i) $\angle PZS = 90^\circ$ $(AP \perp SZ)$

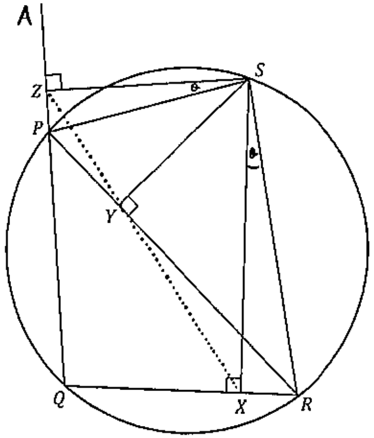
$\angle SYP = 90^\circ$ $(PR \perp SY)$ 1

$\therefore$ $\angle SYP + \angle PZS = 180^\circ$

$\therefore$ PYSZ is a cyclic quadrilateral

(opposite angles are supplementary) 1

$\therefore$ P, S, Y & Z are concyclic, as required.



(ii) $\angle RXS = 90^\circ$ $(QR \perp SX)$ 1

$\therefore$ $\angle RXS = \angle RYS$

$\therefore$ $\angle RXS$ & $\angle RYS$ are angles in the same segment

standing on the same arc RS of circle RXYS 1

(note: not arc RS of circle PQRS)

$\therefore$ R, S, X & Y are concyclic, as required.