



Caringbah High School

Year 12 2024

Mathematics Extension 1

HSC Course

Assessment Task 1

General Instructions

- Reading time – 5 minutes
- Working time – 60 minutes
- Write using black or blue pen
- NESA-approved calculators may be used
- A reference sheet is provided
- In Questions 6-10, show relevant mathematical reasoning and/or calculations
- Marks may not be awarded for partial or incomplete answers

Total marks – 45

Section I 5 marks

Attempt Questions 1-5
Mark your answers on the answer sheet provided. You may detach the sheet and write your name on it.

Section II 40 marks

Attempt Questions 6-10
Write your answers in the answer booklets provided. Ensure your name or student number is clearly visible.

Name: _____

Class: _____

Marker's Use Only							
Section I	Section II					Total	
Q 1-5	Q6	Q7	Q8	Q9	Q10		
/5	/8	/8	/8	/8	/8	/45	%

Section I

5 marks

Attempt Questions 1-5

Allow about 7 minutes for this section

Use the multiple-choice answer sheet for Questions 1–5

- 1** The rate of decay of a radioactive substance is proportional to the mass of the substance present at any time. If a half of the substance decayed in 100 years, what percentage of the original amount will remain after 500 years?
- (A) 13%
(B) 3.125%
(C) 6.5%
(D) 96.875%
- 2** The coefficient of x^4 in $(2x-3)^{13}$ is
- (A) $-\binom{13}{9}2^93^4$
(B) $\binom{13}{4}3^42^9$
(C) $\binom{13}{9}2^43^9$
(D) $-\binom{13}{4}2^43^9$
- 3** After t years the number N of individuals is given by $N = 400 + 100e^{-0.1t}$. What is the difference between the initial population size and the limiting population size?
- (A) 100
(B) 300
(C) 400
(D) 500

4 The solution to $|2x - 1| \leq |x - 2|$ is

- (A) $x \geq 1$
- (B) $x \leq 1$
- (C) $-1 \leq x \leq 1$
- (D) $x \leq -1$, or $x \geq 1$

5 A circular metal disc of 5 cm radius expands when heated. If the radius is increasing at a rate of 0.02 cm/sec, the rate at which the area of one of the faces is increasing, when $r = 5$ cm, is given by

- (A) $\frac{\pi}{10}$ cm²/sec
- (B) $\frac{\pi}{5}$ cm²/sec
- (C) $\frac{2\pi}{5}$ cm²/sec
- (D) $\frac{5\pi}{2}$ cm²/sec

End of Section I

Section II

40 marks

Attempt Questions 6–10

Allow about 53 minutes for this section

Answer each question in a SEPARATE writing booklet.

Your responses should include relevant mathematical reasoning and/or calculations.

Question 6 (8 marks) Use a SEPARATE writing booklet.

- (a) Solve the inequality $\frac{x^2 - 4}{x + 3} < x - 4$ **3**
- (b) Show by simplifying that $\frac{{}^{n+1}C_r}{{}^nC_{r-1}} = \frac{n+1}{r}$ **2**
- (c) The temperature T° of a body after t minutes is given by $T = 20 + 60e^{-0.1t}$
- (i) Find the initial temperature of the body. **1**
- (ii) Find the time taken for the temperature of the body to fall to half of its original value giving your answer correct to the nearest minute. **2**

End of Question 6

Question 7 (8 marks) Use a SEPARATE writing booklet.

(a) Find the term independent of x in the expansion of $\left(x^2 - \frac{3}{x^3}\right)^{10}$ **3**

(b) Solve the inequality **3**

$$\frac{1}{|x-3|} \geq \frac{1}{2}$$

(c) The population of bacteria in a medium increases at a rate proportional to its population present represented by the equation $P = P_0 e^{kt}$. **2**

Initially the population (P_0) was 1200 bacteria and it doubles in 3 hours.
Find the population after 4 hours.

End of Question 7

Question 8 (8 marks) Use a SEPARATE writing booklet.

- (a) The coefficient of x in the expansion of $\left(x + \frac{1}{ax^2}\right)^7$ is $\frac{7}{3}$ **3**

Find all possible values of a .

- (b) By considering the expansion of $x(1+x)^n$ and using differentiation prove **3**

$$\sum_{r=0}^n (r+1) {}^n C_r = 2^n \left(\frac{n}{2} + 1\right)$$

- (c) Sketch the region represented by the following inequality showing any points of intercepts with the axes: **2**

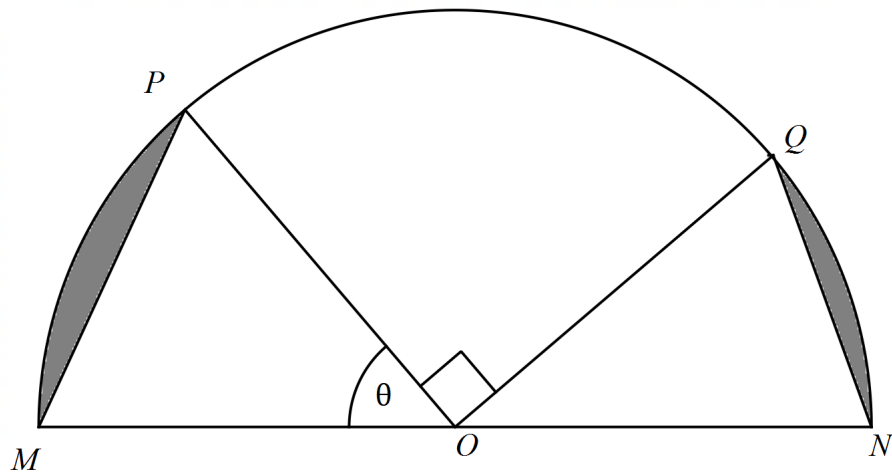
$$y \leq \frac{2}{x-2}$$

Use the template provided to write your answer to this question.

End of Question 8

Question 9 (8 marks) Use a SEPARATE writing booklet.

(a)



In the diagram, MN is a diameter of a semi-circle with centre O and radius 1 metre. P and Q are variable points which move on the semi circle so that $\angle MOP = \theta$ and $\angle POQ = \frac{\pi}{2}$.

- (i) Show that the area A of the shaded regions is given by 2

$$A = \frac{\pi}{4} - \frac{1}{2}(\sin \theta + \cos \theta)$$

- (ii) If θ is increasing at a rate of 0.1 radians/sec, find the rate at which the shaded area is changing when $\theta = 1$ radian. 2

- (b) Graph the region of intersection of the following 4

$$y \leq \sqrt{25 - x^2}$$

$$2y - x \leq 5$$

$$x + 5y \geq -5$$

Clearly show all points of intersection.

Use the template provided to write your answer to this question.

End of Question 9

Question 10 (8 marks) Use a SEPARATE writing booklet.

- (a) Wilhemy's law states that the rate of transformation of a substance in a chemical reaction is proportional to its concentration, that is

$$\frac{dx}{dt} = k(x - c),$$

where x is the amount of substance transformed and c is the initial concentration of the substance.

(Note: Initially none of the substance has been transformed)

- (i) Show that $x = c + Ae^{kt}$, where A is a constant, satisfies the equation. 1
- (ii) If the initial concentration is 7.9 mm^3 and the amount transformed after 2 minutes is 2.7 mm^3 , find the values of A and k . 2
- (iii) How much substance will be transformed after 5 minutes? 1

- (b) (i) Differentiate both sides of the expansion 2

$$(1+x)^{2n} = \sum_{k=0}^{2n} {}^{2n}C_k x^k$$

Note : $\sum_{k=0}^{2n} {}^{2n}C_k x^k = \binom{2n}{0} + \binom{2n}{1}x + \binom{2n}{2}x^2 + \dots + \binom{2n}{2n-1}x^{2n-1} + \binom{2n}{2n}x^{2n}$

- (ii) Hence show that 2

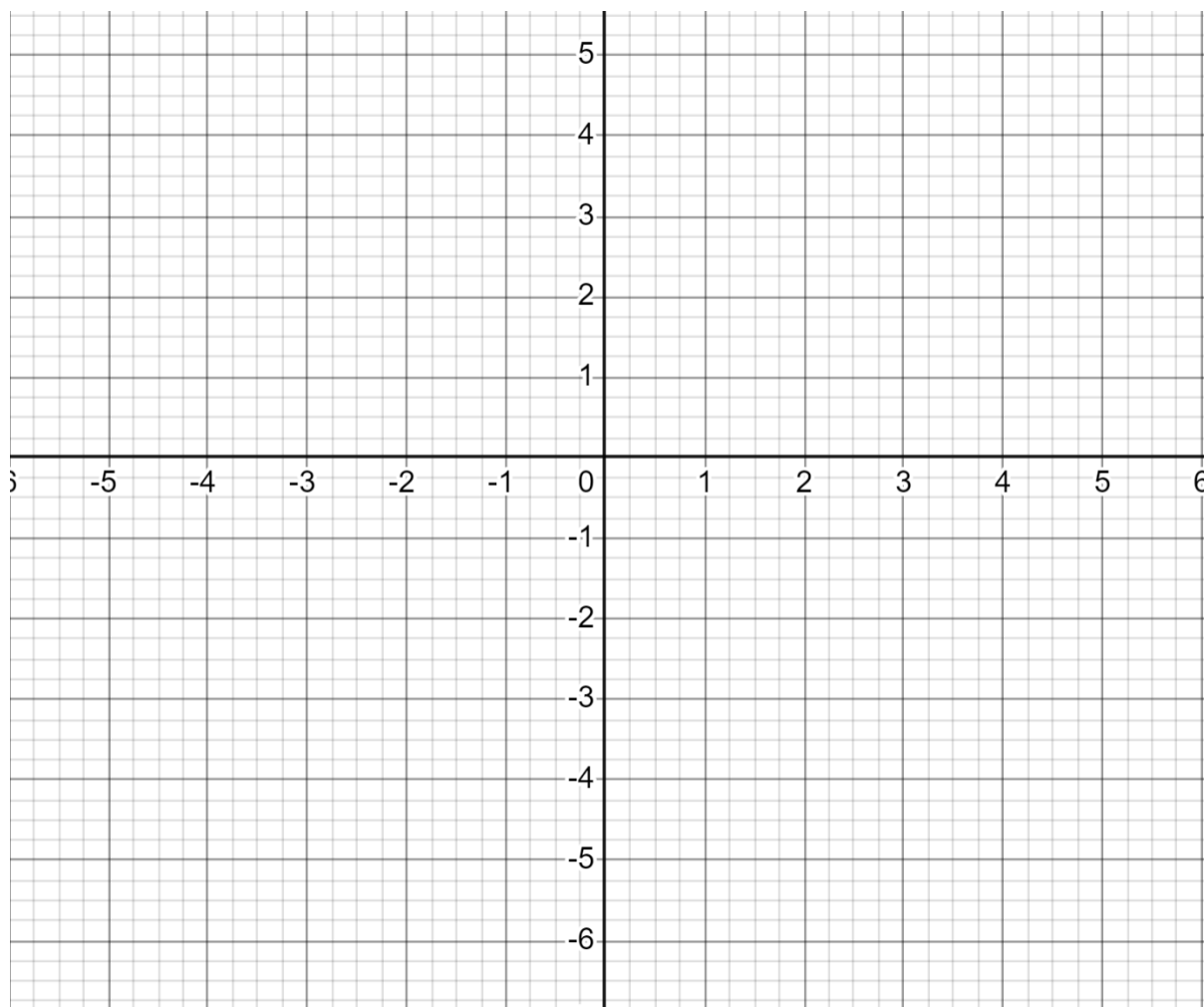
$$\sum_{k=1}^{2n} k {}^{2n}C_k = n \times 4^n$$

End of Exam

Answer Template For Question 8 (c)

Sketch the region represented by the following inequality showing any points of intercepts with the axes:

$$y \leq \frac{2}{x-2}$$



Answer Template For Question 9 (b)

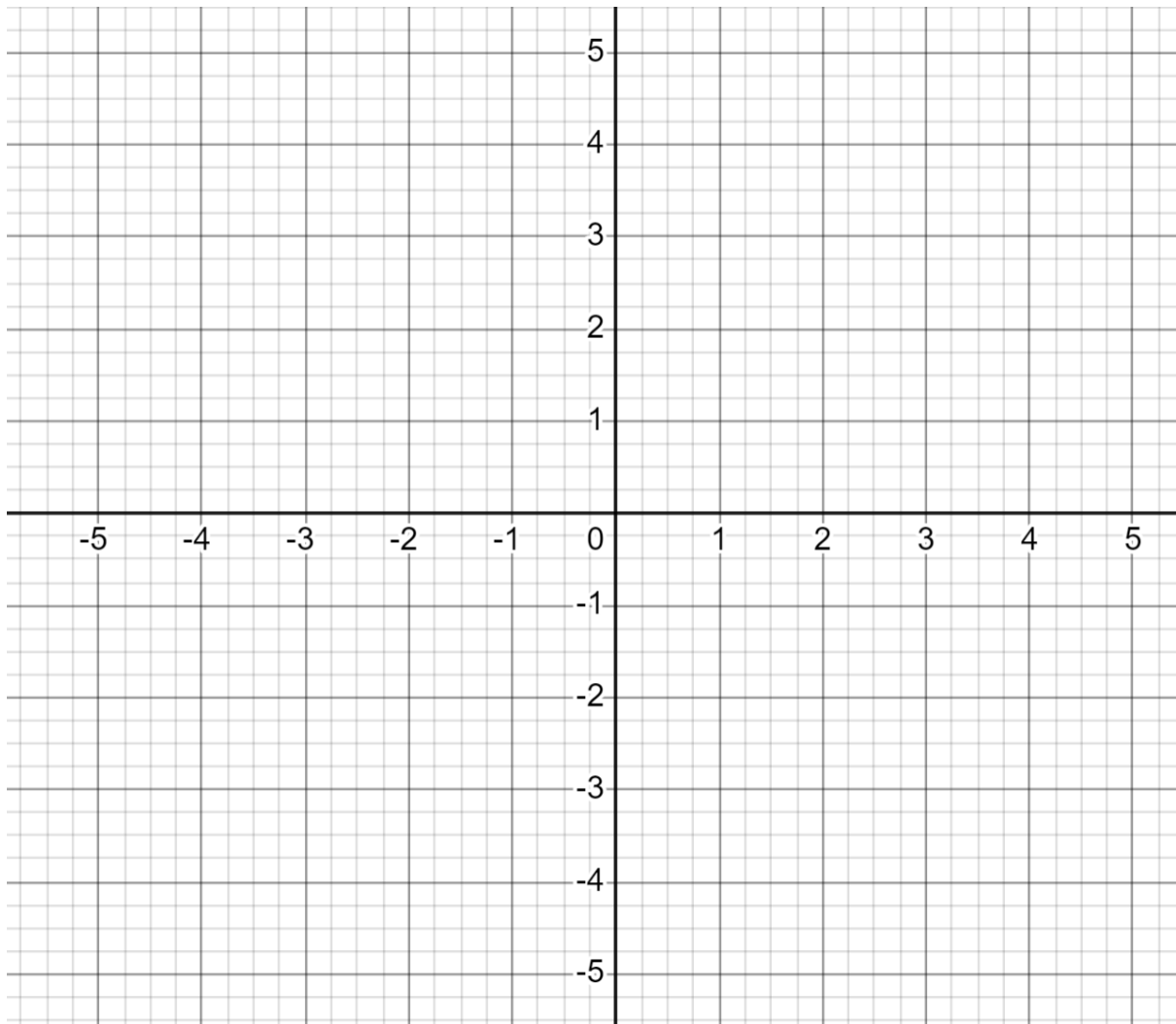
Graph the region of intersection of the following inequalities :

$$y \leq \sqrt{25 - x^2}$$

$$2y - x \leq 5$$

$$x + 5y \geq -5$$

Clearly show all points of intersection.



Section I Multiple Choice Answer Sheet

Name _____

Class _____

If you detach this sheet, please make sure your name is written above.

Completely fill the response oval representing the most correct answer.

Sample:

$2 + 4 = ?$ (A) 2 (B) 6 (C) 8 (D) 9

A ☐ B ☒ C ☐ D ☐

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ☒ B ☒ C ☐ D ☐

If you change your mind and have crossed out what you consider to be the correct answer, then indicate this by writing the word *correct* and drawing an arrow as follows:

correct
A ☒ B ☒ C ☐ D ☐

1 A ☐ B ☐ C ☐ D ☐

2 A ☐ B ☐ C ☐ D ☐

3 A ☐ B ☐ C ☐ D ☐

4 A ☐ B ☐ C ☐ D ☐

5 A ☐ B ☐ C ☐ D ☐

2024 Yr12 Ext1 AT1 solutions

Q1 $\frac{A_0}{2} = A_0 e^{-100k}$

$$k = \frac{-\ln \frac{1}{2}}{100}$$

$$= \frac{\ln 2}{100}$$

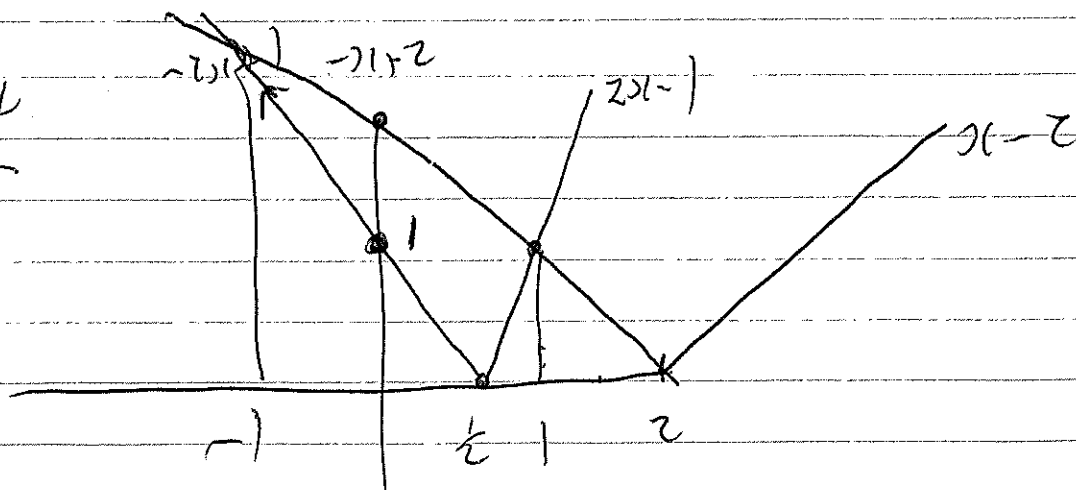
$$A = A_0 e^{-\frac{\ln 2}{100} \times 500}$$

$$\frac{A}{A_0} = \frac{1}{2} = \frac{1}{32} = 3.125\% \quad \boxed{B}$$

Q2 $(2x-3)^{13} : \binom{13}{4} (2)^4 (-3)^9 = \dots \quad \boxed{D}$

Q3 $500 - 400 = 100 \quad \boxed{A}$

Q4



$$-1 \leq x \leq 1$$

$\boxed{C}$

Q5

$$A = \pi r^2$$

$$\frac{dA}{dr} = 2\pi r$$

$$\frac{dA}{dt} = \frac{dA}{dr} \times \frac{dr}{dt}$$

B

$$= 2 \times \pi \times 5 \times 0.02$$

$$= \frac{\pi}{5} \text{ cm}^2/\text{sec}$$

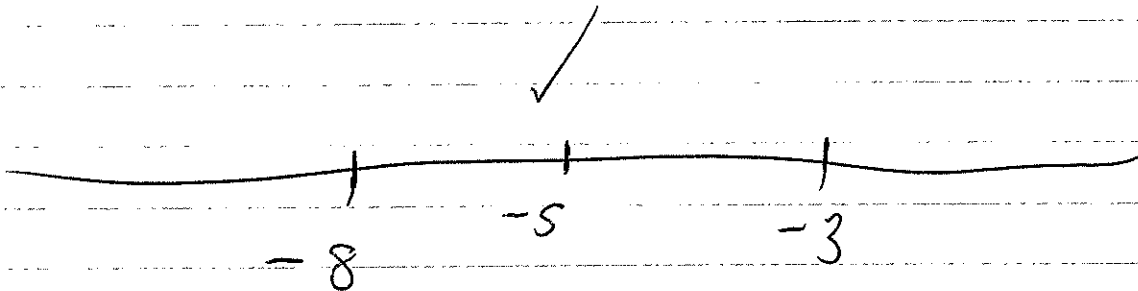
Q6 a) $\frac{x^2-4}{x+3} < x-4$ $x \neq -3$ ✓

$$x^2-4 < (x-4)(x+3)$$

$$x^2-4 < x^2-x-12$$

$$x < -8$$

✓



Test $x = -5$

$$\frac{25-4}{-2} < -9$$

$$\frac{-21}{2} < -9$$

✓

Test $x = -10$

$$\frac{96}{-7} < -14$$

$$-13\frac{6}{7} < -14 \times$$

Test 0 $-\frac{4}{3} < -4 \times$

$$\therefore -8 < x < -3$$

✓

Q6

$$\begin{aligned}
 b) \quad \frac{{}^{n+1}C_r}{{}^nC_{r-1}} &= \frac{(n+1)!}{r!(n+1-r)!} \times \frac{(r-1)!(n-(r-1))!}{n!} \\
 &= \frac{(n+1)! \times (r-1)!(n+1-r)!}{r! \times (n+1-r)! \times n!} \\
 &= \frac{n+1}{r} \quad \downarrow
 \end{aligned}$$

$$c) \quad T = 20 + 60e^{-0.1t}$$

$$i) \quad t = 0 \quad T = 20 + 60 = 80^\circ\text{C} \quad \downarrow$$

$$ii) \quad 40 = 20 + 60e^{-0.1t}$$

$$e^{-0.1t} = \frac{1}{3} \quad \downarrow$$

$$-0.1t = \ln \frac{1}{3} = -\ln 3$$

$$t = 10 \ln 3$$

$$= 11 \text{ min.} \quad \downarrow$$

⑦ a) $(x^2 - 3x^{-3})^{10}$

$$\binom{10}{k} x^{2k} (-3)^{10-k} x^{-3(10-k)}$$

$$= \binom{10}{k} (-3)^{10-k} x^{5k-30} \quad \downarrow$$

$$\therefore 5k - 30 = 0$$

$$k = 6 \quad \downarrow$$

$$\binom{10}{6} (-3)^4 = 81 \times \binom{10}{6} = 17010 \quad \downarrow$$

b) $\frac{1}{|x-3|} \geq \frac{1}{2} \quad x \neq 3$

$$|x-3| \leq 2 \quad \downarrow$$

$$-2 \leq x-3 \leq 2$$

$$1 \leq x \leq 5 \quad \downarrow$$

$$1 \leq x \leq 3 \cup 3 < x \leq 5$$

$$\text{or } [1, 3) \cup (3, 5] \quad \downarrow$$

$$c) \quad p = p_0 e^{kt}$$

$$p = 1200 e^{kt}$$

$$2400 = 1200 e^{3k}$$

$$3k = \ln 2$$

$$k = \frac{\ln 2}{3}$$

$$\frac{4 \ln 2}{3}$$

$$p = 1200 e$$

$$= 3024$$

Q8 a) $\left(x + \frac{1}{ax^2}\right)^7$

General term: $\binom{7}{r} x^{7-r} \left(\frac{1}{a}\right)^r x^{-2r}$

$= \binom{7}{r} (a)^{r-7} x^{3r-14}$

$3r - 14 = 1$

$3r = 15$

$r = 5$

$\therefore \binom{7}{5} a^{-2} = \frac{7}{3}$

$\frac{21}{a^2} = \frac{7}{3}$

$\frac{63}{7} = 4^2$

$a^2 = \pm \sqrt{63}$

$= \pm 3$

b) $x(1+x)^n = \binom{n}{0}x + \binom{n}{1}x^2 + \binom{n}{2}x^3 + \dots + \binom{n}{n}x^{n+1}$

$\sum_{r=0}^n (r+1) \binom{n}{r} = \binom{n}{0} + 2\binom{n}{1} + 3\binom{n}{2} + \dots$

Differentiate

$(1+x)^n + nx(1+x)^{n-1} = \binom{n}{0} + 2\binom{n}{1}x + 3\binom{n}{2}x^2 + \dots$

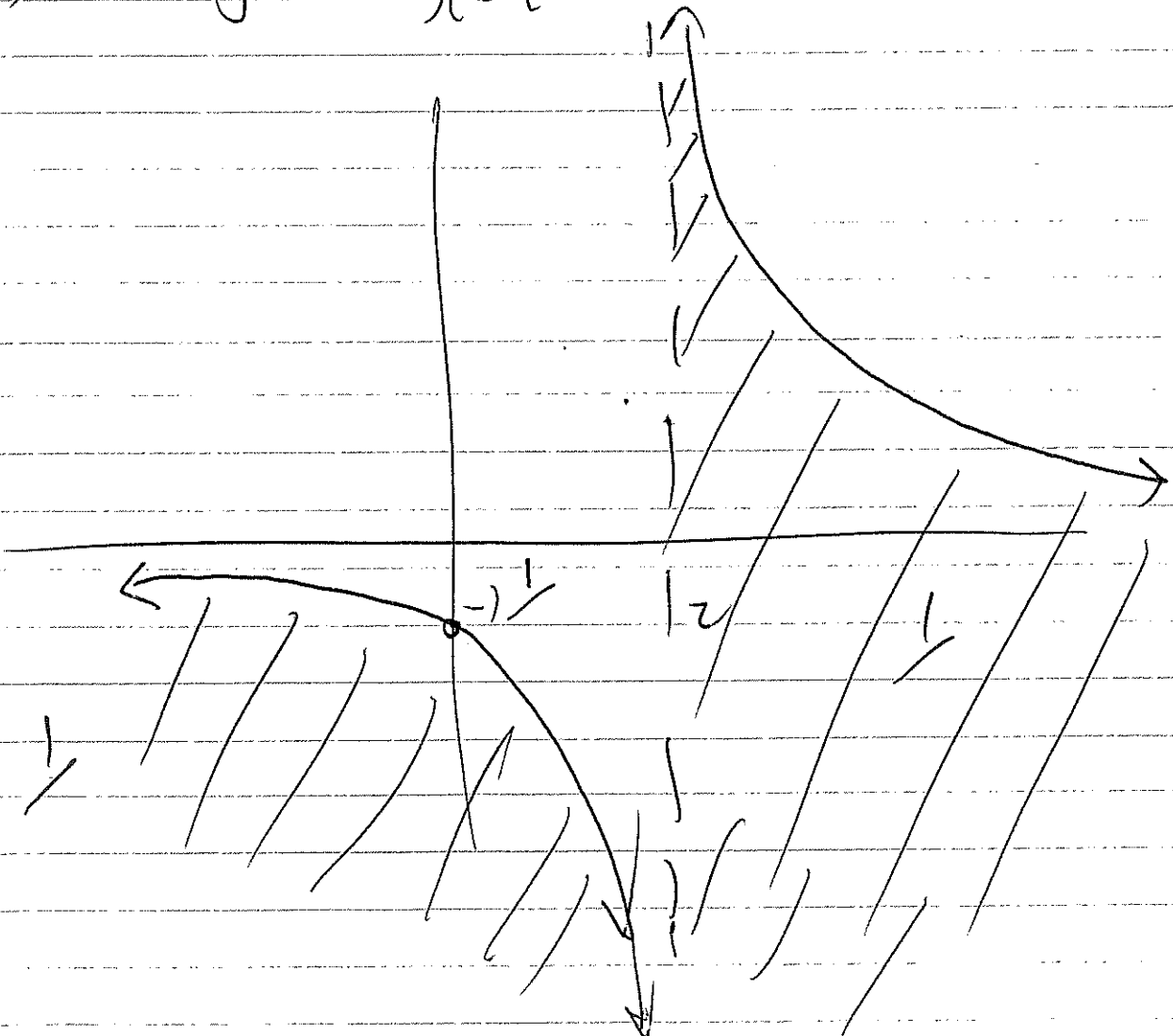
Let $x = 1$

$$1, 2^n + n(2)^{n-1} = \sum_{r=0}^n (r+1)^n C_r$$

$$= 2^{n+1} (2+n)$$

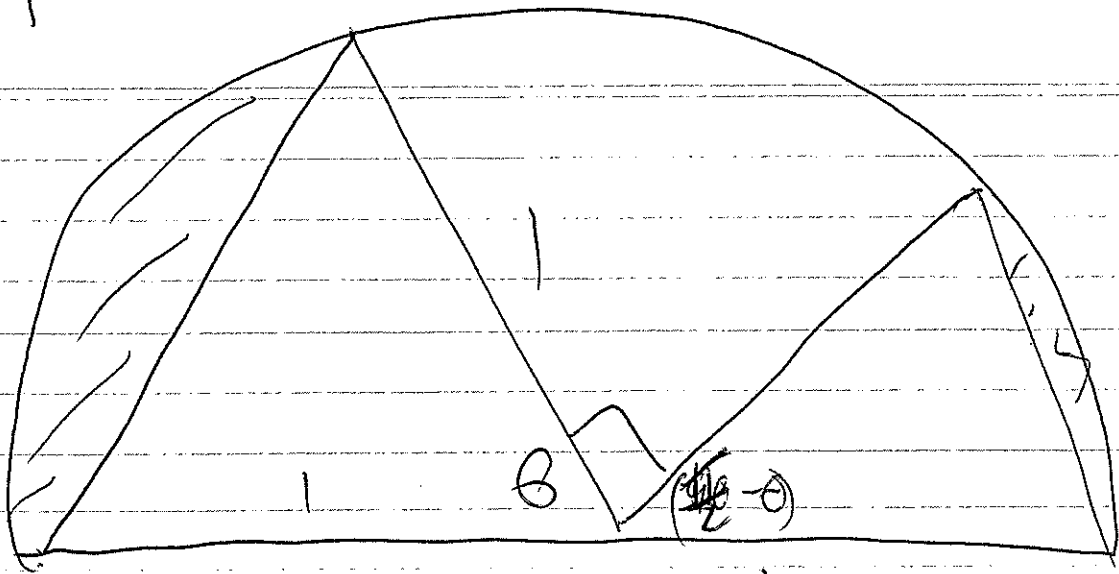
$$= 2^n \left(1 + \frac{n}{2}\right) \quad \underline{\quad}$$

c) $y \leq \frac{2}{x-2}$



Q9

a)



$$\begin{aligned}
 \text{i) segment} &= \frac{1}{2}(6 - \sin \theta) + \frac{1}{2}\left(\frac{\pi}{2} - \theta - \sin\left(\frac{\pi}{2} - \theta\right)\right) \\
 &= \frac{1}{2}\theta + \frac{\pi}{4} - \frac{1}{2}\theta - \frac{1}{2}\sin \theta - \frac{1}{2}\cos \theta \\
 &= \frac{\pi}{4} - \frac{1}{2}(\sin \theta + \cos \theta)
 \end{aligned}$$

$$\text{ii) } \frac{d\theta}{dt} = 0.1 \text{ radians/sec}$$

$$\begin{aligned}
 \frac{dA}{d\theta} &= -\frac{1}{2}(\cos \theta - \sin \theta) \\
 &= \frac{\sin \theta - \cos \theta}{2}
 \end{aligned}$$

$$\frac{dA}{dt} = \frac{dA}{d\theta} \times \frac{d\theta}{dt}$$

$$= \frac{\sin \theta - \cos \theta}{2} \times 0.1$$

$$= \frac{\sin 1 - \cos 1}{2} \times 0.1$$

$$= 0.15 \text{ m}^2/\text{sec}$$

Answer Template For Question 9 (b)

Graph the region of intersection of the following inequalities :

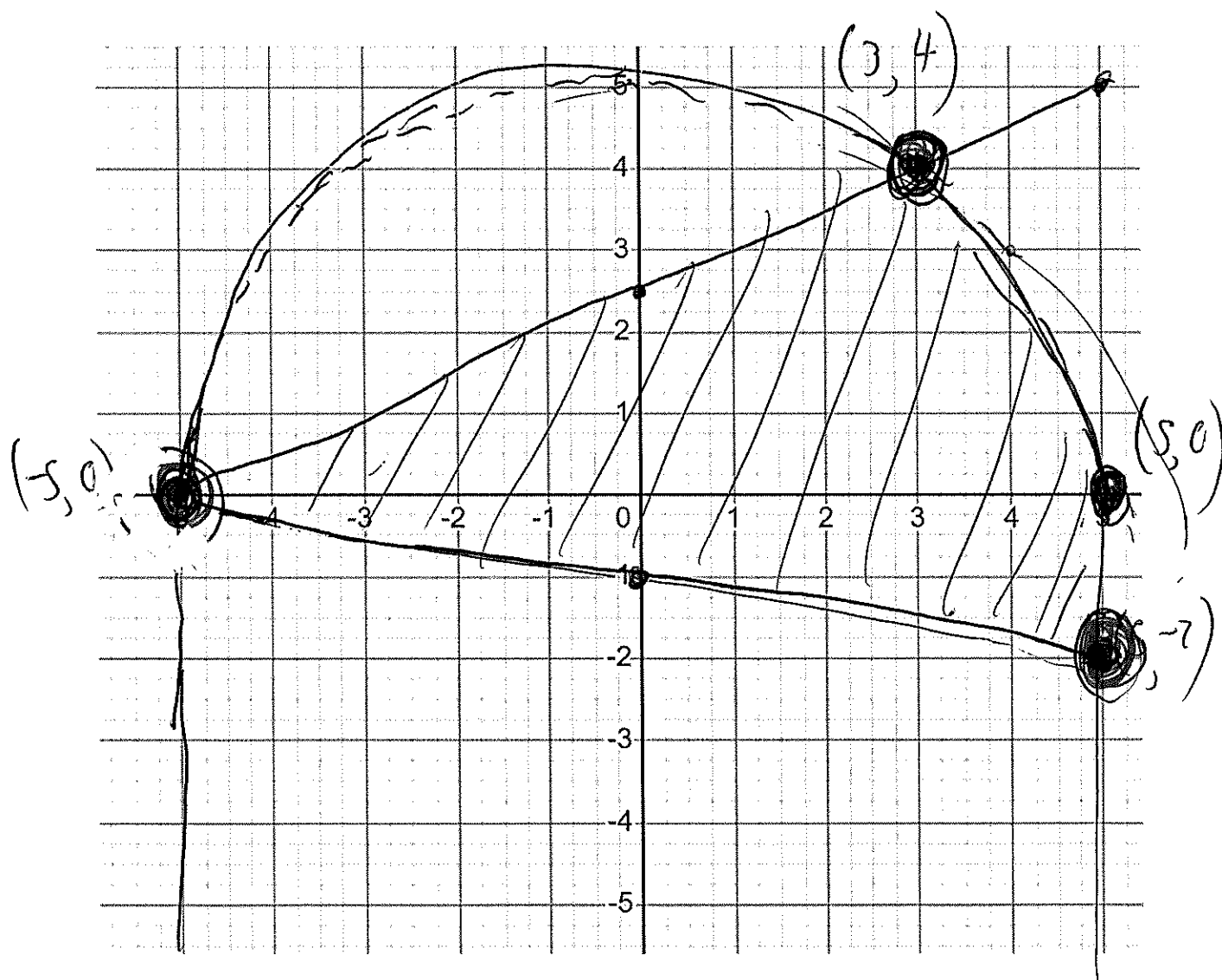
$$y \leq \sqrt{25-x^2} \quad \swarrow$$

$$2y - x \leq 5 \quad \swarrow$$

$$x + 5y \geq -5 \quad \swarrow$$

+ $\swarrow$ for final answer

Clearly show all points of intersection.



$$10) a) x = c + Ae^{kt} \quad \therefore Ae^{kt} = x - c$$

$$i) \quad \frac{dx}{dt} = k(Ae^{kt}) \\ = k(x - c) \quad \checkmark$$

$$ii) \quad t = 0 \quad x = 0 \quad \text{and} \quad c = 7.9$$

$$\therefore x = 7.9 + Ae^{kt}$$

$$0 = 7.9 + A$$

$$\therefore A = -7.9 \quad \checkmark$$

$$x = 7.9 - 7.9e^{kt}$$

$$2.7 = 7.9 - 7.9e^{2k}$$

$$e^{2k} = \frac{-5.2}{-7.9}$$

$$k = \frac{\ln \frac{5.2}{7.9}}{2} \quad \checkmark$$

$$iii) \quad \therefore x = 7.9 - 7.9e^{\frac{5.2}{2} \ln \frac{5.2}{7.9}} \\ = 5.12 \text{ mm}^3 \quad \checkmark$$

$$b) 1) \frac{d(1+x)^{2n}}{dx} = 2n(1+x)^{2n-1} \quad /$$

$$d\left(\sum_{k=0}^{2n} {}^{2n}C_k x^k\right) = \sum_{k=1}^{2n} k {}^{2n}C_k x^{k-1} \quad /$$

$\therefore \text{let } x=1$

$$2n(2)^{2n-1} = n(2 \times 2^{2n-1})$$

$$= n \times 2^{2n} \quad /$$

$$= n \times (2^2)^n$$

$$= n \times 4^n$$

$$\underline{\text{RHS}} = \sum_{k=1}^{2n} k {}^{2n}C_k \quad /$$

$$\therefore \sum_{k=1}^{2n} k {}^{2n}C_k = n \times 4^n$$

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correct

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3 A ☒ B ☐ C ☐ D ☐

4 A ☐ B ☐ C ☒ D ☐

5 A ☐ B ☒ C ☐ D ☐