NSW Education Standards Authority

2021 HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Extension 2

General Instructions

- Reading time – 10 minutes
- Working time – 3 hours
- Write using black pen
- Calculators approved by NESA may be used
- A reference sheet is provided at the back of this paper
- For questions in Section II, show relevant mathematical reasoning and/or calculations
- Write your Centre Number and Student Number on the Question 13 Writing Booklet attached

Total marks: 100

Section I – 10 marks (pages 2–6)

- Attempt Questions 1–10
- Allow about 15 minutes for this section

Section II – 90 marks (pages 7–16)

- Attempt Questions 11–16
- Allow about 2 hours and 45 minutes for this section

Section I

10 marks
Attempt Questions 1–10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1–10.

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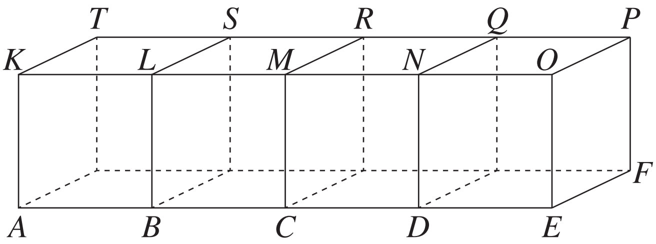
1 Four cubes are placed in a line as shown on the diagram.



Which of the following vectors is equal to $\overrightarrow{AB} + \overrightarrow{CQ}$?

A. $\overrightarrow{AQ}$

B. $\overrightarrow{CP}$

C. $\overrightarrow{PB}$

D. $\overrightarrow{RA}$

2 Which expression is equal to $\int x^5 e^{7x}\,dx$?

A. $\frac{1}{7}x^5e^{7x} - \frac{5}{7}\int x^4e^{7x}\,dx$

B. $\frac{1}{7}x^5e^{7x} - \frac{5}{7}\int x^5e^{7x}\,dx$

C. $\frac{5}{7}x^4e^{7x} - \frac{5}{7}\int x^4e^{7x}\,dx$

D. $\frac{5}{7}x^4e^{7x} - \frac{5}{7}\int x^5e^{7x}\,dx$

3 Which of the following is a vector equation of the line joining the points $A(4, 2, 5)$ and $B(-2, 2, 1)$?

A. $\underset{\sim}{r} = \begin{pmatrix} 4 \\ 2 \\ 5 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$

B. $\underset{\sim}{r} = \begin{pmatrix} 4 \\ 2 \\ 5 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ 0 \\ 2 \end{pmatrix}$

C. $\underset{\sim}{r} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 4 \\ 2 \\ 5 \end{pmatrix}$

D. $\underset{\sim}{r} = \begin{pmatrix} 3 \\ 0 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 4 \\ 2 \\ 5 \end{pmatrix}$

4 Consider the statement:

'For all integers n, if n is a multiple of 6, then n is a multiple of 2'.

Which of the following is the contrapositive of the statement?

A. There exists an integer n such that n is a multiple of 6 and not a multiple of 2.

B. There exists an integer n such that n is a multiple of 2 and not a multiple of 6.

C. For all integers n, if n is not a multiple of 2, then n is not a multiple of 6.

D. For all integers n, if n is not a multiple of 6, then n is not a multiple of 2.

5 Which of the following statements is FALSE?

A. $\forall\, a, b \in \mathbb{R}, \quad a < b \Rightarrow a^3 < b^3$

B. $\forall\, a, b \in \mathbb{R}, \quad a < b \Rightarrow e^{-a} > e^{-b}$

C. $\forall\, a, b \in (0, +\infty), \quad a < b \Rightarrow \ln a < \ln b$

D. $\forall\, a, b \in \mathbb{R}$, with $a, b \neq 0, \quad a < b \Rightarrow \dfrac{1}{a} > \dfrac{1}{b}$

6 Which polynomial could have $2 + i$ as a zero, given that k is a real number?

A. $x^3 - 4x^2 + kx$

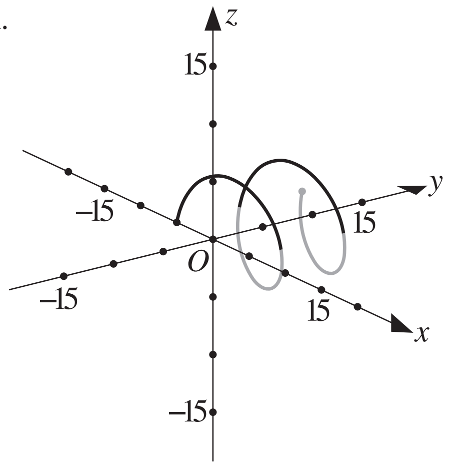
B. $x^3 - 4x^2 + kx + 5$

C. $x^3 - 5x^2 + kx$

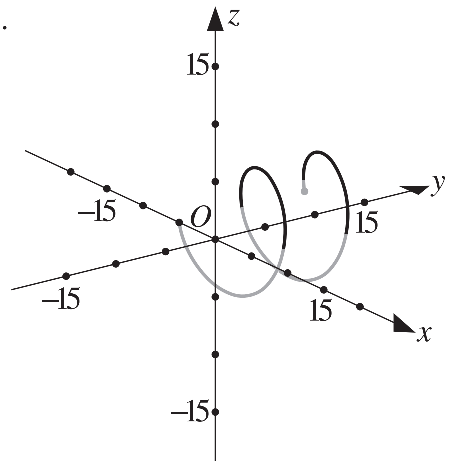
D. $x^3 - 5x^2 + kx + 5$

7 Which diagram best shows the curve described by the position vector $\underset{\sim}{r}(t) = -5\cos(t)\underset{\sim}{i} + 5\sin(t)\underset{\sim}{j} + t\underset{\sim}{k}$ for $0 \le t \le 4\pi$?

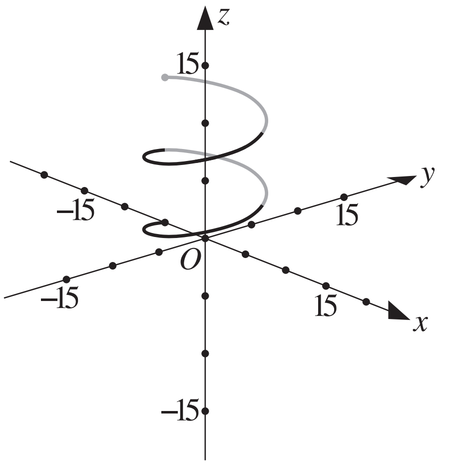
A.



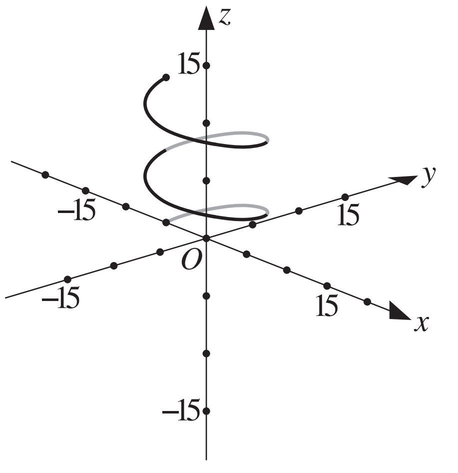
B.



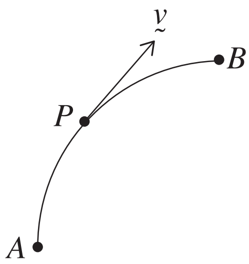
C.



D.



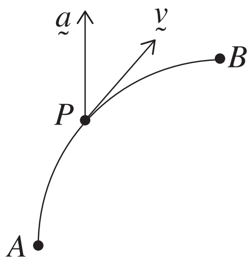
8 A particle is travelling from A on the curve joining A to B. At a particular time, the particle is at point P and has velocity $\underset{\sim}{v}$, as shown in the diagram.



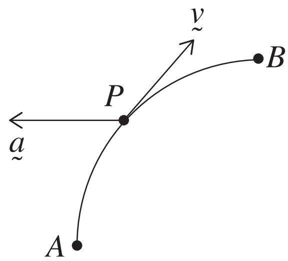
The speed of the particle is increasing.

Which of the following diagrams shows an acceleration, $\underset{\sim}{a}$, which would allow the particle to follow the curve to B?

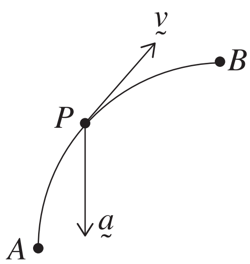
A.



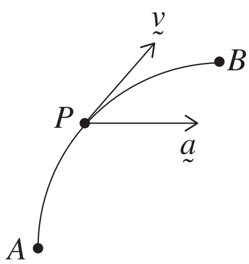
B.



C.



D.



9 Four cards have either RED or BLACK on one side and either WIN or TRY AGAIN on the other side.

Sam places the four cards on the table as shown below.

WIN	BLACK	TRY AGAIN	RED
Card 1	Card 2	Card 3	Card 4

A statement is made: 'If a card is RED, then it has WIN written on the other side'.

Sam wants to check if the statement is true by turning over the minimum number of cards.

Which cards should Sam turn over?

A. 1 and 4

B. 3 and 4

C. 1, 2 and 4

D. 1, 3 and 4

10 Consider the two non-zero complex numbers z and w as vectors.

Which of the following expressions is the projection of z onto w?

A. $\dfrac{\operatorname{Re}(zw)}{|w|}w$

B. $\left|\dfrac{z}{w}\right|w$

C. $\operatorname{Re}\left(\dfrac{z}{w}\right)w$

D. $\dfrac{\operatorname{Re}(z)}{|w|}w$

Section II

90 marks
Attempt Questions 11–16
Allow about 2 hours and 45 minutes for this section

Answer each question in the appropriate writing booklet. Extra writing booklets are available.

For questions in Section II, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (16 marks) Use the Question 11 Writing Booklet

(a) The complex numbers $z = 2e^{i\frac{\pi}{2}}$ and $w = 6e^{i\frac{\pi}{6}}$ are given. **2**

Find the value of zw, giving the answer in the form $re^{i\theta}$.

(b) Find $\sum_{n=1}^{5}(i)^n$. **2**

(c) Find the angle between the vectors $\underset{\sim}{a} = \begin{pmatrix} 2 \\ 0 \\ 4 \end{pmatrix}$ and $\underset{\sim}{b} = \begin{pmatrix} -3 \\ 1 \\ 2 \end{pmatrix}$, giving the angle in degrees correct to 1 decimal place. **3**

(d) (i) Find the two square roots of $-i$, giving the answers in the form $x + iy$, where x and y are real numbers. **2**

(ii) Hence, or otherwise, solve $z^2 + 2z + 1 + i = 0$ giving your solutions in the form $a + ib$ where a and b are real numbers. **2**

(e) The complex numbers $z = 5 + i$ and $w = 2 - 4i$ are given. **2**

Find $\frac{\overline{z}}{w}$, giving your answer in Cartesian form.

(f) Express $\dfrac{3x^2 - 5}{(x-2)(x^2 + x + 1)}$ as a sum of partial fractions over $\mathbb{R}$. **3**

Question 12 (15 marks) Use the Question 12 Writing Booklet

(a) Find $\displaystyle\int \frac{2x+3}{x^2+2x+2}\,dx$. **3**

(b) Consider Statement A.

Statement A: 'If n^2 is even, then n is even.'

(i) What is the converse of Statement A? **1**

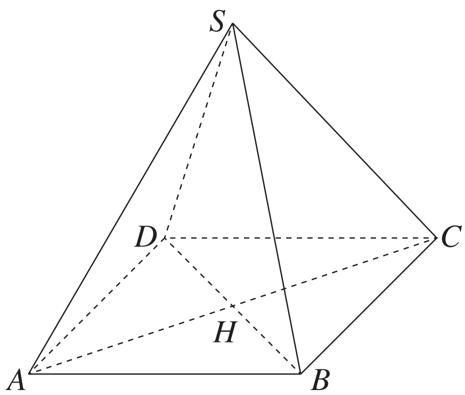
(ii) Show that the converse of Statement A is true. **1**

(c) Two lines are given by $\mathbf{r}_1 = \begin{pmatrix} -2 \\ 1 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}$ and $\mathbf{r}_2 = \begin{pmatrix} 4 \\ -2 \\ q \end{pmatrix} + \mu \begin{pmatrix} p \\ 3 \\ -1 \end{pmatrix}$, where p and q are real numbers. These lines intersect and are perpendicular. **3**

Find the values of p and q.

(d) Prove by mathematical induction that $\sqrt{n!} > 2^n$, for integers $n \geq 9$. **3**

(e) The diagram shows the pyramid $ABCDS$ where $ABCD$ is a square. The diagonals of the square bisect each other at H.



(i) Show that $\overrightarrow{HA} + \overrightarrow{HB} + \overrightarrow{HC} + \overrightarrow{HD} = \underset{\sim}{0}$. **1**

Let G be the point such that $\overrightarrow{GA} + \overrightarrow{GB} + \overrightarrow{GC} + \overrightarrow{GD} + \overrightarrow{GS} = \underset{\sim}{0}$.

(ii) Using part (i), or otherwise, show that $4\overrightarrow{GH} + \overrightarrow{GS} = \underset{\sim}{0}$. **2**

(iii) Find the value of λ such that $\overrightarrow{HG} = \lambda\overrightarrow{HS}$. **1**

Question 13 (15 marks) Use the Question 13 Writing Booklet

(a) The location of the complex number $a + ib$ is shown on the diagram on page 1 of the Question 13 Writing Booklet. **2**

On the diagram provided in the writing booklet, indicate the locations of all of the fourth roots of the complex number $a + ib$.

(b) Use an appropriate substitution to evaluate $\displaystyle\int_{\sqrt{10}}^{\sqrt{13}} x^3\sqrt{x^2 - 9}\,dx$. **3**

Question 13 continues on page 10

Question 13 (continued)

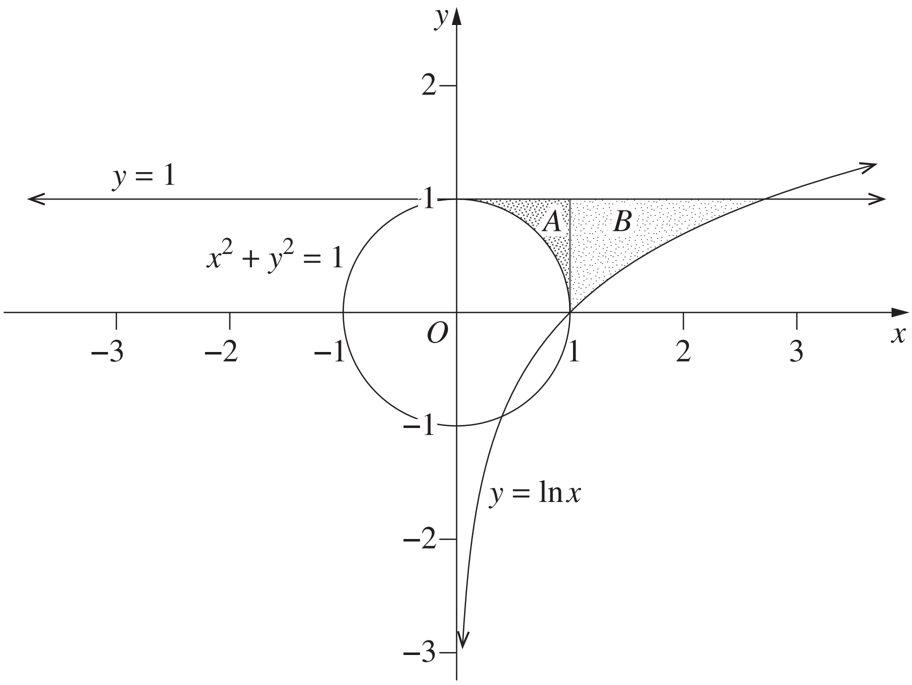
(c) (i) The integral I_n is defined by $I_n = \int_1^e (\ln x)^n \, dx$ for integers $n \geq 0$. **2**

Show that $I_n = e - nI_{n-1}$ for $n \geq 1$.

(ii) The diagram shows two regions. **4**

Region A is bounded by $y = 1$ and $x^2 + y^2 = 1$ between $x = 0$ and $x = 1$.

Region B is bounded by $y = 1$ and $y = \ln x$ between $x = 1$ and $x = e$.



The volume of the solid created when the region between the curve $y = f(x)$ and the x-axis, between $x = a$ and $x = b$, is rotated about the x-axis is given by $V = \pi \int_a^b [f(x)]^2 \, dx$.

The volume of the solid of revolution formed when region A is rotated about the x-axis is V_A.

The volume of the solid of revolution formed when region B is rotated about the x-axis is V_B.

Using part (i), or otherwise, show that the ratio $V_A : V_B$ is $1 : 3$.

Question 13 continues on page 11

(d) An object is moving in simple harmonic motion along the x-axis. The acceleration of the object is given by $\ddot{x} = -4(x-3)$ where x is its displacement from the origin, measured in metres, after t seconds.

Initially, the object is 5.5 metres to the right of the origin and moving towards the origin. The object has a speed of 8 m s^{-1} as it passes through the origin.

(i) Between which two values of x is the particle oscillating? **2**

(ii) Find the first value of t for which $x = 0$, giving the answer correct to 2 decimal places. **2**

End of Question 13

Please turn over

Question 14 (14 marks) Use the Question 14 Writing Booklet

(a) Evaluate $\displaystyle\int_0^{\frac{\pi}{2}} \frac{1}{3+5\cos x}\,dx$. **4**

(b) An object of mass 5 kg is on a slope that is inclined at an angle of 60° to the horizontal. The acceleration due to gravity is g m s^{-2} and the velocity of the object down the slope is v m s^{-1}.

As well as the force due to gravity, the object is acted on by two forces, one of magnitude $2v$ newtons and one of magnitude $2v^2$ newtons, both acting up the slope.

(i) Show that the resultant force down the slope is **2**

$\dfrac{5\sqrt{3}}{2}g - 2v - 2v^2$ newtons.

(ii) There is one value of v such that the object will slide down the slope at a constant speed. **2**

Find this value of v in m s^{-1}, correct to 1 decimal place, given that $g = 10$.

(c) (i) Using de Moivre's theorem and the binomial expansion of $(\cos\theta + i\sin\theta)^5$, or otherwise, show that **3**

$$\cos 5\theta = 16\cos^5\theta - 20\cos^3\theta + 5\cos\theta.$$

(ii) By using part (i), or otherwise, show that $\mathrm{Re}\left(e^{\frac{i\pi}{10}}\right) = \sqrt{\dfrac{5+\sqrt{5}}{8}}$. **3**

Question 15 (15 marks) Use the Question 15 Writing Booklet

(a) For all non-negative real numbers x and y, $\sqrt{xy} \le \dfrac{x+y}{2}$. (Do NOT prove this.)

(i) Using this fact, show that for all non-negative real numbers a, b and c, **2**

$$\sqrt{abc} \le \frac{a^2 + b^2 + 2c}{4}.$$

(ii) Using part (i), or otherwise, show that for all non-negative real numbers a, b and c, **2**

$$\sqrt{abc} \le \frac{a^2 + b^2 + c^2 + a + b + c}{6}.$$

(b) For integers $n \ge 1$, the triangular numbers t_n are defined by $t_n = \dfrac{n(n+1)}{2}$, giving $t_1 = 1$, $t_2 = 3$, $t_3 = 6$, $t_4 = 10$ and so on.

For integers $n \ge 1$, the hexagonal numbers h_n are defined by $h_n = 2n^2 - n$, giving $h_1 = 1$, $h_2 = 6$, $h_3 = 15$, $h_4 = 28$ and so on.

(i) Show that the triangular numbers t_1, t_3, t_5, and so on, are also hexagonal numbers. **2**

(ii) Show that the triangular numbers t_2, t_4, t_6, and so on, are not hexagonal numbers. **1**

Question 15 continues on page 14

Question 15 (continued)

(c) An object of mass 1 kg is projected vertically upwards with an initial velocity of u m/s. It experiences air resistance of magnitude kv^2 newtons where v is the velocity of the object, in m/s, and k is a positive constant. The height of the object above its starting point is x metres. The time since projection is t seconds and acceleration due to gravity is g m/s^2.

(i) Show that the time for the object to reach its maximum height is $\frac{1}{\sqrt{gk}}\arctan\left(u\sqrt{\frac{k}{g}}\right)$ seconds. **3**

(ii) Find an expression for the maximum height reached by the object, in terms of k, g and u. **3**

(d) Prove that $2^n + 3^n \neq 5^n$ for all integers $n \geq 2$. **2**

End of Question 15

Question 16 (15 marks) Use the Question 16 Writing Booklet

(a) (i) The point $P(x, y, z)$ lies on the sphere of radius 1 centred at the origin O. **2**

Using the position vector of P, $\overrightarrow{OP} = x\underset{\sim}{i} + y\underset{\sim}{j} + z\underset{\sim}{k}$, and the triangle inequality, or otherwise, show that $|x| + |y| + |z| \geq 1$.

(ii) Given the vectors $\underset{\sim}{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $\underset{\sim}{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$, show that **3**

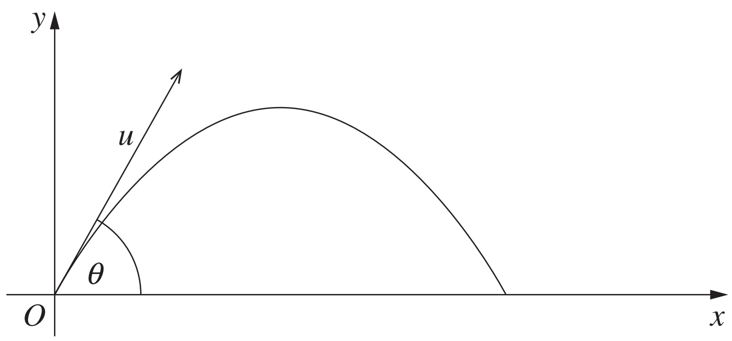
$$|a_1 b_1 + a_2 b_2 + a_3 b_3| \leq \sqrt{a_1^2 + a_2^2 + a_3^2}\sqrt{b_1^2 + b_2^2 + b_3^2}\,.$$

(iii) As in part (i), the point $P(x, y, z)$ lies on the sphere of radius 1 centred at the origin O. **2**

Using part (ii), or otherwise, show that $|x| + |y| + |z| \leq \sqrt{3}$.

Question 16 continues on page 16

(b) A particle which is projected from the origin with initial speed $u \text{ m s}^{-1}$ at an angle of θ to the positive x-axis lands on the x-axis, as shown in the diagram. The particle is subject to an acceleration due to gravity of $g \text{ m s}^{-2}$. **5**



The position vector of the particle, $\underset{\sim}{r}(t)$, where t is the time in seconds after the particle is projected, is given by

$$\underset{\sim}{r}(t) = \begin{pmatrix} ut\cos\theta \\ -\dfrac{gt^2}{2} + ut\sin\theta \end{pmatrix}. \qquad \text{(Do NOT prove this.)}$$

For some value(s) of θ there will be two times during the time of flight when the particle's position vector is perpendicular to its velocity vector.

Find the value(s) of θ for which this occurs, justifying that both times occur during the time of flight.

(c) Sketch the region of the complex plane defined by $\operatorname{Re}(z) \geq \operatorname{Arg}(z)$ where $\operatorname{Arg}(z)$ is the principal argument of z. **3**

End of paper

NSW GOVERNMENT

Centre Number

Student Number

NSW Education Standards Authority

2021 HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Extension 2

Writing Booklet

Question 13

Instructions

- Use this Writing Booklet to answer Question 13.
- Write the number of this booklet and the total number of booklets that you have used for this question (eg: 1 of 3).
- Write your Centre Number and Student Number at the top of this page.
- Write using black pen.
- You may ask for an extra writing booklet if you need more space.
- If you have not attempted the question(s), you must still hand in the writing booklet, with 'NOT ATTEMPTED' written clearly on the front cover.
- You may NOT take any writing booklets, used or unused, from the examination room.

13

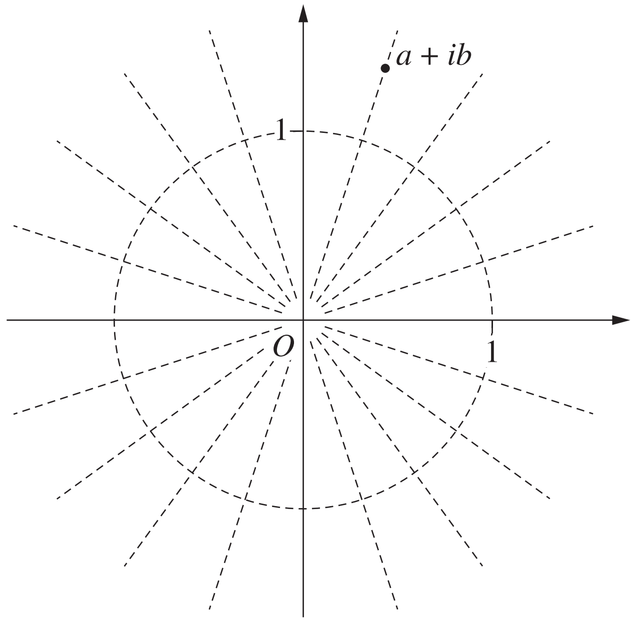
of

this booklet

number of booklets for this question

Start here for Question Number: **13**

(a) Indicate the locations of all of the fourth roots of the complex number $a + ib$.



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Tick this box if you have continued this answer in another writing booklet.

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2021 HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Advanced
Mathematics Extension 1
Mathematics Extension 2

REFERENCE SHEET

Measurement

Length

$$l = \frac{\theta}{360} \times 2\pi r$$

Area

$$A = \frac{\theta}{360} \times \pi r^2$$

$$A = \frac{h}{2}(a + b)$$

Surface area

$$A = 2\pi r^2 + 2\pi rh$$

$$A = 4\pi r^2$$

Volume

$$V = \frac{1}{3}Ah$$

$$V = \frac{4}{3}\pi r^3$$

Financial Mathematics

$$A = P(1+r)^n$$

Sequences and series

$$T_n = a + (n-1)d$$

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right] = \frac{n}{2}(a + l)$$

$$T_n = ar^{n-1}$$

$$S_n = \frac{a(1-r^n)}{1-r} = \frac{a(r^n - 1)}{r-1}, \ r \neq 1$$

$$S = \frac{a}{1-r}, \ |r| < 1$$

Functions

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

For $ax^3 + bx^2 + cx + d = 0$:

$$\alpha + \beta + \gamma = -\frac{b}{a}$$

$$\alpha\beta + \alpha\gamma + \beta\gamma = \frac{c}{a}$$

and $\alpha\beta\gamma = -\frac{d}{a}$

Relations

$$(x-h)^2 + (y-k)^2 = r^2$$

Logarithmic and Exponential Functions

$$\log_a a^x = x = a^{\log_a x}$$

$$\log_a x = \frac{\log_b x}{\log_b a}$$

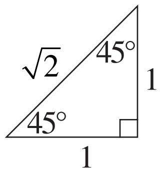
$$a^x = e^{x \ln a}$$

Trigonometric Functions

$$\sin A = \frac{\text{opp}}{\text{hyp}}, \quad \cos A = \frac{\text{adj}}{\text{hyp}}, \quad \tan A = \frac{\text{opp}}{\text{adj}}$$

$$A = \frac{1}{2}ab\sin C$$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

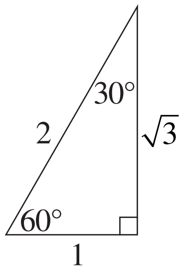


$$c^2 = a^2 + b^2 - 2ab\cos C$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

$$l = r\theta$$

$$A = \frac{1}{2}r^2\theta$$



Trigonometric identities

$$\sec A = \frac{1}{\cos A}, \ \cos A \neq 0$$

$$\operatorname{cosec} A = \frac{1}{\sin A}, \ \sin A \neq 0$$

$$\cot A = \frac{\cos A}{\sin A}, \ \sin A \neq 0$$

$$\cos^2 x + \sin^2 x = 1$$

Compound angles

$$\sin(A + B) = \sin A\cos B + \cos A\sin B$$

$$\cos(A + B) = \cos A\cos B - \sin A\sin B$$

$$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}$$

If $t = \tan\dfrac{A}{2}$ then $\sin A = \dfrac{2t}{1+t^2}$

$$\cos A = \frac{1-t^2}{1+t^2}$$

$$\tan A = \frac{2t}{1-t^2}$$

$$\cos A\cos B = \frac{1}{2}\left[\cos(A - B) + \cos(A + B)\right]$$

$$\sin A\sin B = \frac{1}{2}\left[\cos(A - B) - \cos(A + B)\right]$$

$$\sin A\cos B = \frac{1}{2}\left[\sin(A + B) + \sin(A - B)\right]$$

$$\cos A\sin B = \frac{1}{2}\left[\sin(A + B) - \sin(A - B)\right]$$

$$\sin^2 nx = \frac{1}{2}(1 - \cos 2nx)$$

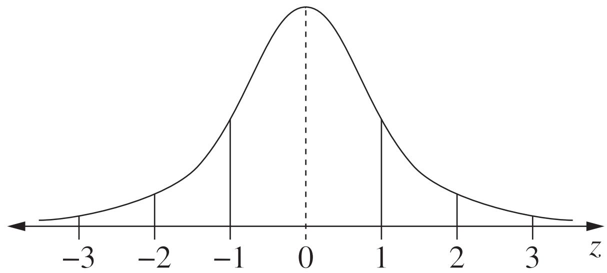
$$\cos^2 nx = \frac{1}{2}(1 + \cos 2nx)$$

Statistical Analysis

$$z = \frac{x - \mu}{\sigma}$$

An outlier is a score
less than $Q_1 - 1.5 \times IQR$
or
more than $Q_3 + 1.5 \times IQR$

Normal distribution



- approximately 68% of scores have z-scores between –1 and 1
- approximately 95% of scores have z-scores between –2 and 2
- approximately 99.7% of scores have z-scores between –3 and 3

$$E(X) = \mu$$

$$\operatorname{Var}(X) = E\left[(X - \mu)^2\right] = E\left(X^2\right) - \mu^2$$

Probability

$$P(A \cap B) = P(A)P(B)$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}, \ P(B) \neq 0$$

Continuous random variables

$$P(X \leq r) = \int_a^r f(x)\,dx$$

$$P(a < X < b) = \int_a^b f(x)\,dx$$

Binomial distribution

$$P(X = r) = {}^nC_r p^r(1 - p)^{n-r}$$

$$X \sim \operatorname{Bin}(n, p)$$

$$\Rightarrow \ P(X = x)$$

$$= \binom{n}{x}p^x(1 - p)^{n-x}, \ x = 0, 1, \ldots, n$$

$$E(X) = np$$

$$\operatorname{Var}(X) = np(1 - p)$$

Differential Calculus

Function	**Derivative**
$y = f(x)^n$	$\frac{dy}{dx} = nf'(x)[f(x)]^{n-1}$
$y = uv$	$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$
$y = g(u)$ where $u = f(x)$	$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$
$y = \frac{u}{v}$	$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$
$y = \sin f(x)$	$\frac{dy}{dx} = f'(x)\cos f(x)$
$y = \cos f(x)$	$\frac{dy}{dx} = -f'(x)\sin f(x)$
$y = \tan f(x)$	$\frac{dy}{dx} = f'(x)\sec^2 f(x)$
$y = e^{f(x)}$	$\frac{dy}{dx} = f'(x)e^{f(x)}$
$y = \ln f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{f(x)}$
$y = a^{f(x)}$	$\frac{dy}{dx} = (\ln a)f'(x)a^{f(x)}$
$y = \log_a f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{(\ln a)f(x)}$
$y = \sin^{-1} f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{\sqrt{1-[f(x)]^2}}$
$y = \cos^{-1} f(x)$	$\frac{dy}{dx} = -\frac{f'(x)}{\sqrt{1-[f(x)]^2}}$
$y = \tan^{-1} f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{1+[f(x)]^2}$

Integral Calculus

$$\int f'(x)[f(x)]^n dx = \frac{1}{n+1}[f(x)]^{n+1} + c$$

where $n \neq -1$

$$\int f'(x)\sin f(x)\,dx = -\cos f(x) + c$$

$$\int f'(x)\cos f(x)\,dx = \sin f(x) + c$$

$$\int f'(x)\sec^2 f(x)\,dx = \tan f(x) + c$$

$$\int f'(x)e^{f(x)}dx = e^{f(x)} + c$$

$$\int \frac{f'(x)}{f(x)}dx = \ln|f(x)| + c$$

$$\int f'(x)a^{f(x)}dx = \frac{a^{f(x)}}{\ln a} + c$$

$$\int \frac{f'(x)}{\sqrt{a^2-[f(x)]^2}}dx = \sin^{-1}\frac{f(x)}{a} + c$$

$$\int \frac{f'(x)}{a^2+[f(x)]^2}dx = \frac{1}{a}\tan^{-1}\frac{f(x)}{a} + c$$

$$\int u\frac{dv}{dx}dx = uv - \int v\frac{du}{dx}dx$$

$$\int_a^b f(x)\,dx$$

$$\approx \frac{b-a}{2n}\left\{f(a) + f(b) + 2\left[f(x_1) + \cdots + f(x_{n-1})\right]\right\}$$

where $a = x_0$ and $b = x_n$

Combinatorics

$${}^nP_r = \frac{n!}{(n-r)!}$$

$$\binom{n}{r} = {}^nC_r = \frac{n!}{r!(n-r)!}$$

$$(x+a)^n = x^n + \binom{n}{1}x^{n-1}a + \cdots + \binom{n}{r}x^{n-r}a^r + \cdots + a^n$$

Vectors

$$|\underset{\sim}{u}| = \left|x\underset{\sim}{i} + y\underset{\sim}{j}\right| = \sqrt{x^2 + y^2}$$

$$\underset{\sim}{u} \cdot \underset{\sim}{v} = |\underset{\sim}{u}||\underset{\sim}{v}|\cos\theta = x_1x_2 + y_1y_2,$$

where $\underset{\sim}{u} = x_1\underset{\sim}{i} + y_1\underset{\sim}{j}$

and $\underset{\sim}{v} = x_2\underset{\sim}{i} + y_2\underset{\sim}{j}$

$$\underset{\sim}{r} = \underset{\sim}{a} + \lambda\underset{\sim}{b}$$

Complex Numbers

$$\begin{aligned} z = a + ib &= r(\cos\theta + i\sin\theta) \\ &= re^{i\theta} \end{aligned}$$

$$\begin{aligned} [r(\cos\theta + i\sin\theta)]^n &= r^n(\cos n\theta + i\sin n\theta) \\ &= r^n e^{in\theta} \end{aligned}$$

Mechanics

$$\frac{d^2x}{dt^2} = \frac{dv}{dt} = v\frac{dv}{dx} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$$

$$x = a\cos(nt + \alpha) + c$$

$$x = a\sin(nt + \alpha) + c$$

$$\ddot{x} = -n^2(x - c)$$

2021 HSC Mathematics Extension 2 Marking Guidelines

Section I

Multiple-choice Answer Key

Question	Answer
1	B
2	A
3	B
4	C
5	D
6	A
7	D
8	D
9	B
10	C

Section II

Question 11 (a)

Criteria	Marks
• Provides correct solution	2
• Finds the correct argument	1

Sample answer:

$$zw = (2 \times 6)e^{i\left(\frac{\pi}{2}+\frac{\pi}{6}\right)}$$
$$= 12e^{\frac{i2\pi}{3}}$$

Question 11 (b)

Criteria	Marks
• Provides correct solution	2
• Expands the notation and provides the correct expression with 5 terms OR • Correctly evaluates i^3 or i^4 or i^5	1

Sample answer:

$$\sum_{n=1}^{n}(i)^n = i + (i)^2 + (i)^3 + (i)^4 + (i)^5$$
$$= i - 1 - i + 1 + i$$
$$= i$$

Question 11 (c)

Criteria	Marks
• Provides correct solution	3
• Correctly finds the dot product and both magnitudes OR • Uses the correct dot product or a correct magnitude in $a \cdot b = \lvert a \rvert \lvert b \rvert \cos\theta$	2
• Correctly finds the dot product of the two vectors OR • Correctly finds the magnitude of one vector	1

Sample answer:

Dot product $\begin{pmatrix} 2 \\ 0 \\ 4 \end{pmatrix} \cdot \begin{pmatrix} -3 \\ 1 \\ 2 \end{pmatrix} = -6 + 0 + 8$

$= 2$

Magnitudes:

$$\left| \begin{pmatrix} 2 \\ 0 \\ 4 \end{pmatrix} \right| = \sqrt{2^2 + 0^2 + 4^2}$$

$$= \sqrt{20}$$

$$\left| \begin{pmatrix} -3 \\ 1 \\ 2 \end{pmatrix} \right| = \sqrt{(-3)^2 + 1^2 + 2^2}$$

$$= \sqrt{14}$$

$\therefore\ \sqrt{20}\sqrt{14}\cos\theta = 2$

$$\cos\theta = 0.1195\ldots$$

$$\theta = 83.135°\ldots$$

$$\approx 83.1° \qquad \text{(1 decimal place)}$$

Question 11 (d) (i)

Criteria	Marks
• Provides correct solution	2
• Attempts to use the square of $(x + iy)$, or equivalent merit OR • Attempts to use the modulus/argument form of $-i$ OR • Plots $-i$ on an Argand diagram and attempts to locate the square roots	1

Sample answer:

Let $(x+iy)^2 = -i$ where $x, y \in \mathbb{R}$

$\therefore x^2 - y^2 = 0$ and $2xy = -1$

$$\therefore y = \frac{-1}{2x}$$

$$\therefore x^2 - \left(\frac{-1}{2x}\right)^2 = 0$$

$$4x^4 - 1 = 0$$

$$x^4 = \frac{1}{4}$$

$$x^2 = \frac{1}{2} \quad (x \in \mathbb{R})$$

$$\therefore \quad x = \pm\frac{1}{\sqrt{2}}$$

$\therefore$ square roots are $\dfrac{1}{\sqrt{2}} - \dfrac{i}{\sqrt{2}}$ and $\dfrac{-1}{\sqrt{2}} + \dfrac{i}{\sqrt{2}}$

Question 11 (d) (ii)

Criteria	Marks
• Provides correct answers	2
• Uses the quadratic formula to obtain $z = -1 \pm \sqrt{-i}$, or equivalent merit	1

Sample answer:

$z^2 + 2z + 1 + i = 0$

$$\begin{aligned} \therefore \quad z &= \frac{-2 \pm \sqrt{(2)^2 - 4(1)(1+i)}}{2} \\ &= \frac{-2 \pm 2\sqrt{1-1-i}}{2} \\ &= -1 \pm \sqrt{-i} \end{aligned}$$

From part (i):

$$z = -1 + \frac{1}{\sqrt{2}} - \frac{i}{\sqrt{2}} \quad \text{or} \quad z = -1 - \frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}}$$

that is,

$$z = \frac{\left(1-\sqrt{2}\right)}{\sqrt{2}} - \frac{1}{\sqrt{2}}i \quad \text{or} \quad z = \frac{-\left(1+\sqrt{2}\right)}{\sqrt{2}} + \frac{1}{\sqrt{2}}i$$

Question 11 (e)

Criteria	Marks
• Provides correct solution	2
• Identifies $\overline{z}$, or equivalent merit	1

Sample answer:

$z = 5 + i \qquad w = 2 - 4i$

$\therefore \overline{z} = 5 - i$

$$\frac{\overline{z}}{w} = \frac{(5-i)}{(2-4i)} \times \frac{(2+4i)}{(2+4i)}$$

$$= \frac{10 + 20i - 2i + 4}{4 + 16}$$

$$= \frac{14 + 18i}{20}$$

$$= \frac{14}{20} + \frac{18}{20}i$$

$$= \frac{7}{10} + \frac{9}{10}i$$

Question 11 (f)

Criteria	Marks
• Provides correct solution	3
• Evaluates one of the three coefficients	2
• Provides a correct expression for the sum of fractions with unknown coefficients, or equivalent merit	1

Sample answer:

Let $\dfrac{3x^2-5}{(x-2)(x^2+x+1)} = \dfrac{A}{x-2} + \dfrac{Bx+C}{x^2+x+1}$

$$= \frac{A(x^2+x+1)+(Bx+C)(x-2)}{(x-2)(x^2+x+1)}$$

$\therefore 3x^2-5 = A\left(x^2+x+1\right)+(Bx+C)(x-2)$

when $x=2$

$$3(2)^2-5 = A(2^2+2+1)$$
$$7 = 7A$$
$$A = 1$$

Equating coefficients of x^2

$3 = A+B$

$\therefore B = 2$

Equating constants:

$-5 = A-2C$

$2C = 6$

$C = 3$

$$\therefore \frac{3x^2-5}{(x-2)(x^2+x+1)} = \frac{1}{x-2} + \frac{2x+3}{x^2+x+1}$$

Question 12 (a)

Criteria	Marks
• Provides correct solution	3
• Integrates one of the correct fractions OR • Writes the integrand as the correct sum of fractions and completes the square in a denominator, or equivalent merit	2
• Attempts to write the integrand as a sum of two suitable fractions, or equivalent merit	1

Sample answer:

$$\int \frac{2x+3}{x^2+2x+2}\,dx = \int \frac{2x+2}{x^2+2x+2}\,dx + \int \frac{1}{x^2+2x+2}\,dx$$

$$= \ln\left(x^2+2x+2\right) + \int \frac{1}{1+(x+1)^2}\,dx + C$$

$$= \ln\left(x^2+2x+2\right) + \tan^{-1}(x+1) + k$$

Question 12 (b) (i)

Criteria	Marks
• States the converse	1

Sample answer:

The converse is 'If n is even, then n^2 is even'.

Question 12 (b) (ii)

Criteria	Marks
• Provides correct proof	1

Sample answer:

If n is even then $n = 2k$ for some integer k, then $n^2 = (2k)^2 = 2(2k^2)$ which is also even.

Question 12 (c)

Criteria	Marks
• Provides correct solution	3
• Finds the value of p or μ	2
• Evaluates the dot product of the direction vectors OR • Recognises that $\mathbf{r}_1 \cdot \mathbf{r}_2 = 0$ OR • Equates component(s) of $\mathbf{r}_1$ and $\mathbf{r}_2$	1

Sample answer:

$$\mathbf{r}_1 = \begin{pmatrix} -2 \\ 1 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix} \qquad \mathbf{r}_2 = \begin{pmatrix} 4 \\ -2 \\ q \end{pmatrix} + \mu \begin{pmatrix} p \\ 3 \\ -1 \end{pmatrix}$$

They are perpendicular $\therefore\ \mathbf{r}_1 \cdot \mathbf{r}_2 = 0$

$$\begin{aligned} 1 \times p + 0 \times 3 + 2 \times -1 &= 0 \\ p - 2 &= 0 \\ p &= 2 \end{aligned}$$

They intersect $\therefore$ components are equal

① $-2 + \lambda = 4 + p\mu$

② $1 = -2 + 3\mu \quad \Rightarrow 3 = 3\mu$ so $\mu = 1$

③ $3 + 2\lambda = q - \mu$

Substitute $\mu = 1$ and $p = 2$ into ①

$$\begin{aligned} -2 + \lambda &= 4 + 2 \times 1 \\ \lambda &= 8 \end{aligned}$$

Substitute $\lambda = 8$ and $\mu = 1$ into ③

$$\begin{aligned} 3 + 2 \times 8 &= q - 1 \\ 19 &= q - 1 \\ q &= 20 \end{aligned}$$

$\therefore\ p = 2$ and $q = 20$.

Question 12 (d)

Criteria	Marks
• Provides correct solution	3
• Shows that $p(k) \Rightarrow p(k+1)$ is a true statement, or equivalent merit	2
• Establishes initial case, or equivalent merit	1

Sample answer:

$\sqrt{n!} > 2^n, \quad n \geq 9$

Prove it's true for $n = 9$

$$\sqrt{9!} = 602.39\ldots$$
$$2^9 = 512$$
$$\therefore \sqrt{9!} > 2^9$$

$\therefore$ It's true for $n = 9$

Assume it's true for $n = k$: Assume $\sqrt{k!} > 2^k$

Prove it's true for $n = k + 1$: Required to prove $\sqrt{(k+1)!} > 2^{k+1}$

$$\begin{aligned} \text{LHS} &= \sqrt{(k+1)!} \\ &= \sqrt{k!} \times \sqrt{(k+1)} \\ &> 2^k \times \sqrt{(k+1)} && \text{using the assumption} \\ &> 2^k \times 2 && \text{because } \sqrt{k+1} \text{ is greater than 2 for } k \geq 9 \\ &= 2^{k+1} \\ &= \text{RHS} \end{aligned}$$

$\therefore \sqrt{(k+1)!} > 2^{k+1}$ as required

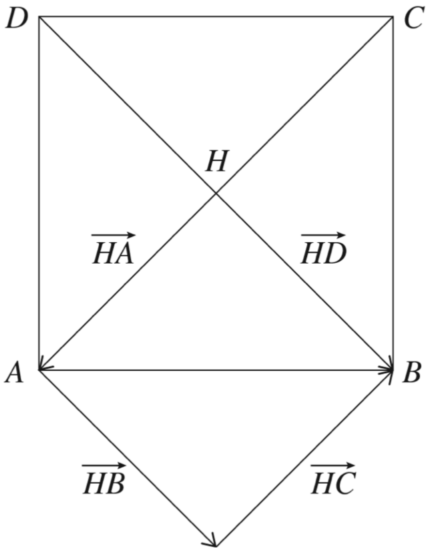
$\therefore$ By principle of mathematical induction, the inequality is true for $n \geq 9$.

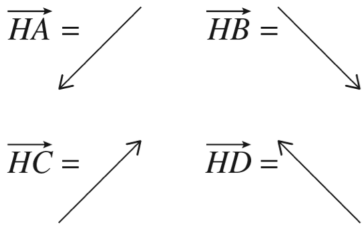
Question 12 (e) (i)

Criteria	Marks
• Provides correct solution	1

Sample answer:

Consider the square base





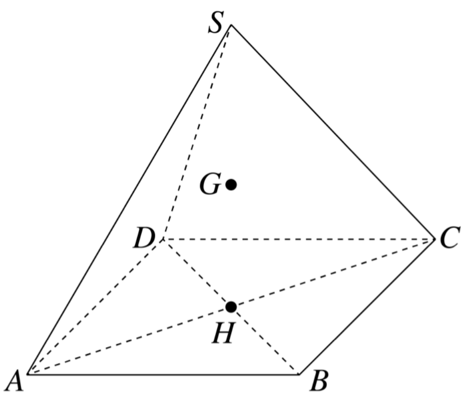
As H is the midpoint of AC and BD, when you add them head to tail, you end up back where you started.

$$\therefore\ \overrightarrow{HA} + \overrightarrow{HB} + \overrightarrow{HC} + \overrightarrow{HD} = \underset{\sim}{0}$$

Question 12 (e) (ii)

Criteria	Marks
• Provides correct solution	2
• Writes $\overrightarrow{GA} = \overrightarrow{GH} + \overrightarrow{HA}$, or equivalent merit	1

Sample answer:



$$\overrightarrow{GA} = \overrightarrow{GH} + \overrightarrow{HA}$$
$$\overrightarrow{GB} = \overrightarrow{GH} + \overrightarrow{HB}$$
$$\overrightarrow{GC} = \overrightarrow{GH} + \overrightarrow{HC}$$
$$\overrightarrow{GD} = \overrightarrow{GH} + \overrightarrow{HD}$$

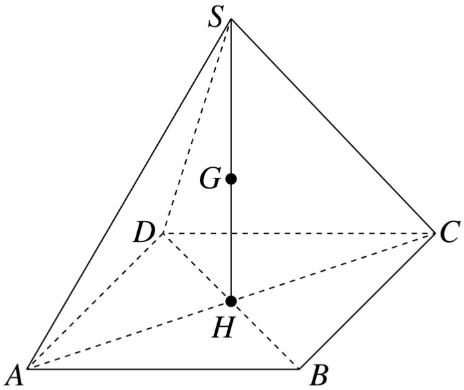
Adding these

$$\overrightarrow{GA} + \overrightarrow{GB} + \overrightarrow{GC} + \overrightarrow{GD} = 4\overrightarrow{GH} + \overrightarrow{HA} + \overrightarrow{HB} + \overrightarrow{HC} + \overrightarrow{HD}$$
$$-\overrightarrow{GS} = 4\overrightarrow{GH} + \underset{\sim}{0}$$
$$\therefore\ 4\overrightarrow{GH} + \overrightarrow{GS} = \underset{\sim}{0}$$

Question 12 (e) (iii)

Criteria	Marks
• Provides correct solution	1

Sample answer:



From part (ii)

$$4\overrightarrow{GH} + \overrightarrow{GS} = \underset{\sim}{0}$$

$$4\overrightarrow{GH} + \overrightarrow{GH} + \overrightarrow{HS} = \underset{\sim}{0}$$

$$5\overrightarrow{GH} + \overrightarrow{HS} = \underset{\sim}{0}$$

$$\overrightarrow{HS} = -5\overrightarrow{GH}$$

$$= 5\overrightarrow{HG}$$

$$\overrightarrow{HG} = \frac{1}{5}\overrightarrow{HS}$$

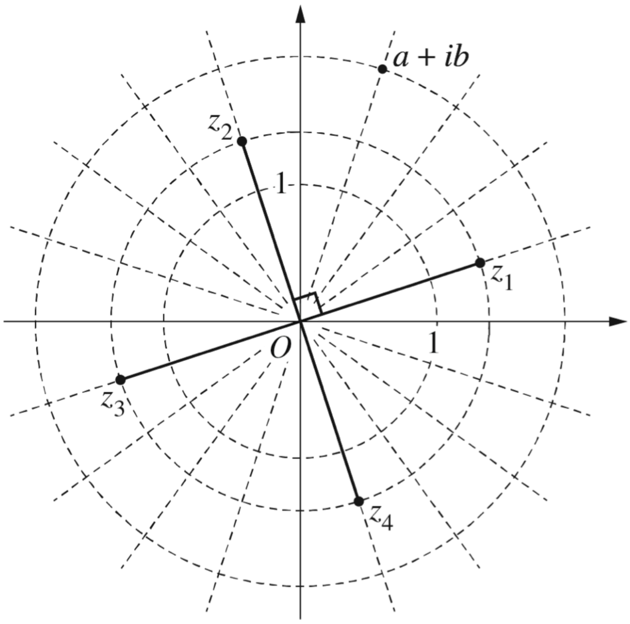
$$\therefore \lambda = \frac{1}{5}$$

Question 13 (a)

Criteria	Marks
• Provides appropriate sketch	2
• Indicates one correct argument OR • Indicates correct modulus	1

Sample answer:

z_1, z_2, z_3 and z_4 represent the fourth roots.



Question 13 (b)

Criteria	Marks
• Provides correct solution	3
• Obtains correct primitive OR correct definite integral in terms of u, or equivalent merit	2
• Attempts to use a suitable substitution	1

Sample answer:

Let $u = \sqrt{x^2 - 9}$

$$du = \frac{x}{\sqrt{x^2-9}}\,dx$$

and $u^2 = x^2 - 9$

When $x = \sqrt{10}$, $u = \sqrt{10-9} = 1$

When $x = \sqrt{13}$, $u = \sqrt{13-9} = 2$

$$I = \int_{\sqrt{10}}^{\sqrt{13}} x^3\sqrt{x^2-9}\,dx$$

$$= \int_{\sqrt{10}}^{\sqrt{13}} x^2\left(x^2-9\right)\frac{x}{\sqrt{x^2-9}}\,dx$$

$$= \int_1^2 \left(u^2+9\right)u^2\,du$$

$$= \int_1^2 u^4 + 9u^2\,du$$

$$= \left[\frac{u^5}{5} + 3u^3\right]_1^2$$

$$= \frac{2^5}{5} + 3\times 2^3 - \left(\frac{1}{5}+3\right)$$

$$= \frac{32}{5} - \frac{1}{5} + 24 - 3$$

$$= \frac{31}{5} + 21$$

$$= \frac{136}{5}$$

Question 13 (c) (i)

Criteria	Marks
• Provides correct solution	2
• Uses integration by parts, or equivalent	1

Sample answer:

$$I_n = \int_1^e (\ln x)^n \, dx, \qquad n \geq 0$$

Let $\quad u = (\ln x)^n \qquad$ and $\quad v' = 1$

$$\therefore \quad u' = \frac{n(\ln x)^{n-1}}{x} \qquad\qquad v = x$$

$$\therefore I_n = \Big[x(\ln x)^n \Big]_1^e - \int_1^e n(\ln x)^{n-1} \, dx$$

$$= e(\ln e)^n - (\ln 1)^n - nI_{n-1}$$

$$= e - 0 - nI_{n-1}$$

$$= e - nI_{n-1}$$

Question 13 (c) (ii)

Criteria	Marks
• Provides correct solution	4
• Shows $V_B = \pi$, or equivalent	3
• Obtains volume V_A • Writes volume V_A or V_B as a difference of volumes involving an integral expression	2
• States volume of a relevant sphere OR hemisphere OR cylinder OR • Provides correct integral expression for a relevant volume	1

Sample answer:

$$V_A = \pi \int_0^1 (1)^2 - \left(1 - x^2\right) dx$$

$$= \pi \int_0^1 x^2 \, dx$$

$$= \pi \left[\frac{x^3}{3} \right]_0^1$$

$$= \frac{\pi}{3}$$

$$V_B = \pi \int_1^e (1)^2 - (\ln x)^2 \, dx$$

$$= \pi \Big[x \Big]_1^e - \pi \int_1^e (\ln x)^2 \, dx$$

$$= \pi (e-1) - \pi I_2 \qquad \text{from part (i)}$$

Now $I_2 = e - 2I_1$

$$I_1 = e - I_0$$

$$I_0 = \int_1^e 1 \, dx$$

$$= \Big[x \Big]_1^e$$

$$= e - 1$$

$$\therefore I_2 = e - 2\big[e - (e-1)\big] = e - 2$$

$$\therefore V_B = \pi(e-1) - \pi(e-2)$$

$$= -\pi + 2\pi$$

$$= \pi$$

$$\therefore V_A : V_B = \frac{\pi}{3} : \pi$$

$$= 1 : 3$$

Question 13 (d) (i)

Criteria	Marks
• Provides correct solution	2
• States the amplitude of the motion, or equivalent merit	1

Sample answer:

$\ddot{x} = -4(x-3)$

$\therefore n^2 = 4, \quad \text{centre is } x = 3$

$n = 2$

$\text{period} = \dfrac{2\pi}{2} = \pi$

$v^2 = n^2\left(a^2 - (x-c)^2\right)$

when $\quad v = 8, x = 0$

$64 = 4\left(a^2 - (0-3)^2\right)$

$16 = a^2 - 9$

$a^2 = 25$

$a = 5$

$\therefore$ The particle oscillates between

$x = 3 + 5 \quad$ and $\quad x = 3 - 5$

$= 8 \qquad\qquad = -2$

Question 13 (d) (ii)

Criteria	Marks
• Provides correct solution	2
• Finds the displacement function, or equivalent merit	1

Sample answer:

Consider $x = a\cos(nt + \alpha) + c$, where

$a = 5, \quad n = 2, \quad c = 3$

$\therefore \; x = 5\cos(2t + \alpha) + 3$

When $\quad t = 0, \quad x = 5.5$

$$5.5 = 5\cos\alpha + 3$$

$$\frac{2.5}{5} = \cos\alpha$$

$$\therefore \quad \alpha = \cos^{-1}\left(\frac{2.5}{5}\right)$$

$$= 1.04719\ldots \text{radians}$$

$$0 = 5\cos(2t + 1.04719\ldots) + 3$$

$$\cos(2t + 1.04719\ldots) = \frac{-3}{5}$$

$$2t + 1.04719\ldots = 2.214\ldots \text{radians}$$

$$2t = 1.167\ldots$$

$$t = 0.583\ldots$$

$\therefore$ First value of t when $x = 0$ is $t = 0.58$ seconds (2 decimal places).

Question 14 (a)

Criteria	Marks
• Provides correct solution	4
• Applies partial fraction to the correct integral, or equivalent merit	3
• Completes *t*-substitution, including the limits of integration, or equivalent merit	2
• Attempts to use a *t*-substitution, or equivalent merit	1

Sample answer:

$$\int_0^{\frac{\pi}{2}} \frac{1}{3+5\cos x}\,dx \qquad \text{let } t = \tan\frac{x}{2} \quad \therefore \cos x = \frac{1-t^2}{1+t^2}$$

$$\text{when} \quad x = 0, \quad t = 0$$

$$x = \frac{\pi}{2}, \quad t = 1$$

$$dt = \frac{1}{2}\sec^2\left(\frac{x}{2}\right)dx$$

$$= \frac{1}{2}\left(1+t^2\right)dx$$

$$\therefore dx = \frac{2}{1+t^2}\,dt$$

$$\therefore \text{Integral} = \int_0^1 \frac{1}{3+\frac{5\left(1-t^2\right)}{1+t^2}} \cdot \frac{2}{1+t^2}\,dt$$

$$= \int_0^1 \frac{2}{3+3t^2+5-5t^2}\,dt$$

$$= \int_0^1 \frac{2}{8-2t^2}\,dt$$

$$= \int_0^1 \frac{1}{4-t^2}\,dt$$

$$= \int_0^1 \frac{1}{(2-t)(2+t)}\,dt$$

$$= \frac{1}{4}\int_0^1 \frac{1}{2-t} + \frac{1}{2+t}\,dt$$

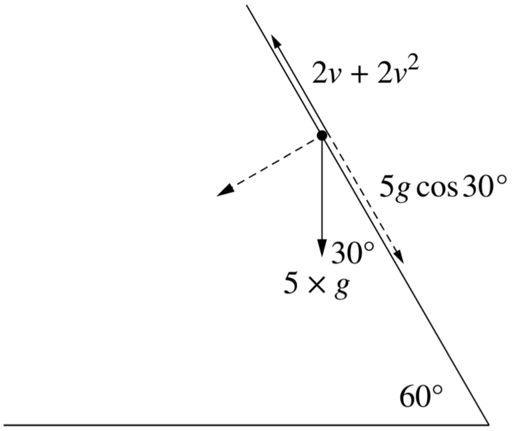
$$= \frac{1}{4}\left[\ln\left|\frac{2+t}{2-t}\right|\right]_0^1$$

$$= \frac{1}{4}(\ln 3 - \ln 1) = \frac{1}{4}\ln 3$$

Question 14 (b) (i)

Criteria	Marks
• Provides correct solution	2
• Finds appropriate component of force due to gravity OR • Provides appropriate diagram, or equivalent merit	1

Sample answer:



$$\text{Resultant force down slope} = 5g\cos 30° - 2v - 2v^2$$

$$= \frac{5\sqrt{3}}{2}g - 2v - 2v^2$$

Question 14 (b) (ii)

Criteria	Marks
• Provides correct solution	2
• States that the resultant force is $\underset{\sim}{0}$, or equivalent merit	1

Sample answer:

Constant speed ⇒ Resultant force = 0

$$\therefore \frac{5\sqrt{3}}{2} g = 2v + 2v^2$$

$$g = 10 \quad \therefore 25\sqrt{3} = 2v + 2v^2$$

$$2v^2 + 2v - 25\sqrt{3} = 0$$

$$\therefore v = \frac{-2 \pm \sqrt{4 + 4(2)\left(25\sqrt{3}\right)}}{4}$$

$$= 4.1798\ldots \text{ or } -5.1798\ldots$$

$$\therefore \text{speed} = 4.1798\ldots \text{ as moving down the slope}$$

$$\simeq 4.2 \text{ m s}^{-1} \quad \text{(1 decimal place)}$$

Question 14 (c) (i)

Criteria	Marks
• Provides correct solution	3
• Expands using binomial theorem AND • Expands using De Moivre's theorem	2
• Expands using binomial theorem OR • Expands using De Moivre's theorem	1

Sample answer:

Using De Moivre:

$$(\cos\theta + i\sin\theta)^5 = \cos 5\theta + i\sin 5\theta$$

Using binomial expansion

$$\begin{aligned}(\cos\theta + i\sin\theta)^5 &= (\cos\theta)^5 + 5(\cos\theta)^4(i\sin\theta)\\ &\quad +10(\cos\theta)^3(i\sin\theta)^2\\ &\quad +10(\cos\theta)^2(i\sin\theta)^3\\ &\quad +5(\cos\theta)(i\sin\theta)^4\\ &\quad +(i\sin\theta)^5\\ &= \cos^5\theta + 5\cos^4\theta\sin\theta i\\ &\quad -10\cos^3\theta\sin^2\theta\\ &\quad -10\cos^2\theta\sin^3\theta i\\ &\quad +5\cos\theta\sin^4\theta\\ &\quad +\sin^5\theta i\end{aligned}$$

Equating real parts:

$$\begin{aligned}\cos 5\theta &= \cos^5\theta - 10\cos^3\theta\sin^2\theta + 5\cos\theta\sin^4\theta\\ &= \cos^5\theta - 10\cos^3\theta\left(1-\cos^2\theta\right) + 5\cos\theta\left(1-\cos^2\theta\right)^2\\ &= \cos^5\theta - 10\cos^3\theta + 10\cos^5\theta + 5\cos\theta\left(1-2\cos^2\theta+\cos^4\theta\right)\\ &= \cos^5\theta - 10\cos^3\theta + 10\cos^5\theta + 5\cos\theta - 10\cos^3\theta + 5\cos^5\theta\\ &= 16\cos^5\theta - 20\cos^3\theta + 5\cos\theta\end{aligned}$$

Question 14 (c) (ii)

Criteria	Marks
• Provides correct solution	3
• Finds the solutions to the degree 5 polynomial, or equivalent merit	2
• Uses $\theta = \frac{\pi}{10}$ in the equation from part (i), or equivalent merit	1

Sample answer:

$$\mathrm{Re}\left(e^{i\frac{\pi}{10}}\right) = \mathrm{Re}\left(\cos\frac{\pi}{10} + i\sin\frac{\pi}{10}\right)$$

$$= \cos\frac{\pi}{10}$$

When $\theta = \frac{\pi}{10}$, $\cos 5\theta = \cos\frac{\pi}{2} = 0$

$\therefore \cos\frac{\pi}{10}$ is a root of $16\cos^5\theta - 20\cos^3\theta + 5\cos\theta = 0$

$$16\cos^5\theta - 20\cos^3\theta + 5\cos\theta = 0$$

$$\cos\theta\left(16\cos^4\theta - 20\cos^2\theta + 5\right) = 0$$

$$\cos\frac{\pi}{10} \neq 0$$

$\therefore \cos\frac{\pi}{10}$ is a root of $16\cos^4\theta - 20\cos^2\theta + 5 = 0$

$$\therefore \cos^2\theta = \frac{20 \pm \sqrt{400 - 4(16)(5)}}{32}$$

$$= \frac{20 \pm 4\sqrt{5}}{32}$$

$$= \frac{5 \pm \sqrt{5}}{8}$$

$$\therefore \cos\frac{\pi}{10} = \pm\sqrt{\frac{5 \pm \sqrt{5}}{8}}$$

But roots of $\cos 5\theta = 0$ are:

$$5\theta = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}, \frac{9\pi}{2}$$

$$\therefore \theta = \frac{\pi}{10}, \frac{3\pi}{10}, \frac{\pi}{2}, \frac{7\pi}{10}, \frac{9\pi}{10}$$

so roots of $16\cos^5\theta - 20\cos^3\theta + 5\cos\theta = 0$ are $\cos\frac{\pi}{10}, \cos\frac{3\pi}{10}, \cos\frac{\pi}{2} = 0, \cos\frac{7\pi}{10}$ and $\cos\frac{9\pi}{10}$

From the graph of $y = \cos x$, we see that $\cos\frac{\pi}{10}$ is the largest of these.

$$\therefore \cos\frac{\pi}{10} = \sqrt{\frac{5+\sqrt{5}}{8}}$$

$$\therefore \operatorname{Re}\left(e^{i\frac{\pi}{10}}\right) = \sqrt{\frac{5+\sqrt{5}}{8}}$$

Question 15 (a) (i)

Criteria	Marks
• Provides correct solution	2
• Attempts to use appropriate expressions for x and y	1

Sample answer:

$$\begin{aligned}\sqrt{abc} = \sqrt{(ab)c} &\le \frac{ab+c}{2} && \text{using the result provided}\\ &= \frac{\sqrt{a^2b^2}+c}{2}\\ &\le \frac{\frac{a^2+b^2}{2}+c}{2} && \text{using the same result a second time}\\ &= \frac{a^2+b^2+2c}{4}\end{aligned}$$

Answers could include:

Let $x = \dfrac{a^2+b^2}{2}$ and $y = c$

$$\therefore \sqrt{\frac{a^2+b^2}{2}\cdot c} \le \frac{\frac{a^2+b^2}{2}+c}{2}$$

$$\sqrt{\frac{a^2c+b^2c}{2}} \le \frac{a^2+b^2+2c}{4}$$

Let $m = a^2c$ and $n = b^2c$

$$\therefore \sqrt{mn} = \sqrt{a^2c\cdot b^2c} = \sqrt{a^2b^2c^2} = abc$$

$$\sqrt{\frac{m+n}{2}} = \frac{a^2c+b^2c}{2}$$

$$\therefore \quad abc \le \frac{a^2c+b^2c}{2}$$

$$\therefore \sqrt{abc} \le \sqrt{\frac{a^2c+b^2c}{2}} \le \frac{a^2+b^2+2c}{4}$$

Question 15 (a) (ii)

Criteria	Marks
• Provides correct solution	2
• Permutes the result in (i), or equivalent merit	1

Sample answer:

$$\sqrt{abc} \le \frac{a^2+b^2+2c}{4}$$

$$\sqrt{acb} \le \frac{a^2+c^2+2b}{4}$$

$$\sqrt{bca} \le \frac{b^2+c^2+2a}{4}$$

Adding

$$3\sqrt{abc} \le \frac{2a^2+2b^2+2c^2+2a+2b+2c}{4}$$

$$3\sqrt{abc} \le \frac{a^2+b^2+c^2+a+b+c}{2}$$

$$\therefore \sqrt{abc} \le \frac{a^2+b^2+c^2+a+b+c}{6} \quad \text{as required}$$

Question 15 (b) (i)

Criteria	Marks
• Provides correct solution	2
• Attempts to use an expression for an odd number in the formula for t_n, or equivalent merit	1

Sample answer:

$$t_n = \frac{n(n+1)}{2} \qquad h_n = 2n^2 - n$$

The odd numbers 1, 3, 5, … can be expressed as $2m - 1$ where m is an integer.

$\therefore$ the odd triangular numbers are t_{2m-1}

$$\begin{aligned} t_{2m-1} &= \frac{(2m-1)(2m-1+1)}{2} \\ &= \frac{(2m-1)2m}{2} \\ &= m(2m-1) \\ &= 2m^2 - m \\ &= h_m \end{aligned}$$

$\therefore$ The odd triangular numbers are hexagonal.

Question 15 (b) (ii)

Criteria	Marks
• Provides correct solution	1

Sample answer:

The even numbers 2, 4, 6 … can be expressed as $2m$ where m is an integer.

$$\begin{aligned} t_{2m} &= \frac{2m(2m+1)}{2} \\ &= m(2m+1) \\ &= 2m^2 + m \end{aligned}$$

Use proof by contradiction to show it is not hexagonal. Assume it is hexagonal.

$$\begin{aligned} 2m^2 + m &= 2k^2 - k \qquad \text{where } k \text{ is an integer} \\ m + k &= 2k^2 - 2m^2 \\ m + k &= 2\left(k^2 - m^2\right) \\ m + k &= 2(k-m)(k+m) \\ \therefore\ 1 &= 2(k-m) \qquad \text{since } k + m \neq 0 \end{aligned}$$

So, 1 is even, which is not true $\therefore$ the original statement is false.

Question 15 (c) (i)

Criteria	Marks
• Provides correct solution	3
• Integrates to obtain an expression for t in terms of v, or equivalent merit	2
• Provides an expression for the resultant force, or equivalent merit	1

Sample answer:

$$\text{Force} = -mg - kv^2$$

$$ma = -mg - kv^2$$

$$a = -g - \frac{kv^2}{m}$$

$$a = -g - kv^2 \qquad \text{given } m = 1$$

$$\frac{dv}{dt} = -g - kv^2 \qquad \text{need } t \text{ in terms of } v\text{, so use } a = \frac{dv}{dt}$$

$$\frac{dt}{dv} = -\frac{1}{g + kv^2}$$

$$t = -\frac{1}{\sqrt{gk}}\tan^{-1}\left(v\sqrt{\frac{k}{g}}\right) + c$$

When $t = 0$, $v = u$

$$0 = -\frac{1}{\sqrt{gk}}\tan^{-1}\left(u\sqrt{\frac{k}{g}}\right) + c$$

$$c = \frac{1}{\sqrt{gk}}\tan^{-1}\left(u\sqrt{\frac{k}{g}}\right)$$

$$\therefore \quad t = \frac{1}{\sqrt{gk}}\tan^{-1}\left(u\sqrt{\frac{k}{g}}\right) - \frac{1}{\sqrt{gk}}\tan^{-1}\left(v\sqrt{\frac{k}{g}}\right)$$

Max height, $v = 0 \therefore \ t = \dfrac{1}{\sqrt{gk}}\tan^{-1}\left(u\sqrt{\dfrac{k}{g}}\right)$ as required.

(Either notation, arctan or $\tan^{-1}$, is acceptable.)

Question 15 (c) (ii)

Criteria	Marks
• Provides correct solution	3
• Integrates to obtain an expression for x in terms of v, or equivalent merit	2
• Provides an integral expression for x in terms of v, or equivalent merit	1

Sample answer:

$$v\frac{dv}{dx} = -g - kv^2$$

$$\frac{dv}{dx} = -\frac{g + kv^2}{v}$$

$$\frac{dx}{dv} = -\frac{v}{g + kv^2}$$

$$x = -\frac{1}{2k}\ln\left(g + kv^2\right) + c$$

When $x = 0,\ v = u$

$$0 = -\frac{1}{2k}\ln\left(g + ku^2\right) + c$$

$$c = \frac{1}{2k}\ln\left(g + ku^2\right)$$

$$\therefore\ x = \frac{1}{2k}\left(\ln\left(g + ku^2\right) - \ln\left(g + kv^2\right)\right) = \frac{1}{2k}\ln\frac{g + ku^2}{g + kv^2}$$

Maximum height, sub in $v = 0$

$\therefore$ maximum height $x = \dfrac{1}{2k}\ln\left(\dfrac{g + ku^2}{g}\right)$

Question 15 (d)

Criteria	Marks
• Provides correct solution	2
• Writes $5^n = (2+3)^n$ or equivalent merit	1

Sample answer:

$$5 = 2 + 3$$

so $5^n = (2+3)^n$

$$= 2^n + 3^n + \binom{n}{1} 2 \times 3^{n-1} + \text{other terms}$$

$$> 2^n + 3^n$$

so $2^n + 3^n \neq 5^n$ if $n \geq 2$

Question 16 (a) (i)

Criteria	Marks
• Provides correct solution	2
• States that $\overrightarrow{OP}$ is a unit vector, or equivalent merit.	1

Sample answer:

Point P is on the unit sphere so

$$\begin{aligned}1 &= \left|\overrightarrow{OP}\right| \\ &= \left|x\underset{\sim}{i} + y\underset{\sim}{j} + z\underset{\sim}{k}\right| \\ &\le \left|x\underset{\sim}{i}\right| + \left|y\underset{\sim}{j} + z\underset{\sim}{k}\right| && \text{(triangular inequality)} \\ &\le \left|x\underset{\sim}{i}\right| + \left|y\underset{\sim}{j}\right| + \left|z\underset{\sim}{k}\right| && \text{(triangular inequality)} \\ &= |x| + |y| + |z| && \underset{\sim}{i},\ \underset{\sim}{j},\ \underset{\sim}{k} \text{ unit vectors}\end{aligned}$$

so $|x| + |y| + |z| \ge 1$

Answers could include:

P is on the unit sphere $\therefore \sqrt{x^2 + y^2 + z^2} = 1$

$\therefore |x|^2 + |y|^2 + |z|^2 = 1$

But $|x| \le 1$, $|y| \le 1$ and $|z| \le 1$

$\therefore |x|^2 \le |x|$, $|y|^2 \le |y|$ and $|z|^2 \le |z|$

$$\begin{aligned}\therefore 1 &= |x|^2 + |y|^2 + |z|^2 \\ &\le |x| + |y| + |z|\end{aligned}$$

Question 16 (a) (ii)

Criteria	Marks
• Provides correct solution	3
• Obtains $\left\lvert a_1b_1 + a_2b_2 + a_3b_3 \right\rvert = \sqrt{a_1{}^2 + a_2{}^2 + a_3{}^2}\sqrt{b_1{}^2 + b_2{}^2 + b_3{}^2}\cos\theta \ldots$	2
• Attempts to apply the dot product, or equivalent merit	1

Sample answer:

$$\underset{\sim}{a} \cdot \underset{\sim}{b} = \lvert \underset{\sim}{a} \rvert \lvert \underset{\sim}{b} \rvert \cos\theta$$

$$a_1b_1 + a_2b_2 + a_3b_3 = \lvert \underset{\sim}{a} \rvert \lvert \underset{\sim}{b} \rvert \cos\theta$$

$$-1 \le \cos\theta \le 1$$

$$-\lvert \underset{\sim}{a} \rvert \lvert \underset{\sim}{b} \rvert \le a_1b_1 + a_2b_2 + a_3b_3 \le \lvert \underset{\sim}{a} \rvert \lvert \underset{\sim}{b} \rvert$$

so

$$\left\lvert a_1b_1 + a_2b_2 + a_3b_3 \right\rvert \le \lvert \underset{\sim}{a} \rvert \lvert \underset{\sim}{b} \rvert$$

$$= \sqrt{a_1{}^2 + a_2{}^2 + a_3{}^2}\sqrt{b_1{}^2 + b_2{}^2 + b_3{}^2}$$

Question 16 (a) (iii)

Criteria	Marks
• Provides correct solution	2
• Chooses one suitable vector to use with the result from part (ii), or equivalent merit	1

Sample answer:

If $P(x,y,z)$ is on the unit sphere $x^2+y^2+z^2=1$, $|x|^2+|y|^2+|z|^2=1$.
Hence $Q(|x|,|y|,|z|)$ is on the unit sphere.

In part (ii) let $\underset{\sim}{a}=\begin{pmatrix}|x|\\|y|\\|z|\end{pmatrix}$

And $\underset{\sim}{b}=\begin{pmatrix}1\\1\\1\end{pmatrix}$

Then

$$\left||x|+|y|+|z|\right|\le\sqrt{|x|^2+|y|^2+|z|^2}\sqrt{1^2+1^2+1^2}$$
$$=\sqrt{3}$$

$|x|,|y|$ and $|z|$ are not negative, so
$|x|+|y|+|z|=\left||x|+|y|+|z|\right|\le\sqrt{3}$

Answers could include:

$P(x,y,z)$ on unit sphere so $x^2+y^2+z^2=1$.
Choose b_1 to be 1 or –1 so that $xb_1\ge 0$ so $xb_1=|x|$, and similarly choose b_2 and b_3

so $yb_2\ge 0$ and $zb_3\ge 0$.

Let $\underset{\sim}{a}=\begin{pmatrix}x\\y\\z\end{pmatrix}$, $\underset{\sim}{b}=\begin{pmatrix}b_1\\b_2\\b_3\end{pmatrix}$

By part (ii)

$$\left|xb_1+yb_2+zb_3\right|\le\sqrt{x^2+y^2+z^2}\sqrt{b_1{}^2+b_2{}^2+b_3{}^2}$$
$$=1\times\sqrt{1+1+1}$$
$$=\sqrt{3}$$

$\left||x|+|y|+|z|\right|\le\sqrt{3}$

$|x|,|y|,|z|$ are not negative so

$|x|+|y|+|z|\le\sqrt{3}$

Question 16 (b)

Criteria	Marks
• Provides correct solution, with justification	5
• Shows that the relevant times are positive OR • Shows that the relevant times occur before the particle lands, or equivalent merit	4
• Obtains a quadratic that will identify all possible angles, or equivalent merit	3
• Uses the fact that the dot product of the two vectors is 0, or equivalent merit	2
• Obtains velocity vector, or equivalent merit	1

Sample answer:

Position $\quad \underset{\sim}{r}(t) = \begin{pmatrix} ut\cos\theta \\ -\dfrac{gt^2}{2} + ut\sin\theta \end{pmatrix}$

Velocity $\quad \dot{\underset{\sim}{r}}(t) = \begin{pmatrix} u\cos\theta \\ -gt + u\sin\theta \end{pmatrix}$

If position vector is perpendicular to velocity vector then

$$\begin{aligned}
0 &= \underset{\sim}{r}(t) \cdot \dot{\underset{\sim}{r}}(t) \\
&= \begin{pmatrix} ut\cos\theta \\ -\dfrac{gt^2}{2} + ut\sin\theta \end{pmatrix} \cdot \begin{pmatrix} u\cos\theta \\ -gt + u\sin\theta \end{pmatrix} \\
&= u^2 t\cos^2\theta + \left(-\frac{gt^2}{2} + ut\sin\theta\right)(-gt + u\sin\theta) \\
&= u^2 t\cos^2\theta + \frac{g^2t^3}{2} - \frac{gt^2u}{2}\sin\theta - gt^2u\sin\theta + u^2t\sin^2\theta \\
&= u^2 t - \frac{3ugt^2\sin\theta}{2} + \frac{g^2t^3}{2} \\
&= \frac{t}{2}\left(g^2t^2 - 3ugt\sin\theta + 2u^2\right)
\end{aligned}$$

During time of flight $t > 0$ so above can only be zero when $g^2t^2 - 3ugt\sin\theta + 2u^2 = 0$.

$y = g^2t^2 - 3ugt\sin\theta + 2u^2$ is the graph of a concave up parabola where y is a function of t.

Want two zeros so $\triangle = b^2 - 4ac > 0$ and we also know $0 < \theta < \frac{\pi}{2}$

$$9u^2g^2\sin^2\theta - 4 \times g^2 \times 2u^2 > 0$$

$$u^2g^2\left(9\sin^2\theta - 8\right) > 0$$

$$9\sin^2\theta - 8 > 0 \qquad \text{as } u^2g^2 > 0$$

$$\sin^2\theta > \frac{8}{9}$$

$$\sin\theta > \frac{\sqrt{8}}{3} \qquad \left(0 < \theta < \frac{\pi}{2} \text{ so } \sin\theta > 0\right)$$

$$\theta > 1.23 \qquad \text{(2 decimal places)}$$

(70.52°)

This shows that we may have two points during the time of flight if $\frac{\pi}{2} > \theta > 1.23$

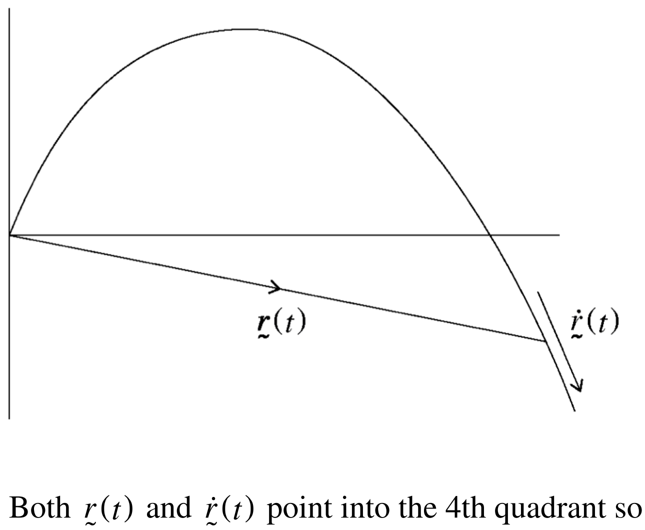
$$t = \frac{3ug\sin\theta \pm ug\sqrt{9\sin^2\theta - 8}}{2g^2} = \frac{u}{2g}\left(3\sin\theta \pm \sqrt{9\sin^2\theta - 8}\right)$$

$$9\sin^2\theta - 8 < 9\sin^2\theta = (3\sin\theta)^2$$

so $t > 0$

so both points occur after projection.

If we ignore the ground, and consider points on the trajectory below the point of projection.



Both $\underset{\sim}{r}(t)$ and $\dot{\underset{\sim}{r}}(t)$ point into the 4th quadrant so the angle between them is less than $\frac{\pi}{2}$.

Thus the two points must occur after projection but before the projectile lands.

Question 16 (c)

Criteria	Marks
• Provides correct sketch	3
• Considers inequalities involving $x\tan(x)$ AND has included or excluded at least one section of the Argand plane • Obtains the region below <diag>	2
• States that a nominated section of the Argand plane is included in the required region OR • States that a nominated section of the Argand plane is excluded from the required region	1

Sample answer:

If $z = x + iy$ and $Arg(z) = \theta$

then $-\pi < \theta \leq \pi$

$\tan\theta = \dfrac{y}{x}$ when $x \neq 0$

$\text{Re}(z) = x$

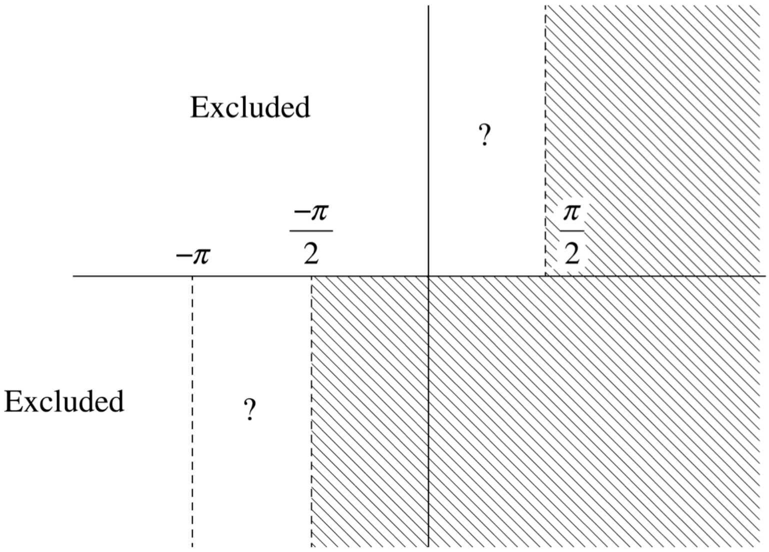
In 2nd quadrant $\text{Re}(z) = x < 0 < \dfrac{\pi}{2} < Arg(z)$ so 2nd quadrant not in region

In 4th quadrant $\text{Re}(z) > 0 > Arg(z)$ so 4th quadrant included in region

In 1st quadrant $Arg(z) < \dfrac{\pi}{2}$ so if $x \geq \dfrac{\pi}{2}$ then z is included in region

In 3rd quadrant $-\pi < Arg(z) < -\dfrac{\pi}{2}$ so $x < -\pi$ excluded and $x > -\dfrac{\pi}{2}$ included.

So far



If $0 \le x < \dfrac{\pi}{2}$ then $\tan x$ is an increasing function

$\operatorname{Re}(z) \ge Arg(z)$

$x \ge \theta$

$\tan x \ge \tan\theta = \dfrac{y}{x} \quad \left(\text{tan increases on} \left[0, \dfrac{\pi}{2}\right]\right)$

$x\tan x \ge y \qquad (x > 0)$

If $-\pi < x < \dfrac{-\pi}{2}$ tan also increasing

$Arg(z) = \arctan\left(\dfrac{y}{x}\right) - \pi$

$x \ge \arctan\left(\dfrac{y}{x}\right) - \pi$

$x + \pi \ge \arctan\left(\dfrac{y}{x}\right)$

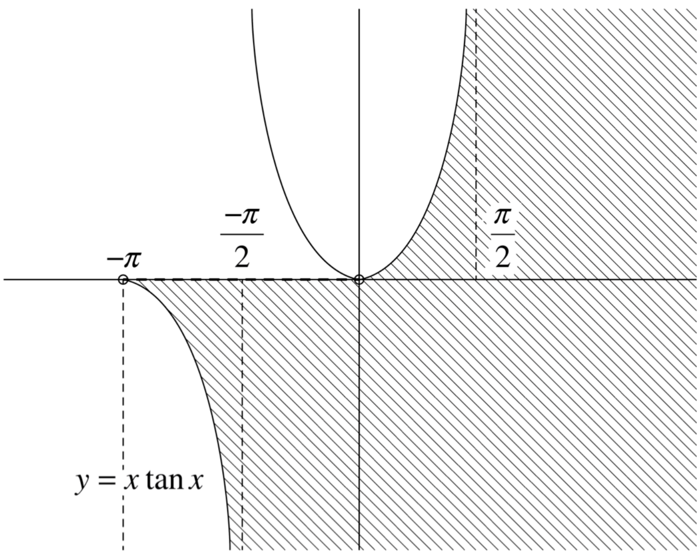
$\tan(x+\pi) \ge \dfrac{y}{x}$

$x\tan(x) \le y$

as $\tan(x+\pi) = \tan x$ and $x < 0$

Also, if $z = 0$ and if $y = 0, x < 0, Arg(z) = \pi > x$

$\therefore$ not included.



2021 HSC Mathematics Extension 2 Mapping Grid

Section I

Question	Marks	Content	Syllabus outcomes
1	1	MEX V1 Further work with vectors	MEX 12 3
2	1	MEX C1 Further integration	MEX 12–5
3	1	MEX V1 Further work with vectors	MEX 12–3
4	1	MEX P1 The nature of proof	MEX 12–2
5	1	MEX P1 The nature of proof	MEX 12–1
6	1	MEX N2 Using complex numbers	MEX 12–4
7	1	MEX V1 Further work with vectors	MEX 12–3
8	1	MEX M1 Applications of calculus to mechanics	MEX 12–6
9	1	MEX P1 The nature of proof	MEX 12–2
10	1	MEX N2 Using complex numbers	MEX 12–4

Section II

Question	Marks	Content	Syllabus outcomes
11(a)	2	MEX N1 Introduction to complex numbers	MEX 12–4
11 (b)	2	MEX P2 Further proof by mathematical induction MEX N1 Introduction to complex numbers	MEX 12–1
11 (c)	3	MEX V1 Further work with vectors	MEX 12–3
11 (d) (i)	2	MEX N2 Using complex numbers	MEX 12–4
11 (d) (ii)	2	MEX N2 Using complex numbers	MEX 12–4
11 (e)	2	MEX N1 Introduction to complex numbers	MEX 12–4
11 (f)	3	MEX C1 Further integration	MEX 12–1
12 (a)	3	MEX C1 Further integration	MEX 12–5
12 (b) (i)	1	MEX P1 The nature of proof	MEX 12–2
12 (b) (ii)	1	MEX P1 The nature of proof	MEX 12–2
12 (c)	3	MEX V1 Further work with vectors	MEX 12–3
12 (d)	3	MEX P2 Further proof by mathematical induction	MEX 12–2
12 (e) (i)	1	MEX V1 Further work with vectors	MEX 12–3
12 (e) (ii)	2	MEX V1 Further work with vectors	MEX 12–3

Question	Marks	Content	Syllabus outcomes
12 (e) (iii)	1	MEX V1 Further work with vectors	MEX 12–3
13 (a)	2	MEX N2 Using complex numbers	MEX 12–4
13 (b)	3	MEX C1 Further integration	MEX 12–5
13 (c) (i)	2	MEX C1 Further integration	MEX 12–5
13 (c) (ii)	4	MEX C1 Further integration	MEX 12–5
13 (d) (i)	2	MEX M1 Application of calculus to mechanics	MEX 12–6
13 (d) (ii)	2	MEX M1 Application of calculus to mechanics	MEX 12–6, MEX 12–7
14 (a)	4	MEX C1 Further integration	MEX 12–5
14 (b) (i)	2	MEX M1 Application of calculus to mechanics	MEX 12–6
14 (b) (ii)	2	MEX M1 Application of calculus to mechanics	MEX 12–6, MEX 12–7
14 (c) (i)	3	MEX N2 Using complex numbers	MEX 12–4
14 (c) (ii)	3	MEX N2 Using complex numbers	MEX 12–4
15 (a) (i)	2	MEX P1 The nature of proof	MEX 12–2
15 (a) (ii)	2	MEX P1 The nature of proof	MEX 12–2
15 (b) (i)	2	MEX P1 The nature of proof	MEX 12–1, MEX 12–2
15 (b) (ii)	1	MEX P1 The nature of proof	MEX 12–1, MEX 12–2
15 (c) (i)	3	MEX M1 Application of calculus to mechanics	MEX 12–6
15 (c) (ii)	3	MEX M1 Application of calculus to mechanics	MEX 12–6
15 (d)	2	MEX P1 The nature of proof	MEX 12–2
16 (a) (i)	2	MEX P1 The nature of proof	MEX 12–2
16 (a) (ii)	3	MEX V1 Further work with vectors	MEX 12–3
16 (a) (iii)	2	MEX V1 Further work with vectors	MEX 12–3
16 (b)	5	MEX M1 Application of calculus to mechanics	MEX 12–6, MEX 12–7
16 (c)	3	MEX N1 Introduction to complex numbers	MEX 12–4