NSW Education Standards Authority

Centre Number

Student Number

2024 HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Extension 2

General Instructions

- Reading time – 10 minutes
- Working time – 3 hours
- Write using black pen
- Calculators approved by NESA may be used
- A reference sheet is provided at the back of this paper
- For questions in Section II, show relevant mathematical reasoning and/or calculations
- Write your Centre Number and Student Number at the top of this page

Total marks: 100

Section I – 10 marks (pages 2–5)

- Attempt Questions 1–10
- Allow about 15 minutes for this section

Section II – 90 marks (pages 6–14)

- Attempt Questions 11–16
- Allow about 2 hours and 45 minutes for this section

Section I

10 marks
Attempt Questions 1–10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1–10.

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1 Which of the following vectors is perpendicular to $3\underset{\sim}{i} + 2\underset{\sim}{j} - 5\underset{\sim}{k}$?

A. $-\underset{\sim}{i} - \underset{\sim}{j} + \underset{\sim}{k}$

B. $\underset{\sim}{i} + \underset{\sim}{j} - \underset{\sim}{k}$

C. $-2\underset{\sim}{i} + 3\underset{\sim}{j} + \underset{\sim}{k}$

D. $3\underset{\sim}{i} - 2\underset{\sim}{j} + \underset{\sim}{k}$

2 Consider the following statement written in the formal language of proof

$$\forall\, \theta \in \left(\frac{\pi}{2}, \pi\right)\ \exists\, \phi \in \left(\pi, \frac{3\pi}{2}\right);\ \sin\theta = -\cos\phi.$$

Which of the following best represents this statement?

A. There exists a θ in the second quadrant such that for all ϕ in the third quadrant $\sin\theta = -\cos\phi$.

B. There exists a ϕ in the third quadrant such that for all θ in the second quadrant $\sin\theta = -\cos\phi$.

C. For all θ in the second quadrant there exists a ϕ in the third quadrant such that $\sin\theta = -\cos\phi$.

D. For all ϕ in the third quadrant there exists a θ in the second quadrant such that $\sin\theta = -\cos\phi$.

3 Consider the statement:

'If a polygon is a square, then it is a rectangle.'

Which of the following is the converse of the statement above?

A. If a polygon is a rectangle, then it is a square.

B. If a polygon is a rectangle, then it is not a square.

C. If a polygon is not a rectangle, then it is not a square.

D. If a polygon is not a square, then it is not a rectangle.

4 A monic polynomial, $f(x)$, of degree 3 with real coefficients has 3 and $2 + i$ as two of its roots.

Which of the following could be $f(x)$?

A. $f(x) = x^3 - 7x^2 - 17x + 15$

B. $f(x) = x^3 - 7x^2 + 17x - 15$

C. $f(x) = x^3 + 7x^2 - 17x + 15$

D. $f(x) = x^3 + 7x^2 + 17x - 15$

5 A particle is moving in simple harmonic motion with period 10 seconds and an amplitude of 8 m. The particle starts at the central point of motion and is initially moving to the left with a speed of $V \text{ m s}^{-1}$, where $V > 0$.

What will be the position and velocity of the particle after 7.5 seconds?

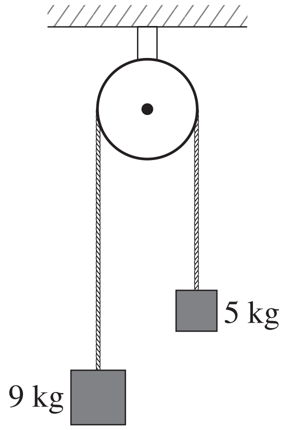
A. At the central point of motion with a velocity of $V \text{ m s}^{-1}$

B. At the central point of motion with a velocity of $-V \text{ m s}^{-1}$

C. 8 m to the left of the central point of motion with a velocity of 0 m s^{-1}

D. 8 m to the right of the central point of motion with a velocity of 0 m s^{-1}

6 A light string passes over a smooth pulley. Attached to the ends of the string are masses of 9 kg and 5 kg, as shown.



The acceleration due to gravity is g m s^{-2}.

What is the acceleration of the 9 kg mass?

A. $\frac{2}{7}g$

B. $1g$

C. $\frac{7}{2}g$

D. $4g$

7 It is given that $|z - 1 + i| = 2$.

What is the maximum possible value of $|z|$?

A. $\sqrt{2}$

B. $\sqrt{10}$

C. $2 + \sqrt{2}$

D. $2 - \sqrt{2}$

8 Which of the following is equal to $e^{\bar{z}}$, where $z = x + iy$ with x and y real numbers?

A. $\overline{e^{z}}$

B. e^{-z}

C. $e^{2x}e^{z}$

D. $e^{-2x}e^{z}$

9 Consider the solutions of the equation $z^4 = -9$.

What is the product of all of the solutions that have a positive principal argument?

A. 3

B. -3

C. $3i$

D. $-3i$

10 Three unit vectors $\underset{\sim}{a}$, $\underset{\sim}{b}$ and $\underset{\sim}{c}$, in 3 dimensions, are to be chosen so that $\underset{\sim}{a} \perp \underset{\sim}{b}$, $\underset{\sim}{b} \perp \underset{\sim}{c}$ and the angle θ between $\underset{\sim}{a}$ and $\underset{\sim}{a} + \underset{\sim}{b} + \underset{\sim}{c}$ is as small as possible.

What is the value of $\cos\theta$?

A. 0

B. $\dfrac{1}{\sqrt{3}}$

C. $\dfrac{1}{\sqrt{2}}$

D. $\dfrac{2}{\sqrt{5}}$

Section II

90 marks
Attempt Questions 11–16
Allow about 2 hours and 45 minutes for this section

Answer each question in the appropriate writing booklet. Extra writing booklets are available.

For questions in Section II, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (16 marks) Use the Question 11 Writing Booklet

(a) Find $\int xe^x \, dx$. **2**

(b) Let $z = 2 + 3i$ and $w = 1 - 5i$.

(i) Find $z + \bar{w}$. **1**

(ii) Find z^2. **1**

(c) Find the angle between the two vectors $\underset{\sim}{u} = \begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix}$ and $\underset{\sim}{v} = \begin{pmatrix} 4 \\ -4 \\ 7 \end{pmatrix}$, giving your answer in radians, correct to 1 decimal place. **2**

(d) Evaluate $\displaystyle\int_0^{\frac{\pi}{2}} \frac{1}{\sin\theta + 1} \, d\theta$. **3**

(e) (i) Write the number $\sqrt{3} + i$ in modulus-argument form. **2**

(ii) Hence, or otherwise, write $\left(\sqrt{3} + i\right)^7$ in exact Cartesian form. **2**

(f) Sketch the region defined by $|z| < 3$ and $0 \le \arg(z - i) \le \dfrac{\pi}{2}$. **3**

Question 12 (14 marks) Use the Question 12 Writing Booklet

(a) The vector $\underset{\sim}{a}$ is $\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ and the vector $\underset{\sim}{b}$ is $\begin{pmatrix} 2 \\ 0 \\ -4 \end{pmatrix}$.

(i) Find $\dfrac{\underset{\sim}{a} \cdot \underset{\sim}{b}}{\underset{\sim}{b} \cdot \underset{\sim}{b}} \underset{\sim}{b}$. **1**

(ii) Show that $\underset{\sim}{a} - \dfrac{\underset{\sim}{a} \cdot \underset{\sim}{b}}{\underset{\sim}{b} \cdot \underset{\sim}{b}} \underset{\sim}{b}$ is perpendicular to $\underset{\sim}{b}$. **2**

(b) Use partial fractions to find **3**

$$\int \frac{3x^2 + 2x + 1}{(x-1)(x^2+1)}\, dx.$$

(c) Consider the equation $|z| = z + 8 + 12i$, where $z = a + bi$ is a complex number and a, b are real numbers.

(i) Explain why $b = -12$. **1**

(ii) Hence, or otherwise, find z. **2**

(d) Explain why there is no integer n such that $(n+1)^{41} - 79n^{40} = 2$. **2**

(e) The line ℓ passes through the points $A(3, 5, -4)$ and $B(7, 0, 2)$.

(i) Find a vector equation of the line ℓ. **1**

(ii) Determine, giving reasons, whether the point $C(10, 5, -2)$ lies on the line ℓ. **2**

Question 13 (16 marks) Use the Question 13 Writing Booklet

(a) The point A has position vector $8\underset{\sim}{i} - 6\underset{\sim}{j} + 5\underset{\sim}{k}$. The line ℓ has vector equation

$$x\underset{\sim}{i} + y\underset{\sim}{j} + z\underset{\sim}{k} = t\left(\underset{\sim}{i} + \underset{\sim}{j} + 2\underset{\sim}{k}\right).$$

The point B lies on ℓ and has position vector $p\underset{\sim}{i} + p\underset{\sim}{j} + 2p\underset{\sim}{k}$.

(i) Show that $\left|AB\right|^2 = 6p^2 - 24p + 125$. **1**

(ii) Hence, or otherwise, determine the shortest distance between the point A and the line ℓ. **2**

(b) A particle is moving in simple harmonic motion, described by $\ddot{x} = -4(x+1)$. **3**

When the particle passes through the origin, the speed of the particle is 4 m s^{-1}.

What distance does the particle travel during a full period of its motion?

(c) A particle of unit mass moves horizontally in a straight line. It experiences a resistive force proportional to v^2, where $v \text{ m s}^{-1}$ is the speed of the particle, so that the acceleration is given by $-kv^2$.

Initially the particle is at the origin and has a velocity of 40 m s^{-1} to the right. After the particle has moved 15 m to the right, its velocity is 10 m s^{-1} (to the right).

(i) Show that $v = 40e^{-kx}$. **3**

(ii) Show that $k = \dfrac{\ln 4}{15}$. **1**

(iii) At what time will the particle's velocity be 30 m s^{-1} to the right? **3**

(d) It is known that for all positive real numbers x, y **3**

$$x + y \geq 2\sqrt{xy}. \qquad \text{(Do NOT prove this.)}$$

Show that if a, b, c are positive real numbers with $\dfrac{1}{a} + \dfrac{1}{b} + \dfrac{1}{c} = 1$ then $a\sqrt{bc} + b\sqrt{ac} + c\sqrt{ab} \leq abc$.

Question 14 (15 marks) Use the Question 14 Writing Booklet

(a) Prove that if a is any odd integer, then $a^2 - 1$ is divisible by 8. **2**

(b) Use mathematical induction to prove that ${}^{2n}C_n < 2^{2n-2}$, for all integers $n \geq 5$. **3**

(c) For the complex numbers z and w, it is known that $\arg\left(\frac{z}{w}\right) = -\frac{\pi}{2}$. **2**

Find $\left|\frac{z-w}{z+w}\right|$.

(d) The following argument attempts to prove that $0 = 1$. **2**

We evaluate $\int \frac{1}{x}\,dx$ using the method of integration by parts.

$$\begin{aligned}\int \frac{1}{x}\,dx &= \int \frac{1}{x} \times 1\,dx \\ &= \frac{1}{x} \times x - \int -\frac{1}{x^2}x\,dx \\ &= 1 + \int \frac{1}{x}\,dx\end{aligned}$$

So we have

$$\int \frac{1}{x}\,dx = 1 + \int \frac{1}{x}\,dx$$

We may now subtract $\int \frac{1}{x}\,dx$ from both sides to show that $0 = 1$.

Explain what is wrong with this argument.

Question 14 continues on page 10

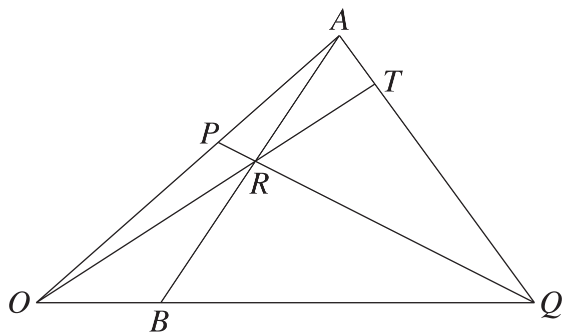
(e) The diagram shows triangle OQA.

The point P lies on OA so that $OP : OA = 3 : 5$.

The point B lies on OQ so that $OB : OQ = 1 : 3$.

The point R is the intersection of AB and PQ.

The point T is chosen on AQ so that O, R and T are collinear.



NOT TO SCALE

Let $\underset{\sim}{a} = \overrightarrow{OA}$, $\underset{\sim}{b} = \overrightarrow{OB}$ and $\overrightarrow{PR} = k\overrightarrow{PQ}$ where k is a real number.

(i) Show that $\overrightarrow{OR} = \dfrac{3}{5}(1-k)\underset{\sim}{a} + 3k\underset{\sim}{b}$. **2**

Writing $\overrightarrow{AR} = h\overrightarrow{AB}$, where h is a real number, it can be shown that $\overrightarrow{OR} = (1-h)\underset{\sim}{a} + h\underset{\sim}{b}$. (Do NOT prove this.)

(ii) Show that $k = \dfrac{1}{6}$. **2**

(iii) Find $\overrightarrow{OT}$ in terms of $\underset{\sim}{a}$ and $\underset{\sim}{b}$. **2**

End of Question 14

Question 15 (15 marks) Use the Question 15 Writing Booklet

(a) Consider the three vectors $\underset{\sim}{a} = \overrightarrow{OA}$, $\underset{\sim}{b} = \overrightarrow{OB}$ and $\underset{\sim}{c} = \overrightarrow{OC}$, where O is the origin and the points A, B and C are all different from each other and the origin.

The point M is the point such that $\frac{1}{2}\left(\underset{\sim}{a} + \underset{\sim}{b}\right) = \overrightarrow{OM}$.

(i) Show that M lies on the line passing through A and B. **1**

(ii) The point G is the point such that $\frac{1}{3}\left(\underset{\sim}{a} + \underset{\sim}{b} + \underset{\sim}{c}\right) = \overrightarrow{OG}$. **2**

Show that G lies on the line passing through M and C, and lies between M and C.

(iii) The complex numbers x, w and z are all different and all have modulus 1. **2**

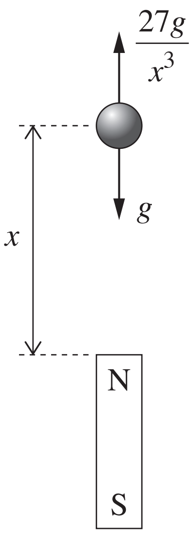
Using part (ii), or otherwise, show that $\frac{1}{3}(x + w + z)$ is never a cube root of xwz.

(b) Let $I_n = \int_0^a x^{n+\frac{1}{2}} (a - x)^{\frac{1}{2}}\, dx$, where $n \geq 0$. **3**

Show that $(2n + 4)I_n = a(2n + 1)I_{n-1}$, for $n > 0$.

Question 15 continues on page 12

(c) A bar magnet is held vertically. An object that is repelled by the magnet is to be dropped from directly above the magnet and will maintain a vertical trajectory. Let x be the distance of the object above the magnet.



The object is subject to acceleration due to gravity, g, and an acceleration due to the magnet $\frac{27g}{x^3}$, so that the total acceleration of the object is given by

$$a = \frac{27g}{x^3} - g.$$

The object is released from rest at $x = 6$.

(i) Show that $v^2 = g\left(\frac{51}{4} - 2x - \frac{27}{x^2}\right)$. **2**

(ii) Find where the object next comes to rest, giving your answer correct to 1 decimal place. **2**

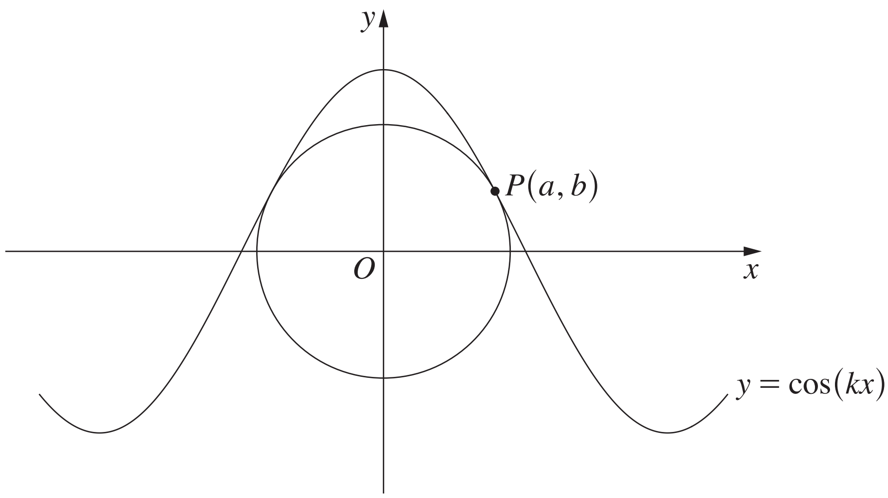
(d) Using a suitable substitution, find **3**

$$\int \frac{2x^2}{\sqrt{2x - x^2}}\,dx.$$

End of Question 15

Question 16 (14 marks) Use the Question 16 Writing Booklet

(a) Consider the function $y = \cos(kx)$, where $k > 0$. The value of k has been chosen so that a circle can be drawn, centred at the origin, which has exactly two points of intersection with the graph of the function and so that the circle is never above the graph of the function. The point $P(a, b)$ is the point of intersection in the first quadrant, so $a > 0$ and $b > 0$, as shown in the diagram. **4**



The vector joining the origin to the point $P(a, b)$ is perpendicular to the tangent to the graph of the function at that point. (Do NOT prove this.)

Show that $k > 1$.

(b) The number $w = e^{\frac{2\pi i}{3}}$ is a complex cube root of unity. The number γ is a cube root of w.

(i) Show that $\gamma + \bar{\gamma}$ is a real root of $z^3 - 3z + 1 = 0$. **3**

(ii) By using part (i) to find the exact value of $\cos\frac{2\pi}{9}\cos\frac{4\pi}{9}\cos\frac{8\pi}{9}$, deduce the value(s) of $\cos\frac{2^n\pi}{9}\cos\frac{2^{n+1}\pi}{9}\cos\frac{2^{n+2}\pi}{9}$ for all integers $n \geq 1$. Justify your answer. **3**

Question 16 continues on page 14

(c) Two particles, A and B, each have mass 1 kg and are in a medium that exerts a resistance to motion equal to kv, where $k > 0$ and v is the velocity of any particle. Both particles maintain vertical trajectories. **4**

The acceleration due to gravity is $g \text{ m s}^{-2}$, where $g > 0$.

The two particles are simultaneously projected towards each other with the same speed, $v_0 \text{ m s}^{-1}$, where $0 < v_0 < \dfrac{g}{k}$.

The particle A is initially d metres directly above particle B, where $d < \dfrac{2v_0}{k}$.

Find the time taken for the particles to meet.

End of paper

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2024 HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Advanced
Mathematics Extension 1
Mathematics Extension 2

REFERENCE SHEET

Measurement

Length

$$l = \frac{\theta}{360} \times 2\pi r$$

Area

$$A = \frac{\theta}{360} \times \pi r^2$$

$$A = \frac{h}{2}(a + b)$$

Surface area

$$A = 2\pi r^2 + 2\pi rh$$

$$A = 4\pi r^2$$

Volume

$$V = \frac{1}{3}Ah$$

$$V = \frac{4}{3}\pi r^3$$

Functions

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

For $ax^3 + bx^2 + cx + d = 0$:

$$\alpha + \beta + \gamma = -\frac{b}{a}$$

$$\alpha\beta + \alpha\gamma + \beta\gamma = \frac{c}{a}$$

and $\alpha\beta\gamma = -\frac{d}{a}$

Relations

$$(x - h)^2 + (y - k)^2 = r^2$$

Financial Mathematics

$$A = P(1 + r)^n$$

Sequences and series

$$T_n = a + (n - 1)d$$

$$S_n = \frac{n}{2}\left[2a + (n - 1)d\right] = \frac{n}{2}(a + l)$$

$$T_n = ar^{n-1}$$

$$S_n = \frac{a(1 - r^n)}{1 - r} = \frac{a(r^n - 1)}{r - 1}, r \neq 1$$

$$S = \frac{a}{1 - r}, |r| < 1$$

Logarithmic and Exponential Functions

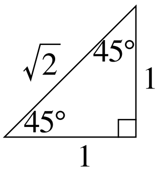
$$\log_a a^x = x = a^{\log_a x}$$

$$\log_a x = \frac{\log_b x}{\log_b a}$$

$$a^x = e^{x \ln a}$$

Trigonometric Functions

$$\sin A = \frac{\text{opp}}{\text{hyp}}, \quad \cos A = \frac{\text{adj}}{\text{hyp}}, \quad \tan A = \frac{\text{opp}}{\text{adj}}$$

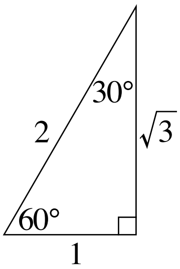


$$A = \frac{1}{2}ab\sin C$$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$c^2 = a^2 + b^2 - 2ab\cos C$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$



$$l = r\theta$$

$$A = \frac{1}{2}r^2\theta$$

Trigonometric identities

$$\sec A = \frac{1}{\cos A}, \ \cos A \neq 0$$

$$\operatorname{cosec} A = \frac{1}{\sin A}, \ \sin A \neq 0$$

$$\cot A = \frac{\cos A}{\sin A}, \ \sin A \neq 0$$

$$\cos^2 x + \sin^2 x = 1$$

Compound angles

$$\sin(A + B) = \sin A\cos B + \cos A\sin B$$

$$\cos(A + B) = \cos A\cos B - \sin A\sin B$$

$$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}$$

If $t = \tan\frac{A}{2}$ then $\sin A = \frac{2t}{1 + t^2}$

$$\cos A = \frac{1 - t^2}{1 + t^2}$$

$$\tan A = \frac{2t}{1 - t^2}$$

$$\cos A\cos B = \frac{1}{2}\left[\cos(A - B) + \cos(A + B)\right]$$

$$\sin A\sin B = \frac{1}{2}\left[\cos(A - B) - \cos(A + B)\right]$$

$$\sin A\cos B = \frac{1}{2}\left[\sin(A + B) + \sin(A - B)\right]$$

$$\cos A\sin B = \frac{1}{2}\left[\sin(A + B) - \sin(A - B)\right]$$

$$\sin^2 nx = \frac{1}{2}(1 - \cos 2nx)$$

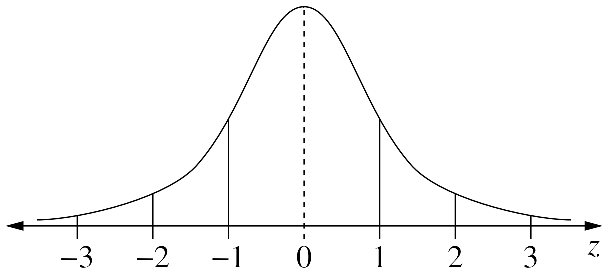
$$\cos^2 nx = \frac{1}{2}(1 + \cos 2nx)$$

Statistical Analysis

$$z = \frac{x - \mu}{\sigma}$$

An outlier is a score
less than $Q_1 - 1.5 \times IQR$
or
more than $Q_3 + 1.5 \times IQR$

Normal distribution



- approximately 68% of scores have z-scores between –1 and 1
- approximately 95% of scores have z-scores between –2 and 2
- approximately 99.7% of scores have z-scores between –3 and 3

$$E(X) = \mu$$

$$\text{Var}(X) = E\left[(X - \mu)^2\right] = E\left(X^2\right) - \mu^2$$

Probability

$$P(A \cap B) = P(A)P(B)$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}, P(B) \neq 0$$

Continuous random variables

$$P(X \leq r) = \int_a^r f(x)\,dx$$

$$P(a < X < b) = \int_a^b f(x)\,dx$$

Binomial distribution

$$P(X = r) = {}^nC_r p^r(1 - p)^{n-r}$$

$$X \sim \text{Bin}(n, p)$$

$$\Rightarrow \quad P(X = x) = \binom{n}{x}p^x(1 - p)^{n-x}, \ x = 0, 1, \ldots, n$$

$$E(X) = np$$

$$\text{Var}(X) = np(1 - p)$$

Differential Calculus

Function	Derivative
$y = f(x)^n$	$\dfrac{dy}{dx} = n f'(x)\left[f(x)\right]^{n-1}$
$y = uv$	$\dfrac{dy}{dx} = u\dfrac{dv}{dx} + v\dfrac{du}{dx}$
$y = g(u)$ where $u = f(x)$	$\dfrac{dy}{dx} = \dfrac{dy}{du} \times \dfrac{du}{dx}$
$y = \dfrac{u}{v}$	$\dfrac{dy}{dx} = \dfrac{v\dfrac{du}{dx} - u\dfrac{dv}{dx}}{v^2}$
$y = \sin f(x)$	$\dfrac{dy}{dx} = f'(x)\cos f(x)$
$y = \cos f(x)$	$\dfrac{dy}{dx} = -f'(x)\sin f(x)$
$y = \tan f(x)$	$\dfrac{dy}{dx} = f'(x)\sec^2 f(x)$
$y = e^{f(x)}$	$\dfrac{dy}{dx} = f'(x)e^{f(x)}$
$y = \ln f(x)$	$\dfrac{dy}{dx} = \dfrac{f'(x)}{f(x)}$
$y = a^{f(x)}$	$\dfrac{dy}{dx} = (\ln a) f'(x) a^{f(x)}$
$y = \log_a f(x)$	$\dfrac{dy}{dx} = \dfrac{f'(x)}{(\ln a) f(x)}$
$y = \sin^{-1} f(x)$	$\dfrac{dy}{dx} = \dfrac{f'(x)}{\sqrt{1 - \left[f(x)\right]^2}}$
$y = \cos^{-1} f(x)$	$\dfrac{dy}{dx} = -\dfrac{f'(x)}{\sqrt{1 - \left[f(x)\right]^2}}$
$y = \tan^{-1} f(x)$	$\dfrac{dy}{dx} = \dfrac{f'(x)}{1 + \left[f(x)\right]^2}$

Integral Calculus

$$\int f'(x)\left[f(x)\right]^n dx = \frac{1}{n+1}\left[f(x)\right]^{n+1} + c$$

where $n \neq -1$

$$\int f'(x)\sin f(x)\,dx = -\cos f(x) + c$$

$$\int f'(x)\cos f(x)\,dx = \sin f(x) + c$$

$$\int f'(x)\sec^2 f(x)\,dx = \tan f(x) + c$$

$$\int f'(x)e^{f(x)}dx = e^{f(x)} + c$$

$$\int \frac{f'(x)}{f(x)}dx = \ln\left|f(x)\right| + c$$

$$\int f'(x)a^{f(x)}dx = \frac{a^{f(x)}}{\ln a} + c$$

$$\int \frac{f'(x)}{\sqrt{a^2 - \left[f(x)\right]^2}}dx = \sin^{-1}\frac{f(x)}{a} + c$$

$$\int \frac{f'(x)}{a^2 + \left[f(x)\right]^2}dx = \frac{1}{a}\tan^{-1}\frac{f(x)}{a} + c$$

$$\int u\frac{dv}{dx}dx = uv - \int v\frac{du}{dx}dx$$

$$\int_a^b f(x)\,dx$$

$$\approx \frac{b-a}{2n}\left\{f(a) + f(b) + 2\left[f\left(x_1\right) + \cdots + f\left(x_{n-1}\right)\right]\right\}$$

where $a = x_0$ and $b = x_n$

Combinatorics

$$^{n}P_{r} = \frac{n!}{(n-r)!}$$

$$\binom{n}{r} = {}^{n}C_{r} = \frac{n!}{r!(n-r)!}$$

$$(x+a)^n = x^n + \binom{n}{1} x^{n-1} a + \cdots + \binom{n}{r} x^{n-r} a^r + \cdots + a^n$$

Vectors

$$\left| \underset{\sim}{u} \right| = \left| x\underset{\sim}{i} + y\underset{\sim}{j} \right| = \sqrt{x^2 + y^2}$$

$$\underset{\sim}{u} \cdot \underset{\sim}{v} = \left| \underset{\sim}{u} \right| \left| \underset{\sim}{v} \right| \cos\theta = x_1 x_2 + y_1 y_2,$$

where $\underset{\sim}{u} = x_1 \underset{\sim}{i} + y_1 \underset{\sim}{j}$

and $\underset{\sim}{v} = x_2 \underset{\sim}{i} + y_2 \underset{\sim}{j}$

$$\underset{\sim}{r} = \underset{\sim}{a} + \lambda \underset{\sim}{b}$$

Complex Numbers

$$\begin{aligned} z = a + ib &= r(\cos\theta + i\sin\theta) \\ &= re^{i\theta} \end{aligned}$$

$$\begin{aligned} \left[r(\cos\theta + i\sin\theta) \right]^n &= r^n (\cos n\theta + i\sin n\theta) \\ &= r^n e^{in\theta} \end{aligned}$$

Mechanics

$$\frac{d^2x}{dt^2} = \frac{dv}{dt} = v\frac{dv}{dx} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$$

$$x = a\cos(nt + \alpha) + c$$

$$x = a\sin(nt + \alpha) + c$$

$$\ddot{x} = -n^2(x - c)$$

2024 HSC Mathematics Extension 2 Marking Guidelines

Section I

Multiple-choice Answer Key

Question	Answer
1	D
2	C
3	A
4	B
5	D
6	A
7	C
8	A
9	B
10	D

Section II

Question 11 (a)

Criteria	Marks
• Provides correct primitive	2
• Attempts to use integration by parts	1

Sample answer:

$$\int xe^x\,dx = xe^x - \int e^x\,dx \qquad \begin{array}{ll} u = x & v' = e^x \\ u' = 1 & v = e^x \end{array}$$

$$= xe^x - e^x + C$$

Question 11 (b) (i)

Criteria	Marks
• Provides correct solution	1

Sample answer:

$$z = 2 + 3i, \quad w = 1 - 5i$$

$$\begin{aligned} z + \bar{w} &= 2 + 3i + 1 + 5i \\ &= 3 + 8i \end{aligned}$$

Question 11 (b) (ii)

Criteria	Marks
• Provides correct solution	1

Sample answer:

$$\begin{aligned} z^2 &= (2 + 3i)^2 \\ &= 4 + 12i + 9i^2 \\ &= -5 + 12i \end{aligned}$$

Question 11 (c)

Criteria	Marks
• Provides correct solution	2
• Finds $\underset{\sim}{u} \cdot \underset{\sim}{v}$, or equivalent merit	1

Sample answer:

$$\begin{aligned} \underset{\sim}{u} \cdot \underset{\sim}{v} &= \left|\underset{\sim}{u}\right|\left|\underset{\sim}{v}\right|\cos\theta \\ 4-8-14 &= \sqrt{1+4+4}\cdot\sqrt{16+16+49}\cos\theta \\ -18 &= 3\cdot 9\cos\theta \\ \cos\theta &= -\frac{2}{3} \\ \theta &= 2.3 \end{aligned}$$

Question 11 (d)

Criteria	Marks
• Provides correct solution	3
• Obtains correct integral, or equivalent merit	2
• Attempts to use the t substitution, or equivalent merit	1

Sample answer:

$$\begin{aligned} \int_0^{\frac{\pi}{2}} \frac{1}{\sin\theta+1}\,d\theta &= \int_0^1 \frac{\frac{2dt}{1+t^2}}{\frac{2t}{1+t^2}+\frac{1+t^2}{1+t^2}} \\ &= 2\int_0^1 \frac{1}{(t+1)^2}\,dt \\ &= 2\int_0^1 (t+1)^{-2}\,dt \\ &= 2\left[\frac{(t+1)^{-1}}{-1}\right]_0^1 \\ &= -2\left[\frac{1}{t+1}\right]_0^1 \\ &= -2\left(\frac{1}{2}\right)+2\left(\frac{1}{1}\right) \\ &= -1+2 \\ &= 1 \end{aligned}$$

$$t = \tan\frac{\theta}{2}$$

$$\theta = 0 \quad t = 0$$

$$\theta = \frac{\pi}{2} \quad t = \tan\frac{\pi}{4} = 1$$

Question 11 (e) (i)

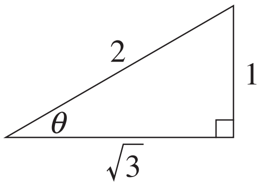
Criteria	Marks
• Provides correct solution	2
• Obtains correct modulus or argument, or equivalent merit	1

Sample answer:

$$\sqrt{3} + i = 2(\cos\theta + i\sin\theta)$$

$$= 2\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)$$

$$= 2e^{i\frac{\pi}{6}}$$



$$\tan\theta = \frac{1}{\sqrt{3}}$$

$$\theta = \frac{\pi}{6}$$

Question 11 (e) (ii)

Criteria	Marks
• Provides correct solution	2
• Obtains correct number in modulus-argument form, or equivalent merit	1

Sample answer:

$$\left(\sqrt{3} + i\right)^7 = \left(2e^{i\frac{\pi}{6}}\right)^7$$

$$= 128e^{i\frac{7\pi}{6}}$$

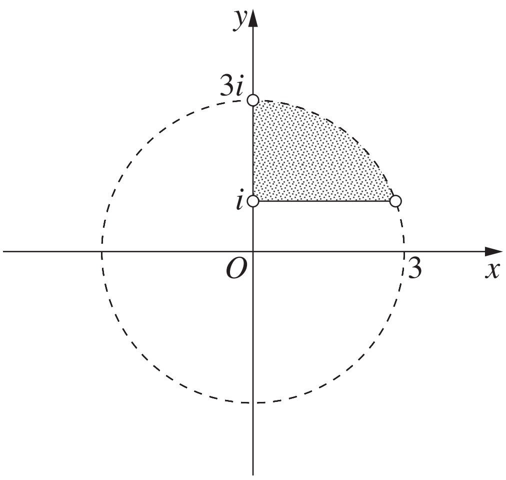
$$= \frac{128}{2}\left(-\sqrt{3} - i\right)$$

$$= -64\sqrt{3} - 64i$$

Question 11 (f)

Criteria	Marks
• Provides correct sketch	3
• Provides correct shading of the interior of the region, or equivalent merit	2
• Provides correct sketch of interior of either $\lvert z \rvert < 3$ or $0 \le \arg(z - i) \le \frac{\pi}{2}$, or equivalent merit	1

Sample answer:



Question 12 (a) (i)

Criteria	Marks
• Provides correct answer	1

Sample answer:

$$\frac{\underset{\sim}{a} \cdot \underset{\sim}{b}}{\underset{\sim}{b} \cdot \underset{\sim}{b}} \cdot \underset{\sim}{b} = -\frac{10}{20}\begin{pmatrix} 2 \\ 0 \\ -4 \end{pmatrix}$$

$$= -\frac{1}{2}\begin{pmatrix} 2 \\ 0 \\ -4 \end{pmatrix}$$

$$= \begin{pmatrix} -1 \\ 0 \\ 2 \end{pmatrix}$$

Question 12 (a) (ii)

Criteria	Marks
• Provides correct solution	2
• Finds $\underset{\sim}{a} - \frac{\underset{\sim}{a} \cdot \underset{\sim}{b}}{\underset{\sim}{b} \cdot \underset{\sim}{b}} \underset{\sim}{b}$, or equivalent merit	1

Sample answer:

$$\left(\underset{\sim}{a} - \begin{pmatrix} -1 \\ 0 \\ 2 \end{pmatrix}\right) \cdot \underset{\sim}{b} = \begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 0 \\ -4 \end{pmatrix} = 4 + 0 - 4 = 0$$

$\therefore$ Vectors are perpendicular.

Question 12 (b)

Criteria	Marks
• Provides correct primitive	3
• Obtains correct integrand, or equivalent merit	2
• Attempts partial fractions using $\dfrac{A}{x-1}+\dfrac{Bx+C}{x^2+1}$, or equivalent merit	1

Sample answer:

$$\int \frac{3x^2+2x+1}{(x-1)(x^2+1)}\,dx$$

$$\frac{3x^2+2x+1}{(x-1)(x^2+1)} = \frac{A}{x-1}+\frac{Bx+C}{x^2+1}$$

$$3x^2+2x+1 = A(x^2+1)+(Bx+C)(x-1)$$

$x=1 \qquad 2A=6$

$\therefore \quad A=3$

$x=0 \qquad A-C=1$

$\therefore \quad C=2$

$x=-1 \qquad 2A+(-B+C)(-2)=2$

$6+2B-4=2$

$2B=0$

$\therefore \quad B=0$

$$\therefore \quad \int \frac{3x^2+2x+1}{(x-1)(x^2+1)}\,dx = \int \frac{3}{x-1}+\int\frac{2}{x^2+1}\,dx$$

$$= 3\ln|x-1| + 2\tan^{-1}x + C$$

Question 12 (c) (i)

Criteria	Marks
• Provides correct explanation	1

Sample answer:

$|z| = z + 8 + 12i$

LHS real so $a + bi + 8 + 12i$ real

$\therefore \quad (b + 12)i = 0$

$b = -12$

Question 12 (c) (ii)

Criteria	Marks
• Provides correct solution	2
• Obtains quadratic equation for a, or equivalent merit	1

Sample answer:

$|a - 12i|^2 = (a + 8)^2$

$\cancel{a^2} + 144 = \cancel{a^2} + 16a + 64$

$80 = 16a$

$a = 5$

$z = 5 - 12i$

Question 12 (d)

Criteria	Marks
• Provides correct explanation	2
• Recognises that considering n odd/even is a correct approach OR • Proves the result for n odd or even, or equivalent merit	1

Sample answer:

$(n+1)^{41} - 79n^{40} = 2$

If n is odd, then $(n+1)$ is even.

$\therefore \quad (n+1)^{41}$ is even and $79n^{40}$ is odd.

But, even − odd $\neq$ even

If n is even, then $(n+1)$ is odd.

$\therefore \quad (n+1)^{41}$ is odd and $79n^{40}$ is even.

But, odd − even $\neq$ even

$\therefore \quad$ Original statement cannot be true.

Question 12 (e) (i)

Criteria	Marks
• Provides correct equation	1

Sample answer:

$A(3, 5, -4)$ and $B(7, 0, 2)$

$$\text{A direction vector } = \begin{pmatrix} 7 \\ 0 \\ 2 \end{pmatrix} - \begin{pmatrix} 3 \\ 5 \\ -4 \end{pmatrix}$$

$$= \begin{pmatrix} 4 \\ -5 \\ 6 \end{pmatrix}$$

$$\ell: \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3 \\ 5 \\ -4 \end{pmatrix} + \lambda \begin{pmatrix} 4 \\ -5 \\ 6 \end{pmatrix}$$

Question 12 (e) (ii)

Criteria	Marks
• Provides correct solution	2
• Obtains one correct λ value for one coordinate, or equivalent merit	1

Sample answer:

$$\begin{pmatrix} 10 \\ 5 \\ -2 \end{pmatrix} = \begin{pmatrix} 3 \\ 5 \\ -4 \end{pmatrix} + \lambda \begin{pmatrix} 4 \\ -5 \\ 6 \end{pmatrix}$$

$10 = 3 + 4\lambda \qquad \lambda = \dfrac{7}{4}$

$5 = 5 - 5\lambda \qquad \lambda = 0$

Inconsistent values for λ

$\therefore$ Point $(10, 5, -2)$ is not on ℓ.

Question 13 (a) (i)

Criteria	Marks
• Provides correct proof	1

Sample answer:

$$AB = \begin{pmatrix} p-8 \\ p+8 \\ 2p-5 \end{pmatrix}$$

$$\begin{aligned} |AB|^2 &= p^2 - 16p + 64 + p^2 + 12p + 36 + 4p^2 - 20p + 25 \\ &= 6p^2 - 24p + 125 \end{aligned}$$

Question 13 (a) (ii)

Criteria	Marks
• Provides correct solution	2
• Finds the correct value of p, or equivalent merit	1

Sample answer:

$|AB|^2$ is a quadratic equation.

Axis of symmetry $\quad x = \dfrac{-b}{2a}$

$$\begin{aligned} \therefore \quad p &= \frac{-(-24)}{2 \times 6} \\ &= 2 \end{aligned}$$

The parabola is concave up.

Shortest distance (minimum) when $p = 2$

$$\begin{aligned} \therefore \quad \text{dist}^2 &= 6(2)^2 - 24(2) + 125 \\ &= 24 - 48 + 125 \\ &= 101 \end{aligned}$$

$\therefore$ Shortest distance is $\sqrt{101}$. $\quad$ (distance will be positive)

Question 13 (b)

Criteria	Marks
• Provides correct solution	3
• Integrates to obtain v^2, or equivalent merit	2
• Find correct period or n, or equivalent merit	1

Sample answer:

$$\ddot{x} = -4\big(x - (-1)\big)$$

$\therefore\ n^2 = 4$

$\therefore\ \ n = 2 \quad \text{and} \quad c = -1$

$$\frac{d}{dx}\left(\frac{1}{2}v^2\right) = -4\,(x+1)$$

Integrate from $x = 0,\ r = 4$ to $x = a - 1,\ v = 0$

$$v^2 = 4\left(A^2 - (x+1)^2\right)$$

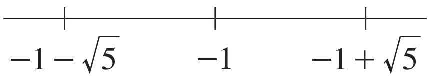
$x = 0, \qquad v = \pm 4$

$$16 = 4\left(A^2 - 1^2\right)$$

$$4 = A^2 - 1$$

$$A^2 = 5$$

$$A = \pm\sqrt{5}$$



Distance travelled

$= \left[\left(-1+\sqrt{5}\right) - \left(-1-\sqrt{5}\right)\right] \times 2$

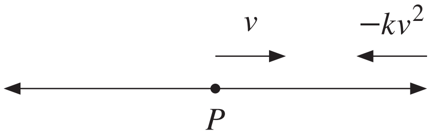
$= 2\sqrt{5} \times 2$

$= 4\sqrt{5}$ metres

Question 13 (c) (i)

Criteria	Marks
• Provides correct proof	3
• Integrates the equation of motion, or equivalent merit	2
• Obtains equation of motion, or equivalent merit	1

Sample answer:



$$m\ddot{x} = -kv^2$$

$$m = 1, \quad \ddot{x} = -kv^2$$

$$v\frac{dv}{dx} = -kv^2$$

$$\int \frac{dv}{v} = \int -k\,dx$$

$$\ln v = -kx + C \quad \text{since} \quad v = 0$$

When $t = 0,\ x = 0$ and $v = 40$

$$\therefore \quad \ln 40 = 0 + C \qquad C = \ln 40$$

$$\ln v - \ln 40 = -kx$$

$$\ln\left(\frac{v}{40}\right) = -kx$$

$$\therefore \quad v = 40e^{-kx}$$

Question 13 (c) (ii)

Criteria	Marks
• Provides correct solution	1

Sample answer:

$$x = 15, \quad v = 10$$

$$10 = 40e^{-k15}$$

$$\frac{1}{4} = e^{-15k}$$

$$-15k = -\ln 4$$

$$k = \frac{\ln 4}{15}$$

Question 13 (c) (iii)

Criteria	Marks
• Provides correct solution	3
• Integrates equation of motion with respect to time, or equivalent merit	2
• Considers equation of motion with respect to time, or equivalent merit	1

Sample answer:

$$\ddot{x} = -kv^2$$

$$\frac{dv}{dt} = -kv^2$$

$$\int \frac{dv}{v^2} = \int -k\,dt$$

$$\frac{v^{-1}}{-1} = -kt + C$$

$$-\frac{1}{v} = -kt + C$$

When $t = 0 \quad v = 40$

$$-\frac{1}{40} = C$$

$$\therefore \quad -\frac{1}{v} = -kt - \frac{1}{40}$$

When will $v = 30$?

$$kt = \frac{1}{30} - \frac{1}{40} = \frac{1}{120}$$

$$t = \frac{1}{120} \times \frac{15}{\ln 4}$$

$$= \frac{1}{8\ln 4}$$

$$(\doteqdot 0.09 \text{ seconds})$$

Question 13 (d)

Criteria	Marks
• Provides correct proof	3
• Correctly applies the AM-GM inequality to $\frac{1}{a}+\frac{1}{b}+\frac{1}{c}=1$, or equivalent merit	2
• Attempts to apply the AM-GM inequality to a relevant expression, or equivalent merit	1

Sample answer:

$$x+y \geq 2\sqrt{xy} \quad \Rightarrow \quad \sqrt{xy} \leq \frac{1}{2}(x+y) \qquad ①$$

$$\frac{1}{a}+\frac{1}{b}+\frac{1}{c}=1$$

$$\therefore \quad bc+ac+ab=abc$$

Now $a\sqrt{bc}+b\sqrt{ac}+c\sqrt{ab}$

$$\leq a\cdot\frac{1}{2}(b+c)+b\cdot\frac{1}{2}(a+c)+c\cdot\frac{1}{2}(a+b),$$

Using $AM-GM$ inequality from ①

$$\leq \frac{1}{2}[ab+ac+ab+bc+ac+bc]$$

$$\leq \frac{1}{\cancel{2}}[\cancel{2}(ab+ac+bc)]$$

$$\leq ab+ac+bc$$

$$\leq abc$$

Question 14 (a)

Criteria	Marks
• Provides correct proof	2
• Shows that $a^2 - 1$ is divisible by 4, or equivalent merit	1

Sample answer:

a is odd so let $a = 2p + 1$ where p is an integer

$$\begin{aligned} a^2 - 1 &= (2p+1)^2 - 1 \\ &= 4p^2 + 4p + 1 - 1 \\ &= 4p^2 + 4p \\ &= 4p(p+1) \end{aligned}$$

One of p and $p + 1$ will be even.

<u>Case 1</u> $p = 2q$ is even, where q is an integer

then $\quad a^2 - 1 = 4 \times 2q(p+1)$

$\qquad\qquad = 8q(p+1)$

<u>Case 2</u> $p + 1 = 2r$ is even, where r is an integer

then $\quad a^2 - 1 = 4p \times 2r$

$\qquad\qquad = 8p\mathrm{r}$

Both of which are divisible by 8.

$\therefore$ If a is any odd integer, then $a^2 - 1$ is divisible by 8.

Question 14 (b)

Criteria	Marks
• Provides correct proof	3
• Proves that $p(k) \Rightarrow p(k+1)$, or equivalent merit	2
• Showing true for $n = 5$	1

Sample answer:

Prove: $\quad {}^{2n}C_n < 2^{2n-2} \qquad n \geq 5$

Initial case: $\quad n = 5 \qquad {}^{10}C_5 = 252 \qquad 2^8 = 256$

$$252 < 256$$

$\therefore$ True for $n = 5$.

Inductive step: Suppose statement is true for $n = k$

Assume $\quad {}^{2k}C_k < 2^{2k-2} \quad$ for $\quad k \geq 5$

ie $\quad \dfrac{(2k)!}{k! \cdot k!} < 2^{2k-2}$

Prove that $\quad {}^{2k+2}C_{k+1} < 2^{2k}$

$$\begin{aligned}
\text{LHS} &= {}^{2k+2}C_{k+1} \\
&= \frac{(2k+2)!}{(k+1)!\,(k+1)!} \\
&= \frac{(2k+2)(2k+1)(2k)!}{(k+1)^2 \cdot (k!)^2} \\
&= \frac{2(k+1)(2k+1)(2k)!}{(k+1)^2 \cdot (k)!^2} \\
&= \frac{2(2k+1)(2k)!}{(k+1)(k!)^2} \\
&= \frac{2(2k+1)}{(k+1)} \cdot {}^{2k}C_k \\
&< \frac{2(2k+1)}{(k+1)} \cdot 2^{2k-2} \qquad \text{using assumption } n = k \\
&= \frac{2k+1}{(k+1)} \cdot 2^{2k-1} \\
&< \frac{2k+2}{k+1} \cdot 2^{2k-1} \\
&= 2 \cdot 2^{2k-1} \\
&= 2^{2k} \\
&= \text{RHS}
\end{aligned}$$

${}^{2k+2}C_{k+1} < 2^{2k} \qquad$ as required

Hence, the statement is true by mathematical induction for all integers $n \geq 5$.

Question 14 (c)

Criteria	Marks
• Provides correct solution	2
• Recognises that $\frac{z}{w}$ is purely imaginary, or equivalent merit	1

Sample answer:

$\arg\left(\frac{z}{w}\right) = -\frac{\pi}{2}$ z, w are complex numbers.

$\therefore\ \frac{z}{w}$ is purely imaginary.

$$\left|\frac{z-w}{z+w}\right| = \left|\frac{\frac{z}{w}-1}{\frac{z}{w}+1}\right|$$

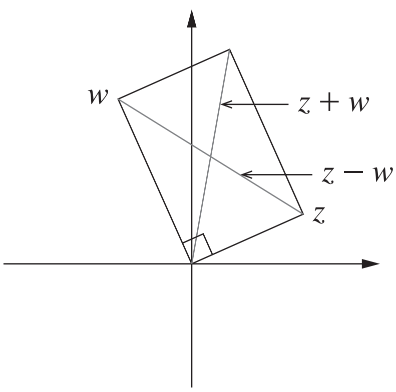
$\therefore$ Let $\frac{z}{w} = ki$

$$= \left|\frac{ki-1}{ki+1}\right|$$

$$= \frac{\sqrt{k^2+1}}{\sqrt{k^2+1}}$$

$$= 1$$

Answers could include:



$z-w$ and $z+w$ are the diagonals of a rectangle.

$\therefore\ |z-w| = |z+w|$

Hence $\left|\frac{z-w}{z+w}\right| = 1$

Question 14 (d)

Criteria	Marks
• Provides correct explanation	2
• Recognises that constants of integration are involved, or equivalent merit	1

Sample answer:

Using product rule

$$\frac{d(uv)}{dx} = u'v + uv'$$

$$u'v = \frac{d(uv)}{dx} - uv'$$

$$\int u'v\,dx = \int \frac{d(uv)}{dx}\,dx - \int uv'\,dx$$

But $\displaystyle\int \frac{d(uv)}{dx}\,dx = uv + C$ not just uv

So should have

$$\int \frac{1}{x}\,dx = 1 + C + \int \frac{1}{x}\,dx$$

$$0 = 1 + C \qquad \text{for some constant } C$$

Answers could include:

$$\int \frac{1}{x}\,dx = 1 + \int \frac{1}{x}\,dx$$

involves two indefinite integrals, and so two different constants

$$\ln|x| + C_1 = 1 + \ln|x| + C_2$$

$$C_1 = 1 + C_2 \qquad \text{for some constants } C_1 \text{ and } C_2$$

Question 14 (e) (i)

Criteria	Marks
• Provides correct solution	2
• Obtains $\overrightarrow{OR}$ in terms of $\underset{\sim}{a}$ and $\overrightarrow{PQ}$, or equivalent merit	1

Sample answer:

$\overrightarrow{OP} = \frac{3}{5}\underset{\sim}{a}$ $\qquad \overrightarrow{PR} = k\overrightarrow{PQ}$ $\qquad \overrightarrow{OQ} = 3\underset{\sim}{b}$

$$\begin{aligned}\overrightarrow{OR} &= \overrightarrow{OP} + \overrightarrow{PR}\\ &= \frac{3}{5}\underset{\sim}{a} + k\overrightarrow{PQ}\\ &= \frac{3}{5}\underset{\sim}{a} + k\left(\frac{-3}{5}\underset{\sim}{a} + 3\underset{\sim}{b}\right)\\ &= \frac{3}{5}(1-k)\underset{\sim}{a} + 3k\underset{\sim}{b}\end{aligned}$$

Question 14 (e) (ii)

Criteria	Marks
• Provides correct solution	2
• Equates coefficients for $\underset{\sim}{a}$ or $\underset{\sim}{b}$, or equivalent merit	1

Sample answer:

Given $\overrightarrow{OR} = (1-h)\underset{\sim}{a} + h\underset{\sim}{b}$

And, from part (i) $\overrightarrow{OR} = \frac{3}{5}(1-k)\underset{\sim}{a} + 3k\underset{\sim}{b}$

Equating coefficients

$$1-h = \frac{3}{5}(1-k) \text{ and } 3k = h$$

$$1-3k = \frac{3}{5}(1-k)$$

$$5-15k = 3-3k$$

$$2 = 12k$$

$$k = \frac{1}{6}$$

Question 14 (e) (iii)

Criteria	Marks
• Provides correct solution	2
• Obtains a correct expression for $\overrightarrow{OT}$ in terms of $\underset{\sim}{a}$ and $\underset{\sim}{b}$, or equivalent merit	1

Sample answer:

Using part (ii), $\quad h = 3k$

$$\therefore \quad h = 3\left(\frac{1}{6}\right) = \frac{1}{2}$$

$$\overrightarrow{OT} = \lambda \overrightarrow{OR} = \lambda\left(\frac{1}{2}\underset{\sim}{a} + \frac{1}{2}\underset{\sim}{b}\right)$$

$$= \frac{\lambda}{2}\underset{\sim}{a} + \frac{\lambda}{2}\underset{\sim}{b}$$

$$\overrightarrow{OT} = \overrightarrow{OA} + \mu\overrightarrow{AQ} = \underset{\sim}{a} + \mu\left(-\underset{\sim}{a} + 3\underset{\sim}{b}\right)$$

$$= (1-\mu)\underset{\sim}{a} + 3\mu\underset{\sim}{b}$$

Equating coefficients of $\underset{\sim}{b}$

$$\frac{\lambda}{2} = 3\mu$$

$$\therefore \quad \lambda = 6\mu$$

Equating coefficients of $\underset{\sim}{a}$

$$\frac{\lambda}{2} = 1 - \mu$$

$$\lambda = 2 - 2\mu$$

$$\therefore \quad 6\mu = 2 - 2\mu$$

$$8\mu = 2$$

$$\mu = \frac{1}{4} \quad \text{and} \quad \lambda = \frac{3}{2}$$

$$\therefore \quad \overrightarrow{OT} = \frac{3}{2}\overrightarrow{OR}$$

$$= \frac{3}{4}\underset{\sim}{a} + \frac{3}{4}\underset{\sim}{b}$$

Question 15 (a) (i)

Criteria	Marks
• Provides correct solution	1

Sample answer:

Line ℓ through A and B has equation $\overrightarrow{OX} = \overrightarrow{OA} + \lambda\overrightarrow{AB}$ for points X on the line.

$$\begin{aligned}\overrightarrow{OM} &= \frac{1}{2}\underset{\sim}{a} + \frac{1}{2}\underset{\sim}{b}\\ &= \left(\underset{\sim}{a} - \frac{1}{2}\underset{\sim}{a}\right) + \frac{1}{2}\underset{\sim}{b}\\ &= \overrightarrow{OA} + \frac{1}{2}\left(-\underset{\sim}{a} + \underset{\sim}{b}\right)\\ &= \overrightarrow{OA} + \frac{1}{2}\overrightarrow{AB}\end{aligned}$$

$\therefore$ M lies on ℓ.

Question 15 (a) (ii)

Criteria	Marks
• Provides correct solution	2
• Obtains $\overrightarrow{OG}$ in terms of $\overrightarrow{OC}$ and $\overrightarrow{CM}$, or equivalent merit	1

Sample answer:

Line k through M and C has equation

$$\overrightarrow{OX} = \overrightarrow{OC} + \mu\overrightarrow{CM}, \text{ for points } X \text{ on the line.}$$

$$\begin{aligned}\overrightarrow{OG} &= \frac{1}{3}\left(\underset{\sim}{a} + \underset{\sim}{b} + \underset{\sim}{c}\right)\\ &= \left(\underset{\sim}{c} - \frac{2}{3}\underset{\sim}{c}\right) + \frac{1}{3}\underset{\sim}{a} + \frac{1}{3}\underset{\sim}{b}\\ &= \underset{\sim}{c} + \frac{2}{3}\left(\frac{1}{2}\underset{\sim}{a} + \frac{1}{2}\underset{\sim}{b} - \underset{\sim}{c}\right)\\ &= \overrightarrow{OC} + \frac{2}{3}\overrightarrow{CM}\end{aligned}$$

$\therefore$ G lies on line k.

$\overrightarrow{OG} \neq \overrightarrow{OC}$ and $\overrightarrow{OG} \neq \overrightarrow{OM}$

So G lies between C and M.

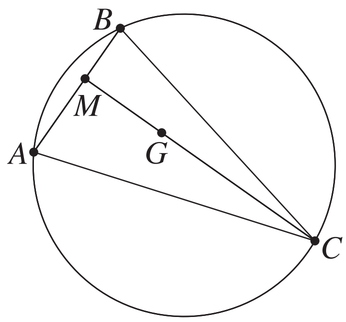
Question 15 (a) (iii)

Criteria	Marks
• Provides correct solution	2
• Shows that G lies within the unit circle, or equivalent merit	1

Sample answer:

The three points are on the unit circle.

$\therefore \quad |z| = 1$



The point equivalent to G, $\frac{1}{3}(x + w + z)$, lies on line CM and so is inside the unit circle.

And hence, $\left|\frac{1}{3}(x + w + z)\right| < 1$.

But, $|xwz| = 1$ and so all cube roots have modulus 1.

Therefore, $\frac{1}{3}(x + w + z)$ can't be a cube root of xwz.

Question 15 (b)

Criteria	Marks
• Provides correct solution	3
• Correctly uses integration by parts to obtain a single integral term, or equivalent merit	2
• Attempts to use integration by parts, or equivalent merit	1

Sample answer:

$$I_n = \int_0^a x^{n+\frac{1}{2}} (a-x)^{\frac{1}{2}}\, dx$$

Using integration by parts,

$$u = x^{n+\frac{1}{2}} \qquad v' = (a-x)^{\frac{1}{2}}$$

$$u' = \left(n+\frac{1}{2}\right)x^{n-\frac{1}{2}} \qquad v = \frac{(a-x)^{\frac{3}{2}}}{-\frac{3}{2}}$$

$$I_n = \left[x^{n+\frac{1}{2}}\left(-\frac{2}{3}\right)(a-x)^{\frac{3}{2}}\right]_0^a + \frac{2}{3}\left(n+\frac{1}{2}\right)\int_0^a x^{n-\frac{1}{2}} (a-x)^{\frac{3}{2}}\, dx$$

$$= \frac{2}{3}\left(n+\frac{1}{2}\right)\int_0^a x^{n-\frac{1}{2}} (a-x)^{\frac{3}{2}}\, dx$$

$$= \frac{1}{3}(2n+1)\int_0^a x^{n-\frac{1}{2}} (a-x)^1 (a-x)^{\frac{1}{2}}\, dx$$

$$= \frac{1}{3}(2n+1)\int_0^a ax^{n-\frac{1}{2}} (a-x)^{\frac{1}{2}} - x^{n+\frac{1}{2}} (a-x)^{\frac{1}{2}}\, dx$$

$$= \frac{1}{3}(2n+1)\left(aI_{n-1} - I_n\right)$$

$$I_n + \frac{1}{3}(2n+1)I_n = \frac{a}{3}(2n+1)I_{n-1}$$

$$\frac{(3+2n+1)}{3}I_n = \frac{a}{3}(2n+1)I_{n-1}$$

$$(2n+4)I_n = a(2n+1)I_{n-1}$$

Question 15 (c) (i)

Criteria	Marks
• Provides correct solution	2
• Correctly integrates the equation of motion, or equivalent merit	1

Sample answer:

$$a = \frac{27g}{x^3} - g \qquad t = 0 \quad v = 0 \quad x = 6$$

$$\frac{d}{dx}\left(\frac{1}{2}v^2\right) = \frac{27g}{x^3} - g$$

$$\frac{d\left(\frac{1}{2}v^2\right)}{dx} = 27gx^{-3} - g$$

$$\frac{1}{2}v^2 = \frac{27gx^{-2}}{-2} - gx + C$$

$$\frac{1}{2}v^2 = \frac{-27g}{2x^2} - gx + C$$

$$v^2 = \frac{-27g}{x^2} - 2gx + D \qquad (D = 2C)$$

$$x = 6 \quad v = 0$$

$$0 = \frac{-27g}{36} - 12g + D$$

$$D = \frac{3}{4}g + 12g$$

$$= \frac{51}{4}g$$

$$v^2 = \frac{-27g}{x^2} - 2gx + \frac{51}{4}g$$

$$= g\left(\frac{51}{4} - 2x - \frac{27}{x^2}\right)$$

Question 15 (c) (ii)

Criteria	Marks
• Provides correct solution	2
• Obtains a correct quadratic equation, or equivalent merit	1

Sample answer:

$$\frac{51}{4} - 2x - \frac{27}{x^2} = 0$$

$$51x^2 - 8x^3 - 108 = 0$$

$$8x^3 - 51x^2 + 108 = 0$$

Since $x = 6$ is a solution, $x - 6$ is a factor.

$$(x-6)\left(8x^2 + bx - 18\right) = 0$$

Equating terms in x^2,

$$b - 48 = -51$$

$$b = -3$$

$$(x-6)\left(8x^2 - 3x - 18\right) = 0$$

$$x = \frac{3 \pm \sqrt{9 - 4(8)(-18)}}{16}$$

$$x = \frac{3 \pm \sqrt{585}}{16}$$

$x \approx 1.699$ or -1.324

Since $x > 0$

$$x \approx 1.7$$

Question 15 (d)

Criteria	Marks
• Provides correct solution in terms of x	3
• Provides correct integral in terms of θ, or equivalent merit	2
• Completes the square OR • Attempts to substitute, or equivalent merit	1

Sample answer:

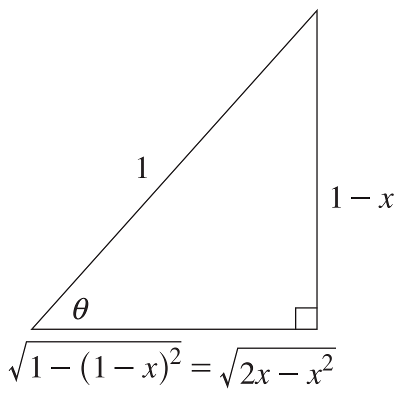
Let $1 - x = \sin\theta$

$$-dx = \cos\theta\, d\theta$$

$$dx = -\cos\theta\, d\theta$$

$$x = 1 - \sin\theta$$

$$\begin{aligned}
\int \frac{2x^2}{\sqrt{2x - x^2}}\,dx &= \int \frac{2x^2}{\sqrt{1 - \left(1 - 2x + x^2\right)}}\,dx \\
&= \int \frac{2x^2}{\sqrt{1 - (1 - x)^2}}\,dx \\
&= \int \frac{2(1 - \sin\theta)^2}{\sqrt{1 - \sin^2\theta}}\,(-\cos\theta)\,d\theta \\
&= -2\int \frac{1 - 2\sin\theta + \sin^2\theta}{\cos\theta}\,(\cos\theta)\,d\theta \\
&= -2\int 1 - 2\sin\theta + \frac{1}{2}(1 - \cos 2\theta)\,d\theta \\
&= \int -2 + 4\sin\theta - 1 + \cos 2\theta\,d\theta \\
&= -3\theta - 4\cos\theta + \frac{1}{2}\sin 2\theta + C \\
&= -3\sin^{-1}(1 - x) - 4\sqrt{2x - x^2} + \frac{1}{2}\,2\sin\theta\cos\theta + C \\
&= -3\sin^{-1}(1 - x) - 4\sqrt{2x - x^2} + (1 - x)\sqrt{2x - x^2} + C \\
&= -3\sin^{-1}(1 - x) - 3\sqrt{2x - x^2} - x\sqrt{2x - x^2} + C \\
&= -3\sin^{-1}(1 - x) - (x + 3)\sqrt{2x - x^2} + C
\end{aligned}$$



Question 15 (d) (continued)

Answers could include:

Using $\cos\theta = 1 - x$ as substitution

$$2x - x^2 = 1 - (1 - x)^2$$

Let $\cos\theta = 1 - x$

$$x = 1 - \cos\theta$$

$$\frac{dx}{d\theta} = \sin\theta$$

$$dx = \sin\theta\, d\theta$$

$$\int \frac{2x^2}{\sqrt{2x - x^2}}\,dx = 2\int \frac{x^2}{\sqrt{1 - (1 - x)^2}}\,dx$$

$$= 2\int \frac{(1 - \cos\theta)^2}{\sqrt{1 - \cos^2\theta}} \sin\theta\, d\theta$$

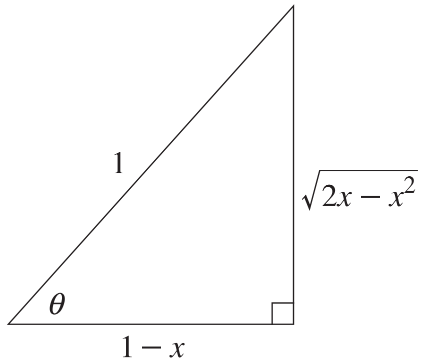
$$= 2\int \frac{1 - 2\cos\theta + \cos^2\theta}{\cancel{\sin\theta}} \cancel{\sin\theta}\, d\theta$$

$$= 2\int 1 - 2\cos\theta + \frac{1}{2} + \frac{\cos 2\theta}{2}\, d\theta$$

$$= \int 2 - 4\cos\theta + 1 + \cos 2\theta\, d\theta$$

$$= \int 3 - 4\cos\theta + \cos 2\theta\, d\theta$$

$$= 3\theta - 4\sin\theta + \frac{1}{2}\sin 2\theta + D$$



$$= 3\cos^{-1}(1 - x) - 4\sqrt{2x - x^2} + \sqrt{2x - x^2}\,(1 - x) + D$$

$$= 3\cos^{-1}(1 - x) - (x + 3)\sqrt{2x - x^2} + D$$

Question 16 (a)

Criteria	Marks
• Provides correct solution	4
• Attempts to use a relevant trigonometric fact, with $2a = k\sin 2ka$ to obtain result, or equivalent merit	3
• Obtains a correct equation involving $\sin 2ka$, or equivalent merit	2
• Provides a correct expression using the slope of OP, or equivalent merit	1

Sample answer:

At $P(a, b)$ the slope of the curve is $-k\sin ka$.

Since OP is perpendicular to the tangent at P

$$\frac{b}{a} \times (-k\sin ka) = -1$$

$$-\frac{a}{b} = -k\sin ka$$

But $\quad b = \cos ka$

$$a = k\sin ka \cos ka$$
$$2a = k\sin 2ka$$

Let $\quad X = 2a, \quad$ then we have

$$X = k\sin kX$$

If $\quad k \le 1, \quad$ then, as $\quad X > 0$

$$k\sin kX \le \sin kX < kX \le X$$

But $\quad X = k\sin kX$

So by contradiction $k > 1$

Question 16 (b) (i)

Criteria	Marks
• Provides correct solution	3
• Obtains $(\gamma + \overline{\gamma})^3 - 3(\gamma + \overline{\gamma}) + 1 = \gamma^3 + \overline{\gamma}^3 + 1$, or equivalent merit	2
• Substitutes $\gamma + \overline{\gamma}$ into $z^3 - 3z + 1 = 0$, or equivalent merit	1

Sample answer:

$w = e^{\frac{2\pi i}{3}}$ as $|\gamma| = |\overline{\gamma}| = 1, \quad \gamma = e^{i\theta}, \quad \overline{\gamma} = e^{-i\theta}, \quad \gamma\overline{\gamma} = 1$

$$
\begin{aligned}
(\gamma + \overline{\gamma})^3 - 3(\gamma + \overline{\gamma}) + 1 &= \gamma^3 + 3\gamma^2\overline{\gamma} + 3\gamma\overline{\gamma}^2 + \overline{\gamma}^3 - 3\gamma - 3\overline{\gamma} + 1 \\
&= \gamma^3 + \overline{\gamma}^3 + 3\gamma + 3\overline{\gamma} - 3\gamma - 3\overline{\gamma} + 1 && \text{as } \gamma\overline{\gamma} = 1 \\
&= \gamma^3 + \overline{\gamma}^3 + 1 && \gamma^3 = e^{\frac{2\pi i}{3}} \\
&= 2\cos\frac{2\pi}{3} + 1 && \overline{\gamma}^3 = e^{-\frac{2\pi i}{3}} \\
&= 2\left(-\frac{1}{2}\right) + 1 \\
&= 0
\end{aligned}
$$

$\therefore \quad \gamma + \overline{\gamma}$ is a real root of $z^3 - 3z + 1 = 0$.

Question 16 (b) (ii)

Criteria	Marks
• Provides correct solution	3
• Attempts to use inductive reasoning, or equivalent merit	2
• Shows $\cos\frac{2\pi}{9}\cos\frac{4\pi}{9}\cos\frac{8\pi}{9} = -\frac{1}{8}$, or equivalent merit	1

Sample answer:

The cube roots of $w = e^{\frac{2\pi i}{3}}$ are $e^{\frac{1}{3}\left(\frac{2\pi i}{3}\right)}$, $e^{\frac{1}{3}\left(\frac{2\pi}{3}+2\pi\right)i}$ and $e^{\frac{1}{3}\left(\frac{2\pi}{3}+4\pi\right)i}$ that is $e^{\frac{2\pi i}{9}}$, $e^{\frac{8\pi i}{9}}$ and $e^{\frac{14\pi i}{9}}$. By part (i) each of these plus its conjugate will be a root of $z^3 - 3z + 1 = 0$.

So $2\cos\frac{2\pi}{9}$, $2\cos\frac{8\pi}{9}$ and $2\cos\frac{14\pi}{9}$ are roots of $z^3 - 3z + 1 = 0$.

Noting that
$$\begin{aligned}\cos\frac{14\pi}{9} &= \cos\left(-\frac{14\pi}{9}\right)\\ &= \cos\left(2\pi - \frac{14\pi}{9}\right)\\ &= \cos\frac{4\pi}{9}\end{aligned}$$

The roots of $z^3 - 3z + 1 = 0$ are $2\cos\frac{2\pi}{9}$, $2\cos\frac{4\pi}{9}$ and $2\cos\frac{8\pi}{9}$.
(These are different as $\cos\theta$ is decreasing for $0 < \theta < \pi$.)
The product of the roots is -1.

Hence
$$\begin{aligned}8\cos\frac{2\pi}{9}\cos\frac{4\pi}{9}\cos\frac{8\pi}{9} &= -1\\ \cos\frac{2\pi}{9}\cos\frac{4\pi}{9}\cos\frac{8\pi}{9} &= -\frac{1}{8}\end{aligned}$$

This is the base case of this induction.

Noting that
$$\begin{aligned}\cos\frac{2^{k+3}\pi}{9} &= \cos\left(\frac{8}{9}\times 2^k\pi\right)\\ &= \cos\left(-\frac{8}{9}\times 2^k\pi\right), \text{ cos even}\\ &= \cos\left(2^k\pi - \frac{8}{9}2^k\pi\right), \text{ for } k \ge 1\\ &= \cos\frac{2^k\pi}{9}\end{aligned}$$

Hence $\cos\frac{2^{k+1}\pi}{9}\cos\frac{2^{k+2}\pi}{9}\cos\frac{2^{k+3}\pi}{9} = \cos\frac{2^k\pi}{9}\cos\frac{2^{k+1}\pi}{9}\cos\frac{2^{k+2}\pi}{9}$ (verifying the induction step).

So $\cos\frac{2^n\pi}{9}\cos\frac{2^{n+1}\pi}{9}\cos\frac{2^{n+2}\pi}{9} = -\frac{1}{8}$ for all integers $n \ge 1$.

Question 16 (c)

Criteria	Marks
• Provides correct solution	4
• Finds position of either particle or $\ell = x_A - x_B$ in terms of t, or equivalent merit	3
• Finds velocity of either particle or $\dfrac{d\ell}{dt}$, or equivalent merit	2
• Obtains correct equations of motion, or equivalent merit	1

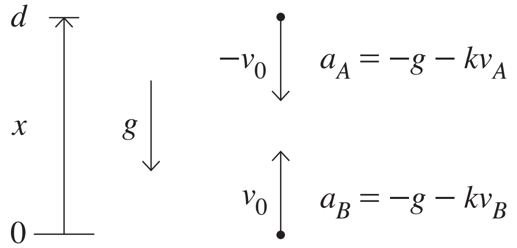
Sample answer:

Let

$$\ell = x_A - x_B$$

$$\begin{aligned}
\frac{d^2\ell}{dt^2} &= a_A - a_B \\
&= \cancel{-g} - kv_A \cancel{+g} + kv_B \\
&= -k(v_A - v_B) \\
&= -k\left(\frac{dx_A}{dt} - \frac{dx_B}{dt}\right) \\
&= -k\frac{d}{dt}(x_A - x_B) \\
&= -k\frac{d\ell}{dt}
\end{aligned}$$

$$\frac{d\ell}{dt} = Le^{-kt}, \text{ where } L \text{ is a constant}$$



$$\left[t = 0, \quad \frac{d\ell}{dt} = -2v_0\right]$$

$$\frac{d\ell}{dt} = -2v_0 e^{-kt}$$

$$\ell = \frac{2v_0}{k}e^{-kt} + C$$

$$[t = 0, \quad \ell = d]$$

$$d = \frac{2v_0}{k} + C$$

$$C = \frac{kd - 2v_0}{k}$$

$$\ell = \frac{2v_0}{k}e^{-kt} + \frac{kd - 2v_0}{k}$$

$$[\ell = 0]$$

$$2v_0 e^{-kt} + kd - 2v_0 = 0$$

$$\frac{2v_0 - kd}{2v_0} = e^{-kt}$$

$$\frac{2v_0}{2v_0 - kd} = e^{kt}$$

$$t = \frac{1}{k}\log\left(\frac{2v_0}{2v_0 - kd}\right)$$

Question 16 (c) (continued)

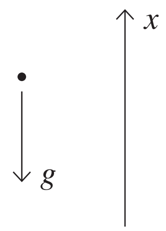
Answers could include:

The equation of motion for both particles is the same.

$$\frac{dv}{dt} = -(g + kv)$$

$$\int \frac{dv}{g + kv} = \int -dt$$

$$\frac{1}{k}\log\left| g + kv \right| = -t + C_1$$



$g + kV$ can never be 0, so can't change sign.

For A, v is initially $-v_0 > -\frac{g}{k}$

So $v > -\frac{g}{k}$

Hence $g + kv > 0$

For B, v is $v_0 > 0$

So $g + kv_0 > 0$

Hence $g + kv > 0$ for all t.

So, in both cases, $g + kv > 0$ for all t.

$$\frac{1}{k}\log(g + kv) = -t + C_1$$

$$g + kv = L_1 e^{-kt}$$

$$v = -\frac{g}{k} + L_1 e^{-kt}$$

Question 16 (c), answers could include (continued)

For Particle A,

When $t = 0,\ v_A = -v_0$

$$-v_0 = -\frac{g}{k} + \frac{L_1}{k}$$

$$\frac{L_1}{k} = \frac{g}{k} - v_0$$

$$v_A = -\frac{g}{k} + \left(\frac{g}{k} - v_0\right)e^{-kt}$$

$$x_A = -\frac{gt}{k} - \frac{1}{k}\left(\frac{g}{k} - v_0\right)e^{-kt} + L_2$$

When $t = 0,\ x_A = d$

$$d = -\frac{1}{k}\left(\frac{g}{k} - v_0\right) + L_2$$

$$L_2 = d + \frac{1}{k}\left(\frac{g}{k} - v_0\right)$$

$$x_A = -\frac{gt}{k} - \frac{1}{k}\left(\frac{g}{k} - v_0\right)e^{-kt} + d + \frac{1}{k}\left(\frac{g}{k} - v_0\right)$$

For Particle B,

When $t = 0,\ v_B = v_0$

$$v_0 = -\frac{g}{k} + \frac{L_3}{k}$$

$$v_B = -\frac{g}{k} + \left(\frac{g}{k} + v_0\right)e^{-kt}$$

$$x_B = -\frac{gt}{k} - \frac{1}{k}\left(\frac{g}{k} + v_0\right)e^{-kt} + L_4$$

When $t = 0,\ x_B = 0$

$$0 = -\frac{1}{k}\left(\frac{g}{k} + v_0\right) + L_4$$

$$x_B = -\frac{gt}{k} - \frac{1}{k}\left(\frac{g}{k} + v_0\right)e^{-kt} + \frac{1}{k}\left(\frac{g}{k} + v_0\right)$$

Particles meet when $x_A = x_B$

$$-\frac{gt}{k} - \frac{1}{k}\left(\frac{g}{k} - v_0\right)e^{-kt} + d + \frac{1}{k}\left(\frac{g}{k} - v_0\right) = -\frac{gt}{k} - \frac{1}{k}\left(\frac{g}{k} + v_0\right)e^{-kt} + \frac{1}{k}\left(\frac{g}{k} + v_0\right)$$

$$\frac{v_0}{k}e^{-kt} + d - \frac{v_0}{k} = -\frac{v_0}{k}e^{-kt} + \frac{v_0}{k}$$

$$\frac{2v_0}{k}e^{-kt} = \frac{2v_0}{k} - d$$

$$e^{-kt} = \frac{2v_0 - kd}{2v_0} > 0$$

$$t = \frac{1}{k}\log\left(\frac{2v_0}{2v_0 - kd}\right)$$

2024 HSC Mathematics Extension 2

Mapping Grid

Section I

Question	Marks	Content	Syllabus outcomes
1	1	MEX-V1 Further Work with Vectors	MEX12-3
2	1	MEX-P1 The Nature of Proof	MEX12-8
3	1	MEX-P1 The Nature of Proof	MEX12-2
4	1	MEX-N2 Using Complex Numbers	MEX12-4
5	1	MEX-M1 Applications of Calculus to Mechanics	MEX12-6
6	1	MEX-M1 Applications of Calculus to Mechanics	MEX12-6
7	1	MEX-N2 Using Complex Numbers	MEX12-4
8	1	MEX-N1 Introduction to Complex Numbers	MEX12-4
9	1	MEX-N2 Using Complex Numbers	MEX12-4
10	1	MEX-V1 Further Work with Vectors	MEX12-3

Section II

Question	Marks	Content	Syllabus outcomes
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11 (b) (i)	1	MEX-N1 Introduction to Complex Numbers	MEX12-4
11 (b) (ii)	1	MEX-N1 Introduction to Complex Numbers	MEX12-4
11 (c)	2	MEX-V1 Further Work with Vectors	MEX12-3
11 (d)	3	MEX-C1 Further Integration	MEX12-5
11 (e) (i)	2	MEX-N1 Introduction to Complex Numbers	MEX12-4
11 (e) (ii)	2	MEX-N1 Introduction to Complex Numbers	MEX12-4
11 (f)	3	MEX-N2 Using Complex Numbers	MEX12-4
12 (a) (i)	1	MEX-V1 Further Work with Vectors	MEX12-3
12 (a) (ii)	2	MEX-V1 Further Work with Vectors	MEX12-3
12 (b)	3	MEX-C1 Further Integration	MEX12-5
12 (c) (i)	1	MEX-N1 Introduction to Complex Numbers	MEX12-4
12 (c) (ii)	2	MEX-N1 Introduction to Complex Numbers	MEX12-4
12 (d)	2	MEX-P1 The Nature of Proof	MEX12-8
12 (e) (i)	1	MEX-V1 Further Work with Vectors	MEX12-3
12 (e) (ii)	2	MEX-V1 Further Work with Vectors	MEX12-3
13 (a) (i)	1	MEX-V1 Further Work with Vectors	MEX12-3
13 (a) (ii)	2	MEX-V1 Further Work with Vectors	MEX12-3
13 (b)	3	MEX-M1 Applications of Calculus to Mechanics	MEX12-6
13 (c) (i)	3	MEX-M1 Applications of Calculus to Mechanics	MEX12-6
13 (c) (ii)	1	MEX-M1 Applications of Calculus to Mechanics	MEX12-6
13 (c) (iii)	3	MEX-M1 Applications of Calculus to Mechanics	MEX12-6
13 (d)	3	MEX-P1 The Nature of Proof	MEX12-8

Question	Marks	Content	Syllabus outcomes
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14 (b)	3	MEX-P2 Further Proof by Mathematical Induction	MEX12-8
14 (c)	2	MEX-N2 Using Complex Numbers	MEX12-4
14 (d)	2	MEX-P1 The Nature of Proof	MEX12-2
14 (e) (i)	2	MEX-V1 Further Work with Vectors	MEX12-3
14 (e) (ii)	2	MEX-V1 Further Work with Vectors	MEX12-3
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15 (a) (i)	1	MEX-V1 Further Work with Vectors	MEX12-3
15 (a) (ii)	2	MEX-V1 Further Work with Vectors	MEX12-3
15 (a) (iii)	2	MEX-V1 Further Work with Vectors	MEX12-3
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