NSW GOVERNMENT

NSW Education Standards Authority

Centre Number

Student Number

2025 **HIGHER SCHOOL CERTIFICATE EXAMINATION**

Mathematics Extension 2

General Instructions

- Reading time – 10 minutes
- Working time – 3 hours
- Write using black pen
- Calculators approved by NESA may be used
- A reference sheet is provided at the back of this paper
- For questions in Section II, show relevant mathematical reasoning and/or calculations
- Write your Centre Number and Student Number at the top of this page and on the Question 11 Writing Booklet attached

Total marks: 100

Section I – 10 marks (pages 2–6)

- Attempt Questions 1–10
- Allow about 15 minutes for this section

Section II – 90 marks (pages 7–14)

- Attempt Questions 11–16
- Allow about 2 hours and 45 minutes for this section

Section I

10 marks
Attempt Questions 1–10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1–10.

---

1 Points A and B are $(-3, 1)$ and $(1, 4)$ respectively.

Which of the following is a vector equation of the line AB with parameter λ?

A. $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ 4 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ 4 \end{pmatrix}$

B. $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 3 \\ 4 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 4 \end{pmatrix}$

C. $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 4 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} -3 \\ 1 \end{pmatrix}$

D. $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -3 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 4 \\ 3 \end{pmatrix}$

2 Consider the statement:

$$\exists\, x \in \mathbb{Z}, \text{ such that } x^2 \text{ is odd.}$$

Which of the following is the negation of the statement?

A. $\forall\, x \in \mathbb{Z}, x^2$ is odd

B. $\forall\, x \in \mathbb{Z}, x^2$ is even

C. x^2 is even $\Rightarrow x \in \mathbb{Z}$

D. $\exists\, x \in \mathbb{Z}$, such that x^2 is even

3 What are the square roots of $3 - 4i$?

A. $1 - 2i$ and $-1 + 2i$

B. $1 + 2i$ and $-1 - 2i$

C. $2 - i$ and $-2 + i$

D. $-2 - i$ and $2 + i$

4 A particle in simple harmonic motion has speed $v \text{ m s}^{-1}$, given by $v^2 = -x^2 + 2x + 8$ where x is the displacement from the origin in metres.

What is the amplitude of the motion?

A. 1 m

B. 3 m

C. 6 m

D. 9 m

5 Consider the statement:

$$\text{If } x^2 - 2x \geq 0, \text{ then } x \leq 0.$$

Which of the following is the contrapositive of the statement?

A. If $x > 0$, then $x^2 - 2x < 0$.

B. If $x \leq 0$, then $x^2 - 2x \geq 0$.

C. If $x^2 - 2x < 0$, then $x < 0$.

D. If $x^2 - 2x \leq 0$, then $x > 0$.

6 The complex numbers z and w lie on the unit circle. The modulus of $z + w$ is $\frac{3}{2}$.

What is the modulus of $z - w$?

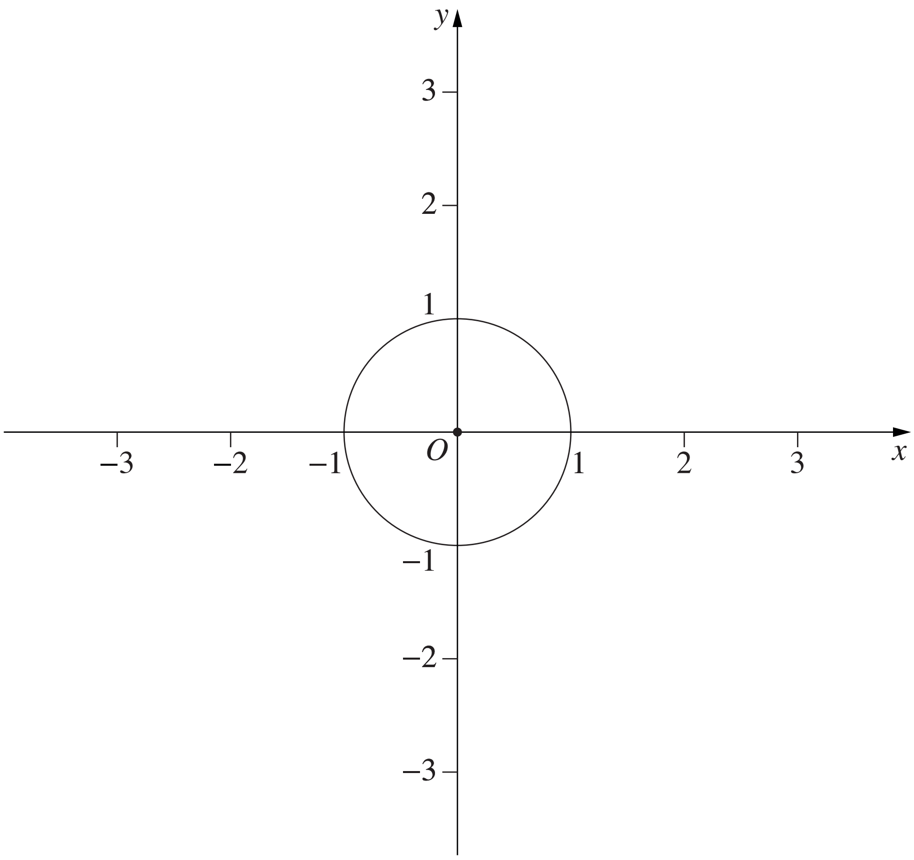
A. $\frac{1}{8}$

B. $\frac{\sqrt{7}}{2}$

C. $\frac{3}{2}$

D. $\frac{7}{4}$

7 The complex number z lies on the unit circle.



What is the range of $\text{Arg}(z - 2i)$?

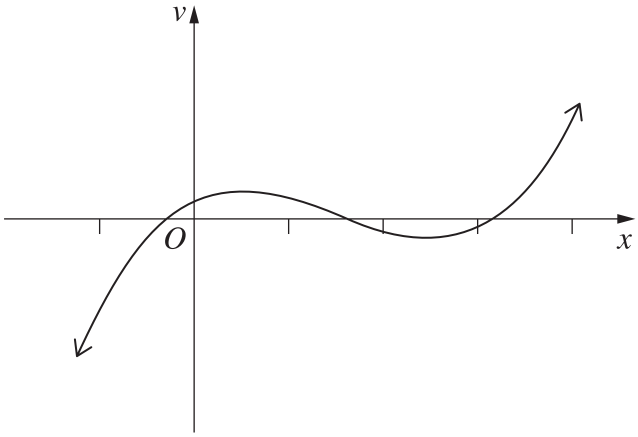
A. $\frac{\pi}{6} \leq \text{Arg}(z - 2i) \leq \frac{5\pi}{6}$

B. $\frac{\pi}{3} \leq \text{Arg}(z - 2i) \leq \frac{2\pi}{3}$

C. $-\frac{5\pi}{6} \leq \text{Arg}(z - 2i) \leq -\frac{\pi}{6}$

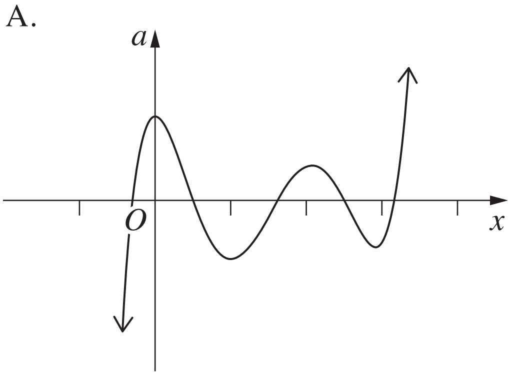
D. $-\frac{2\pi}{3} \leq \text{Arg}(z - 2i) \leq -\frac{\pi}{3}$

8 The graph shows the velocity of a particle as a function of its displacement.

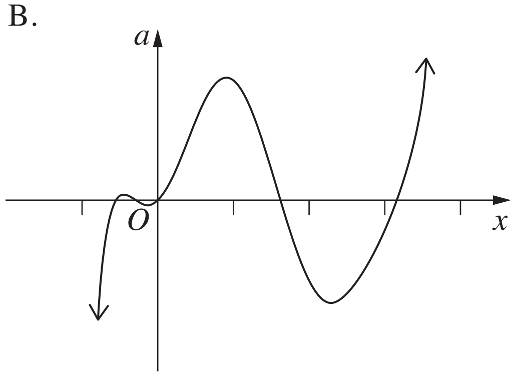


Which of the following graphs best shows the acceleration of the particle as a function of its displacement?

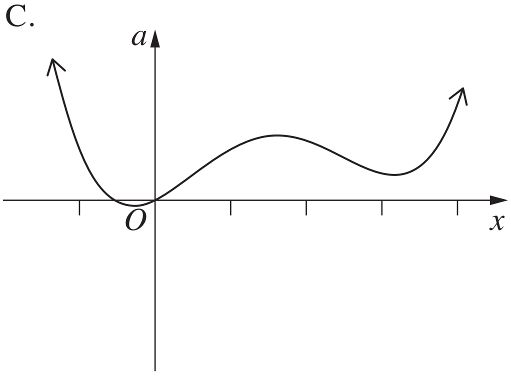
A.



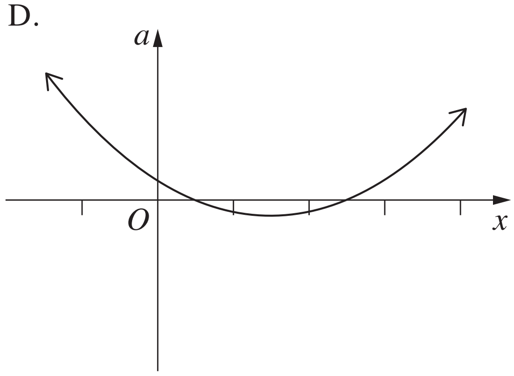
B.



C.



D.



9 The points U, V, W and Z represent the complex numbers u, v, w and z respectively. It is given that $v + z = u + w$ and $u + kiz = w + kiv$ where $k \in \mathbb{R}$, $k > 1$.

Which quadrilateral best describes $UVWZ$?

A. Parallelogram

B. Rectangle

C. Rhombus

D. Square

10 Which of the following gives the same curve as $\begin{pmatrix} \cos(t) \\ -t \\ \sin(t) \end{pmatrix}$ for $t \in \mathbb{R}$?

A. $\begin{pmatrix} \cos(2t) \\ 2t \\ \sin(2t) \end{pmatrix}$

B. $\begin{pmatrix} \cos\left(t^2 + \frac{\pi}{2}\right) \\ t^2 + \frac{\pi}{2} \\ \sin\left(t^2 + \frac{\pi}{2}\right) \end{pmatrix}$

C. $\begin{pmatrix} \cos\left(t^2\right) \\ -t^2 \\ \sin\left(t^2\right) \end{pmatrix}$

D. $\begin{pmatrix} \cos\left(2t + \frac{\pi}{2}\right) \\ 2t + \frac{\pi}{2} \\ -\sin\left(2t + \frac{\pi}{2}\right) \end{pmatrix}$

Section II

90 marks
Attempt Questions 11–16
Allow about 2 hours and 45 minutes for this section

Answer each question in the appropriate writing booklet. Extra writing booklets are available.

For questions in Section II, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (14 marks) Use the Question 11 Writing Booklet

(a) The location of the complex number z is shown on the diagram on page 1 of the Question 11 Writing Booklet. **2**

On the diagram provided in the writing booklet, indicate the locations of $\overline{z}$ and $i\overline{z}$.

(b) The complex numbers w and z are given by $w = 2e^{\frac{i\pi}{6}}$ and $z = 3e^{\frac{i\pi}{6}}$. **2**

Find the modulus and argument of wz.

(c) The complex number z is given by $x + iy$.

Find, in Cartesian form:

(i) z^2 **1**

(ii) $\dfrac{1}{z}$. **2**

Question 11 continues on page 8

(d) (i) Force $\underset{\sim}{F}_1$ has magnitude 12 newtons in the direction of vector $2\underset{\sim}{i} - 2\underset{\sim}{j} + \underset{\sim}{k}$. **1**

Show that $\underset{\sim}{F}_1 = 8\underset{\sim}{i} - 8\underset{\sim}{j} + 4\underset{\sim}{k}$.

(ii) Force $\underset{\sim}{F}_1$ from part (i) and a second force, $\underset{\sim}{F}_2 = -6\underset{\sim}{i} + 12\underset{\sim}{j} + 4\underset{\sim}{k}$, both act upon a particle. **1**

Show that the resultant force acting on the particle is given by:

$$\underset{\sim}{F}_3 = 2\underset{\sim}{i} + 4\underset{\sim}{j} + 8\underset{\sim}{k}.$$

(iii) Calculate $\underset{\sim}{F}_3 \cdot \underset{\sim}{d}$, where $\underset{\sim}{F}_3$ is the resultant force from part (ii) and $\underset{\sim}{d} = \underset{\sim}{i} + \underset{\sim}{j} + 2\underset{\sim}{k}$. **1**

(e) Prove by contradiction that $\sqrt{3} + \sqrt{5} > \sqrt{11}$. **2**

(f) Find $\displaystyle\int \frac{5}{\sqrt{7 - x^2 - 6x}}\,dx$. **2**

End of Question 11

Question 12 (16 marks) Use the Question 12 Writing Booklet

(a) Using integration by parts, evaluate $\displaystyle\int_0^{\frac{\pi}{2}} x \sin x \, dx$. **3**

(b) Given the function $y = xe^{2x}$, use mathematical induction to prove that $\dfrac{d^n y}{dx^n} = \left(2^n x + n2^{n-1}\right)e^{2x}$ for all positive integers n, where $\dfrac{d^n y}{dx^n}$ is the nth derivative of y and $\dfrac{d}{dx}\left(\dfrac{d^n y}{dx^n}\right) = \dfrac{d^{n+1} y}{dx^{n+1}}$. **3**

(c) Sketch the region of the complex plane defined by $|z + 5 - i| > |z - 3 + 3i|$. **3**

(d) Find $\displaystyle\int \frac{x^2 - 2x + 9}{(4 - x)\left(x^2 + 1\right)} dx$. **4**

(e) A particle of mass m kg moves along a horizontal line with an initial velocity of V_0 m s^{-1}. **3**

The motion of the particle is resisted by a constant force of mk newtons and a variable force of mv^2 newtons, where k is a positive constant and v m s^{-1} is the velocity of the particle at t seconds.

Show that the distance travelled when the particle is brought to rest is $\dfrac{1}{2}\ln\left(\dfrac{k + V_0^{\,2}}{k}\right)$ metres.

Question 13 (15 marks) Use the Question 13 Writing Booklet

(a) It is given that $A = \displaystyle\int_2^4 \frac{e^x}{x-1}\,dx$. **3**

Show that $\displaystyle\int_{m-4}^{m-2} \frac{e^{-x}}{x-m+1}\,dx = kA$, where k and m are constants.

(b) Let $\underset{\sim}{c} = x\underset{\sim}{i} + y\underset{\sim}{j} + z\underset{\sim}{k}$ be a unit vector that is perpendicular to both $\underset{\sim}{a} = 2\underset{\sim}{i} + 4\underset{\sim}{j} - 3\underset{\sim}{k}$ and $\underset{\sim}{b} = -4\underset{\sim}{i} - 5\underset{\sim}{j} + 3\underset{\sim}{k}$. **4**

Find all possible vectors $\underset{\sim}{c}$.

(c) (i) For positive real numbers a and b, prove that $\dfrac{a+b}{2} \geq \sqrt{ab}$. **2**

(ii) Hence, or otherwise, show that $\dfrac{2n+1}{2n+2} < \dfrac{\sqrt{2n+1}}{\sqrt{2n+3}}$ for any integer $n \geq 0$. **2**

(d) Evaluate $\displaystyle\int_0^{\frac{\pi}{2}} \frac{u}{1+\sin u+\cos u}\,du$, by first using the substitution $u = \dfrac{\pi}{2} - x$. **4**

Question 14 (15 marks) Use the Question 14 Writing Booklet

(a) Let $I_n = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cot^{2n}\theta \, d\theta$ for integers $n \geq 0$.

(i) Show that $I_n = \dfrac{1}{2n-1} - I_{n-1}$ for $n > 0$, given that $\dfrac{d}{d\theta}\cot\theta = -\mathrm{cosec}^2\,\theta$. **3**

(ii) Hence, or otherwise, calculate I_2. **1**

(b) The acceleration of a particle is given by $\ddot{x} = 32x\left(x^2 + 3\right)$, where x is the displacement of the particle from a fixed-point O after t seconds, in metres. Initially the particle is at O and has a velocity of $12\ \mathrm{m\,s^{-1}}$ in the negative direction.

(i) Show that the velocity of the particle is given by $v = -4\left(x^2 + 3\right)$. **2**

(ii) Find the time taken for the particle to travel 3 metres from the origin. **2**

(c) Let w be a complex number such that $1 + w + w^2 + \cdots + w^6 = 0$.

(i) Show that w is a 7th root of unity. **1**

The complex number $\alpha = w + w^2 + w^4$ is a root of the equation $x^2 + bx + c = 0$, where b and c are real and α is not real.

(ii) Find the other root of $x^2 + bx + c = 0$ in terms of positive powers of w. **2**

(iii) Find the numerical value of c. **1**

(d) Positive real numbers a, b, c and d are chosen such that $\dfrac{1}{a}$, $\dfrac{1}{b}$, $\dfrac{1}{c}$ and $\dfrac{1}{d}$ are consecutive terms in an arithmetic sequence with common difference k, where $k \in \mathbb{R}$, $k > 0$. **3**

Show that $b + c < a + d$.

Question 15 (15 marks) Use the Question 15 Writing Booklet

(a) The adjacent sides of a parallelogram are represented by the vectors $\underset{\sim}{a} = 4\underset{\sim}{i} + 3\underset{\sim}{j} - \underset{\sim}{k}$ and $\underset{\sim}{b} = 2\underset{\sim}{i} - \underset{\sim}{j} + 3\underset{\sim}{k}$. **4**

Show that the area of the parallelogram is $6\sqrt{10}$ square units.

(b) A particle moves in simple harmonic motion about the origin with amplitude A, and it completes two cycles per second. When it is $\frac{1}{4}$ metres from the origin, its speed is half its maximum speed. **4**

Find the maximum positive acceleration of the particle during its motion.

(c) (i) Show that **3**

$$\frac{1 + \cos\theta + i\sin\theta}{1 - \cos\theta - i\sin\theta} = i\cot\frac{\theta}{2}.$$

(ii) Use De Moivre's theorem to show that the sixth roots of -1 are given by $\cos\left(\frac{(2k+1)\pi}{6}\right) + i\sin\left(\frac{(2k+1)\pi}{6}\right)$ for $k = 0, 1, 2, 3, 4, 5$. **2**

(iii) Hence, or otherwise, show the solutions to $\left(\frac{z-1}{z+1}\right)^6 = -1$ are $z = i\cot\left(\frac{\pi}{12}\right)$, $i\cot\left(\frac{3\pi}{12}\right)$, $i\cot\left(\frac{5\pi}{12}\right)$, $i\cot\left(\frac{7\pi}{12}\right)$, $i\cot\left(\frac{9\pi}{12}\right)$, and $i\cot\left(\frac{11\pi}{12}\right)$. **2**

Question 16 (15 marks) Use the Question 16 Writing Booklet

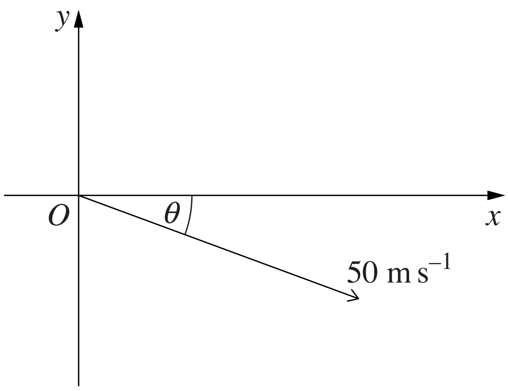
(a) Consider the equation **4**

$$z^n \cos[n\theta] + z^{n-1}\cos[(n-1)\theta] + z^{n-2}\cos[(n-2)\theta] + \cdots + z\cos[\theta] = 1$$

where $z \in \mathbb{C}$, $\theta \in \mathbb{R}$, and n is a positive integer.

Using a proof by contradiction and the triangle inequality, or otherwise, prove that all the solutions to the equation lie outside the circle $|z| = \frac{1}{2}$ on the complex plane.

(b) A particle of mass 1 kg is projected from the origin with a speed of 50 m s^{-1}, at an angle of θ below the horizontal into a resistive medium. **5**



The position of the particle t seconds after projection is (x, y), and the velocity of the particle at that time is $\underset{\sim}{v} = \begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix}$.

The resistive force, $\underset{\sim}{R}$, is proportional to the velocity of the particle, so that $\underset{\sim}{R} = -k\underset{\sim}{v}$, where k is a positive constant.

Taking the acceleration due to gravity to be 10 m s^{-2}, and the upwards vertical direction to be positive, the acceleration of the particle at time t is given by:

$$\underset{\sim}{a} = \begin{pmatrix} -k\dot{x} \\ -k\dot{y} - 10 \end{pmatrix}. \qquad \text{(Do NOT prove this.)}$$

Derive the Cartesian equation of the motion of the particle, given $\sin\theta = \frac{3}{5}$.

Question 16 continues on page 14

(c) Consider the point B with three-dimensional position vector $\underset{\sim}{b}$ and the line ℓ: $\underset{\sim}{a} + \lambda \underset{\sim}{d}$, where $\underset{\sim}{a}$ and $\underset{\sim}{d}$ are three-dimensional vectors, $|\underset{\sim}{d}| = 1$ and λ is a parameter.

Let $f(\lambda)$ be the distance between a point on the line ℓ and the point B.

(i) Find λ_0, the value of λ that minimises f, in terms of $\underset{\sim}{a}$, $\underset{\sim}{b}$ and $\underset{\sim}{d}$. **2**

(ii) Let P be the point with position vector $\underset{\sim}{a} + \lambda_0 \underset{\sim}{d}$. **1**

Show that PB is perpendicular to the direction of the line ℓ.

(iii) Hence, or otherwise, find the shortest distance between the line ℓ and the sphere of radius 1 unit, centred at the origin O, in terms of $\underset{\sim}{d}$ and $\underset{\sim}{a}$. **3**

You may assume that if B is the point on the sphere closest to ℓ, then OBP is a straight line.

End of paper

BLANK PAGE

BLANK PAGE

NSW GOVERNMENT

Centre Number

Student Number

NSW Education Standards Authority

2025 HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Extension 2

Writing Booklet

Question 11

Instructions

- Use this Writing Booklet to answer Question 11
- Write the number of this booklet and the total number of booklets that you have used for this question (eg: 1 of 3)
- Write your Centre Number and Student Number at the top of this page
- Write using black pen
- You may ask for an extra writing booklet if you need more space
- If you have not attempted the question(s), you must still hand in the writing booklet, with 'NOT ATTEMPTED' written clearly on the front cover
- You may NOT take any writing booklets, used or unused, from the examination room

11

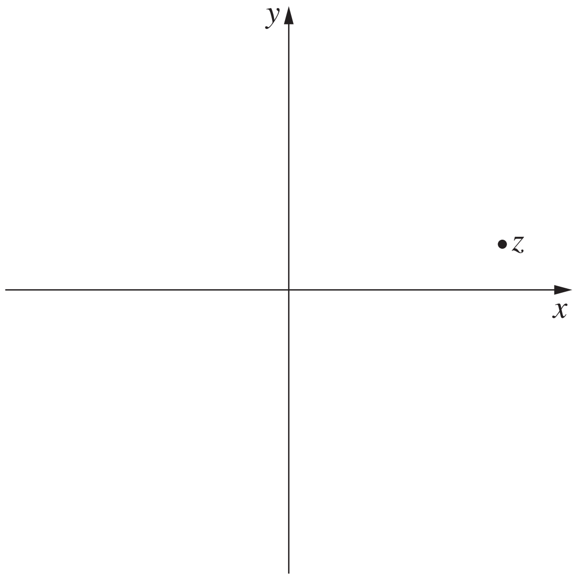
of

this booklet

number of booklets for this question

Start here for
Question Number: **11**

(a) The location of the complex number z is shown on the diagram. Indicate the locations of $\overline{z}$ and $i\overline{z}$.



8287310054

Additional writing space on back page.

Tick this box if you have continued this answer in another writing booklet.

NSW Education Standards Authority

2025 HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Advanced
Mathematics Extension 1
Mathematics Extension 2

REFERENCE SHEET

Measurement

Length

$$l = \frac{\theta}{360} \times 2\pi r$$

Area

$$A = \frac{\theta}{360} \times \pi r^2$$

$$A = \frac{h}{2}(a + b)$$

Surface area

$$A = 2\pi r^2 + 2\pi rh$$

$$A = 4\pi r^2$$

Volume

$$V = \frac{1}{3}Ah$$

$$V = \frac{4}{3}\pi r^3$$

Functions

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

For $ax^3 + bx^2 + cx + d = 0$:

$$\alpha + \beta + \gamma = -\frac{b}{a}$$

$$\alpha\beta + \alpha\gamma + \beta\gamma = \frac{c}{a}$$

and $\alpha\beta\gamma = -\frac{d}{a}$

Relations

$$(x - h)^2 + (y - k)^2 = r^2$$

Financial Mathematics

$$A = P(1 + r)^n$$

Sequences and series

$$T_n = a + (n - 1)d$$

$$S_n = \frac{n}{2}\left[2a + (n - 1)d\right] = \frac{n}{2}(a + l)$$

$$T_n = ar^{n-1}$$

$$S_n = \frac{a(1 - r^n)}{1 - r} = \frac{a(r^n - 1)}{r - 1}, r \neq 1$$

$$S = \frac{a}{1 - r}, |r| < 1$$

Logarithmic and Exponential Functions

$$\log_a a^x = x = a^{\log_a x}$$

$$\log_a x = \frac{\log_b x}{\log_b a}$$

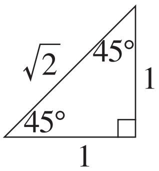
$$a^x = e^{x \ln a}$$

Trigonometric Functions

$$\sin A = \frac{\text{opp}}{\text{hyp}}, \quad \cos A = \frac{\text{adj}}{\text{hyp}}, \quad \tan A = \frac{\text{opp}}{\text{adj}}$$

$$A = \frac{1}{2}ab\sin C$$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

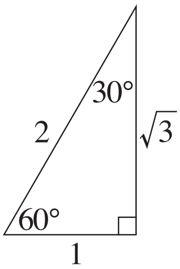


$$c^2 = a^2 + b^2 - 2ab\cos C$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

$$l = r\theta$$

$$A = \frac{1}{2}r^2\theta$$



Trigonometric identities

$$\sec A = \frac{1}{\cos A}, \ \cos A \neq 0$$

$$\operatorname{cosec} A = \frac{1}{\sin A}, \ \sin A \neq 0$$

$$\cot A = \frac{\cos A}{\sin A}, \ \sin A \neq 0$$

$$\cos^2 x + \sin^2 x = 1$$

Compound angles

$$\sin(A + B) = \sin A\cos B + \cos A\sin B$$

$$\cos(A + B) = \cos A\cos B - \sin A\sin B$$

$$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}$$

If $t = \tan\frac{A}{2}$ then $\sin A = \frac{2t}{1 + t^2}$

$$\cos A = \frac{1 - t^2}{1 + t^2}$$

$$\tan A = \frac{2t}{1 - t^2}$$

$$\cos A\cos B = \frac{1}{2}\left[\cos(A - B) + \cos(A + B)\right]$$

$$\sin A\sin B = \frac{1}{2}\left[\cos(A - B) - \cos(A + B)\right]$$

$$\sin A\cos B = \frac{1}{2}\left[\sin(A + B) + \sin(A - B)\right]$$

$$\cos A\sin B = \frac{1}{2}\left[\sin(A + B) - \sin(A - B)\right]$$

$$\sin^2 nx = \frac{1}{2}(1 - \cos 2nx)$$

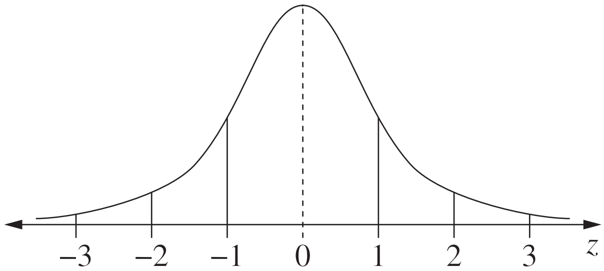
$$\cos^2 nx = \frac{1}{2}(1 + \cos 2nx)$$

Statistical Analysis

$$z = \frac{x - \mu}{\sigma}$$

An outlier is a score
less than $Q_1 - 1.5 \times IQR$
or
more than $Q_3 + 1.5 \times IQR$

Normal distribution



- approximately 68% of scores have z-scores between –1 and 1
- approximately 95% of scores have z-scores between –2 and 2
- approximately 99.7% of scores have z-scores between –3 and 3

$$E(X) = \mu$$

$$\operatorname{Var}(X) = E\left[(X - \mu)^2\right] = E\left(X^2\right) - \mu^2$$

Probability

$$P(A \cap B) = P(A)P(B)$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}, \ P(B) \neq 0$$

Continuous random variables

$$P(X \le r) = \int_a^r f(x)\,dx$$

$$P(a < X < b) = \int_a^b f(x)\,dx$$

Binomial distribution

$$P(X = r) = {}^nC_r p^r (1 - p)^{n-r}$$

$$X \sim \operatorname{Bin}(n, p)$$

$$\Rightarrow \quad P(X = x)$$

$$= \binom{n}{x} p^x (1 - p)^{n-x}, \ x = 0, 1, \ldots, n$$

$$E(X) = np$$

$$\operatorname{Var}(X) = np(1 - p)$$

Differential Calculus

Function	Derivative
$y = f(x)^n$	$\dfrac{dy}{dx} = n f'(x)\left[f(x)\right]^{n-1}$
$y = uv$	$\dfrac{dy}{dx} = u\dfrac{dv}{dx} + v\dfrac{du}{dx}$
$y = g(u)$ where $u = f(x)$	$\dfrac{dy}{dx} = \dfrac{dy}{du} \times \dfrac{du}{dx}$
$y = \dfrac{u}{v}$	$\dfrac{dy}{dx} = \dfrac{v\dfrac{du}{dx} - u\dfrac{dv}{dx}}{v^2}$
$y = \sin f(x)$	$\dfrac{dy}{dx} = f'(x)\cos f(x)$
$y = \cos f(x)$	$\dfrac{dy}{dx} = -f'(x)\sin f(x)$
$y = \tan f(x)$	$\dfrac{dy}{dx} = f'(x)\sec^2 f(x)$
$y = e^{f(x)}$	$\dfrac{dy}{dx} = f'(x)e^{f(x)}$
$y = \ln f(x)$	$\dfrac{dy}{dx} = \dfrac{f'(x)}{f(x)}$
$y = a^{f(x)}$	$\dfrac{dy}{dx} = (\ln a) f'(x) a^{f(x)}$
$y = \log_a f(x)$	$\dfrac{dy}{dx} = \dfrac{f'(x)}{(\ln a) f(x)}$
$y = \sin^{-1} f(x)$	$\dfrac{dy}{dx} = \dfrac{f'(x)}{\sqrt{1 - \left[f(x)\right]^2}}$
$y = \cos^{-1} f(x)$	$\dfrac{dy}{dx} = -\dfrac{f'(x)}{\sqrt{1 - \left[f(x)\right]^2}}$
$y = \tan^{-1} f(x)$	$\dfrac{dy}{dx} = \dfrac{f'(x)}{1 + \left[f(x)\right]^2}$

Integral Calculus

$$\int f'(x)\left[f(x)\right]^n dx = \frac{1}{n+1}\left[f(x)\right]^{n+1} + c$$

where $n \neq -1$

$$\int f'(x)\sin f(x)\,dx = -\cos f(x) + c$$

$$\int f'(x)\cos f(x)\,dx = \sin f(x) + c$$

$$\int f'(x)\sec^2 f(x)\,dx = \tan f(x) + c$$

$$\int f'(x)e^{f(x)}dx = e^{f(x)} + c$$

$$\int \frac{f'(x)}{f(x)}dx = \ln\left|f(x)\right| + c$$

$$\int f'(x)a^{f(x)}dx = \frac{a^{f(x)}}{\ln a} + c$$

$$\int \frac{f'(x)}{\sqrt{a^2 - \left[f(x)\right]^2}}dx = \sin^{-1}\frac{f(x)}{a} + c$$

$$\int \frac{f'(x)}{a^2 + \left[f(x)\right]^2}dx = \frac{1}{a}\tan^{-1}\frac{f(x)}{a} + c$$

$$\int u\frac{dv}{dx}dx = uv - \int v\frac{du}{dx}dx$$

$$\int_a^b f(x)\,dx$$

$$\approx \frac{b-a}{2n}\left\{f(a) + f(b) + 2\left[f\left(x_1\right) + \cdots + f\left(x_{n-1}\right)\right]\right\}$$

where $a = x_0$ and $b = x_n$

Combinatorics

$$^{n}P_{r} = \frac{n!}{(n-r)!}$$

$$\binom{n}{r} = {}^{n}C_{r} = \frac{n!}{r!(n-r)!}$$

$$(x+a)^{n} = x^{n} + \binom{n}{1}x^{n-1}a + \cdots + \binom{n}{r}x^{n-r}a^{r} + \cdots + a^{n}$$

Vectors

$$\left|\underset{\sim}{u}\right| = \left|x\underset{\sim}{i} + y\underset{\sim}{j}\right| = \sqrt{x^2 + y^2}$$

$$\underset{\sim}{u} \cdot \underset{\sim}{v} = \left|\underset{\sim}{u}\right|\left|\underset{\sim}{v}\right|\cos\theta = x_1x_2 + y_1y_2,$$

where $\underset{\sim}{u} = x_1\underset{\sim}{i} + y_1\underset{\sim}{j}$

and $\underset{\sim}{v} = x_2\underset{\sim}{i} + y_2\underset{\sim}{j}$

$$\underset{\sim}{r} = \underset{\sim}{a} + \lambda\underset{\sim}{b}$$

Complex Numbers

$$z = a + ib = r(\cos\theta + i\sin\theta)$$
$$= re^{i\theta}$$

$$\left[r(\cos\theta + i\sin\theta)\right]^{n} = r^{n}(\cos n\theta + i\sin n\theta)$$
$$= r^{n}e^{in\theta}$$

Mechanics

$$\frac{d^2x}{dt^2} = \frac{dv}{dt} = v\frac{dv}{dx} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$$

$$x = a\cos(nt + \alpha) + c$$

$$x = a\sin(nt + \alpha) + c$$

$$\ddot{x} = -n^2(x - c)$$

2025 HSC Mathematics Extension 2 Marking Guidelines

Section I

Multiple-choice Answer Key

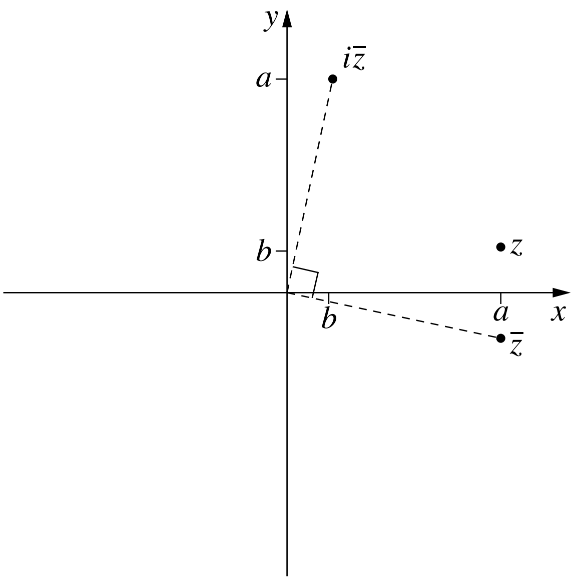
Question	Answer
1	D
2	B
3	C
4	B
5	A
6	B
7	D
8	A
9	C
10	D

Section II

Question 11 (a)

Criteria	Marks
• Indicates the locations of $\overline{z}$ and $i\overline{z}$	2
• Indicates the location of $\overline{z}$, or equivalent merit	1

Sample answer:



Let $z = a + ib$

$\overline{z} = a - ib$

$$\begin{aligned} i\overline{z} &= i(a - ib) \\ &= -i^2 b + ia \\ &= b + ia \end{aligned}$$

Question 11 (b)

Criteria	Marks
• Provides correct solution	2
• Provides the modulus or argument	1

Sample answer:

$w = 2e^{\frac{i\pi}{6}}$, $z = 3e^{\frac{i\pi}{6}}$

$\therefore \quad wz = (2 \times 3)e^{i\pi\left(\frac{1}{6}+\frac{1}{6}\right)}$

$wz = 6e^{\frac{i\pi}{3}}$

$\therefore \quad \text{Modulus} = 6$

$\text{Argument} = \dfrac{\pi}{3}$

Question 11 (c) (i)

Criteria	Marks
• Provides correct answer	1

Sample answer:

$$\begin{aligned} z^2 &= (x+iy)(x+iy) \\ &= x^2 + 2ixy - y^2 \\ &= \left(x^2 - y^2\right) + 2xyi \end{aligned}$$

Question 11 (c) (ii)

Criteria	Marks
• Provides correct solution	2
• Attempts to realise the denominator of $\frac{1}{x+iy}$	1

Sample answer:

$$\begin{aligned} \frac{1}{z} &= \frac{1}{x+iy} \\ &= \frac{x-iy}{(x+iy)(x-iy)} \\ &= \frac{x-iy}{x^2+y^2} \\ &= \frac{x}{x^2+y^2} - \frac{y}{x^2+y^2}\,i \end{aligned}$$

Question 11 (d) (i)

Criteria	Marks
• Provides correct solution	1

Sample answer:

$$\left|2\underset{\sim}{i} - 2\underset{\sim}{j} + \underset{\sim}{k}\right| = \sqrt{2^2 + (-2)^2 + 1^2} = 3$$

$$\left|\underset{\sim}{F}_1\right| = 12$$

$$\therefore \quad \underset{\sim}{F}_1 = 4\left(2\underset{\sim}{i} - 2\underset{\sim}{j} + \underset{\sim}{k}\right) = 8\underset{\sim}{i} - 8\underset{\sim}{j} + 4\underset{\sim}{k}$$

Question 11 (d) (ii)

Criteria	Marks
• Provides correct solution	1

Sample answer:

$$\begin{aligned}\underset{\sim}{F}_3 &= \underset{\sim}{F}_1 + \underset{\sim}{F}_2 \\ &= (8-6)\underset{\sim}{i} + (-8+12)\underset{\sim}{j} + (4+4)\underset{\sim}{k} \\ &= 2\underset{\sim}{i} + 4\underset{\sim}{j} + 8\underset{\sim}{k}\end{aligned}$$

Question 11 (d) (iii)

Criteria	Marks
• Provides correct answer	1

Sample answer:

$$\begin{aligned}&\underset{\sim}{F}_3 \cdot \underset{\sim}{d} \\ &= \left(2\underset{\sim}{i} + 4\underset{\sim}{j} + 8\underset{\sim}{k}\right)\cdot\left(\underset{\sim}{i} + \underset{\sim}{j} + 2\underset{\sim}{k}\right) \\ &= (2\times 1) + (4\times 1) + (8\times 2) \\ &= 22\end{aligned}$$

Question 11 (e)

Criteria	Marks
• Provides correct proof	2
• Assumes negation is true	1

Sample answer:

Assume

$$\sqrt{3} + \sqrt{5} \le \sqrt{11}$$

Squaring,

$$\left(\sqrt{3} + \sqrt{5}\right)^2 \le \left(\sqrt{11}\right)^2$$

$$8 + 2\sqrt{15} \le 11$$

$$2\sqrt{15} \le 3$$

Squaring,

$$60 \le 9$$

which is not true, contradicting the assumption.

$$\therefore \quad \sqrt{3} + \sqrt{5} > \sqrt{11}$$

Question 11 (f)

Criteria	Marks
• Provides correct solution	2
• Completes the square	1

Sample answer:

$$\begin{aligned}\int \frac{5}{\sqrt{7 - x^2 - 6x}}\,dx &= \int \frac{5}{\sqrt{-\left[x^2 + 6x - 7\right]}}\,dx \\ &= \int \frac{5}{\sqrt{-\left[(x+3)^2 - 16\right]}}\,dx \\ &= \int \frac{5}{\sqrt{16 - (x+3)^2}}\,dx \\ &= 5\sin^{-1}\left(\frac{x+3}{4}\right) + C\end{aligned}$$

Question 12 (a)

Criteria	Marks
• Provides correct solution	3
• Uses integration by parts	2
• Attempts to use integration by parts	1

Sample answer:

$$\int_0^{\frac{\pi}{2}} x \sin x \, dx$$

Using integration by parts

Let $u = x, \quad u' = 1$

Let $v' = \sin x, \quad v = -\cos x$

$$\therefore \int_0^{\frac{\pi}{2}} x \sin x \, dx = \left[-x \cos x\right]_0^{\frac{\pi}{2}} + \int_0^{\frac{\pi}{2}} \cos x \, dx$$

$$= \left(-\frac{\pi}{2} \cdot \cos\left(\frac{\pi}{2}\right) + 0 \cdot \cos(0)\right) + \left[\sin x\right]_0^{\frac{\pi}{2}}$$

$$= (0 + 0) + \left(\sin\left(\frac{\pi}{2}\right) - \sin(0)\right)$$

$$= 1$$

Question 12 (b)

Criteria	Marks
• Provides correct proof	3
• Establishes that $\dfrac{d^k y}{dx^k} = P_k(x) \Rightarrow \dfrac{d^{k+1}y}{dx^{k+1}} = P_{k+1}(x)$, or equivalent merit	2
• Establishes initial statement, or equivalent merit	1

Sample answer:

Let $P_n(x) = \left(2^n x + n2^{n-1}\right)e^{2x}$.

The first derivative is $\dfrac{dy}{dx} = 2xe^{2x} + e^{2x} = \left(2x + 1 \times 2^0\right)e^{2x} = P_1(x)$. So the statement is true for $n = 1$.

If the statement is true for $n = k$, that is $\dfrac{d^k y}{dx^k} = P_k(x)$,

then $\dfrac{d^{k+1}y}{dx^{k+1}} = \dfrac{d}{dx}\left(\dfrac{d^k y}{dx^k}\right) = \dfrac{d}{dx}\left(P_k(x)\right) = \dfrac{d}{dx}\left(\left(2^k x + k2^{k-1}\right)e^{2x}\right)$

$= 2\left(2^k x + k2^{k-1}\right)e^{2x} + 2^k e^{2x}$

$= \left(2^{(k+1)}x + k2^k + 2^k\right)e^{2x}$

$= \left(2^{(k+1)}x + (k+1)\,2^k\right)e^{2x} = P_{k+1}(x)$.

So the statement is also true for $n = k + 1$.

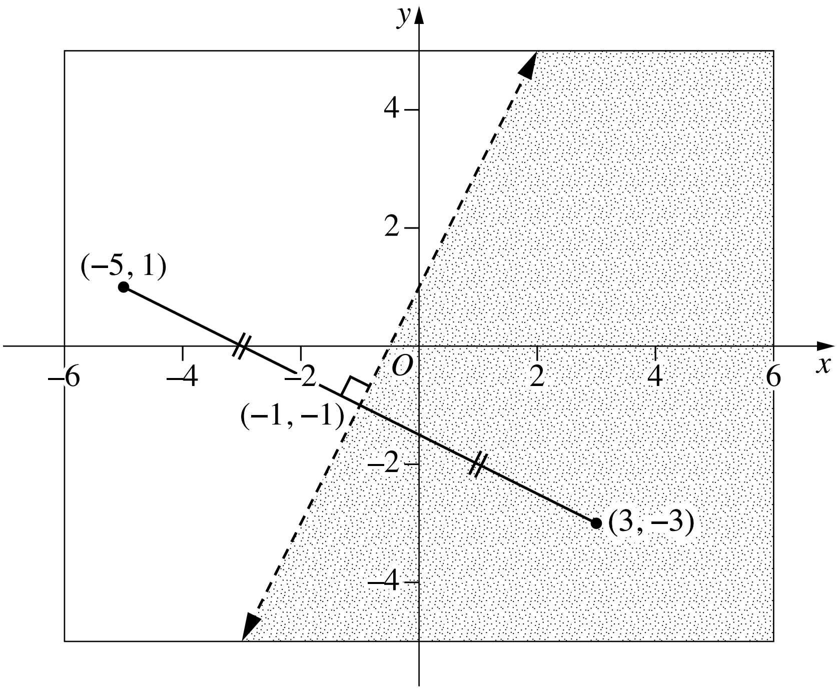
Hence the statement is true for all positive integers by mathematical induction.

Question 12 (c)

Criteria	Marks
• Provides correct sketch	3
• Sketches the perpendicular bisector, or equivalent merit	2
• Plots the point $(-5, 1)$ or $(3, -3)$, or equivalent merit	1

Sample answer:

The region is to the right of the perpendicular bisector of the line joining $(-5, 1)$ and $(3, -3)$, as shown.



Answers could include:

$$|z + 5 - i| > |z - 3 + 3i|$$

$$|(x + 5) + i(y - 1)| > |(x - 3) + i(y + 3)|$$

$$\sqrt{(x + 5)^2 + (y - 1)^2} > \sqrt{(x - 3)^2 + (y + 3)^2}$$

$$y < 2x + 1$$

Question 12 (d)

Criteria	Marks
• Provides correct solution	4
• Obtains one anti-derivative	3
• Shows $\dfrac{x^2-2x+9}{(4-x)(x^2+1)} = \dfrac{1}{4-x} + \dfrac{2}{x^2+1}$, or equivalent merit	2
• Attempts to use partial fractions, or equivalent merit	1

Sample answer:

$$\int \frac{x^2-2x+9}{(4-x)(x^2+1)}\,dx$$

For partial fractions, let

$$\frac{A}{4-x} + \frac{Bx+C}{x^2+1} = \frac{x^2-2x+9}{(4-x)(x^2+1)}$$

$$A(x^2+1) + (Bx+C)(4-x) = x^2-2x+9$$

Put $x = 4$

$$17A = 16 - 8 + 9$$

$$\boxed{A = 1}$$

Put $x = 0$

$$A + 4C = 9$$

$$4C = 8$$

$$\boxed{C = 2}$$

Compare coefficients of x^2

$$A - B = 1$$

$$\boxed{B = 0}$$

$$\int \frac{dx}{4-x} + \int \frac{2}{x^2+1}\,dx$$

$$= -\ln|4-x| + 2\tan^{-1}x + c$$

Question 12 (e)

Criteria	Marks
• Provides correct solution	3
• Integrates to find an expression in x	2
• Obtains force equation, or equivalent merit	1

Sample answer:

The total force on the particle is $-mk - mv^2$

$$m\ddot{x} = -mk - mv^2$$
$$\ddot{x} = -\left(k + v^2\right)$$
$$v\frac{dv}{dx} = -\left(k + v^2\right)$$
$$\int \frac{v\,dv}{k + v^2} = -\int dx$$
$$\frac{1}{2}\ln\left(k + v^2\right) = -x + C$$

When $x = 0,\ v = V_0$

$$\therefore\quad \frac{1}{2}\ln\left(k + {V_0}^2\right) = C$$

$$\therefore\quad \frac{1}{2}\ln\left(k + v^2\right) = -x + \frac{1}{2}\ln\left(k + {V_0}^2\right)$$
$$x = \frac{1}{2}\ln\left(\frac{k + {V_0}^2}{k + v^2}\right)$$

When $v = 0$

$$x = \frac{1}{2}\ln\left(\frac{k + {V_0}^2}{k}\right)$$

Question 12 (e) (continued)

Alternative solution

The total force on the particle is $-mk - mv^2$ newtons.

$$\therefore \quad m\ddot{x} = -mk - mv^2$$

$$\ddot{x} = -\left(k + v^2\right)$$

$$\frac{1}{2}\frac{d}{dx}\left(v^2\right) = -\left(k + v^2\right)$$

$$-\frac{1}{2}\int \frac{d\left(v^2\right)}{k + v^2} = \int 1\,dx$$

Since v^2 goes from ${V_0}^2$ to 0 while x goes from 0 to X:

$$-\frac{1}{2}\int_{{V_0}^2}^{0} \frac{d\left(v^2\right)}{k + v^2} = \int_0^X 1\,dx$$

$$-\frac{1}{2}\left[\ln\left(k + v^2\right)\right]_{{V_0}^2}^{0} = [x]_0^X \qquad \text{Using } v^2 \text{ as the variable of integration}$$

$$-\frac{1}{2}\left[\ln(k) - \ln\left(k + {V_0}^2\right)\right] = X - 0$$

$$\therefore \quad X = \frac{1}{2}\left[\ln\left(k + {V_0}^2\right) - \ln(k)\right]$$

$$= \frac{1}{2}\ln\left(\frac{k + {V_0}^2}{k}\right)$$

Question 13 (a)

Criteria	Marks
• Provides correct solution	3
• Obtains correct integral after substitution	2
• Attempts to integrate by substitution	1

Sample answer:

$$I = \int_{m-4}^{m-2} \frac{e^{-x}}{x - m + 1} dx$$

Let $u = m - x$

When $x = m - 2$ then $u = 2$,
when $x = m - 4$ then $u = 4$,
$\frac{du}{dx} = -1$.

So

$$I = \int_{4}^{2} \frac{e^{u-m}}{1-u}(-1)du$$

$$= -e^{-m}\int_{4}^{2} \frac{e^{u}du}{1-u}$$

$$= -e^{-m}\int_{2}^{4} \frac{e^{u}}{u-1}du$$

$$= -Ae^{-m}$$

$$= kA \text{ where } k = -e^{-m}.$$

Question 13 (b)

Criteria	Marks
• Provides correct solution	4
• Finds one value of $\underset{\sim}{c}$	3
• Obtains a relationship between x and y or z, or equivalent merit	2
• Evaluates $\underset{\sim}{a} \cdot \underset{\sim}{c} = 0$ or $\underset{\sim}{b} \cdot \underset{\sim}{c} = 0$, or equivalent merit	1

Sample answer:

Since $\underset{\sim}{c}$ is perpendicular to both $\underset{\sim}{a}$ and $\underset{\sim}{b}$,

$$\underset{\sim}{a} \cdot \underset{\sim}{c} = 0: \quad 2x + 4y - 3z = 0 \qquad (1)$$
$$\underset{\sim}{b} \cdot \underset{\sim}{c} = 0: \quad -4x - 5y + 3z = 0 \qquad (2)$$

$$(1) + (2): \quad -2x - y = 0 \Rightarrow y = -2x$$

Substitute this into equation (1).

$$2x - 8x - 3z = 0 \Rightarrow z = -2x$$

Since $\underset{\sim}{c}$ is a unit vector, $x^2 + y^2 + z^2 = 1 \qquad (3)$

Substituting into equation (3):

$$\begin{aligned} x^2 + (-2x)^2 + (-2x)^2 &= 1 \\ 9x^2 &= 1 \\ x^2 &= \frac{1}{9} \\ x &= \pm\frac{1}{3} \\ y = z = -2x &= \mp\frac{2}{3} \end{aligned}$$

$$\underset{\sim}{c} = \pm\left(\frac{1}{3}\underset{\sim}{i} - \frac{2}{3}\underset{\sim}{j} - \frac{2}{3}\underset{\sim}{k}\right)$$

Question 13 (c) (i)

Criteria	Marks
• Provides correct solution	2
• Begins proof with $(a-b)^2 \geq 0$, or equivalent merit	1

Sample answer:

$$(a-b)^2 \geq 0$$
$$a^2 - 2ab + b^2 \geq 0$$
$$a^2 + 2ab + b^2 \geq 4ab$$
$$(a+b)^2 \geq 4ab$$
$$a + b \geq 2\sqrt{ab} \qquad \begin{pmatrix}\text{ignore negative root} \\ \text{since } a, b \text{ positive}\end{pmatrix}$$
$$\frac{a+b}{2} \geq \sqrt{ab}$$

Question 13 (c) (ii)

Criteria	Marks
• Provides correct solution	2
• Attempts to use AM−GM inequality, or equivalent merit	1

Sample answer:

Apply AM−GM inequality

$$\sqrt{ab} \leq \frac{a+b}{2} \qquad \text{with } a = 2n+1,\ b = 2n+3.$$

Since $a \neq b$, this becomes $\sqrt{ab} < \dfrac{a+b}{2}$

$$\sqrt{(2n+1)(2n+3)} < \frac{(2n+1)+(2n+3)}{2}$$
$$\frac{\sqrt{(2n+1)(2n+3)}}{2n+1} < \frac{2n+2}{2n+1}$$
$$\frac{\sqrt{2n+3}}{\sqrt{2n+1}} < \frac{2n+2}{2n+1}$$
$$\therefore \qquad \frac{2n+1}{2n+2} < \frac{\sqrt{2n+1}}{\sqrt{2n+3}} \qquad \text{as required.}$$

Question 13 (d)

Criteria	Marks
• Provides correct solution	4
• Attempts to use t-substitution, or equivalent merit	3
• Obtains two times original integral, or equivalent merit	2
• Uses given substitution	1

Sample answer:

Let $u = \frac{\pi}{2} - x$. When $u = \frac{\pi}{2}$ then $x = 0$, when $u = 0$ then $x = \frac{\pi}{2}$ and $\frac{du}{dx} = (-1)$.
Hence,

$$\int_0^{\frac{\pi}{2}} \frac{u}{1 + \sin u + \cos u}\, du = \int_{\frac{\pi}{2}}^{0} \frac{\frac{\pi}{2} - x}{1 + \sin\left(\frac{\pi}{2} - x\right) + \cos\left(\frac{\pi}{2} - x\right)} (-1)\, dx$$

$$= \int_0^{\frac{\pi}{2}} \frac{\frac{\pi}{2} - x}{1 + \cos x + \sin x}\, dx$$

$$= \int_0^{\frac{\pi}{2}} \frac{\frac{\pi}{2}}{1 + \cos x + \sin x}\, dx - \int_0^{\frac{\pi}{2}} \frac{x}{1 + \cos x + \sin x}\, dx$$

$$= \frac{\pi}{2} \int_0^{\frac{\pi}{2}} \frac{dx}{1 + \cos x + \sin x} - \int_0^{\frac{\pi}{2}} \frac{u}{1 + \cos u + \sin u}\, du$$

$$\therefore \quad 2 \int_0^{\frac{\pi}{2}} \frac{u}{1 + \sin u + \cos u}\, du = \frac{\pi}{2} \int_0^{\frac{\pi}{2}} \frac{dx}{1 + \cos x + \sin x}$$

$$\therefore \quad \int_0^{\frac{\pi}{2}} \frac{u}{1 + \sin u + \cos u}\, du = \frac{\pi}{4} \int_0^{\frac{\pi}{2}} \frac{dx}{1 + \cos x + \sin x}$$

Question 13 (d) sample answer (continued)

Putting $t = \tan\frac{x}{2}$, $\cos(x) = \frac{1-t^2}{1+t^2}$, $\sin(x) = \frac{2t}{1+t^2}$, $\frac{dt}{dx} = \frac{1+t^2}{2}$.

When $x = \frac{\pi}{2}$, $t = 1$, and when $x = 0$, $t = 0$.

$$\int_0^{\frac{\pi}{2}} \frac{u}{1+\sin u+\cos u}\,du = \frac{\pi}{4}\int_0^1 \frac{\frac{2dt}{\left(1+t^2\right)}}{\frac{\left(1+t^2\right)+\left(1-t^2\right)+2t}{\left(1+t^2\right)}}$$

$$= \frac{\pi}{4}\int_0^1 \frac{2dt}{2+2t} = \frac{\pi}{4}\int_0^1 \frac{dt}{1+t} = \frac{\pi}{4}\Big[\ln|1+t|\Big]_0^1$$

$$= \frac{\pi}{4}\ln 2$$

Question 14 (a) (i)

Criteria	Marks
• Provides correct solution	3
• Replaces integral with I_{n-1}	2
• Uses the Pythagorean identity, or equivalent merit	1

Sample answer:

$$I_n = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cot^{2n}\theta \, d\theta$$

$$= \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cot^{2(n-1)}\theta \cot^2\theta \, d\theta$$

$$= \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cot^{2(n-1)}\theta \left(\operatorname{cosec}^2\theta - 1\right) d\theta$$

$$= \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cot^{2(n-1)}\theta \operatorname{cosec}^2\theta \, d\theta - \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cot^{2(n-1)}\theta \, d\theta$$

$$= \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cot^{2(n-1)}\theta \operatorname{cosec}^2\theta \, d\theta - I_{n-1}$$

$$= -\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \left(-\operatorname{cosec}^2\theta\right) \cot^{2(n-1)}\theta \, d\theta - I_{n-1}$$

$$= -\left[\frac{\cot^{2(n-1)+1}\theta}{2n-1}\right]_{\frac{\pi}{4}}^{\frac{\pi}{2}} - I_{n-1}$$

$$= -\left(\frac{0-1}{2n-1}\right) - I_{n-1}$$

$$= \frac{1}{2n-1} - I_{n-1}$$

Question 14 (a) (ii)

Criteria	Marks
• Provides correct answer	1

Sample answer:

Since $I_0 = \frac{\pi}{4}$, using the recurrence relation we have

$I_1 = 1 - I_0 = 1 - \frac{\pi}{4}$

$I_2 = \frac{1}{3} - I_1 = -\frac{2}{3} + \frac{\pi}{4}$.

Question 14 (b) (i)

Criteria	Marks
• Provides correct solution	2
• Integrates to get an expression for v^2 in terms of x	1

Sample answer:

Since $v\frac{dv}{dx} = 32x\left(x^2 + 3\right)$ we have $\int v\, dv = \int 32x\left(x^2 + 3\right) dx$.

Hence,

$$\frac{v^2}{2} = 8\left(x^2 + 3\right)^2 + C.$$

Since the initial velocity is -12, we have

$$\frac{144}{2} = 72 + C \text{ and so } C = 0.$$

Hence $v^2 = 16\left(x^2 + 3\right)^2$ and $v = \pm 4\left(x^2 + 3\right)$.

Since the initial velocity is negative, $v = -4\left(x^2 + 3\right)$.

Question 14 (b) (ii)

Criteria	Marks
• Provides correct solution	2
• Integrates to find a relation between time and displacement, or equivalent merit	1

Sample answer:

Given $\dfrac{dx}{dt} = -4\left(x^2 + 3\right)$ we have

$$\int \frac{dx}{x^2 + 3} = \int -4\,dt\ .$$

Hence $\dfrac{1}{\sqrt{3}} \tan^{-1}\left(\dfrac{x}{\sqrt{3}}\right) = -4t + C$

Using initial conditions, $C = 0$.

Since $x < 0$ at all times we let $x = -3$. Then

$$\frac{1}{\sqrt{3}} \tan^{-1}\left(\frac{-3}{\sqrt{3}}\right) = -4t$$

$$\frac{1}{\sqrt{3}} \cdot \left(-\frac{\pi}{3}\right) = -4t$$

and $t = \dfrac{\pi}{12\sqrt{3}}$ seconds.

Question 14 (c) (i)

Criteria	Marks
• Provides correct solution	1

Sample answer:

$$1 + w + w^2 + \cdots + w^6 = 0$$

$$\therefore \quad (1 - w)\left(1 + w + w^2 + \cdots + w^6\right) = 0$$

$$1 + w + w^2 + \cdots + w^6 - \left(w + w^2 + w^3 + \cdots + w^7\right) = 0$$

$$1 - w^7 = 0$$

$$\therefore \quad w^7 = 1$$

Question 14 (c) (ii)

Criteria	Marks
• Provides correct solution	2
• Recognises that the complex conjugate of $w + w^2 + w^4$ is the other root	1

Sample answer:

w is a 7th root of unity, $w = e^{\frac{2i\pi k}{7}}$ $(k \in \mathbb{Z})$

$$\begin{aligned}\therefore \quad \bar{w} &= e^{\frac{-2i\pi k}{7}} \\ &= \frac{1}{w} \\ &= \frac{w^7}{w} \\ &= w^6\end{aligned}$$

$$\begin{aligned}\therefore \quad \bar{\alpha} &= \overline{w + w^2 + w^4} \\ &= \bar{w} + \left(\bar{w}\right)^2 + \left(\bar{w}\right)^4 \\ &= w^6 + \left(w^6\right)^2 + \left(w^6\right)^4 \\ &= w^6 + w^{12} + w^{24} \\ &= w^6 + w^{7+5} + w^{(3\times 7)+3} \\ &= w^6 + w^5 + w^3\end{aligned}$$

Question 14 (c) (iii)

Criteria	Marks
• Provides correct solution	1

Sample answer:

$c = \alpha\bar{\alpha}$

$$\begin{aligned}\left(w + w^2 + w^4\right)\left(w^3 + w^5 + w^6\right) &= w^4 + w^6 + w^7 + w^5 + w^7 + w^8 + w^7 + w^9 + w^{10} \\ &= w^4 + w^6 + 1 + w^5 + 1 + w + 1 + w^2 + w^3 \\ &= 3 + (-1) \\ \therefore \quad c &= 2\end{aligned}$$

Question 14 (d)

Criteria	Marks
• Provides correct solution	3
• Writes $a + d$ and $b + c$ in terms of a and k, or equivalent merit	2
• Writes $b = \dfrac{a}{1 + ak}$, or equivalent merit	1

Sample answer:

Since $\dfrac{1}{b} = \dfrac{1}{a} + k = \dfrac{1 + ak}{a}$ then $b = \dfrac{a}{1 + ak}$. Similarly $c = \dfrac{a}{1 + 2ak}$, and $d = \dfrac{a}{1 + 3ak}$

$$\begin{aligned} a + d &= a + \frac{a}{1 + 3ak} \\ &= \frac{2a + 3a^2k}{1 + 3ak} \end{aligned}$$

$$\begin{aligned} b + c &= \frac{a}{1 + ak} + \frac{a}{1 + 2ak} \\ &= \frac{2a + 3a^2k}{(1 + ak)(1 + 2ak)} \\ &= \frac{2a + 3a^2k}{1 + 3ak + 2a^2k^2} \end{aligned}$$

Since $k \in \mathbb{R}$, $k > 0$

$$2a^2k^2 > 0$$

$$\therefore \quad 1 + 3ak + 2a^2k^2 > 1 + 3ak$$

$$\therefore \quad \frac{1}{1 + 3ak + 2a^2k^2} < \frac{1}{1 + 3ak}$$

Multiply by $2a + 3a^2k$

$$\frac{2a + 3a^2k}{1 + 3ak + 2a^2k^2} < \frac{2a + 3a^2k}{1 + 3ak}$$

$$\therefore \quad b + c < a + d$$

Alternate solution:

$$\frac{1}{a} < \frac{1}{b} < \frac{1}{c} < \frac{1}{d}$$

$$\therefore \quad a > b > c > d$$

$$\therefore \quad ab > cd$$

$$\frac{ab}{cd} > 1$$

Now, $\dfrac{1}{b} - \dfrac{1}{a} = k$

and $\dfrac{1}{d} - \dfrac{1}{c} = k$

$$\therefore \quad \frac{1}{b} - \frac{1}{a} = \frac{1}{d} - \frac{1}{c}$$

$$\frac{a - b}{ab} = \frac{c - d}{cd}$$

$$(a - b) = (c - d) \, . \, \frac{ab}{cd} > (c - d)$$

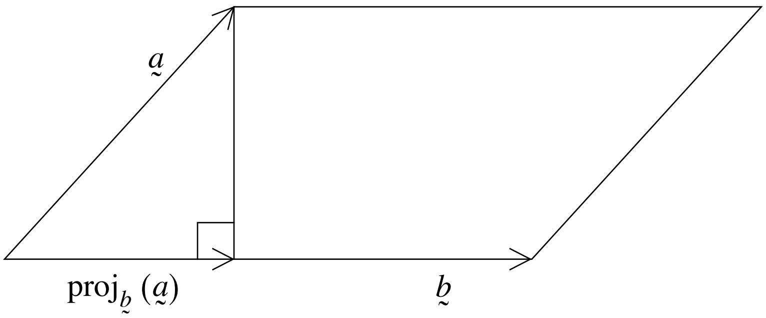
$$\therefore \quad a - b > c - d$$

$$\therefore \quad a + d > b + c$$

Question 15 (a)

Criteria	Marks
• Provides correct solution	4
• Calculates the perpendicular height, or equivalent merit	3
• Calculates a projection, or equivalent merit	2
• Finds $\lvert \underset{\sim}{a} \rvert$ or $\lvert \underset{\sim}{b} \rvert$, or equivalent merit	1

Sample answer:



Area of parallelogram is base × perpendicular height

$$\text{base} = \lvert \underset{\sim}{b} \rvert = \sqrt{2^2 + (-1)^2 + 3^2} = \sqrt{14}$$

$$\left\lvert \text{proj}_{\underset{\sim}{b}}\,(\underset{\sim}{a}) \right\rvert = \left\lvert \frac{\underset{\sim}{b} \cdot \underset{\sim}{a}}{\lvert \underset{\sim}{b} \rvert^2}\, \underset{\sim}{b} \right\rvert = \frac{\lvert 8 - 3 - 3 \rvert}{14} \cdot \sqrt{14} = \frac{1}{7} \cdot \sqrt{14}$$

Hence perpendicular height $= \sqrt{\lvert \underset{\sim}{a} \rvert^2 - \left\lvert \text{proj}_{\underset{\sim}{b}}\,(\underset{\sim}{a}) \right\rvert^2}$

$$= \sqrt{(16 + 9 + 1) - \frac{14}{49}}$$

$$= \sqrt{26 - \frac{2}{7}} = 3\sqrt{\frac{20}{7}}$$

$$\text{Area} = \sqrt{14} \times 3\sqrt{\frac{20}{7}} = 3 \times \sqrt{2} \times \sqrt{20} = 6\sqrt{10} \text{ square units.}$$

Question 15 (b)

Criteria	Marks
• Provides correct solution	4
• Finds $A = \dfrac{1}{2\sqrt{3}}$	3
• Finds maximum speed in terms of A, or equivalent merit	2
• Finds period is $\dfrac{1}{2}$, or equivalent merit	1

Sample answer:

Two cycles per second $\Rightarrow$ period $T = \dfrac{1}{2}$

$$T = \frac{2\pi}{n} \Rightarrow n = 4\pi$$

Let A be the amplitude

$$\ddot{x} = -n^2 x \quad \therefore \text{ maximum positive acceleration } = \left| -(4\pi)^2 \cdot A \right| = 16\pi^2 A$$

Let $x(t) = A\cos(4\pi t + \alpha)$

$\therefore \quad v(t) = -4\pi A \sin(4\pi t + \alpha)$

$\therefore \quad$ Maximum speed $= \left| -4\pi A \right| = 4\pi A$

Let $t = 0$ be the time when $x = \dfrac{1}{4}$ and $v = \dfrac{1}{2} \times 4\pi A = 2\pi A$

$\therefore \quad A\cos\alpha = \dfrac{1}{4}$

and $\quad \left| -4\pi A \sin\alpha \right| = 2\pi A$

$$4\pi A \sin\alpha = 2\pi A$$

$$\therefore \quad \sin\alpha = \frac{1}{2}$$

$$\therefore \quad \alpha = \frac{\pi}{6}$$

$\therefore \quad A\cos\dfrac{\pi}{6} = \dfrac{1}{4}$

$$A = \frac{1}{2\sqrt{3}}$$

$\therefore \quad$ Maximum positive acceleration $= 16\pi^2 \times \dfrac{1}{2\sqrt{3}}$

$$= \frac{8\pi^2}{\sqrt{3}} \text{ m s}^{-2}$$

Question 15 (b) sample answer (continued)

Alternative solution

The particle is in simple harmonic motion about the origin with period $\frac{1}{2}$ and amplitude A, so the displacement at time t is

$$x(t) = A\cos(4\pi t + \alpha).$$

The velocity is

$$\dot{x}(t) = -4\pi A\sin(4\pi t + \alpha). \quad \text{The maximum speed is } 4\pi A.$$

Let t_0 be the time when the particle is $\frac{1}{4}$ m from the origin

$$x(t_0) = A\cos(4\pi t_0 + \alpha) = \frac{1}{4}$$

and the speed is half its maximum

$$\dot{x}(t_0) = \frac{4\pi A}{2} = 2\pi A = -4\pi A\sin(4\pi t_0 + \alpha).$$

This means $\sin(4\pi t_0 + \alpha) = -\frac{1}{2}$ and so $\cos(4\pi t_0 + \alpha) = \pm\frac{\sqrt{3}}{2}$.

Hence $A\cos(4\pi t_0 + \alpha) = \pm\frac{A\sqrt{3}}{2} = \pm\frac{1}{4}$ and so $A = \frac{1}{2\sqrt{3}}$. (Since amplitude is not negative.)

The acceleration is $\ddot{x}(t) = -16\pi^2 A\cos(4\pi t + \alpha)$. The maximum positive acceleration is $16\pi^2 A = \frac{8\pi^2}{\sqrt{3}}$ m s^{-2}.

Question 15 (c) (i)

Criteria	Marks
• Provides correct solution	3
• Uses double angle formulas	2
• Realises denominator, or equivalent merit	1

Sample answer:

$$\text{LHS} = \frac{1+\cos\theta+i\sin\theta}{1-\cos\theta-i\sin\theta}\times\frac{1-\cos\theta+i\sin\theta}{1-\cos\theta+i\sin\theta}$$

$$= \frac{2i\sin\theta}{2-2\cos\theta}$$

$$= \frac{4i\sin\dfrac{\theta}{2}\cos\dfrac{\theta}{2}}{4\sin^2\dfrac{\theta}{2}}$$

$$= i\cot\frac{\theta}{2}$$

Question 15 (c) (ii)

Criteria	Marks
• Provides correct solution	2
• Uses De Moivre's theorem	1

Sample answer:

$w^6 = -1 \qquad \therefore\ |w| = 1$

Let $w = \cos\theta + i\sin\theta$

$$(\cos\theta + i\sin\theta)^6 = 1$$
$$\cos 6\theta + i\sin 6\theta = \cos\pi + i\sin\pi$$

Equating real parts,

$$6\theta = \pi + 2k\pi, \qquad k = 0, 1, 2, 3, 4, 5$$
$$\theta = \frac{\pi}{6} + \frac{2k\pi}{6}$$
$$= \frac{(2k+1)\pi}{6}$$

$$\therefore\ w = \cos\left(\frac{(2k+1)\pi}{6}\right) + i\sin\left(\frac{(2k+1)\pi}{6}\right), \qquad k = 0, 1, 2, 3, 4, 5$$

Question 15 (c) (iii)

Criteria	Marks
• Provides correct solution	2
• Finds an expression for z in terms of $\cos$ and $\sin$, or equivalent merit	1

Sample answer:

Let $\alpha_k = \cos\left(\frac{(2k+1)\pi}{6}\right) + i\sin\left(\frac{(2k+1)\pi}{6}\right)$ for $k = 0, 1, 2, 3, 4, 5$

From part (ii) $\quad \frac{z-1}{z+1} = \alpha_k$

$$\therefore \quad z - 1 = z\alpha_k + \alpha_k$$

$$z(1-\alpha_k) = 1 + \alpha_k$$

$$z = \frac{1+\alpha_k}{(1-\alpha_k)}$$

$$= \frac{1+\cos\left(\frac{(2k+1)\pi}{6}\right) + i\sin\left(\frac{(2k+1)\pi}{6}\right)}{1-\cos\left(\frac{(2k+1)\pi}{6}\right) - i\sin\left(\frac{(2k+1)\pi}{6}\right)}$$

$$= i\cot\left(\frac{(2k+1)\pi}{2\times 6}\right) \qquad \text{from part (i)}$$

$$= i\cot\left(\frac{(2k+1)\pi}{12}\right) \qquad \text{for } k = 0, 1, 2, 3, 4, 5$$

$$= i\cot\frac{\pi}{12},\ i\cot\frac{3\pi}{12},\ i\cot\frac{5\pi}{12},\ i\cot\frac{7\pi}{12},$$

$$i\cot\frac{9\pi}{12} \text{ and } i\cot\frac{11\pi}{12}$$

Question 16 (a)

Criteria	Marks
• Provides correct solution	4
• States and uses the assumption that at least one solution lies inside the circle	3
• Obtains a geometric series, or equivalent merit	2
• Uses triangle inequality, or equivalent merit	1

Sample answer:

Assume for contradiction that w is a solution to the equation and $|w| \le \frac{1}{2}$.

$$\therefore \quad w^n \cos n\theta + w^{n-1}\cos(n-1)\theta + w^{n-2}\cos(n-2)\theta + \cdots + w\cos\theta = 1$$

$$\therefore \left| w^n \cos n\theta + w^{n-1}\cos(n-1)\theta + w^{n-2}\cos(n-2)\theta + \cdots + w\cos\theta \right| = |1| = 1$$

$$\therefore 1 \le \left| w^n \cos n\theta \right| + \left| w^{n-1}\cos(n-1)\theta \right| + \left| w^{n-2}\cos(n-2)\theta \right| + \cdots + |w\cos\theta| \qquad \left(\begin{matrix}\text{by triangle}\\ \text{inequality}\end{matrix}\right)$$

$$= |w^n||\cos n\theta| + |w^{n-1}||\cos(n-1)\theta| + |w^{n-2}||\cos(n-2)\theta| + \cdots + |w||\cos\theta| \qquad \left(\begin{matrix}\text{property of}\\ \text{modulus}\end{matrix}\right)$$

$$\le |w^n| + |w^{n-1}| + |w^{n-2}| + \cdots + |w| \qquad (\text{since } |\cos\alpha| \le 1 \ \forall\ \alpha)$$

$$= |w|^n + |w|^{n-1} + |w|^{n-2} + \cdots + |w| \qquad (\text{property of modulus})$$

$$\le \left(\frac{1}{2}\right)^n + \left(\frac{1}{2}\right)^{n-1} + \left(\frac{1}{2}\right)^{n-2} + \cdots + \frac{1}{2} \qquad (\text{by assumption})$$

$$= \frac{1}{2}\left(\frac{1-\left(\frac{1}{2}\right)^n}{1-\frac{1}{2}} \right) \qquad (\text{sum of GP})$$

$$= 1 - \frac{1}{2^n}$$

$$< 1$$

This is a contradiction, so original assumption must be false, and all the solutions lie outside the circle $|z| = \frac{1}{2}$.

Question 16 (b)

Criteria	Marks
• Provides correct solution	5
• Obtains x and y, or equivalent merit	4
• Obtains $(x$ and $\dot{y})$ or $(y$ and $\dot{x})$, or equivalent merit	3
• Obtains x or y or both $\dot{x}$ and $\dot{y}$, or equivalent merit	2
• Obtains $\dot{x}$ or $\dot{y}$, or equivalent merit	1

Sample answer:

Horizontal motion:

$$\ddot{x} = -k\dot{x}$$

$$\frac{d(\dot{x})}{dt} = -k\dot{x}$$

$$\therefore \quad \dot{x} = Ae^{-kt}$$

When $t = 0,\ \dot{x} = 50\cos\theta = 50 \times \frac{4}{5} = 40 \quad \therefore\ A = 40$

$$\therefore \quad \dot{x} = 40e^{-kt}$$

Integrating,

$$\therefore \quad x = -\frac{40}{k}e^{-kt} + C$$

When $t = 0,\ x = 0 \quad \therefore\ C = \frac{40}{k}$

$$\therefore \quad x = -\frac{40}{k}e^{-kt} + \frac{40}{k}$$

$$x = \frac{40}{k}\left(1 - e^{-kt}\right)$$

$*$ Rearranging,

$$1 - \frac{kx}{40} = e^{-kt}$$

$$\therefore \quad e^{kt} = \frac{40}{40 - kx}$$

$$\therefore \quad t = \frac{1}{k}\ln\left(\frac{40}{40 - kx}\right)$$

Question 16 (b) sample answer (continued)

<u>Vertical motion</u>:

$$\ddot{y} = -k\dot{y} - 10$$

$$\frac{d(\dot{y})}{dt} = -k\dot{y} - 10$$

$$\int \frac{d(\dot{y})}{k\dot{y} + 10} = -\int dt$$

$$\therefore \quad t = -\frac{1}{k}\ln\left|(k\dot{y} + 10)\right| + C$$

$$Ae^{-kt} = k\dot{y} + 10$$

When $t = 0$, $\dot{y} = -50 \times \sin\theta$

$$= -50 \times \frac{3}{5}$$

$$= -30$$

$$\therefore \quad A = -30k + 10$$

$$\therefore \quad \dot{y} = \frac{1}{k}\left\{(10 - 30k)e^{-kt} - 10\right\}$$

$$= \frac{-10}{k}\left\{(3k - 1)e^{-kt} + 1\right\}$$

Integrating,

$$y = \frac{-10}{k}\left\{(3k - 1)\frac{e^{-kt}}{-k} + t\right\} + C$$

$$= \frac{10}{k}\left\{\frac{(3k - 1)e^{-kt}}{k} - t\right\} + C$$

When $t = 0$, $y = 0$

$$\therefore \quad C = \frac{-10}{k^2}(3k - 1)$$

$$\therefore \quad y = \frac{10}{k}\left\{\frac{(3k - 1)}{k}e^{-kt} - t - \frac{(3k - 1)}{k}\right\}$$

$$= \frac{10}{k}\left\{\frac{(3k - 1)}{k}\left[e^{-kt} - 1\right] - t\right\}$$

Question 16 (b) sample answer (continued)

Using $\circledast$, $\quad e^{-kt} - 1 = \dfrac{-kx}{40}$

and $\quad t = \dfrac{1}{k}\ln\left(\dfrac{40}{40 - kx}\right)$

$$\therefore \quad y = \frac{10}{k}\left\{\frac{(3k-1)}{k}\left(\frac{-kx}{40}\right) - \frac{1}{k}\ln\left(\frac{40}{40-kx}\right)\right\}$$

$$= \frac{10}{k^2}\left\{\frac{(1-3k)kx}{40} + \ln\left(\frac{40-kx}{40}\right)\right\}$$

Question 16 (c) (i)

Criteria	Marks
• Provides correct solution	2
• Finds an expression for distance, or equivalent merit	1

Sample answer:

The distance is minimised when the squared distance is minimised. The squared distance is

$$
\begin{aligned}
g(\lambda) &= \left| \underset{\sim}{b} - \left(\underset{\sim}{a} + \lambda \underset{\sim}{d}\right) \right|^2 \\
&= \left| \left(\underset{\sim}{b} - \underset{\sim}{a}\right) - \lambda \underset{\sim}{d} \right|^2 \\
&= \left(\left(\underset{\sim}{b} - \underset{\sim}{a}\right) - \lambda \underset{\sim}{d}\right) \cdot \left(\left(\underset{\sim}{b} - \underset{\sim}{a}\right) - \lambda \underset{\sim}{d}\right) \\
&= \left| \underset{\sim}{b} - \underset{\sim}{a} \right|^2 - 2\left(\underset{\sim}{b} - \underset{\sim}{a}\right) \cdot \underset{\sim}{d}\lambda + \lambda^2 \qquad \text{since, } \underset{\sim}{d} \cdot \underset{\sim}{d} = \left| \underset{\sim}{d} \right|^2 = 1
\end{aligned}
$$

This is a concave-up parabola. The smallest value occurs on the axis of symmetry.

$$
\begin{aligned}
\lambda_0 &= \frac{2\left(\underset{\sim}{b} - \underset{\sim}{a}\right) \cdot \underset{\sim}{d}}{2} \\
&= \left(\underset{\sim}{b} - \underset{\sim}{a}\right) \cdot \underset{\sim}{d}
\end{aligned}
$$

Question 16 (c) (ii)

Criteria	Marks
• Provides correct solution	1

Sample answer:

The direction of line ℓ is $\underset{\sim}{d}$.

$$
\begin{aligned}
\overrightarrow{PB} &= \underset{\sim}{b} - \left(\underset{\sim}{a} + \lambda_0 \underset{\sim}{d}\right) \\
&= \left(\underset{\sim}{b} - \underset{\sim}{a}\right) - \lambda_0 \underset{\sim}{d}
\end{aligned}
$$

Hence

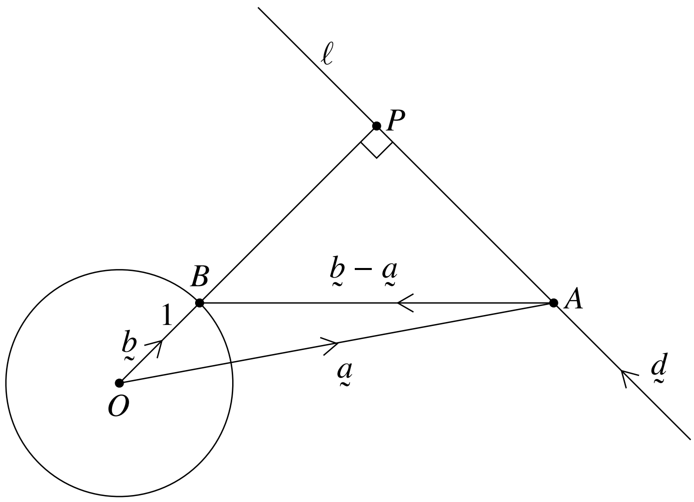
$$
\begin{aligned}
\overrightarrow{PB} \cdot \underset{\sim}{d} &= \left(\left(\underset{\sim}{b} - \underset{\sim}{a}\right) - \lambda_0 \underset{\sim}{d}\right) \cdot \underset{\sim}{d} \\
&= \left(\underset{\sim}{b} - \underset{\sim}{a}\right) \cdot \underset{\sim}{d} - \lambda_0 \underset{\sim}{d} \cdot \underset{\sim}{d} \\
&= \lambda_0 - \lambda_0 \left| \underset{\sim}{d} \right|^2 \\
&= \lambda_0 - \lambda_0 \\
&= 0
\end{aligned}
$$

So the vectors are perpendicular.

Question 16 (c) (iii)

Criteria	Marks
• Provides correct solution	3
• Uses Pythagoras' theorem, or equivalent merit	2
• Uses the projection formula, or equivalent merit	1

Sample answer:



Let A be the point on line ℓ with position vector $\underset{\sim}{a}$ and let B be the point on the sphere closest to the line.

$\overrightarrow{BP} \perp \overrightarrow{AP}$ from part (ii)

$\overrightarrow{AP}$ is the projection of $\overrightarrow{AB}$ onto $\underset{\sim}{d}$. So $\overrightarrow{AP} = \dfrac{\left(\underset{\sim}{b} - \underset{\sim}{a}\right) \cdot \underset{\sim}{d}}{\left|\underset{\sim}{d}\right|^2} \underset{\sim}{d} = \lambda_0 \underset{\sim}{d}$

By Pythagoras,
$$\left|\overrightarrow{OA}\right|^2 = \left|\overrightarrow{OP}\right|^2 + \left|\overrightarrow{AP}\right|^2$$
$$\left|\underset{\sim}{a}\right|^2 = \left(1 + \left|\overrightarrow{BP}\right|\right)^2 + \left|\overrightarrow{AP}\right|^2$$
$$\therefore \left(1 + \left|\overrightarrow{BP}\right|\right)^2 = \left|\underset{\sim}{a}\right|^2 - \left|\overrightarrow{AP}\right|^2$$
$$= \left|\underset{\sim}{a}\right|^2 - \left|\lambda_0 \underset{\sim}{d}\right|^2$$
$$= \left|\underset{\sim}{a}\right|^2 - \lambda_0^2 \left|\underset{\sim}{d}\right|^2$$
$$= \left|\underset{\sim}{a}\right|^2 - \lambda_0^2$$

Question 16 (c) (iii) sample answer (continued)

Now $\underset{\sim}{b} \cdot \underset{\sim}{d} = 0$ from part (ii) since OBP is a straight line. So $\lambda_0 = \underset{\sim}{b} \cdot \underset{\sim}{d} - \underset{\sim}{a} \cdot \underset{\sim}{d} = -\underset{\sim}{a} \cdot \underset{\sim}{d}$.

$$\begin{aligned}
\therefore \quad \left(1 + \left|\overrightarrow{BP}\right|\right)^2 &= \left|\underset{\sim}{a}\right|^2 - \left[0 - \left(\underset{\sim}{a} \cdot \underset{\sim}{d}\right)\right]^2 \\
&= \left|\underset{\sim}{a}\right|^2 - \left(\underset{\sim}{a} \cdot \underset{\sim}{d}\right)^2 \\
1 + \left|\overrightarrow{BP}\right| &= \sqrt{\left|\underset{\sim}{a}\right|^2 - \left(\underset{\sim}{a} \cdot \underset{\sim}{d}\right)^2} \\
\therefore \quad \left|\overrightarrow{BP}\right| &= \sqrt{\left|\underset{\sim}{a}\right|^2 - \left(\underset{\sim}{a} \cdot \underset{\sim}{d}\right)^2} - 1
\end{aligned}$$

unless the line intersects the sphere, in which case the distance is zero.

2025 HSC Mathematics Extension 2 Mapping Grid

Section I

Question	Marks	Content	Syllabus outcomes
1	1	MEX-V1 Further Work with Vectors	MEX12-3
2	1	MEX-P1 The Nature of Proof	MEX12-8
3	1	MEX-N1 Introduction to Complex Numbers	MEX12-4
4	1	MEX-M1 Applications of Calculus to Mechanics	MEX12-6
5	1	MEX-P1 The Nature of Proof	MEX12-8
6	1	MEX-N2 Using Complex Numbers	MEX12-4
7	1	MEX-N2 Using Complex Numbers	MEX12-4
8	1	MEX-M1 Applications of Calculus to Mechanics	MEX12-7
9	1	MEX-N2 Using Complex Numbers	MEX12-4
10	1	MEX-V1 Further Work with Vectors	MEX12-3

Section II

Question	Marks	Content	Syllabus outcomes
11 (a)	2	MEX-N2 Using Complex Numbers	MEX12-4
11 (b)	2	MEX-N1 Introduction to Complex Numbers	MEX12-4
11 (c) (i)	1	MEX-N1 Introduction to Complex Numbers	MEX12-1
11 (c) (ii)	2	MEX-N1 Introduction to Complex Numbers	MEX12-1
11 (d) (i)	1	MEX-M1 Applications of Calculus to Mechanics	MEX12-6
11 (d) (ii)	1	MEX-M1 Applications of Calculus to Mechanics	MEX12-6
11 (d) (iii)	1	MEX-V1 Further Work with Vectors	MEX12-3
11 (e)	2	MEX-P1 The Nature of Proof	MEX12-2
11 (f)	2	MEX-C1 Further Integration	MEX12-5
12 (a)	3	MEX-C1 Further Integration	MEX12-5
12 (b)	3	MEX-P2 Further Proof by Mathematical Induction	MEX12-2
12 (c)	3	MEX-N2 Using Complex Numbers	MEX12-1
12 (d)	4	MEX-C1 Further Integration	MEX12-5
12 (e)	3	MEX-M1 Applications of Calculus to Mechanics	MEX12-6
13 (a)	3	MEX-C1 Further Integration	MEX12-5
13 (b)	4	MEX-V1 Further Work with Vectors	MEX12-3
13 (c) (i)	2	MEX-P1 The Nature of Proof	MEX12-2
13 (c) (ii)	2	MEX-P1 The Nature of Proof	MEX12-2
13 (d)	4	MEX-C1 Further Integration	MEX12-5
14 (a) (i)	3	MEX-C1 Further Integration	MEX12-5
14 (a) (ii)	1	MEX-C1 Further Integration	MEX12-5
14 (b) (i)	2	MEX-M1 Applications of Calculus to Mechanics	MEX12-6

Question	Marks	Content	Syllabus outcomes
14 (b) (ii)	2	MEX-M1 Applications of Calculus to Mechanics	MEX12-6
14 (c) (i)	1	MEX-N2 Using Complex Numbers	MEX12-4
14 (c) (ii)	2	MEX-N2 Using Complex Numbers	MEX12-4
14 (c) (iii)	1	MEX-N2 Using Complex Numbers	MEX12-4
14 (d)	3	MEX-P1 The Nature of Proof	MEX12-1
15 (a)	4	MEX-V1 Further Work with Vectors	MEX12-3
15 (b)	4	MEX-M1 Applications of Calculus to Mechanics	MEX12-6
15 (c) (i)	3	MEX-N1 Introduction to Complex Numbers	MEX12-1
15 (c) (ii)	2	MEX-N2 Using Complex Numbers	MEX12-1
15 (c) (iii)	2	MEX-N2 Using Complex Numbers	MEX12-4
16 (a)	4	MEX-P1 The Nature of Proof MEX-N1 Introduction to Complex Numbers	MEX12-2, MEX12-4
16 (b)	5	MEX-M1 Applications of Calculus to Mechanics	MEX12-6
16 (c) (i)	2	MEX-V1 Further Work with Vectors	MEX12-3, MEX12-7
16 (c) (ii)	1	MEX-V1 Further Work with Vectors	MEX12-3, MEX12-7
16 (c) (iii)	3	MEX-V1 Further Work with Vectors	MEX12-3, MEX12-7