BAULKHAM HILLS
HIGH
SCHOOL

2020

YEAR 12
ASSESSMENT TASK 3

Mathematics Extension 2

General

- Reading time – 5 minutes

Instructions

- Reading time – 5 minutes
- Working time – 90 minutes
- Write using black or blue pen
  Black pen is preferred
- Board- approved calculators may be used
- In Questions 6-8, show relevant mathematical reasoning and/or calculations
- Marks may be deducted for careless or badly arranged work

Total marks: 47

Section I – 5 marks (pages 2 – 3)
- Attempt Questions 1 – 5
- Allow about 8 minutes for this section

Section II – 42 marks (pages 4 – 8)
- Attempt Questions 6-8
- Allow about 82 minutes for this section

Section I

5 marks
Attempt questions 1-5
Allow about 8 minutes for this section

Use the multiple choice answer sheet in your booklet for Questions 1 – 5.

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1. A particle is moving in simple harmonic motion which satisfies $\ddot{x} = -4x$.
The particle starts from $x = 0$ with initial velocity 4m/s.
Which of the following is a possible equation for the displacement, x, of the particle?

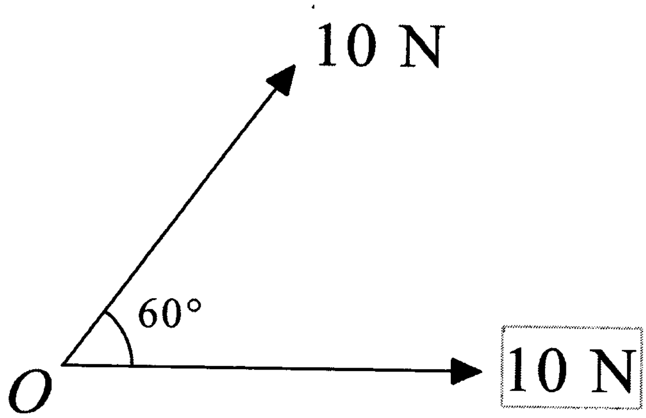
(A) $x = 2 \sin 2t$

(B) $x = 2 \sin 4t$

(C) $x = 4 \sin 2t$

(D) $x = 4 \sin 4t$

2. Two forces of magnitude 10 N act on a particle at O as shown. What is the magnitude of the resultant force?



(A) 5

(B) 10

(C) $10\sqrt{3}$

(D) 20

3. The velocity, v m/s of a particle moving in a straight line is governed by the equation $v = x - 3$, where x is the displacement of the particle at time t seconds. Initially the particle was at $x = 7$. What is the displacement function of this particle?

(A) $x = 7 + e^t$

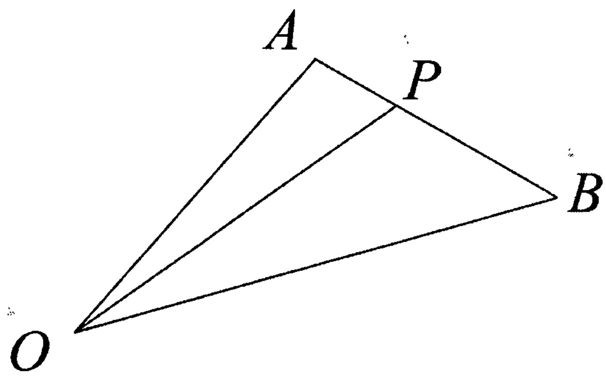
(B) $x = 7e^t$

(C) $x = 3 + e^t$

(D) $x = 3 + 4e^t$

4. In this diagram, $\overrightarrow{OA} = 6\underset{\sim}{i} - \underset{\sim}{j} + 8\underset{\sim}{k}$, $\overrightarrow{OB} = -3\underset{\sim}{i} + 4\underset{\sim}{j} - 2\underset{\sim}{k}$ and $AP{:}PB = 1 : 2$.

The vector $\overrightarrow{OP}$ is equal to



(A) $\frac{7}{3}\underset{\sim}{i} + \frac{4}{3}\underset{\sim}{k}$

(B) $3\underset{\sim}{i} + \frac{7}{3}\underset{\sim}{j} + \frac{4}{3}\underset{\sim}{k}$

(C) $3\underset{\sim}{j} + 4\underset{\sim}{k}$

(D) $3\underset{\sim}{i} + \frac{2}{3}\underset{\sim}{j} + \frac{14}{3}\underset{\sim}{k}$

5. A particle of mass m is moving in a straight line under the action of a force where $F = \frac{m}{x^3}(6 - 10x)$.

Which of the following is an equation for its velocity at any position, if the particle starts from rest at $x = 1$?

(A) $v = \pm\frac{1}{x}\sqrt{-3 + 10x - 7x^2}$

(B) $v = \pm x\sqrt{-3 + 10x - 7x^2}$

(C) $v = \pm\frac{1}{x}\sqrt{2(-3 + 10x - 7x^2)}$

(D) $v = \pm x\sqrt{2(-3 + 10x - 7x^2)}$

Section II

42 marks
Attempt questions 6-8
Allow about 82 minutes for this section

Answer each question on the appropriate page of your answer booklet
In Questions 6-8, your responses should include relevant mathematical reasoning and/or calculations.

Question 6 (13 marks) Answer on the appropriate page

(a) Given $P = (2,3,-1)$ and $Q = (-4,4,2)$, find

i) $\overrightarrow{PQ}$ **1**

ii) The distance between P and Q **1**

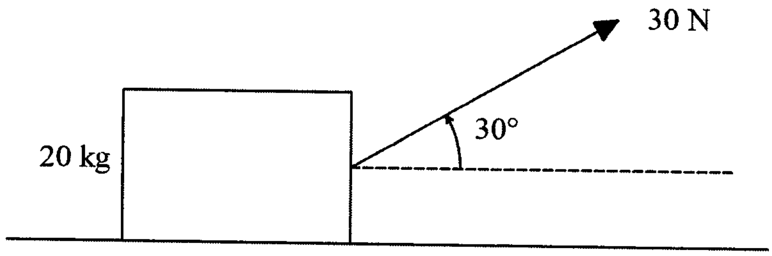
iii) The midpoint of PQ **1**

(b) If $\underset{\sim}{u} = \begin{bmatrix} -4 \\ 2 \\ 1 \end{bmatrix}$ and $\underset{\sim}{v} = \begin{bmatrix} -1 \\ 3 \\ 2 \end{bmatrix}$, find

i) $\underset{\sim}{u} \cdot \underset{\sim}{v}$ **1**

ii) The angle between $\underset{\sim}{u}$ and $\underset{\sim}{v}$ (correct to the nearest minute) **2**

(c) A box of mass 20 kg is pulled along a smooth horizontal table by a force of 30 newtons acting at an angle of $30°$ to the horizontal.



Find the magnitude of the normal reaction of the table on the box. Use $g = 10\text{m/s}^2$. **2**

Question 6 continues on the following page

(d) Find the values of k given that $\begin{bmatrix} 2k \\ \frac{1}{\sqrt{2}} \\ -k \end{bmatrix}$ is a unit vector. **2**

(e) Find all vectors which are perpendicular to both $\begin{bmatrix} 2 \\ -1 \\ -2 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$ **3**

End of Question 6

Question 7 (13 marks) Answer on the appropriate page

(a) Find the projection of $\underset{\sim}{a}$ onto $\underset{\sim}{b}$ given $\underset{\sim}{a} = 3\underset{\sim}{i} - \underset{\sim}{j} + 2\underset{\sim}{k}$ and $\underset{\sim}{b} = \underset{\sim}{i} + 3\underset{\sim}{j} + 5\underset{\sim}{k}$ **2**

(b) (i) Find the vector equation of the line through the points (3,-1,-2) and (5,3,-4) **2**

(ii) Find the point(s) where this line meets the sphere $x^2 + y^2 + z^2 = 26$. **2**

(c) A particle moves so that its displacement, x cm, from the origin, O, at time t seconds is given by $x = \sqrt{3}\cos 3t - \sin 3t$.

i) Show that the particle moves in simple harmonic motion. **2**

ii) Find the period of the motion. **1**

iii) Find the time when the particle first passes through the origin. **2**

iv) What is the particle's velocity the first time it is 1cm from the origin? **2**

End of Question 7

Question 8 (16 marks) Answer on the appropriate page

(a) A particle of mass m kilograms is falling from rest. It experiences air resistance of magnitude mkv^2 where k is a positive constant and v is the velocity of the particle.
The terminal velocity of the particle during the downward motion is V_T.

(i) By considering the forces on the particle during its downward motion, show that $kV_T^2 = g$ (where g is the acceleration due to gravity). **2**

(ii) The particle was then projected vertically upwards from level ground. The initial speed of projection is $\frac{1}{5}$ of the magnitude of the terminal velocity from the downward motion. **2**

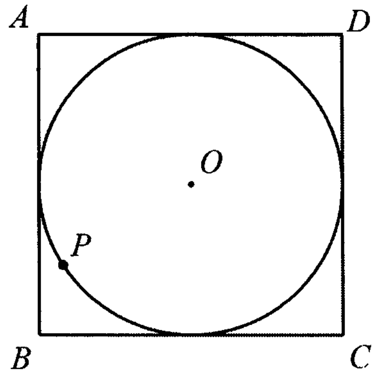
Show that during its upward motion the acceleration of the particle, $\ddot{x}$, is given by $\ddot{x} = -g\left(1 + \dfrac{v^2}{(V_T)^2}\right)$

(iii) Show that the maximum height, H, reached is given by **3**

$$H = \frac{V_T^2}{2g}\ln\left(\frac{26}{25}\right).$$

Question 8 continues on the following page

(b) In the figure, the circle has centre O and radius r. The circle is inscribed in a square $ABCD$ and P is any point on the circle.



i) Show that $\overrightarrow{AP} \cdot \overrightarrow{AP} = 3r^2 - 2\,\overrightarrow{OP} \cdot \overrightarrow{OA}$ **2**

ii) Hence find $AP^2 + BP^2 + CP^2 + DP^2$ in terms of r. **2**

(c) A short-barrelled machine gun stands on horizontal ground. The gun fires bullets, from ground level, at speed u continuously from $t = 0$ to $t = \dfrac{\pi}{6\lambda}$, where λ is a positive constant. During this time period, the angle of elevation α of the barrel decreases from $\dfrac{\pi}{3}$ to $\dfrac{\pi}{6}$ and is given at time t by $\alpha = \dfrac{\pi}{3} - \lambda t$.

You may assume the equations of motion $x = ut\cos\alpha$ and $y = -\dfrac{gt^2}{2} + ut\sin\alpha$

(i) T is the time it takes for a bullet, fired t seconds after the first bullet, to hit the ground. Show that $T = t + \dfrac{2u}{g}\sin\left(\dfrac{\pi}{3} - \lambda t\right)$. **2**

(ii) Show that the last bullet to hit the ground does so at a distance $\dfrac{2ku^2\sqrt{1-k^2}}{g}$ from the gun, where $k = \dfrac{g}{2\lambda u}$. **3**

End of paper

EXTENSION 2 TASK 3 2020

MC. 1. $n^2 = 4$

$n = 2$

Consider (A): $x = 2\sin 2t$ (For B: $\dot{x} = 8\cos 4t$

$\dot{x} = 4\cos 2t$ $\dot{x}(0) = 8$

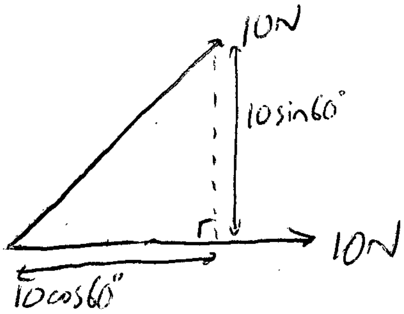
when $t=0$, $x = 0$ C: $\dot{x} = 8\cos 2t$

$\dot{x} = 4$ $\dot{x} = 8$

$\therefore$ (A) D: $\dot{x} = 16\cos 4t$

$\dot{x} = 16$)

2.



$\therefore$ Resultant $10\underset{\sim}{i} + 10\cos 60°\underset{\sim}{i} + 10\sin 60°\underset{\sim}{j}$

$= 15\underset{\sim}{i} + 5\sqrt{3}\underset{\sim}{j}$

Magnitude $= \sqrt{15^2 + (5\sqrt{3})^2}$

$= \sqrt{300}$

$= 10\sqrt{3}$ N

$\therefore$ (C)

3. $\frac{dx}{dt} = x - 3$

$\frac{dt}{dx} = \frac{1}{x-3}$

$\int_0^t dt = \int \frac{1}{x-3}\, dt$

$[t]_0^t = [\ln|x-3|]^x$

$t - 0 = \ln|x-3| - \ln 4$

$t = \ln\left|\frac{x-3}{4}\right|$

$\frac{x-3}{4} = e^t$

$x - 3 = 4e^t$

$x = 3 + 4e^t$

$\therefore$ (D)

4. $\vec{OP} = \vec{OA} + \frac{1}{1+2}\vec{AB}$

$= 6\underset{\sim}{i} - \underset{\sim}{j} + 8\underset{\sim}{k} + \frac{1}{3}\left((-3-6)\underset{\sim}{i} + (4--1)\underset{\sim}{j} + (-2-8)\underset{\sim}{k}\right)$

$= 6\underset{\sim}{i} - \underset{\sim}{j} + 8\underset{\sim}{k} + \frac{1}{3}\left(-9\underset{\sim}{i} + 5\underset{\sim}{j} - 10\underset{\sim}{k}\right)$

$\vec{OP} = 3\underset{\sim}{i} + \frac{2}{3}\underset{\sim}{j} + \frac{14}{3}\underset{\sim}{k} \quad \therefore (D)$

5. $F = \frac{m}{x^3}(6-10x)$

$\cancel{m}a = \frac{\cancel{m}}{x^3}(6-10x)$

$\frac{d}{dx}\left(\frac{1}{2}v^2\right) = 6x^{-3} - 10x^{-2}$

$\frac{v^2}{2} = -3x^{-2} + 10x^{-1} + c$

When $x=1, v=0$

$0 = 7+c$

$c = -7$

$\therefore v^2 = \frac{-6}{x^2} + \frac{20}{x} - 14$

$v^2 = \frac{1}{x^2}\left(2(-3+10x-7x^2)\right)$

$v = \pm\frac{1}{x}\sqrt{2(-3+10x-7x^2)}$

$\therefore$ (C)

6\. a) i) $\vec{PQ} = \begin{bmatrix} -4 \\ 4 \\ 2 \end{bmatrix} - \begin{bmatrix} 2 \\ 3 \\ -1 \end{bmatrix}$

$= \begin{bmatrix} -6 \\ 1 \\ 3 \end{bmatrix}$

① correct answer (can't be expressed as coordinates)

ii) $|\vec{PQ}| = \sqrt{(-6)^2 + 1^2 + 3^2}$

$= \sqrt{46}$

① correct answer

iii) Midpoint $= \left(-\frac{2}{2}, \frac{7}{2}, \frac{1}{2}\right)$

$= \left(-1, \frac{7}{2}, \frac{1}{2}\right)$

① correct answer

b) i) $\underset{\sim}{u} \cdot \underset{\sim}{v} = (-4 \times -1) + (2 \times 3) + (1 \times 2)$

$= 12$

① correct answer

ii) $\cos\theta = \dfrac{\underset{\sim}{u} \cdot \underset{\sim}{v}}{|\underset{\sim}{u}||\underset{\sim}{v}|}$

$= \dfrac{12}{\sqrt{21} \times \sqrt{14}}$

$= \dfrac{12}{7\sqrt{6}}$ or $\dfrac{12\sqrt{6}}{7}$

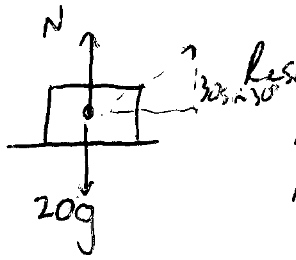
$\therefore \theta = \cos^{-1} \dfrac{12}{7\sqrt{6}}$

$\theta = 45°35'$ nearest minute

② correct answer

① correct expression for $\cos\theta$

c) i)



Resolving vertically:

$N + 30\sin 30° = 20g$

$N + 30 \times \frac{1}{2} = 200$

$N = 185\text{ N}.$

② Correct solution

① Resolves force into vertical component

6 d) $\sqrt{(2k)^2+\left(\frac{1}{\sqrt{2}}\right)^2+(-k)^2} = 1$

$5k^2+\frac{1}{2} = 1$

$5k^2 = \frac{1}{2}$

$k^2 = \frac{1}{10}$

$k = \pm\frac{1}{\sqrt{10}}$

(2) Correct solution
(1) Attempts to find magnitude and equate to 1

e) Let perpendicular vector be $\begin{bmatrix} a \\ b \\ c \end{bmatrix}$. Since perpendicular, dot product must be 0.

$\begin{bmatrix} a \\ b \\ c \end{bmatrix} \cdot \begin{bmatrix} 2 \\ -1 \\ -2 \end{bmatrix} = 0$ and $\begin{bmatrix} a \\ b \\ c \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix} = 0$

$2a - b - 2c = 0$ ① $\qquad a + b + 2c = 0$ ②

$\therefore$ ①+②

$3a = 0$

$a = 0$

From ① $b = -2c$ $\qquad$ From ② $b = -2c$

$\therefore$ vector is of the form: $\lambda\begin{bmatrix} 0 \\ -2 \\ 1 \end{bmatrix}$ will be perpendicular.

(3) correct solution
(2) finds $a=0$ and relationship between b and c
(1) finds $a=0$ or equates dot products to be 0.

7 a) $proj_{\underset{\sim}{b}}\underset{\sim}{a} = \frac{\underset{\sim}{b}\cdot\underset{\sim}{a}}{|b|^2}\underset{\sim}{b}$

$= \frac{3\times1 - 1\times3 + 5\times2}{1+3^2+5^2}\left(\underset{\sim}{i} + 3\underset{\sim}{j} + 5\underset{\sim}{k}\right)$

$= \frac{10}{35}\left(\underset{\sim}{i} + 3\underset{\sim}{j} + 5\underset{\sim}{k}\right)$

$= \frac{2}{7}\underset{\sim}{i} + \frac{6}{7}\underset{\sim}{j} + \frac{10}{7}\underset{\sim}{k}$

(2) Correct solution
(1) progress towards (correct formula with one error in calculation)

7 b) i) Equation of line through points is

$$\underset{\sim}{r} = \begin{pmatrix} 3 \\ -1 \\ -2 \end{pmatrix} + \lambda \begin{pmatrix} 5-3 \\ 3--1 \\ -4+2 \end{pmatrix}$$ where λ is a constant

$$\underset{\sim}{r} = \begin{pmatrix} 3 \\ -1 \\ -2 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 4 \\ -2 \end{pmatrix}$$

or $$\underset{\sim}{r} = \begin{pmatrix} 3 \\ -1 \\ -2 \end{pmatrix} + \lambda_1 \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$$ $\underbrace{\lambda_1 = 2\lambda}$ not penalised if omitted

(3) correct solution
(2) forms correct quadratic equation to find λ
(1) attempts to find point of intersection by substituting coordinates of point on the line into equation of sphere

ii) $$\underset{\sim}{r} = \begin{pmatrix} 3+\lambda \\ -1+2\lambda \\ -2-\lambda \end{pmatrix}$$

Since $\underset{\sim}{r}$ lies on sphere when they intersect

$$(3+\lambda)^2 + (-1+2\lambda)^2 + (-2-\lambda)^2 = 26$$
$$\lambda^2+6\lambda+9 + 4\lambda^2-4\lambda+1 + \lambda^2+4\lambda+4 = 26$$
$$6\lambda^2 + 6\lambda + 14 = 26$$
$$6\lambda^2+6\lambda-12 = 0$$
$$\lambda^2+\lambda-2 = 0$$
$$(\lambda+2)(\lambda-1) = 0$$

$\lambda = -2$: Pt is $(1,-5,0)$
$\lambda = 1$: Pt is $(4,1,-3)$
$\therefore$ Points of intersection are $(1,-5,0)$ and $(4,1,-3)$

7 c) i) $$x = -3\sqrt{3}\sin 3t - 3\cos 3t$$
$$\dot{x} = -9\sqrt{3}\cos 3t + 9\sin 3t$$
$$= -9(\sqrt{3}\cos 3t - \sin 3t)$$
$$\ddot{x} = -9x$$ which is of the form $\ddot{x} = -n^2x$
where $n=3$

(2) correct solution
(1) finds acceleration

ii) $$T = \frac{2\pi}{n}$$ $n^2=9$
Period $= \frac{2\pi}{3}$ seconds $n=3$

(1) correct answer

7 c iii) When $x=0$, $0=\sqrt{3}\cos 3t-\sin 3t$

$$\sin 3t = \sqrt{3}\cos 3t$$
$$\tan 3t = \sqrt{3}$$
$$3t = \frac{\pi}{3}$$
$$t = \frac{\pi}{9} \text{ seconds}$$

② Correct answer (must be exact)
① obtains trig equation $\tan(3t)=\sqrt{3}$ or equivalent

c iv) let $\sqrt{3}\cos 3t - \sin 3t = \sqrt{3+1}\,\cos(3t+\alpha)$

$$\cos\alpha = \frac{\sqrt{3}}{2}$$
$$\alpha = \frac{\pi}{6}$$

When $x=1$,

$$\therefore\ 2\cos\left(3t+\frac{\pi}{6}\right)=1$$
$$\cos\left(3t+\frac{\pi}{6}\right)=\frac{1}{2}$$
$$3t+\frac{\pi}{6}=\frac{\pi}{3}$$
$$3t=\frac{\pi}{6}$$
$$t=\frac{\pi}{18}$$

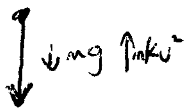
$\therefore$ passes through 1 when $t=\frac{\pi}{18}$

$$\dot{x} = -3\sqrt{3}\sin\frac{\pi}{6} - 3\cos\frac{\pi}{6}$$
$$= -\frac{3\sqrt{3}}{2} - \frac{3\sqrt{3}}{2}$$

$\therefore$ Velocity $= -3\sqrt{3}$ cm/s

② correct solution
① significant progress (eg expresses displacement as $2(\cos 3t+\frac{\pi}{6})$)

8 a) i) Downward



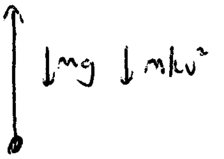
$F=ma$

For terminal velocity $a=0$

$$mg - mkv_T^2 = 0$$
$$kv_T^2 = g \quad \text{as req'd}$$

② correct solution
① correct force diagram (or correct equation for net force)

8 a ii) upward



$$ma = -mg - mkv^2$$
$$a = -(g + kv^2)$$

But from i) $g = kV_T^2$

$$\ddot{x} = -\left(\frac{gv^2}{V_T^2} + g\right)$$
$$\ddot{x} = -g\left(1 + \frac{v^2}{V_T^2}\right)$$

iii) Using $\ddot{x} = v\frac{dv}{dx}$

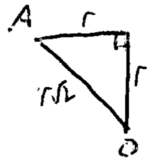
$$v\frac{dv}{dx} = -g\left(\frac{V^2 + V_T^2}{V_T^2}\right)$$
$$\int_0^H dx = \frac{-V_T^2}{g}\int_{V_T/5}^{0}\frac{v\,dv}{v^2 + V_T^2}$$
$$\left[x\right]_0^H = \frac{-V_T^2}{2g}\left[\ln\left(V^2 + V_T^2\right)\right]_{V_T/5}^{0}$$
$$H - 0 = \frac{V_T^2}{2g}\left[\ln\left(V^2 + V_T^2\right)\right]_0^{V_T/5}$$
$$= \frac{V_T^2}{2g}\ln\left(\frac{\frac{V_T^2}{25} + V_T^2}{V_T^2}\right)$$
$$= \frac{V_T^2}{2g}\ln\left(\frac{26\cancel{V_T^2}}{25\cancel{V_T^2}}\right)$$
$$= \frac{V_T^2}{2g}\ln\left(\frac{26}{25}\right)$$

(2) Correct proof
(1) finds correct expression for $\ddot{x}$ in terms of g, k, v

(3) Correct solution
(2) significant progress (eg correctly integrates excluding limits)
(1) uses $v\frac{dv}{dx}$ and expresses as correct integral.

8 b) i) $\overrightarrow{OA} + \overrightarrow{AP} = \overrightarrow{OP}$

$\overrightarrow{AP} = \overrightarrow{OP} - \overrightarrow{OA}$



$|\overrightarrow{OA}| = |\overrightarrow{OB}| = |\overrightarrow{OC}| = |\overrightarrow{OD}| = r\sqrt{2}$

Using cosine rule in ΔAOP

$|\overrightarrow{AP}|^2 = |\overrightarrow{OA}|^2 + |\overrightarrow{OP}|^2 - 2|\overrightarrow{OA}||\overrightarrow{OP}|\cos(\angle AOP)$

$|\overrightarrow{AP}|^2 = 2r^2 + r^2 - 2|\overrightarrow{OA}||\overrightarrow{OP}| \dfrac{\overrightarrow{OA}\cdot\overrightarrow{OP}}{|\overrightarrow{OA}||\overrightarrow{OP}|}$

$|\overrightarrow{AP}|^2 = 3r^2 - 2\,\overrightarrow{OA}\cdot\overrightarrow{OP}$

$\overrightarrow{AP}\cdot\overrightarrow{AP} = 3r^2 - 2\,\overrightarrow{OA}\cdot\overrightarrow{OP}$ since $\underset{\sim}{\ell}\cdot\underset{\sim}{\ell} = |\underset{\sim}{\ell}|^2$

(2) correct solution*
(1) uses cosine rule in ΔOPA or dot product
(note * must define θ)

ii) Similarly

BP^2 $\overrightarrow{BP}\cdot\overrightarrow{BP} = 3r^2 - 2\,\overrightarrow{OB}\cdot\overrightarrow{OP}$

$CP^2 = |\overrightarrow{CP}|^2 = \overrightarrow{CP}\cdot\overrightarrow{CP} = 3r^2 - 2\,\overrightarrow{OC}\cdot\overrightarrow{OP}$

$DP^2 = \overrightarrow{DP}\cdot\overrightarrow{DP} = 3r^2 - 2\,\overrightarrow{OD}\cdot\overrightarrow{OP}$

(2) correct solution
(1) finds expression for sum of squares

$AP^2 + BP^2 + CP^2 + DP^2 = 12r^2 - 2(\overrightarrow{OA}\cdot\overrightarrow{OP} + \overrightarrow{OB}\cdot\overrightarrow{OP} + \overrightarrow{OC}\cdot\overrightarrow{OP} + \overrightarrow{OD}\cdot\overrightarrow{OP})$

But diagonals of a square bisect $\overrightarrow{OB} = -\overrightarrow{OD}$ and $\overrightarrow{OC} = -\overrightarrow{OA}$

$\therefore AP^2 + BP^2 + CP^2 + DP^2 = 12r^2 - 2(\overbrace{\cancel{\overrightarrow{OA}\cdot\overrightarrow{OP}} - \cancel{\overrightarrow{OA}\cdot\overrightarrow{OP}} + \cancel{\overrightarrow{OP}\cdot -\overrightarrow{OD}} + \cancel{\overrightarrow{OP}\cdot\overrightarrow{OD}}}^{0})$

$\therefore AP^2 + BP^2 + CP^2 + DP^2 = 12r^2$

8 a i) Time of flight

$$y = -\frac{gt^2}{2} + ut\sin\alpha = 0$$

$$gt^2 - 2ut\sin\alpha = 0$$

$$t(gt - 2u\sin\alpha) = 0$$

$$t = 0 \quad \text{or} \quad t = \frac{2u\sin\alpha}{g}$$

$\therefore$ time of flight $\frac{2u\sin\alpha}{g}$

Time since : $T_L = t + \frac{2u\sin\alpha}{g}$

$$T_L = t + \frac{2u}{g}\sin\left(\frac{\pi}{3} - \lambda t\right)$$

(2) correct solution
(1) calculates time of flight

ii) For last bullet to hit the ground, T_L will be a maximum

$$\frac{dT_L}{dt} = 1 - \frac{2u\lambda}{g}\cos\left(\frac{\pi}{3} - \lambda t\right)$$

$$= 1 - \frac{1}{k}\cos\left(\frac{\pi}{3} - \lambda t\right)$$

For max value $\frac{dT_L}{dt} = 0$

$$1 - \frac{1}{k}\cos\left(\frac{\pi}{3} - \lambda t\right) = 0$$

$$k - \cos\left(\frac{\pi}{3} - \lambda t\right) = 0$$

$$k = \cos\left(\frac{\pi}{3} - \lambda t\right) \quad *$$

$$\frac{d^2T_L}{dt} = \frac{-2\lambda^2 u}{g}\sin\left(\frac{\pi}{3} - \lambda t\right)$$

$< 0 \quad \therefore$ max value

Horizontal range, $R = ut\cos\alpha$

$$= u\,\frac{2u\sin\alpha}{g}\cos\alpha$$

$$\text{Range} = \frac{2u^2\sin\alpha\cos\alpha}{g}$$

From $*$ $k = \cos\alpha$, $\sin\alpha = \sqrt{1-\cos^2\alpha}$
$= \sqrt{1-k^2}$

$\therefore$ Range $= \frac{2ku^2\sqrt{1-k^2}}{g}$ as req'd.

(3) correct solution
(2) finds range in terms of u, α, g and derives $k = \cos\left(\frac{\pi}{3} - \lambda t\right)$
(1) Finds expression for range in terms of u, α, g
or (1) attempts to find maximum time by differentiating and solving