BAULKHAM HILLS
HIGH
SCHOOL

2022

YEAR 12
H3: ASSESSMENT TASK 3

Mathematics Extension 2

General Instructions

- Reading time – 5 minutes
- Working time – 60 minutes
- Write using black pen
- Calculators approved by NESA may be used
- A reference sheet is provided at the back of this paper
- In all questions, show relevant mathematical reasoning and/or calculations
- Marks may be deducted for careless or badly arranged work

Total marks: 40

- There are 4 Questions in total
- All questions should be attempted

40 marks
Attempt Questions 1 – 4

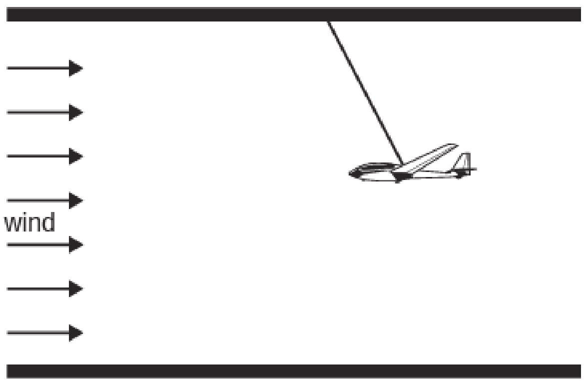
Answer each question on the appropriate answer sheet. Each answer sheet must show your NESA#. Extra paper is available.

For all questions your responses should include relevant mathematical reasoning and/or calculations.

Question 1 (9 marks) Use the pages labelled Question 1 in the answer booklet

(a) A model glider with a mass of 4 kg is suspended from the roof of a wind tunnel by a thin wire. A light wind exerts a horizontal force of magnitude $\frac{g}{2}$ Newtons on the model glider.

(i) Let T Newtons be the magnitude of the tension in the suspending wire. Clearly label all forces acting on the model glider on the copy of the below diagram on your answer sheet. *1*



(ii) Calculate the value of T, giving your answer in the form $\frac{g\sqrt{a}}{b}$ where a and b are positive integers. *2*

(b) The acceleration of a particle P moving in a straight line is given by

$$\ddot{x} = 3x(x-4)$$

where x metres is the displacement from the origin O and t is the time in seconds. Initially the particle is at O and its velocity is $4\sqrt{2}$ m/s.

(i) Show that $v^2 = 2(x^3 - 6x^2 + 16)$, where v is the velocity of P. *2*

(ii) Calculate the velocity and acceleration of P at $x = 2$. *2*

(iii) In which direction does P move from $x = 2$? Give a reason for your answer. *1*

(iv) *Briefly* describe the motion of P after it moves from $x = 2$. *1*

Question 2 (11 marks) Use the pages labelled Question 2 in the answer booklet

(a) The position vectors of points P and Q are $\underset{\sim}{p} = 2\underset{\sim}{i} - 3\underset{\sim}{j} + \underset{\sim}{k}$ and $\underset{\sim}{q} = 2\underset{\sim}{i} + 2\underset{\sim}{j} - 4\underset{\sim}{k}$ respectively.

(i) Find the magnitude of $\underset{\sim}{p}$. *1*

(ii) Determine $\angle POQ$, to the nearest degree, where O is the origin. *2*

(iii) Find the vector equation of the line joining P and Q. *2*

(iv) Which plane does the vector $\overrightarrow{PQ}$ lie on? *1*

(v) Find a vector equation for a line that would intersect $\overrightarrow{PQ}$ and is also perpendicular to $\overrightarrow{PQ}$. *2*

(vi) Find the position vector of any point that lies on your line in part (v), but does not lie on $\overrightarrow{PQ}$. *1*

(b) What is the Cartesian equation of the line $\underset{\sim}{r} = \begin{pmatrix} 3 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 6 \\ -3 \end{pmatrix}$? *2*

Question 3 (10 marks) Use the pages labelled Question 3 in the answer booklet

(a) A body falling from rest experiences resistance directly proportional to its velocity squared. (i.e. resistive force = mkv^2 where k is a constant)

(i) Write the equation of motion for this body. *1*

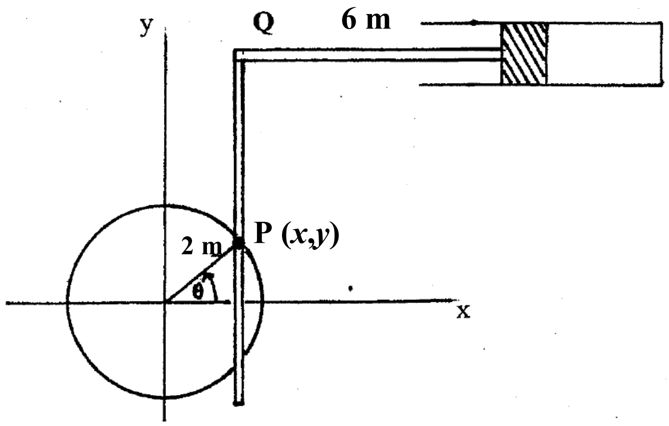
(ii) Show that the distance fallen when the velocity is V is given by *2*

$$x = \frac{1}{2k} \ln \left| \frac{g}{g - kV^2} \right|$$

(iii) Calculate the terminal velocity. *1*

(b) A piston moves horizontally back and forth on the end of a 6 metre shaft.

The other end is attached at Q to a vertical slotted arm fitted to a peg P on the rim of a wheel of radius 2 metres.



Suppose the wheel begins with point P at $\theta = \frac{\pi}{4}$, when $t = 0$ and rotates anticlockwise at 5 radians per second.

(i) Show that $x = 2\cos\left(5t + \frac{\pi}{4}\right)$. *2*

(ii) Show that the motion of the piston is simple harmonic. *1*

(iii) State the amplitude and period of the motion *2*

(iv) Find the initial velocity of the piston. *1*

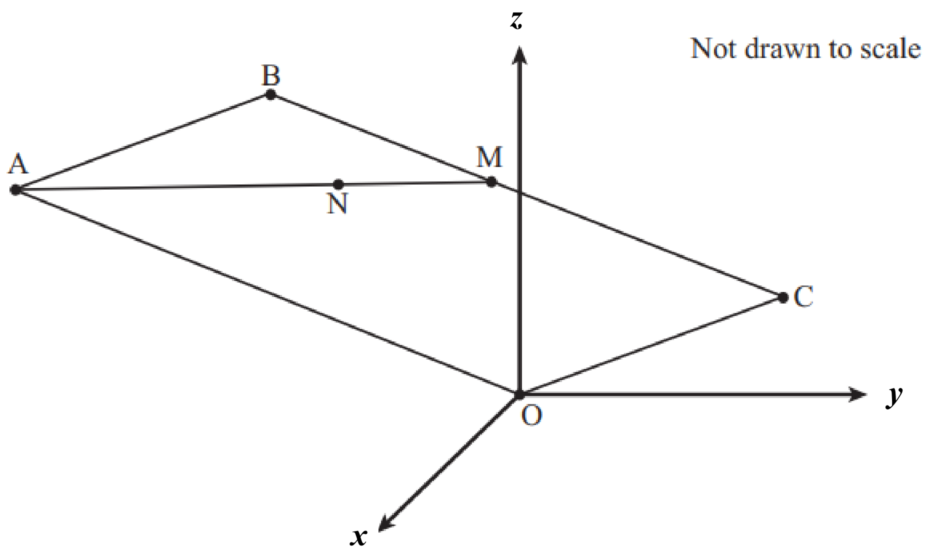
Question 4 (10 marks) Use the pages labelled Question 4 in the answer booklet

(a) Consider two lines in three dimensions given by 3

$$\underset{\sim}{r} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix} \text{ and } \underset{\sim}{r} = \begin{pmatrix} 4 \\ 1 \\ 0 \end{pmatrix} + \mu \begin{pmatrix} 5 \\ 2 \\ 1 \end{pmatrix}$$

By equating components, find the point of intersection of the two lines.

(b) The points $A(-6,2,-2)$ and $C(3,1,2)$ are represented in three-dimensional space in the diagram



OABC forms a parallelogram in three-dimensional space. Using vector techniques;

(i) Determine the coordinates of *B*. 2

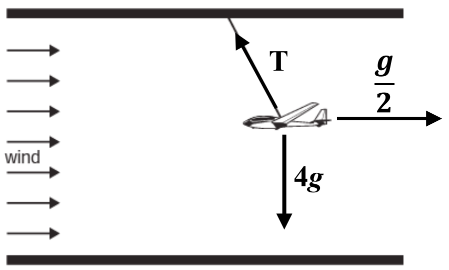
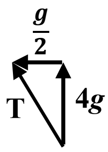
(ii) *M* is the midpoint of *BC*. Determine the vector that represents $\overrightarrow{MO}$. 1

(iii) *N* divides *AM* in the ratio 2:1. Determine the vector that represents $\overrightarrow{ON}$. 2

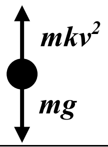
(iv) Show that *O*, *B* and *N* lie on a straight line. 2

End of paper

BAULKHAM HILLS HIGH SCHOOL
YEAR 12 EXTENSION 2 H3: ASSESSMENT TASK 3 2022 SOLUTIONS

Solution	Marks	Comments
QUESTION 1		
1 (a) (i) 	1	**1 mark** • Correct answer with all three forces shown with direction indicated
1 (a) (ii)  As the model is in equilibrium, both the horizontal and vertical forces must balance. $T^2 = \left(\frac{g}{2}\right)^2 + (4g)^2$ $= \frac{g^2}{4} + 16g^2$ $= \frac{65g^2}{4}$ $T = \frac{g\sqrt{65}}{2}$ Newtons	2	**2 marks** • Correct solution **1 mark** • Attempts to resolve forces in a correct approach
1 (b) (i) $v\frac{dv}{dx} = 3x(x-4)$ $\int_{4\sqrt{2}}^{v} v\,dv = \int_0^x (3x^2 - 12x)\,dx$ $\left[v^2\right]_{4\sqrt{2}}^{v} = 2\left[x^3 - 6x^2\right]_0^x$ $v^2 - 32 = 2(x^3 - 6x^2)$ $v^2 = 2(x^3 - 6x^2 + 32)$	2	**2 marks** • Correct solution **1 mark** • Uses $\ddot{x} = v\frac{dv}{dx}$ and separates the variables
1 (b) (ii) When $x = 2$; $v^2 = 2\left(2^3 - 6(2)^2 + 16\right)$ $= 0$ $v = 0$ $\ddot{x} = 3(2)(2-4)$ $= -12$	2	**2 marks** • Correct solution **1 mark** • Finds either velocity or acceleration
1 (b) (iii) As $v = 0$ and $\ddot{x} < 0$, the particle must move towards the origin.	1	**1 mark** • Correct answer with explanation
1 (b) (iv) After the particle passes through the origin it will slow down, as $v < 0$ and $\ddot{x} > 0$, until it stops and returns to the origin	1	**1 mark** • A description of the motion that explains why the particle is slowing down.

Solution	Marks	Comments
QUESTION 2		
2 (a) (i) $\lvert \underset{\sim}{p} \rvert = \sqrt{2^2+3^2+1^2}$ $= \sqrt{14}$	1	**1 mark** • Correct answer
2 (a) (ii) $\cos \angle POQ = \dfrac{\underset{\sim}{p} \cdot \underset{\sim}{q}}{\lvert\underset{\sim}{p}\rvert\lvert\underset{\sim}{q}\rvert}$ $= \dfrac{4-6-4}{\sqrt{14}\sqrt{24}}$ $= -\dfrac{6}{\sqrt{336}}$ $\angle POQ = 109°$ (to nearest degree) $\lvert\underset{\sim}{q}\rvert = \sqrt{2^2+2^2+4^2}$ $= \sqrt{24}$	2	**2 marks** • Correct solution **1 mark** • Finds $\underset{\sim}{p} \cdot \underset{\sim}{q}$
2 (a) (iii) $\underset{\sim}{r} = \begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 2-2 \\ 2+3 \\ -4-1 \end{pmatrix}$ $= \begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 0 \\ 5 \\ -5 \end{pmatrix}$ $= \begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}$	2	**2 marks** • Correct solution **1 mark** • Finds a correct direction vector
2 (a) (iv) $\overrightarrow{PQ}$ lies on the plane $x = 2$	1	**1 mark** • Correct answer
2 (a) (v) let the direction vector of the line be $\begin{pmatrix} x \\ y \\ z \end{pmatrix}$. $\begin{pmatrix} x \\ y \\ z \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} = 0$ $y - z = 0$ $y = z$ $\therefore$ a possible equation would be $\underset{\sim}{r} = \begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix} + \mu \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$	2	**2 marks** • Correct solution **1 mark** • Uses $\underset{\sim}{p} \cdot \underset{\sim}{q} = 0$ to establish perpendicular lines *Note: point must lie on* $\overrightarrow{PQ}$
2 (a) (vi) let $\mu = 1$ $\underset{\sim}{r} = \begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$ $= \begin{pmatrix} 2 \\ -2 \\ 2 \end{pmatrix}$	1	**1 mark** • Correct answer
2 (b) required slope $= -\dfrac{3}{6}$ $= -\dfrac{1}{2}$ $y - 1 = -\dfrac{1}{2}(x-3)$ $2y - 2 = -x + 3$ $x + 2y - 5 = 0$	2	**2 marks** • Correct solution **1 mark** • Finds the slope of the line • Attempts to eliminate the parameter

Solution	Marks	Comments
QUESTION 3		
3 (a) (i)  $m\ddot{x} = mg - mkv^2$ $\ddot{x} = g - kv^2$	**1**	**1 mark** • Correct answer
3 (a) (ii) $v\dfrac{dv}{dx} = g - kv^2$ $\displaystyle\int_0^v \frac{v}{g - kv^2}\,dv = \int_0^x dx$ $x = -\dfrac{1}{2k}\left[\ln\lvert g - kv^2\rvert\ \right]_0^v$ $= -\dfrac{1}{2k}\ln\left\lvert \dfrac{g - kv^2}{g}\right\rvert$ $= \dfrac{1}{2k}\ln\left\lvert \dfrac{g}{g - kv^2}\right\rvert$	**2**	**2 marks** • Correct solution **1 mark** • Separates the differential equation into two valid integrals
3 (a) (iii) terminal velocity occurs when $\ddot{x} = 0$ i.e. $g - kv^2 = 0$ $v^2 = \dfrac{g}{k}$ $v = \sqrt{\dfrac{g}{k}}$	**1**	**1 mark** • Correct answer
3 (b) (i) $\dfrac{d\theta}{dt} = 5$ $\displaystyle\int_{\frac{\pi}{4}}^{\theta} d\theta = 5\int_0^t dt$ $\theta - \dfrac{\pi}{4} = 5t$ $\theta = 5t + \dfrac{\pi}{4}$ $\dfrac{x}{2} = \cos\theta$ $x = 2\cos\theta$ $x = 2\cos\left(5t + \dfrac{\pi}{4}\right)$	**2**	**2 marks** • Correct solution **1 mark** • Establishes $\theta = 5t + \dfrac{\pi}{4}$
3 (b) (ii) $x = 2\cos\left(5t + \dfrac{\pi}{4}\right)$ $\dot{x} = -10\sin\left(5t + \dfrac{\pi}{4}\right)$ $\ddot{x} = -50\cos\left(5t + \dfrac{\pi}{4}\right)$ $\ddot{x} = -25x$ $\therefore$ motion is SHM as it takes the form $\ddot{x} = -n^2x$	**1**	**1 mark** • Correct solution
3 (b) (iii) amplitude = 2 metres period $= \dfrac{2\pi}{n}$ $= \dfrac{2\pi}{5}$ seconds	**2**	**2 marks** • Correct solution **1 mark** • Correctly finds either amplitude or acceleration
3 (b) (iv) when $t = 0$; $\dot{x} = -10\sin\dfrac{\pi}{4}$ $= -10\left(\dfrac{1}{\sqrt{2}}\right)$ $= -5\sqrt{2}$ m/s	**1**	**1 mark** • Correct answer

Solution	Marks	Comments
QUESTION 4		
4 (a) equating the z components: $3+4\lambda=\mu$ equating the y components: $2+3\lambda = 1+2\mu$ $= 7+8\lambda$ $5\lambda=-5$ $\lambda=-1$ $\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}1\\2\\3\end{pmatrix}-\begin{pmatrix}2\\3\\4\end{pmatrix}$ $=\begin{pmatrix}-1\\-1\\-1\end{pmatrix}$ $\therefore$ the points meet at $(-1,-1,-1)$	3	**3 marks** • Correct solution *Note: answer must be a point NOT a vector* **2 marks** • Finds either λ or μ **1 mark** • Establishes a relationship between λ and μ
4 (b) (i) $\overrightarrow{OB}=\overrightarrow{OA}+\overrightarrow{AB}$ $=\overrightarrow{OA}+\overrightarrow{OC}$ $=\begin{pmatrix}-6\\2\\-2\end{pmatrix}+\begin{pmatrix}3\\1\\2\end{pmatrix}$ $=\begin{pmatrix}-3\\3\\0\end{pmatrix}$ $\therefore$ B is the points meet at $(-3,3,0)$	2	**2 marks** • Correct solution **1 mark** • Uses a correct technique
4 (b) (ii) $\overrightarrow{MO}=-\frac{1}{2}(\overrightarrow{OB}+\overrightarrow{OC})$ $=-\frac{1}{2}\left[\begin{pmatrix}-3\\3\\0\end{pmatrix}+\begin{pmatrix}3\\1\\2\end{pmatrix}\right]$ $=\begin{pmatrix}0\\-2\\-1\end{pmatrix}$	1	**1 mark** • Correct answer
4 (b) (iii) $\overrightarrow{ON}=\overrightarrow{OA}+\frac{2}{3}\overrightarrow{AM}$ $=\begin{pmatrix}-6\\2\\-2\end{pmatrix}+\frac{2}{3}\left[\begin{pmatrix}0\\2\\1\end{pmatrix}-\begin{pmatrix}-6\\2\\-2\end{pmatrix}\right]$ $=\begin{pmatrix}-6\\2\\-2\end{pmatrix}+\begin{pmatrix}4\\0\\2\end{pmatrix}$ $=\begin{pmatrix}-2\\2\\0\end{pmatrix}$ **OR** $\overrightarrow{ON}=\frac{1}{3}\begin{pmatrix}-6\\2\\-2\end{pmatrix}+\frac{2}{3}\begin{pmatrix}0\\2\\1\end{pmatrix}$ $=\begin{pmatrix}-2\\2\\0\end{pmatrix}$	2	**2 marks** • Correct solution **1 mark** • Finds an expression for $\overrightarrow{ON}$ involving other vectors
4 (b) (iv) $\overrightarrow{ON}=\begin{pmatrix}-2\\2\\0\end{pmatrix}$ $\overrightarrow{OB}=\begin{pmatrix}-3\\3\\0\end{pmatrix}$ $=\frac{2}{3}\overrightarrow{ON}$ $\therefore$ $\overrightarrow{ON}\parallel\overrightarrow{OB}$ as they are scalar multiples of each other However as the two vectors share the common point O, O, B and N must be collinear.	2	**2 marks** • Correct solution **1 mark** • Shows that two relevant vectors are parallel