2024 **Mathematics Extension 2 Task 3**

Mathematics Extension 2

General Instructions

- Reading time – 5 minutes
- Working time – 1 hour
- Write using black pen
- Calculators approved by NESA may be used
- A reference sheet is provided at the back of this paper
- In Questions 6-8, show relevant mathematical reasoning and/or calculations

Total marks: 37

Section I – 5 marks (pages 2 – 3)

- Attempt Questions 1 – 5
- Allow about 8 minutes for this section.

Section II – 32 marks (pages 4 – 6)

- Attempt Questions 6 – 8
- Allow about 52 minutes for this section.

Section I

5 marks
Attempt Questions 1 – 5
Allow about 8 minutes for this section.

Use the multiple-choice answer page in the writing booklet for Questions 1 – 5.

1 A particle moves in a straight line with a displacement of x. When $t = 0$ the acceleration is $3x^2$, velocity is $-\sqrt{2}$ and displacement is 1. Which of the following is the correct equation for x as a function of t?

A. $x = -\dfrac{2}{(t+\sqrt{2})^2}$

B. $x = -\dfrac{2}{(t-\sqrt{2})^2}$

C. $x = \dfrac{2}{(t+\sqrt{2})^2}$

D. $x = \dfrac{2}{(t-\sqrt{2})^2}$

2 The spheres $x^2 + y^2 + z^2 = 9$ and $x^2 + (y-4)^2 + z^2 = 4$ intersect. What are the coordinates of the centre and radius of the circle in which they intersect?

A. Centre $\left(0, \dfrac{21}{8}, 0\right)$, radius $\dfrac{3\sqrt{15}}{8}$

B. Centre $\left(0, \dfrac{16}{9}, 0\right)$, radius $\dfrac{3\sqrt{15}}{8}$

C. Centre $\left(0, \dfrac{21}{8}, 0\right)$, radius $\dfrac{5\sqrt{15}}{8}$

D. Centre $\left(0, \dfrac{23}{8}, 0\right)$, radius $\dfrac{5\sqrt{15}}{8}$

3 A variable force acts on a particle causing it to move in a straight line. At time t seconds, its velocity v metres per second and position x metres from the origin are such that $v = e^x \cos x$.

Which of the following is an expression for the acceleration of the particle?

A. $-e^x \sin x$

B. $e^{2x} \cos x\,(\cos x - \sin x)$

C. $e^x(\sin x + \cos x)$

D. $2e^{2x}(\cos^2 x)$

4 A particle is moving in a straight line in simple harmonic motion. The displacement of the particle is given by $x = 6\cos^2 t - 1$, where x is the displacement in metres and t is the time in seconds.

Where is the centre of motion?

A. $x = -1$

B. $x = 0$

C. $x = 1$

D. $x = 2$

5 The lines $\underset{\sim}{r_1} = \begin{bmatrix} 1 \\ -6 \\ 4 \end{bmatrix} + \lambda \begin{bmatrix} 1 \\ 2 \\ a \end{bmatrix}$ and $\underset{\sim}{r_2} = \mu \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$, where λ and μ are scalar parameters and a is constant, are skew. Which of the following is NOT correct?

A. a can equal -2

B. a can equal -1

C. a can equal 1

D. a can equal 2

End of Section I

Section II

32 marks

Attempt Questions 6 – 8

Allow about 52 minutes for this section.

Instructions

- Answer the questions in the appropriate page of your answer booklet.
- Your responses should include relevant mathematical reasoning and/or calculations.
- Extra writing paper is available. If you use extra paper, write your name, and your teacher's name on the paper and clearly indicate which question you are answering.

Question 6 (9 marks) Start on the appropriate page of your answer booklet

a) Let A and B be the points on the line $\underset{\sim}{r} = \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ -2 \\ 1 \end{pmatrix}$ corresponding to $\lambda = 1$ and $\lambda = -1$ respectively. Let P be the point (1, 0, 2).

i) Find the projection of $\overrightarrow{AP}$ onto $\overrightarrow{AB}$. **3**

ii) Hence find the perpendicular distance d from P to the line AB. **2**

b) A particle is performing Simple Harmonic Motion in a straight line. At time t seconds it has displacement x metres from a fixed point O on the line, velocity v ms^{-1} given by $v^2 = 32 + 8x - 4x^2$ and acceleration a ms^{-2}.

i) Find an expression for a in terms of x. **1**

ii) Find the centre and amplitude of the motion. **2**

iii) Find the maximum speed of the particle. **1**

Question 7 (12 marks) Start on the appropriate page of your answer booklet

a) Relative to a fixed point at the origin, the points *B, C* and *D* are defined respectively by the position vectors $\underset{\sim}{b} = \underset{\sim}{i} - \underset{\sim}{j} + 2\underset{\sim}{k}$, $\underset{\sim}{c} = 2\underset{\sim}{i} - \underset{\sim}{j} + \underset{\sim}{k}$ and $\underset{\sim}{d} = a\underset{\sim}{i} - 2\underset{\sim}{j}$, where *a* is a real constant.

Given that the magnitude of angle *BCD* is $\frac{\pi}{3}$, find the value of *a*. **4**

b) A line has the Cartesian equation $\frac{x+2}{4} = \frac{y-4}{5} = -\frac{z+1}{3}$. Find the Cartesian equation of a parallel line passing through the point with position vector $-2i - 4j + 5k$. **4**

c) A sphere has a centre at (3, -3, 4) and its radius is 6 units.

A line has equation $\underset{\sim}{r} = \begin{pmatrix} 1 \\ 5 \\ 4 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ -1 \\ -1 \end{pmatrix}$.

i) Write down the vector equation of the sphere. **1**

ii) Determine whether the line is a tangent to the sphere, clearly justifying your conclusion. **3**

Question 8 (11 marks) Start on the appropriate page of your answer booklet

a) From an origin O, a parachutist of mass m kg falls from rest vertically downwards. The forces acting on the parachutist are his weight and a force due to air resistance of magnitude mkv^2 newtons, where k is a constant and v ms^{-1} is the parachutist's velocity. Let x be the parachutist's displacement in metres below O.

i) Show that the parachutist's equation of motion is $\ddot{x} = g - kv^2$ where g is the acceleration due to gravity. **1**

ii) Use integration to show that $v^2 = \frac{g}{k}\left(1 - e^{-2kx}\right)$. **2**

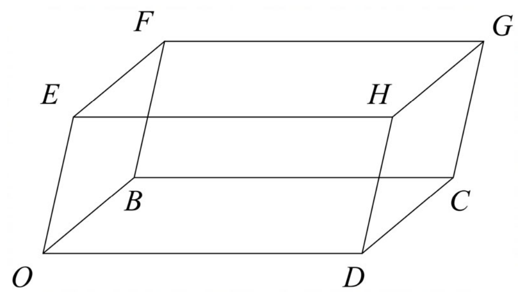
When the parachutist's velocity is $5g$ ms^{-1}, he opens his parachute. The motion is now modelled by assuming that the magnitude of the force due to air resistance instantaneously changes to $\frac{mgv}{10}$ newtons. The time from the parachute opening is t seconds.

iii) Use integration to show that $v = 10 + 5(g-2)e^{\frac{-gt}{10}}$. **3**

iv) It takes T seconds for the parachutist's velocity to decrease to $2g$ ms^{-1}. **1**

Show that $T = \frac{10}{g}\ln\left[\frac{5(g-2)}{2(g-5)}\right]$.

b) A parallelepiped is an oblique prism that has a parallelogram cross-section. It has three pairs of parallel and congruent faces.



$OBCDEFGH$ is a parallelepiped with $\underset{\sim}{b} = \overrightarrow{OB}$, $\underset{\sim}{d} = \overrightarrow{OD}$ and $\underset{\sim}{e} = \overrightarrow{OE}$.

i) Express $\overrightarrow{OG}$ in terms of $\underset{\sim}{b}$, $\underset{\sim}{d}$ and $\underset{\sim}{e}$. **1**

ii) Prove that $|\overrightarrow{OG}|^2 + |\overrightarrow{DF}|^2 + |\overrightarrow{BH}|^2 + |\overrightarrow{CE}|^2 = 4(|\underset{\sim}{b}|^2 + |\underset{\sim}{d}|^2 + |\underset{\sim}{e}|^2)$. **3**

End of Paper

BAULKHAM HILLS HIGH SCHOOL
YEAR 12 H3 TEST 2024
MATHEMATICS EXTENSION 2

MARKING COVER SHEET

FINAL MARK

NESA #:________________________________ Teacher:________________________________

		Outcomes		
Qn	**Parts**	**Vectors**	**Mechanics**	**Total**
Multiple Choice		2,5	1,3,4	
		/2	/3	/5
7	a	/5		
	b		/4	/9
8	a	/4		
	b	/4		
	c	/4		/12
9	a		/7	
	b	/4		/11
	Total	/23	/14	/37

Extension 2 Task 3 2024 solutions

Multiple choice

1. when $t=0$, both A and B have acceleration of -1.

Consider C: $x = 2(t+\sqrt{2})^{-2}$

$$\frac{dx}{dt} = -4(t+\sqrt{2})^{-3}$$

$$v = \frac{-4}{(t+\sqrt{2})^3}$$

when $t=0$, $v = \frac{-4}{2\sqrt{2}}$

$$v = -\sqrt{2}\ ms^{-1}$$

$\therefore$ C

OR

$$a = 3x^2$$

$$v\frac{dv}{dx} = 3x^2$$

$$\int_{-2}^{v} v\,dv = \int_{1}^{x} 3x^2\,dx$$

$$\left[\frac{v^2}{2}\right]_{-\sqrt{2}}^{v} = \left[x^3\right]_1^x$$

$$\frac{v^2}{2} - 1 = x^3 - 1$$

$$\frac{v^2}{2} = x^3$$

$$v^2 = 2x$$

when $x = 1$, $v = -\sqrt{2}$

$$\therefore v = -\sqrt{2}\sqrt{x^3}$$

$$\frac{dx}{dt} = -\sqrt{2}\,x^{3/2}$$

$$\int_1^x x^{-3/2}\,dx = -\sqrt{2}\int_0^t dt$$

$$-2\left[x^{-1/2}\right]_1^x = -\sqrt{2}\left[t\right]_0^t$$

$$\frac{1}{\sqrt{x}} - 1 = \frac{1}{\sqrt{2}}t$$

$$\frac{1}{\sqrt{x}} = \frac{t}{\sqrt{2}} + \frac{\sqrt{2}}{\sqrt{2}}$$

$$\frac{1}{\sqrt{x}} = \frac{\sqrt{2}+t}{\sqrt{2}}$$

$$\sqrt{x} = \frac{\sqrt{2}}{\sqrt{2}+t}$$

$$x = \frac{2}{(t+\sqrt{2})^2}$$

② $x^2+y^2+z^2=9$ $-$

$x^2+(y-4)^2+z^2=4$

$8y-16=5$

$8y=21$

$y=\frac{21}{8}$

when $y=\frac{21}{8}$ $x^2+y^2+z^2=9$

$x^2+z^2=\frac{135}{64}$

$\therefore$ centre is $(0,\frac{21}{8},0)$ and radius is $\frac{3\sqrt{15}}{8}$

$\therefore A$

③ $v=e^x\cos x$

$a=v\frac{dv}{dx}$

$a=e^x\cos x(\cos x e^x-e^x\sin x)$

$a=e^{2x}\cos x(\cos x-\sin x)$

$\therefore$ B

④ $x=6\cos^2 t-1$

$\cos 2\theta=2\cos^2\theta-1$

$x=3(\cos 2t+1)-1$

$x=3\cos 2t+2$

$\therefore$ Centre of motion is $x=2$.

$\therefore$ D

⑤ For lines to meet, (equating x and y)

$1+\lambda=\mu$ ①

$-6+2\lambda=-2\mu$ ②

①×2: $2+2\lambda=2\mu$ ③

②−③: $-8=-4\mu$

$\therefore \mu=2$

$1+\lambda = 2$
$\lambda = 1$
$\therefore \lambda = 1 \;\&\; \mu = 2.$

For lines to meet $\quad 4+\lambda a = \mu$
$4 + a = 2$
$a = -2$

$\therefore$ For lines to be skew $a \neq -2$

$\therefore$ A

6 a i) Point A: $\lambda = 1$ $\quad A = (5,-1,0)$

(2) correct solution
(1) finds $\overrightarrow{AB}$ or $\overrightarrow{AP}$

Point B: $\lambda = -1$ $\quad B = (-1,3,-2)$

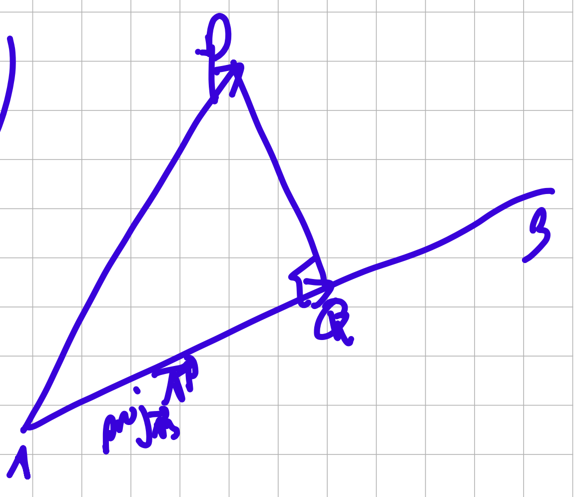
$\overrightarrow{AP} = \begin{pmatrix} -4 \\ 1 \\ 2 \end{pmatrix} \qquad \overrightarrow{AB} = \begin{pmatrix} -6 \\ 4 \\ -2 \end{pmatrix}$

$$\text{proj}_{\overrightarrow{AB}}\overrightarrow{AP} = \frac{\overrightarrow{AP}\cdot\overrightarrow{AB}}{|\overrightarrow{AB}|^2}\overrightarrow{AB}$$

$$= \frac{24+4-4}{(\sqrt{56})^2}\begin{pmatrix} -6 \\ 4 \\ -2 \end{pmatrix}$$

$$= \frac{24}{56} \times 2\begin{pmatrix} -3 \\ 2 \\ -1 \end{pmatrix}$$

$$= \frac{6}{7}\begin{pmatrix} -3 \\ 2 \\ -1 \end{pmatrix}$$



6 a) ii) Perp distance $= \left|\overrightarrow{AP} - \text{proj}_{\overrightarrow{AB}}\overrightarrow{AP}\right|$

$$= \left|\begin{pmatrix} -4 \\ 1 \\ 2 \end{pmatrix} - \begin{pmatrix} -\frac{18}{7} \\ \frac{12}{7} \\ -\frac{6}{7} \end{pmatrix}\right|$$

(2) correct solution
(1) finds expression for perpendicular distance

$$=\sqrt{\frac{75}{7}}$$

$$=\frac{5\sqrt{3}}{\sqrt{7}} \text{ or } \frac{5\sqrt{21}}{7} \text{ units}$$

6 b) i) $a=\frac{d}{dx}\left(\frac{1}{2}v^2\right)$

1 correct solution

$$a=\frac{d}{dx}\left(\frac{1}{2}(32+8x-4x^2)\right)$$

$$=\frac{d}{dx}\left(16+4x-2x^2\right)$$

$$\therefore \ddot{x}=4-4x$$

6 b ii) $\ddot{x}=-4(x-1)$

2 correct solution

1 finds centre of motion or amplitude

$\therefore$ centre of motion is $x=1$

$$v^2=32+8x-4x^2$$

when $v=0$ $\quad 0=8+2x-x^2$

$$x^2-2x-8=0$$

$$(x-4)(x+2)=0$$

$$x=-2 \text{ or } x=4$$

$$\text{Amplitude}=\frac{4--2}{2}$$

$$=3$$

6 b iii) Max speed occurs at centre of motion

1 correct solution

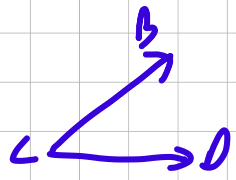
when $x=1$, $v^2=32+8-4$

$$v^2=36$$

max speed $=6\ ms^{-1}$

7 a) $\underset{\sim}{b} = \underset{\sim}{i} - \underset{\sim}{j} + 2\underset{\sim}{k}$ $\quad \underset{\sim}{c} = 2\underset{\sim}{i} - \underset{\sim}{j} + \underset{\sim}{k}$ $\quad \underset{\sim}{d} = a\underset{\sim}{i} - 2\underset{\sim}{j}$

$\angle BCD = \frac{\pi}{3}$



④ correct solution

③ obtains expression in a

② substitutes into $\underset{\sim}{a} \cdot \underset{\sim}{b} = |\underset{\sim}{a}||\underset{\sim}{b}|\cos\theta$

① finds $\overrightarrow{CB}$ or $\overrightarrow{CD}$

$$\overrightarrow{CB} = \overrightarrow{OB} - \overrightarrow{OC}$$
$$\overrightarrow{CB} = -\underset{\sim}{i} + \underset{\sim}{k}$$

$$\overrightarrow{CD} = \overrightarrow{OD} - \overrightarrow{OC}$$
$$\overrightarrow{CD} = (a-2)\underset{\sim}{i} - \underset{\sim}{j} - \underset{\sim}{k}$$

$$\cos \angle BCD = \frac{\overrightarrow{CB} \cdot \overrightarrow{CD}}{|\overrightarrow{CB}||\overrightarrow{CD}|}$$

$$\cos \frac{\pi}{3} = \frac{-a+2-1}{\sqrt{2}\sqrt{(a-2)^2+1+1}}$$

$$\frac{1}{2} = \frac{a-1}{\sqrt{2}\sqrt{a^2-4a+6}}$$

$$\sqrt{2}\sqrt{a^2-4a+6} = 2(a-1)$$

$$2(a^2-4a+6) = 4(a-1)^2$$

$$a^2-4a+6 = 2(a^2-2a+1)$$

$$a^2-4a+6 = 2a^2-4a+2$$

$$4 = a^2$$

$$a = \pm 2$$

7 b) $\frac{x}{4} + \frac{1}{2} = \frac{4-y}{-5} = \frac{z+1}{-3}$

(4) correct solution

(3) finds vector form of line

(2) finds direction vector

(1) finds a parametric equation for x, y or z

$\therefore \frac{x}{4} + \frac{1}{2} = \lambda$

$x + 2 = 4\lambda$

$x = -2 + 4\lambda$

$\frac{4-y}{-5} = \lambda$

$4 - y = -5\lambda$

$y = 4 + 5\lambda$

$\lambda = \frac{z+1}{-3}$

$-3\lambda = z+1$

$z = -1 - 3\lambda$

$\therefore$ Line is $\underset{\sim}{q} = \begin{pmatrix} -2 \\ 4 \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} 4 \\ 5 \\ -3 \end{pmatrix}$

Line passing through $\begin{pmatrix} -2 \\ -4 \\ 5 \end{pmatrix}$ is

$\underset{\sim}{r} = \begin{pmatrix} -2 \\ -4 \\ 5 \end{pmatrix} + \lambda \begin{pmatrix} 4 \\ 5 \\ -3 \end{pmatrix}$

$x = -2 + 4\lambda$ $\quad$ $y = -4 + 5\lambda$ $\quad$ $z = 5 - 3\lambda$

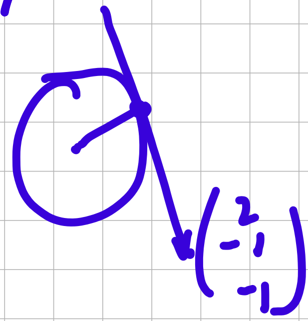
$4\lambda = x + 2$ $\quad$ $5\lambda = y + 4$ $\quad$ $3\lambda = -z + 5$

$\lambda = \frac{x+2}{4}$ $\quad$ $\lambda = \frac{y+4}{5}$ $\quad$ $\lambda = \frac{-z+5}{3}$

$\therefore$ Cartesian form $\frac{x+2}{4} = \frac{y+4}{5} = \frac{z-5}{-3}$

7c i) Let z be a point on the sphere ① Correct equation

$\therefore \left| z - \begin{pmatrix} 3 \\ -3 \\ 4 \end{pmatrix} \right| = 6$ is the equation of the sphere



7c ii) $x = 1+2\lambda$

$y = 5-\lambda$

$z = 4-\lambda$

$\therefore (1+2\lambda-3)^2 + (5-\lambda+3)^2 + (4-\lambda-4)^2 = 36$

$(2\lambda-2)^2 + (8-\lambda)^2 + \lambda^2 = 36$

$4\lambda^2 - 8\lambda + 4 + 64 - 16\lambda + \lambda^2 + \lambda^2 = 36$

$6\lambda^2 - 24\lambda + 68 = 36$

$6\lambda^2 - 24\lambda + 32 = 0$

$3\lambda^2 - 12\lambda + 16 = 0$

$\lambda = \dfrac{12 \pm \sqrt{144 - 4 \times 3 \times 16}}{6}$

$= \dfrac{12 \pm \sqrt{-48}}{6}$

no real solution

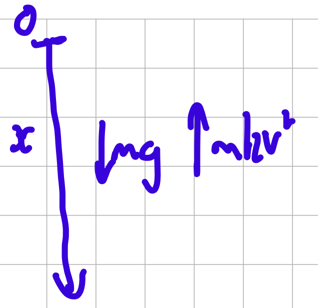
$\therefore$ Line does not intersect the sphere.

③ correct solution

② solves/finds discriminant

① forms equation in λ

8a i)



① correct solution

$$f = ma$$
$$m\ddot{x} = mg - mkv^2$$
$$\ddot{x} = g - kv^2$$

8a ii)

② correct solution

① correct primitive

$$v\frac{dv}{dx} = g - kv^2$$
$$\int_0^v \frac{v\,dv}{g - kv^2} = \int_0^x dx$$
$$[x]_0^x = \frac{-1}{2k}\int_0^v \frac{-2kv\,dv}{g-kv^2}$$
$$x - 0 = \frac{-1}{2k}\Big[\ln|g - kv^2|\Big]_0^v$$
$$-2kx = \ln|g - kv^2| - \ln|g|$$
$$-2kx = \ln\left|\frac{g - kv^2}{g}\right|$$
$$e^{-2kx} = \frac{g - kv^2}{g}$$
$$ge^{-2kx} = g - kv^2$$
$$kv^2 = g(1 - e^{-2kx})$$
$$v^2 = \frac{g}{k}(1 - e^{-2kx})$$

8a iii)

$$m\ddot{x} = mg - \frac{mgv}{10}$$

$$\ddot{x} = g - \frac{gv}{10}$$

$$\frac{dv}{dt} = \frac{10g - gv}{10}$$

$$\frac{1}{g}\int_{5g}^{v} \frac{10\,dv}{10-v} = \int_0^t dt$$

$$\frac{10}{-g}\int_{5g}^{v} \frac{-1}{10-v}\,dv = \Big[t\Big]_0^t$$

$$-\frac{10}{g}\Big[\ln|10-v|\Big]_{5g}^{v} = t - 0$$

$$\frac{-gt}{10} = \ln|10-v| - \ln|10-5g|$$

$$\frac{-gt}{10} = \ln\left|\frac{10-v}{10-5g}\right| \quad (1)$$

$$e^{-\frac{gt}{10}} = \frac{10-v}{10-5g}$$

$$5e^{-\frac{gt}{10}}(2-g) = 10 - v$$

$$v = 10 - 5e^{-\frac{gt}{10}}(2-g)$$

$$\therefore v = 10 + 5(g-2)e^{-\frac{gt}{10}}$$ as required

(3) correct solution

(2) Integrates correctly

(1) correct integral

8a iv) When $t = T$, $v = 2g$

From (1) above $\frac{-gT}{10} = \ln\left|\frac{10-2g}{10-5g}\right|$

$$T = -\frac{10}{g}\ln\left|\frac{2(5-g)}{5(2-g)}\right|$$

$$T = \frac{10}{g}\ln\left|\frac{5(2-g)}{2(5-g)}\right|$$

$$\therefore T = \frac{10}{g}\ln\left[\frac{5(g-2)}{2(g-5)}\right]$$

(1) correct proof

8b i) $\overrightarrow{OG} = \overrightarrow{OB} + \overrightarrow{BC} + \overrightarrow{CG}$

$= \mathbf{b} + \overrightarrow{OD} + \overrightarrow{OE}$ $\quad\overrightarrow{OD} = \overrightarrow{BC}$, $\overrightarrow{CG} = \overrightarrow{OE}$ opposite sides of parallelopiped are equal and parallel

$\overrightarrow{OG} = \mathbf{b} + \mathbf{d} + \mathbf{e}$

① correct expression

ii) Similarly, $\overrightarrow{OF} = \mathbf{b} + \mathbf{e} - \mathbf{d}$

$\overrightarrow{BH} = \mathbf{d} - \mathbf{b} + \mathbf{e}$

$\overrightarrow{CE} = -\mathbf{d} - \mathbf{b} + \mathbf{e}$

③ correct proof

② expresses sum as dot products

① finds expressions for $\overrightarrow{OF}$, $\overrightarrow{BH}$ and $\overrightarrow{CE}$

$|\overrightarrow{OG}|^2 + |\overrightarrow{OF}|^2 + |\overrightarrow{BH}|^2 + |\overrightarrow{CE}|^2$

$= (\mathbf{b}+\mathbf{d}+\mathbf{e})\cdot(\mathbf{b}+\mathbf{d}+\mathbf{e}) + (\mathbf{b}+\mathbf{e}-\mathbf{d})\cdot(\mathbf{b}+\mathbf{e}-\mathbf{d}) + (\mathbf{d}-\mathbf{b}+\mathbf{e})\cdot(\mathbf{d}-\mathbf{b}+\mathbf{e})$
$\quad + (-\mathbf{d}-\mathbf{b}+\mathbf{e})\cdot(-\mathbf{d}-\mathbf{b}+\mathbf{e})$

$= \mathbf{b}\cdot\mathbf{b} + \mathbf{d}\cdot\mathbf{d} + \mathbf{e}\cdot\mathbf{e} + \cancel{\mathbf{b}\cdot\mathbf{d}} + \cancel{\mathbf{b}\cdot\mathbf{e}} + \cancel{\mathbf{d}\cdot\mathbf{b}} + \cancel{\mathbf{d}\cdot\mathbf{e}} + \cancel{\mathbf{e}\cdot\mathbf{b}} + \cancel{\mathbf{e}\cdot\mathbf{d}}$

$\quad + \mathbf{b}\cdot\mathbf{b} + \mathbf{e}\cdot\mathbf{e} + \mathbf{d}\cdot\mathbf{d} + 2\cancel{\mathbf{b}\cdot\mathbf{e}} - 2\cancel{\mathbf{b}\cdot\mathbf{d}} - 2\cancel{\mathbf{d}\cdot\mathbf{e}}$

$\quad + \mathbf{d}\cdot\mathbf{d} - 2\cancel{\mathbf{b}\cdot\mathbf{d}} + 2\cancel{\mathbf{d}\cdot\mathbf{e}} - 2\mathbf{b}\cdot\mathbf{e} + \mathbf{b}\cdot\mathbf{b} + \mathbf{e}\cdot\mathbf{e}$

$\quad + \mathbf{d}\cdot\mathbf{d} + \mathbf{b}\cdot\mathbf{b} + \mathbf{e}\cdot\mathbf{e} + 2\cancel{\mathbf{b}\cdot\mathbf{d}} - 2\cancel{\mathbf{b}\cdot\mathbf{e}} - 2\cancel{\mathbf{d}\cdot\mathbf{e}}$

$= 4|\mathbf{b}|^2 + 4|\mathbf{d}|^2 + 4|\mathbf{e}|^2$

$= 4(|\mathbf{b}|^2 + |\mathbf{d}|^2 + |\mathbf{e}|^2)$