CRANBROOK SCHOOL

Student Number: ____________________

2022 **TASK 1**

Mathematics Extension 2

General Instructions

- Reading time – 5 minutes
- Working time – 90 minutes
- Write using blue or black pen
- Calculators approved by NESA may be used
- A reference sheet is provided at the back of this paper
- For all questions show relevant mathematical reasoning and/or calculations

Total Marks: 50

Section I – 5 marks

- Attempt Questions 1 – 3 on the Multiple Choice Answer sheet.

Section II – 45 marks

- Attempt Questions 6 – 8 in separate booklets.

Multiple Choice

5 Marks
Attempt Questions 1 – 5

Answer each question on your Multiple Choice Answer Sheet

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1 What is the distance between the point $(3, -5, 2)$ and the z axis?

A. $\sqrt{13}$

B. $\sqrt{29}$

C. $\sqrt{34}$

D. $\sqrt{38}$

2 Let $\overrightarrow{OA} = -4\underset{\sim}{i} - 3\underset{\sim}{j} - 2\underset{\sim}{k}$ and $\overrightarrow{OB} = \underset{\sim}{i} - 5\underset{\sim}{j} - \underset{\sim}{k}$.

What the scalar projection of $\overrightarrow{OB}$ onto $\overrightarrow{OA}$?

A. $\dfrac{13}{3\sqrt{3}}$

B. $\dfrac{13}{\sqrt{29}}$

C. $\dfrac{13}{3\sqrt{87}}$

D. 13

3 Which of the following vectors lie within the plane $\begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} \cdot \left(\underset{\sim}{p} - \begin{pmatrix} -5 \\ 0 \\ -1 \end{pmatrix} \right) = 0$?

A. $\begin{pmatrix} 2 \\ -3 \\ 4 \end{pmatrix}$

B. $\begin{pmatrix} -4 \\ 3 \\ -6 \end{pmatrix}$

C. $\begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}$

D. $\begin{pmatrix} -2 \\ 4 \\ 4 \end{pmatrix}$

4 The vectors $\underset{\sim}{m} = -3\underset{\sim}{i} - 2\underset{\sim}{j} + \lambda\underset{\sim}{k}$ and $\underset{\sim}{n} = 3\lambda\underset{\sim}{i} + 10\underset{\sim}{j} - 5\underset{\sim}{k}$ are perpendicular.

What is the value of λ?

A. -2

B. $-\dfrac{5}{7}$

C. $\dfrac{5}{7}$

D. 2

5 Consider the lines $\underset{\sim}{l_1} = \begin{pmatrix} 2 \\ -1 \\ 2 \end{pmatrix} + \lambda_1 \begin{pmatrix} -1 \\ 1 \\ 3 \end{pmatrix}$ and $\underset{\sim}{l_2} = \begin{pmatrix} 4 \\ -4 \\ 4 \end{pmatrix} + \lambda_2 \begin{pmatrix} 3 \\ -3 \\ 9 \end{pmatrix}$.

Which of the following best describes the relationship between l_1 and l_2?

A. Parallel

B. Co-incident

C. Perpendicular

D. Orthogonal

End of Section I

Extended Response

45 Marks
Attempt Questions 6 – 8

Answer each question in a separate writing booklet.

In Questions 6 to 8, your responses should include relevant mathematical reasoning and/or calculations.

Question 6 (15 marks) Begin a new writing booklet.

(a) Let $\underset{\sim}{a} = 5\underset{\sim}{i} + 2\underset{\sim}{j} - \underset{\sim}{k}$ and $\underset{\sim}{b} = -4\underset{\sim}{i} - 4\underset{\sim}{j} + 3\underset{\sim}{k}$.

Find:

(i) $3\underset{\sim}{a} - 4\underset{\sim}{b}$ **1**

(ii) $|\underset{\sim}{a}|$ **1**

(iii) $\underset{\sim}{\hat{a}}$ **1**

(iv) The angle between $\underset{\sim}{a}$ and $\underset{\sim}{b}$. **2**

(v) The vector projection of $\underset{\sim}{b}$ onto $\underset{\sim}{a}$. **2**

(b) A parallelogram $OMPQ$ is created with the vectors $\overrightarrow{OP} = \underset{\sim}{p}$ and $\overrightarrow{OQ} = \underset{\sim}{q}$.

(i) Find the vector $\overrightarrow{OM}$ in terms of $\underset{\sim}{p}$ and $\underset{\sim}{q}$ **1**

(ii) It is known that $OMPQ$ is a rhombus. Prove that the diagonals of $OMPQ$ are perpendicular. **3**

(c) (i) What can be said about the vectors $\underset{\sim}{m}$ and $\underset{\sim}{n}$ if the scalar projection of $\underset{\sim}{m}$ onto $\underset{\sim}{n}$ is the same as the scalar projection of $\underset{\sim}{n}$ onto $\underset{\sim}{m}$? Justify your answer. **2**

(ii) What can be said about the vectors $\underset{\sim}{m}$ and $\underset{\sim}{n}$ if the **vector** projection of $\underset{\sim}{m}$ onto $\underset{\sim}{n}$ is the same as the **vector** projection of $\underset{\sim}{n}$ onto $\underset{\sim}{m}$? Justify your answer. **2**

End of Question 6

Question 7 (15 marks) Begin a new writing booklet.

(a) The parallelogram $ABCD$ has vertices at the end points of the position vectors

$\underset{\sim}{a} = 2\underset{\sim}{i} + 5\underset{\sim}{j} - 4\underset{\sim}{k}$, $\underset{\sim}{b} = -\underset{\sim}{i} - \underset{\sim}{j} - 2\underset{\sim}{k}$, $\underset{\sim}{c}$ and $\underset{\sim}{d} = -5\underset{\sim}{i} + \underset{\sim}{j} + 5\underset{\sim}{k}$

(i) Find the vectors $\overrightarrow{AB}$ and $\overrightarrow{AD}$ **2**

(ii) Hence find the value of $\underset{\sim}{c}$. **2**

(iii) Using vector methods, find the area of $ABCD$ **3**

(b) Two spheres, S_1 and S_2, have equations $x^2 + y^2 + z^2 = 16$ and $(x-3)^2 + y^2 + z^2 = 20$.

(i) Find the value of x at which the two spheres meet. **1**

(ii) Hence find the equation of the circle of intersection between the two spheres. **2**

(c) Let the vector equation of a curve be $\underset{\sim}{r}(t) = \begin{pmatrix} 2\tan t \\ -1 \\ \cot 2t \end{pmatrix}$ **2**

Find the Cartesian equation of the curve.

(d) Determine if the following vectors are linearly independent. **3**

$$\underset{\sim}{a} = \begin{pmatrix} 5 \\ 2 \\ 1 \end{pmatrix} \qquad \underset{\sim}{b} = \begin{pmatrix} -1 \\ 2 \\ -3 \end{pmatrix} \qquad \underset{\sim}{c} = \begin{pmatrix} 8 \\ -3 \\ -2 \end{pmatrix}$$

End of Question 7

Question 8 (15 marks) Begin a new writing booklet.

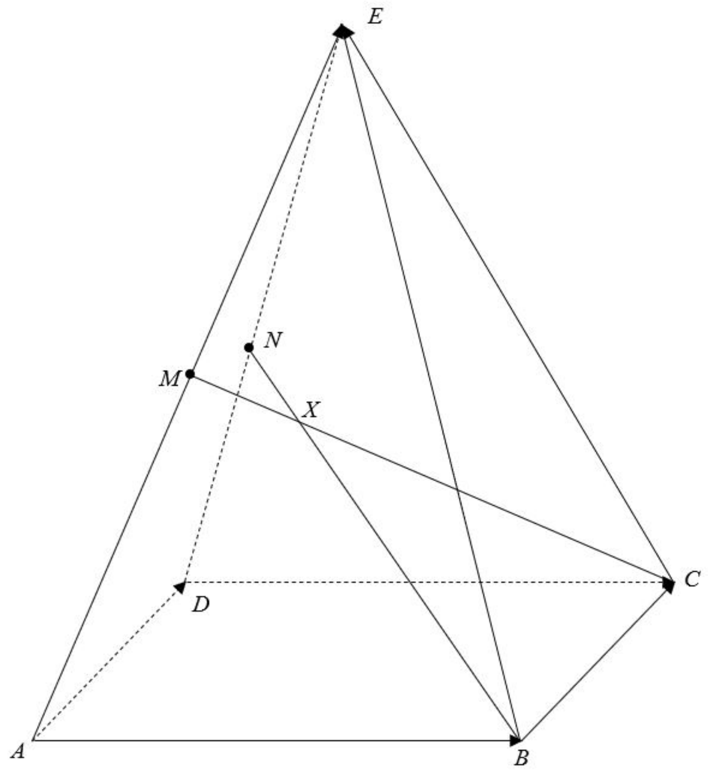
(a) Let the three points A, B and C represent the points $(3,-2,-1)$, $(-5,1,1)$ and $(9,-4,-1)$.

(i) The vector equation of the line AB. **3**

(ii) Find the minimum distance between the line AB and the point C. **3**

(b) The solid $ABCDE$ is a right rectangular pyramid with the point E as the apex. The midpoints of AE and DE are M and N respectively. It is known that the lines MC and NB meet at the point X.

Let $\overrightarrow{AB} = \underset{\sim}{p}$, $\overrightarrow{AD} = \underset{\sim}{q}$ and $\overrightarrow{AE} = \underset{\sim}{r}$.



Question 8 continues on page 10

Question 8 (continued)

(i) Find the vector $\overrightarrow{DE}$ in terms of $\underset{\sim}{p}$, $\underset{\sim}{q}$ and/or $\underset{\sim}{r}$. **1**

(ii) Find the vectors $\overrightarrow{NB}$ and $\overrightarrow{MC}$ in terms of $\underset{\sim}{p}$, $\underset{\sim}{q}$ and/or $\underset{\sim}{r}$. **2**

(iii) Hence or otherwise show that $\dfrac{|\overrightarrow{MX}|}{|\overrightarrow{XC}|} = \dfrac{|\overrightarrow{NX}|}{|\overrightarrow{XB}|} = \dfrac{1}{2}$. **3**

The solid is now place on a three dimensional Cartesian plane such that A is on the origin, $B = (4,0,0)$, $D = (0,2,0)$ and $E = (2,1,5)$.

(iv) Find the smallest angle between the vector $\overrightarrow{EX}$ and the plane containing $\triangle XBC$. **3**

End of Question 8

NSW GOVERNMENT

NSW Education Standards Authority

2021 HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Advanced
Mathematics Extension 1
Mathematics Extension 2

REFERENCE SHEET

Measurement

Length

$$l = \frac{\theta}{360} \times 2\pi r$$

Area

$$A = \frac{\theta}{360} \times \pi r^2$$

$$A = \frac{h}{2}\left(a + b\right)$$

Surface area

$$A = 2\pi r^2 + 2\pi rh$$

$$A = 4\pi r^2$$

Volume

$$V = \frac{1}{3}Ah$$

$$V = \frac{4}{3}\pi r^3$$

Functions

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

For $ax^3 + bx^2 + cx + d = 0$:

$$\alpha + \beta + \gamma = -\frac{b}{a}$$

$$\alpha\beta + \alpha\gamma + \beta\gamma = \frac{c}{a}$$

and $\alpha\beta\gamma = -\frac{d}{a}$

Relations

$$\left(x - h\right)^2 + \left(y - k\right)^2 = r^2$$

Financial Mathematics

$$A = P\left(1 + r\right)^n$$

Sequences and series

$$T_n = a + \left(n - 1\right)d$$

$$S_n = \frac{n}{2}\left[2a + \left(n - 1\right)d\right] = \frac{n}{2}\left(a + l\right)$$

$$T_n = ar^{n-1}$$

$$S_n = \frac{a\left(1 - r^n\right)}{1 - r} = \frac{a\left(r^n - 1\right)}{r - 1}, \; r \neq 1$$

$$S = \frac{a}{1 - r}, \; |r| < 1$$

Logarithmic and Exponential Functions

$$\log_a a^x = x = a^{\log_a x}$$

$$\log_a x = \frac{\log_b x}{\log_b a}$$

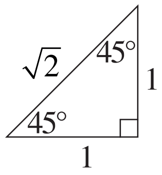
$$a^x = e^{x \ln a}$$

Trigonometric Functions

$$\sin A = \frac{\text{opp}}{\text{hyp}}, \quad \cos A = \frac{\text{adj}}{\text{hyp}}, \quad \tan A = \frac{\text{opp}}{\text{adj}}$$

$$A = \frac{1}{2}ab\sin C$$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

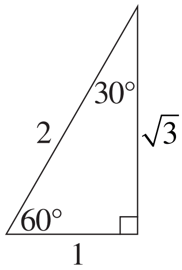


$$c^2 = a^2 + b^2 - 2ab\cos C$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

$$l = r\theta$$

$$A = \frac{1}{2}r^2\theta$$



Trigonometric identities

$$\sec A = \frac{1}{\cos A}, \ \cos A \neq 0$$

$$\operatorname{cosec} A = \frac{1}{\sin A}, \ \sin A \neq 0$$

$$\cot A = \frac{\cos A}{\sin A}, \ \sin A \neq 0$$

$$\cos^2 x + \sin^2 x = 1$$

Compound angles

$$\sin(A + B) = \sin A\cos B + \cos A\sin B$$

$$\cos(A + B) = \cos A\cos B - \sin A\sin B$$

$$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}$$

If $t = \tan\frac{A}{2}$ then $\sin A = \frac{2t}{1+t^2}$

$$\cos A = \frac{1-t^2}{1+t^2}$$

$$\tan A = \frac{2t}{1-t^2}$$

$$\cos A\cos B = \frac{1}{2}\left[\cos(A - B) + \cos(A + B)\right]$$

$$\sin A\sin B = \frac{1}{2}\left[\cos(A - B) - \cos(A + B)\right]$$

$$\sin A\cos B = \frac{1}{2}\left[\sin(A + B) + \sin(A - B)\right]$$

$$\cos A\sin B = \frac{1}{2}\left[\sin(A + B) - \sin(A - B)\right]$$

$$\sin^2 nx = \frac{1}{2}(1 - \cos 2nx)$$

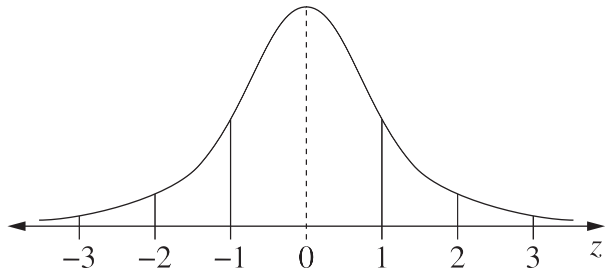
$$\cos^2 nx = \frac{1}{2}(1 + \cos 2nx)$$

Statistical Analysis

$$z = \frac{x - \mu}{\sigma}$$

An outlier is a score
less than $Q_1 - 1.5 \times IQR$
or
more than $Q_3 + 1.5 \times IQR$

Normal distribution



- approximately 68% of scores have z-scores between –1 and 1
- approximately 95% of scores have z-scores between –2 and 2
- approximately 99.7% of scores have z-scores between –3 and 3

$$E(X) = \mu$$

$$\operatorname{Var}(X) = E\left[(X - \mu)^2\right] = E\left(X^2\right) - \mu^2$$

Probability

$$P(A \cap B) = P(A)P(B)$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}, \ P(B) \neq 0$$

Continuous random variables

$$P(X \le r) = \int_a^r f(x)\,dx$$

$$P(a < X < b) = \int_a^b f(x)\,dx$$

Binomial distribution

$$P(X = r) = {}^nC_r p^r (1 - p)^{n-r}$$

$$X \sim \operatorname{Bin}(n, p)$$

$$\Rightarrow \ P(X = x) = \binom{n}{x} p^x (1 - p)^{n-x}, \ x = 0, 1, \ldots, n$$

$$E(X) = np$$

$$\operatorname{Var}(X) = np(1 - p)$$

Differential Calculus

Function	Derivative
$y = f(x)^n$	$\dfrac{dy}{dx} = nf'(x)[f(x)]^{n-1}$
$y = uv$	$\dfrac{dy}{dx} = u\dfrac{dv}{dx} + v\dfrac{du}{dx}$
$y = g(u)$ where $u = f(x)$	$\dfrac{dy}{dx} = \dfrac{dy}{du} \times \dfrac{du}{dx}$
$y = \dfrac{u}{v}$	$\dfrac{dy}{dx} = \dfrac{v\dfrac{du}{dx} - u\dfrac{dv}{dx}}{v^2}$
$y = \sin f(x)$	$\dfrac{dy}{dx} = f'(x)\cos f(x)$
$y = \cos f(x)$	$\dfrac{dy}{dx} = -f'(x)\sin f(x)$
$y = \tan f(x)$	$\dfrac{dy}{dx} = f'(x)\sec^2 f(x)$
$y = e^{f(x)}$	$\dfrac{dy}{dx} = f'(x)e^{f(x)}$
$y = \ln f(x)$	$\dfrac{dy}{dx} = \dfrac{f'(x)}{f(x)}$
$y = a^{f(x)}$	$\dfrac{dy}{dx} = (\ln a)f'(x)a^{f(x)}$
$y = \log_a f(x)$	$\dfrac{dy}{dx} = \dfrac{f'(x)}{(\ln a)f(x)}$
$y = \sin^{-1} f(x)$	$\dfrac{dy}{dx} = \dfrac{f'(x)}{\sqrt{1-[f(x)]^2}}$
$y = \cos^{-1} f(x)$	$\dfrac{dy}{dx} = -\dfrac{f'(x)}{\sqrt{1-[f(x)]^2}}$
$y = \tan^{-1} f(x)$	$\dfrac{dy}{dx} = \dfrac{f'(x)}{1+[f(x)]^2}$

Integral Calculus

$$\int f'(x)[f(x)]^n dx = \frac{1}{n+1}[f(x)]^{n+1} + c$$

where $n \neq -1$

$$\int f'(x)\sin f(x)\,dx = -\cos f(x) + c$$

$$\int f'(x)\cos f(x)\,dx = \sin f(x) + c$$

$$\int f'(x)\sec^2 f(x)\,dx = \tan f(x) + c$$

$$\int f'(x)e^{f(x)} dx = e^{f(x)} + c$$

$$\int \frac{f'(x)}{f(x)} dx = \ln|f(x)| + c$$

$$\int f'(x)a^{f(x)} dx = \frac{a^{f(x)}}{\ln a} + c$$

$$\int \frac{f'(x)}{\sqrt{a^2-[f(x)]^2}} dx = \sin^{-1}\frac{f(x)}{a} + c$$

$$\int \frac{f'(x)}{a^2+[f(x)]^2} dx = \frac{1}{a}\tan^{-1}\frac{f(x)}{a} + c$$

$$\int u\frac{dv}{dx} dx = uv - \int v\frac{du}{dx} dx$$

$$\int_a^b f(x)\,dx$$

$$\approx \frac{b-a}{2n}\left\{f(a) + f(b) + 2\left[f(x_1) + \cdots + f(x_{n-1})\right]\right\}$$

where $a = x_0$ and $b = x_n$

Combinatorics

$${}^{n}P_{r} = \frac{n!}{(n-r)!}$$

$$\binom{n}{r} = {}^{n}C_{r} = \frac{n!}{r!\,(n-r)!}$$

$$(x+a)^n = x^n + \binom{n}{1}x^{n-1}a + \cdots + \binom{n}{r}x^{n-r}a^r + \cdots + a^n$$

Vectors

$$|\underset{\sim}{u}| = \left|x\underset{\sim}{i} + y\underset{\sim}{j}\right| = \sqrt{x^2 + y^2}$$

$$\underset{\sim}{u} \cdot \underset{\sim}{v} = |\underset{\sim}{u}|\,|\underset{\sim}{v}|\cos\theta = x_1x_2 + y_1y_2,$$

where $\underset{\sim}{u} = x_1\underset{\sim}{i} + y_1\underset{\sim}{j}$

and $\underset{\sim}{v} = x_2\underset{\sim}{i} + y_2\underset{\sim}{j}$

$$\underset{\sim}{r} = \underset{\sim}{a} + \lambda\underset{\sim}{b}$$

Complex Numbers

$$\begin{aligned} z = a + ib &= r(\cos\theta + i\sin\theta) \\ &= re^{i\theta} \end{aligned}$$

$$\begin{aligned} \left[r(\cos\theta + i\sin\theta)\right]^n &= r^n(\cos n\theta + i\sin n\theta) \\ &= r^n e^{in\theta} \end{aligned}$$

Mechanics

$$\frac{d^2x}{dt^2} = \frac{dv}{dt} = v\frac{dv}{dx} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$$

$$x = a\cos(nt + \alpha) + c$$

$$x = a\sin(nt + \alpha) + c$$

$$\ddot{x} = -n^2(x - c)$$

12 Ext 2 2023 Task 1 Solutions RDS - MC

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Q1 Distance from z axis $= \sqrt{x^2+y^2}$

$= \sqrt{9+25}$

$= \sqrt{34}$ $\therefore$ C

Q2 $\text{Proj}_{\underset{\sim}{b}}\underset{\sim}{a} = \dfrac{\underset{\sim}{a}\cdot\underset{\sim}{b}}{|\underset{\sim}{b}|}$

$= \dfrac{\overrightarrow{OB}\cdot\overrightarrow{OA}}{|\overrightarrow{OA}|}$

$= \dfrac{(-4)(1)+(-3)(-5)+(-2)(-1)}{\sqrt{4^2+3^2+2^2}}$

$= \dfrac{-4+15+2}{\sqrt{29}}$

$= \dfrac{13}{\sqrt{29}}$ $\therefore$ B

Q3 $\begin{pmatrix}2\\1\\1\end{pmatrix}\cdot\left(\underset{\sim}{p}-\begin{pmatrix}-5\\0\\-1\end{pmatrix}\right)=0$

let $\underset{\sim}{p}=(x,y,z)$

$2(x+5)+1(y)+1(z+1)=0$

$2x+10+y+z+1=0$

$2x+y+z=-11$

let $x=-4,\ y=3,\ z=-6$

$-8+3-6=-11$ $\therefore$ B

Q4 $\underset{\sim}{m}\cdot\underset{\sim}{n}=0$

$-3\times3\lambda-2\times10+\lambda\times-5=0$

$-9\lambda-20-5\lambda=0$

$-14\lambda=20$

$\lambda=-\dfrac{10}{7}$ $\therefore$ B

Q5 $\begin{pmatrix}-1\\1\\3\end{pmatrix} \neq k\begin{pmatrix}3\\-3\\9\end{pmatrix}$ $\therefore$ Not parallel

$\begin{pmatrix}-1\\1\\3\end{pmatrix} \cdot \begin{pmatrix}3\\-3\\9\end{pmatrix} = -3-3+27 \neq 0$ $\therefore$ Not perpendicular

Solve
$$2-\lambda_1 = 4+3\lambda_2 \quad (1)$$
$$-1+\lambda_1 = -4-3\lambda_2 \quad (2)$$
$$2+3\lambda_1 = 4+9\lambda_2$$

Solve

(1) + (2)

$1 = 0$ Not possible $\therefore$ D

Answers :

Q1	Q2	Q3	Q4	Q5
C	B	B	B	D

12 Ext 2 2022 Task 1 Solutions RDS - Q6

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(a) (i) $3\underset{\sim}{a} - 4\underset{\sim}{b} = 3\begin{bmatrix}5\\2\\-1\end{bmatrix} - 4\begin{bmatrix}-4\\-4\\3\end{bmatrix}$

$= 31\underset{\sim}{i} + 22\underset{\sim}{j} - 15\underset{\sim}{k}$

(ii) $|\underset{\sim}{a}| = \sqrt{5^2+2^2+1^2}$

$= \sqrt{30}$

(iii) $\hat{\underset{\sim}{a}} = \dfrac{1}{\sqrt{30}}\begin{bmatrix}5\\2\\-1\end{bmatrix}$

(iv) $\cos\theta = \dfrac{\underset{\sim}{a}\cdot\underset{\sim}{b}}{|\underset{\sim}{a}|\,|\underset{\sim}{b}|}$

$\underset{\sim}{a}\cdot\underset{\sim}{b} = -20-8-3$

$= -31$

$|\underset{\sim}{a}| = \sqrt{30}$ $\quad |\underset{\sim}{b}| = \sqrt{41}$ (1)

$\theta = \cos^{-1}\left[\left|\dfrac{-31}{\sqrt{30}\times\sqrt{41}}\right|\right]$

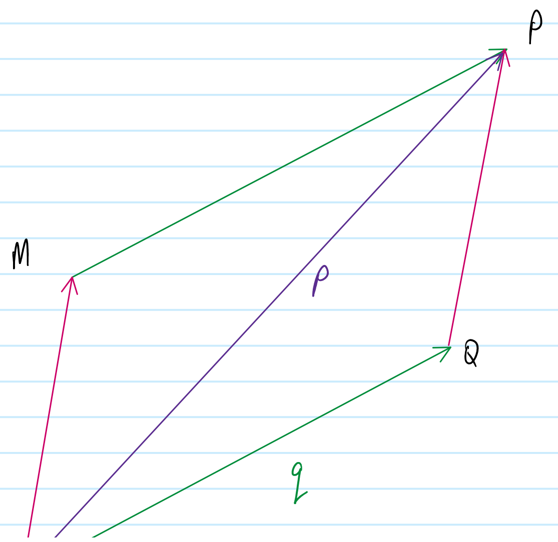
$= 27.862\ldots$

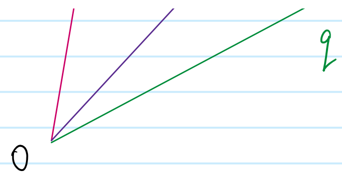
$= 28^\circ$ (Nearest degree) (1)

(v) $\text{Proj}_{\underset{\sim}{a}}\underset{\sim}{b} = \left[\dfrac{\underset{\sim}{a}\cdot\underset{\sim}{b}}{|\underset{\sim}{a}|^2}\right] \times \underset{\sim}{a}$ (1) Scalar projection

$= \left[\dfrac{-31}{30}\right] \times \begin{bmatrix}5\\2\\-1\end{bmatrix}$ (1)

(b)





(i) $\vec{OM} + \vec{MP} = \vec{OP}$ $\qquad$ $\vec{MP} = \underset{\sim}{q}$ (opposite sides of a parallelogram)

$\therefore \vec{OM} = \underset{\sim}{p} - \underset{\sim}{q}$ (1)

(ii) RTP $\vec{MQ} \cdot \vec{OP} = 0$ (1)

let $\vec{OM} = \underset{\sim}{m}$

If OMPQ is a Rhombus then $|q| = |m|$

$\therefore \underset{\sim}{q} \cdot \underset{\sim}{q} = \underset{\sim}{m} \cdot \underset{\sim}{m}$

$\vec{MQ} = \underset{\sim}{q} - \underset{\sim}{m}$

$\vec{OP} = \underset{\sim}{m} + \underset{\sim}{q}$ (1)

$$\therefore \vec{MQ} \cdot \vec{OP} = (\underset{\sim}{q} - \underset{\sim}{m}) \cdot (\underset{\sim}{q} + \underset{\sim}{m})$$
$$= \underset{\sim}{q} \cdot \underset{\sim}{q} - \cancel{\underset{\sim}{q} \cdot \underset{\sim}{m}} + \cancel{\underset{\sim}{q} \cdot \underset{\sim}{m}} - \underset{\sim}{m} \cdot \underset{\sim}{m}$$
$$= |\underset{\sim}{q}| - |\underset{\sim}{m}|$$
$$= 0 \quad (\text{As } |\underset{\sim}{q}| = |\underset{\sim}{m}|)$$
(1)

$\therefore$ Diagonals are perpendicular.

(c) (i) The vectors $\underset{\sim}{m}$ & $\underset{\sim}{n}$ must be the same length or perpendicular

$$\frac{\underset{\sim}{m} \cdot \underset{\sim}{n}}{|\underset{\sim}{m}|} = \frac{\underset{\sim}{m} \cdot \underset{\sim}{n}}{|\underset{\sim}{n}|}$$

if $\underset{\sim}{m} \cdot \underset{\sim}{n} = 0$ then both projections are zero.

$\therefore |\underset{\sim}{m}| = |\underset{\sim}{n}|$ (1)

(ii) The vectors $\underset{\sim}{m}$ & $\underset{\sim}{n}$ must be the same vector or they are perpendicular. (1)

12 Ext 2 2023 Task 1 Solutions RDS - Q7

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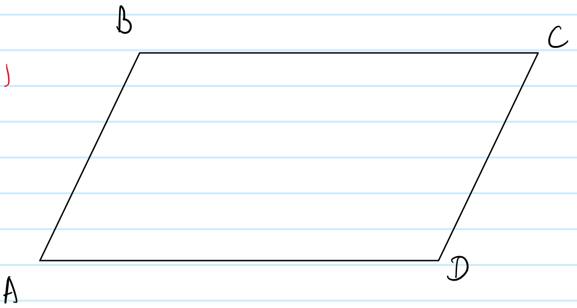
(a) (i)



$$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = \begin{pmatrix} -1-2 \\ -1-5 \\ -2+4 \end{pmatrix} = \begin{pmatrix} -3 \\ -6 \\ 2 \end{pmatrix} \quad 1$$

$$\overrightarrow{AD} = \overrightarrow{OD} - \overrightarrow{OA} = \begin{pmatrix} -5-2 \\ 1-5 \\ 5+4 \end{pmatrix} = \begin{pmatrix} -7 \\ -4 \\ 9 \end{pmatrix} \quad 1$$

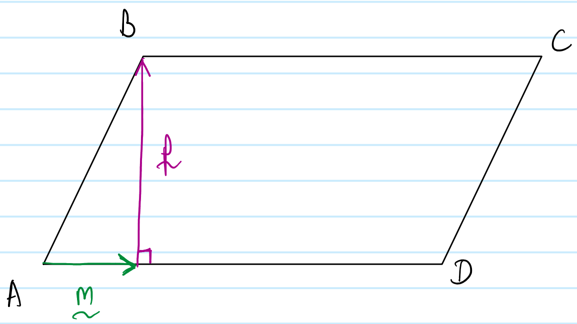
(ii) $\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC}$ ($\overrightarrow{BC} = \overrightarrow{AD}$ As ABCD is a parallelogram)

$$\therefore \overrightarrow{AC} = \begin{pmatrix} -10 \\ -10 \\ 11 \end{pmatrix} \quad 1$$

$$\overrightarrow{OC} = \overrightarrow{OA} + \overrightarrow{AC}$$

$$\therefore \overrightarrow{OC} = \begin{pmatrix} 2-10 \\ 5-10 \\ -4+11 \end{pmatrix} = \begin{pmatrix} -8 \\ -5 \\ 7 \end{pmatrix} \quad 1$$

(iii) Scalar projection of $\overrightarrow{AB}$ onto $\overrightarrow{AD}$ $= |\underset{\sim}{m}|$



$$|\underset{\sim}{m}| = \frac{\overrightarrow{AB} \cdot \overrightarrow{AD}}{|\overrightarrow{AD}|} = \frac{21+24+18}{\sqrt{146}}$$

$$\frac{\ldots}{\sqrt{146}}$$

$$= \frac{63}{\sqrt{146}} \quad (1)$$

$$\therefore |p|^2 = |AB|^2 - |m|^2$$

$$= 49 - \frac{63^2}{146}$$

$$= \frac{3185}{146}$$

$$\therefore |p| = \sqrt{\frac{3185}{146}} \quad (1)$$

$$\therefore \text{Area of } ABCD = \sqrt{3185}$$

$$\doteqdot 56.4358\ldots$$

$$= 56.44 \quad (2dp) \quad (1)$$

12 Ext 2 2023 Task 1 Solutions RDS - Q8

Thursday, 24 November 2022 3:27 PM

(a) (i) $\overrightarrow{AB} = \begin{pmatrix} -5-3 \\ 1-(-2) \\ 1-(-1) \end{pmatrix}$

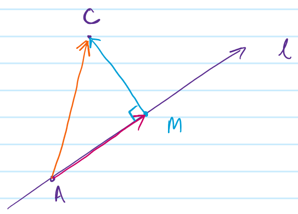
$= \begin{pmatrix} -8 \\ 3 \\ 2 \end{pmatrix}$

let line l go through A & B

$\therefore \; l = \underbrace{\begin{pmatrix} 3 \\ -2 \\ -1 \end{pmatrix}}_{\text{Can be A or B}} + \lambda \begin{pmatrix} -8 \\ 3 \\ 2 \end{pmatrix}$ Any multiple of this.

1 1

(ii)



$\overrightarrow{AB} = \begin{pmatrix} -8 \\ 3 \\ 2 \end{pmatrix}$ $|\overrightarrow{AB}| = \sqrt{77}$

$\overrightarrow{AC} = \begin{pmatrix} 9-3 \\ -4-(-2) \\ -1-(-1) \end{pmatrix} = \begin{pmatrix} 6 \\ -2 \\ 0 \end{pmatrix}$ $|\overrightarrow{AC}| = \sqrt{40}$

1

let M be on the line l so that $\angle AMC = 90°$

$\therefore \; |\overrightarrow{AM}| = \text{Proj}_{\overrightarrow{AB}} \overrightarrow{AC} = \dfrac{\overrightarrow{AC} \cdot \overrightarrow{AB}}{|\overrightarrow{AB}|}$

$= \dfrac{-8\times 6 + 3\times -2 + 0}{\sqrt{77}}$

$= \dfrac{-54}{\sqrt{77}}$

1

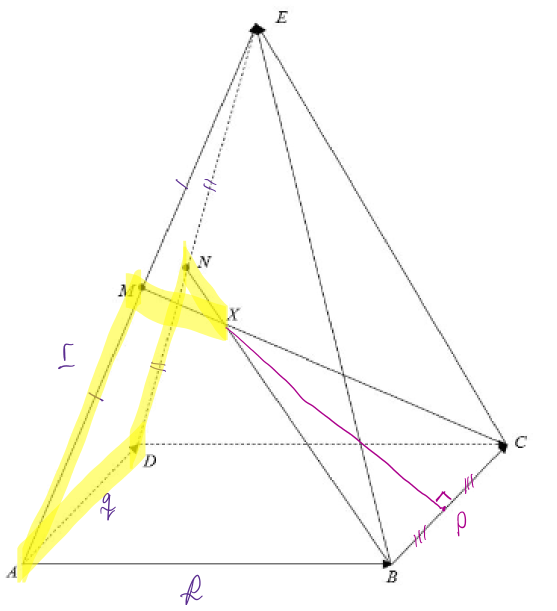
$|MC| = \sqrt{|AC|^2 - |AM|^2}$

$= \sqrt{40 - \dfrac{2916}{77}}$

$= 1.4594\ldots$

$= 1.46$ (2dp) 1

(b)



(i) $\vec{AD} + \vec{DE} = \vec{AE}$

$\therefore \quad \vec{DE} = r - q$

(ii) $\vec{DN} = \frac{1}{2}\vec{DE}$

$= \frac{1}{2}(r - q)$

$\vec{AM} = \frac{1}{2}\vec{AE}$

$= \frac{1}{2}(r)$

$\vec{AD} + \vec{DN} + \vec{NB} + \vec{BA} = 0$

$\vec{NB} = \vec{AB} - \vec{AD} - \vec{DN}$

$= p - q - \frac{1}{2}(r - q)$

$= p - \frac{1}{2}(q + r)$ (1)

$\vec{AM} + \vec{MC} + \vec{CB} + \vec{BA} =$

$\vec{MC} = \vec{AB} + \vec{BC} - \vec{AM}$

$= p + q - \frac{1}{2}r$ (1)

(iii) let $\vec{MX} = \lambda\vec{MC}$ & $\vec{NX} = \sigma\vec{NB}$ (1)

from highlight $\vec{AD} + \vec{DN} + \vec{NX} + \vec{XM} + \vec{MA} = 0$

$q + \frac{1}{2}(r - q) + \sigma\left(p - \frac{1}{2}q - \frac{1}{2}r\right) - \lambda\left(p + q - \frac{1}{2}r\right) - \frac{1}{2}r = 0$

$q\left(1 - \frac{1}{2} - \frac{\sigma}{2} - \lambda\right) + p(\sigma - \lambda) + r\left(\frac{1}{2} - \frac{\sigma}{2} + \frac{\lambda}{2} - \frac{1}{2}\right) = 0$

All coefficients must be zero

$\therefore \quad -\frac{\sigma}{2} - \lambda = -\frac{1}{2} \qquad \sigma = \lambda \qquad \frac{\sigma}{2} = \frac{\lambda}{2}$ (1)

sub in

$\sigma \quad -\sigma \quad = -\frac{1}{}$

sub in

$$-\frac{\sigma}{2} - \sigma = -\frac{1}{2}$$

$$-\frac{3\sigma}{2} = \frac{1}{2}$$

$$\sigma = \frac{1}{3} \quad \therefore \lambda = \frac{1}{3}$$

$$\therefore \overrightarrow{MX} = \frac{1}{3}\overrightarrow{MC} \quad \therefore \overrightarrow{XC} = \frac{2}{3}\overrightarrow{MC}$$

$$\overrightarrow{NX} = \frac{1}{3}\overrightarrow{NB} \quad \therefore \overrightarrow{XB} = \frac{2}{3}\overrightarrow{NB}$$

$$\therefore \frac{\overrightarrow{MX}}{\overrightarrow{XC}} = \frac{\frac{1}{3}\cancel{\overrightarrow{MC}}}{\frac{2}{3}\cancel{\overrightarrow{MC}}} = \frac{1}{2} \qquad \frac{\overrightarrow{NX}}{\overrightarrow{XB}} = \frac{\frac{1}{3}\cancel{\overrightarrow{NB}}}{\frac{2}{3}\cancel{\overrightarrow{NB}}} = \frac{1}{2}$$

1

(iv) $\underset{\sim}{p} = \begin{pmatrix} 4 \\ 0 \\ 0 \end{pmatrix} \quad \underset{\sim}{q} = \begin{pmatrix} 0 \\ 2 \\ 0 \end{pmatrix} \quad \underset{\sim}{r} = \begin{pmatrix} 2 \\ 1 \\ 5 \end{pmatrix}$

$$\overrightarrow{AX} = \overrightarrow{AM} + \overrightarrow{MX}$$
$$= \frac{1}{2}\overrightarrow{AE} + \frac{1}{3}\overrightarrow{MC}$$
$$= \frac{1}{2}\begin{pmatrix} 2 \\ 1 \\ 5 \end{pmatrix} + \frac{1}{3}\left(\begin{pmatrix} 4 \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 2 \\ 0 \end{pmatrix} - \frac{1}{2}\begin{pmatrix} 2 \\ 1 \\ 5 \end{pmatrix}\right)$$
$$= \begin{pmatrix} 1 \\ \frac{1}{2} \\ \frac{5}{2} \end{pmatrix} + \begin{pmatrix} \frac{1}{3} \\ \frac{1}{2} \\ -\frac{5}{6} \end{pmatrix}$$
$$= \begin{pmatrix} \frac{4}{3} \\ 1 \\ \frac{5}{3} \end{pmatrix}$$

$$\overrightarrow{AE} = \overrightarrow{AX} + \overrightarrow{XE}$$
$$\therefore \overrightarrow{XE} = \overrightarrow{AE} - \overrightarrow{AX}$$
$$= \begin{pmatrix} 2 \\ 1 \\ 5 \end{pmatrix} - \begin{pmatrix} \frac{4}{3} \\ 1 \\ \frac{5}{3} \end{pmatrix}$$
$$= \begin{pmatrix} \frac{2}{3} \\ 0 \\ \frac{10}{3} \end{pmatrix} = \frac{2}{3}\begin{pmatrix} 1 \\ 0 \\ 5 \end{pmatrix} \quad 1 \qquad |\overrightarrow{XE}| = \frac{2\sqrt{26}}{3}$$

let P be the midpoint of BC

$$\therefore \overrightarrow{AP} = \begin{pmatrix} 4 \\ 1 \\ 0 \end{pmatrix}$$

$$\overrightarrow{AX} = \overrightarrow{AP} + \overrightarrow{PX}$$

$$\therefore \overrightarrow{PX} = \overrightarrow{AX} - \overrightarrow{AP}$$

$\vec{AX} = \vec{AP} + \vec{PX}$

$\therefore \vec{PX} = \vec{AX} - \vec{AP}$

$= \begin{pmatrix} \frac{4}{3} \\ 1 \\ \frac{5}{3} \end{pmatrix} - \begin{pmatrix} 4 \\ 1 \\ 0 \end{pmatrix}$

$= \begin{pmatrix} -\frac{8}{3} \\ 0 \\ \frac{5}{3} \end{pmatrix} = \frac{1}{3}\begin{pmatrix} -8 \\ 0 \\ 5 \end{pmatrix}$ $\therefore |\vec{PX}| = \frac{\sqrt{89}}{3}$ 1

or finding the plane containg ΔXBC

Angle between $\vec{PX}$ & $\vec{EX}$

$\theta = \cos^{-1}\left[\left|\frac{\vec{PX} \cdot \vec{EX}}{|\vec{PX}||\vec{EX}|}\right|\right]$

$\vec{PX} \cdot \vec{EX} = \frac{2}{3} \times \frac{1}{3}\left[-8 + 0 + 25\right]$

$= \frac{34}{9}$

$\therefore \theta = \cos^{-1}\left[\frac{\frac{34}{\cancel{9}}}{\frac{\sqrt{89}}{\cancel{3}} \times \frac{2\sqrt{26}}{\cancel{3}}}\right]$

$= 69.3045...$

$= 69°$ (Nearest degree) 1