CRANBROOK SCHOOL

Student Number: ____________________

2024 YEAR 12 TASK 1

Mathematics Extension 2

General Instructions

- Reading time – 10 minutes
- Working time – 90 Minutes
- Write using blue or black pen
- Calculators approved by NESA may be used
- A reference sheet is provided at the back of this paper
- For questions in Section II, show relevant mathematical reasoning and/or calculations

Total Marks: 48

Section I – 5 marks

- Attempt Questions 1–5
- Allow about 15 minutes for this section

Section II – 44 marks

- Attempt Questions 6–8
- Allow about 75 minutes for this section

Section I

5 marks
Attempt Questions 1–5

Use the multiple-choice answer sheet for Questions 1–5.

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1 Consider the vectors $\underset{\sim}{a} = 3\underset{\sim}{i} - 2\underset{\sim}{j} + \underset{\sim}{k}$ and $\underset{\sim}{b} = -3\underset{\sim}{i} + \underset{\sim}{j} - 3\underset{\sim}{k}$

Which of the following is closest to the obtuse angle between $\underset{\sim}{a}$ and $\underset{\sim}{b}$?

A. 0.5^c

B. 2.6^c

C. 1.3^c

D. 1.8^c

2 A parallelogram $OABC$ has the vertices at A and B defined by the position vectors

$\begin{bmatrix} -2 \\ 3 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} 3 \\ 1 \\ 4 \end{bmatrix}$. Let M be the midpoint of $\overrightarrow{BC}$.

Which of the following is the vector $\overrightarrow{OM}$?

A. $\dfrac{1}{2}\begin{bmatrix} 8 \\ -1 \\ 7 \end{bmatrix}$

B. $-\dfrac{1}{2}\begin{bmatrix} 8 \\ -1 \\ 7 \end{bmatrix}$

C. $\dfrac{1}{2}\begin{bmatrix} 4 \\ 5 \\ 9 \end{bmatrix}$

D. $-\dfrac{1}{2}\begin{bmatrix} 4 \\ 5 \\ 9 \end{bmatrix}$

3 Gladys is looking at the relationship between two lines, $L_1 = \underset{\sim}{a}_1 + \lambda \underset{\sim}{b}_1$ and $L_2 = \underset{\sim}{a}_2 + \alpha \underset{\sim}{b}_2$ where λ and α are constants. She finds the following pieces of information.

- The ratio of $\underset{\sim}{a}_1$ to $\underset{\sim}{a}_2$ is 3.

- When solved simultaneously, there is no solutions.

- The dot product of $\underset{\sim}{b}_1$ and $\underset{\sim}{b}_2$ is not zero.

Based on her information only, which of the following relationships between L_1 and L_2 is guaranteed to be true?

A. Parallel.

B. Perpendicular.

C. Skew.

D. None of the above.

4 Consider three unique unit vectors, $\underset{\sim}{a}$, $\underset{\sim}{b}$ and $\underset{\sim}{c}$. It is known that $\underset{\sim}{a} \cdot \underset{\sim}{b} = 0$ and $\underset{\sim}{c} \cdot \underset{\sim}{b} = 0$.

Which of the following is false?

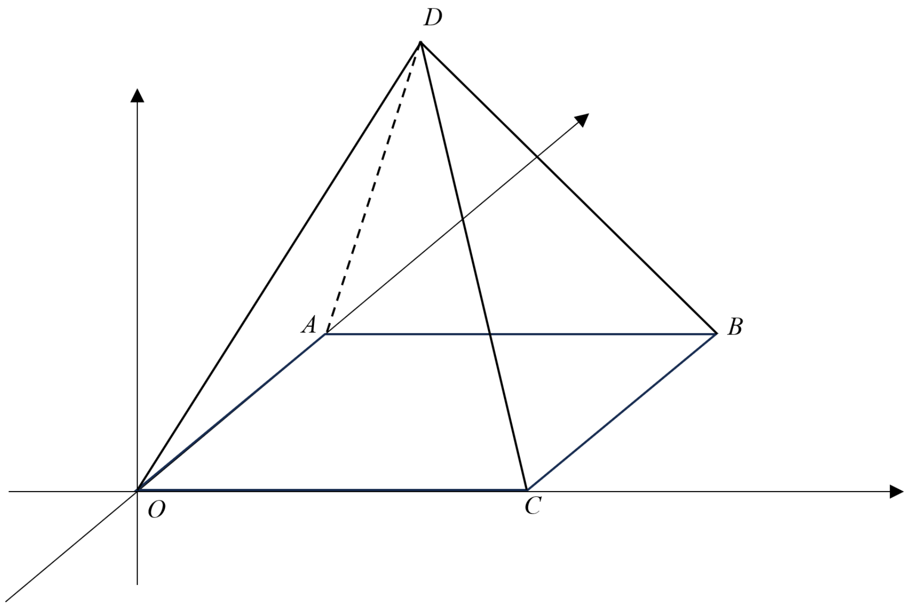
A. $\underset{\sim}{a} \cdot \underset{\sim}{c} = 1$

B. $(\underset{\sim}{a} - \underset{\sim}{b}) \cdot (\underset{\sim}{c} - \underset{\sim}{b}) = 0$

C. $(\underset{\sim}{a} - \underset{\sim}{c}) \cdot \underset{\sim}{b} = 0$

D. $|\underset{\sim}{a} - \underset{\sim}{c}| = 2 \times |\underset{\sim}{b}|$

5 A square based, right pyramid *OABCD* is placed with its square face in the *xy* plane as shown below. All sides of the pyramid are unit length.



Let *M* be the point on the triangular face, *CBD*, that is equidistant from *B*, *C* and *D*.

What is the vector $\overrightarrow{OM}$?

A. $\frac{1}{6}\left(5\underset{\sim}{i}+3\underset{\sim}{j}-\sqrt{2}\underset{\sim}{k}\right)$

B. $\frac{1}{6}\left(5\underset{\sim}{i}-3\underset{\sim}{j}-\sqrt{2}\underset{\sim}{k}\right)$

C. $\frac{1}{6}\left(5\underset{\sim}{i}+3\underset{\sim}{j}+\sqrt{2}\underset{\sim}{k}\right)$

D. $\frac{1}{6}\left(5\underset{\sim}{i}-3\underset{\sim}{j}+\sqrt{2}\underset{\sim}{k}\right)$

End of Section I

Section II

43 marks
Attempt Questions 6 – 8

Answer each question in the appropriate writing booklet. Extra writing booklets are available.

Question 6 (14 marks) Please begin a new Writing Booklet.

(a) Let $\underset{\sim}{a} = 6\underset{\sim}{i} - 3\underset{\sim}{j} + \underset{\sim}{k}$ and $\underset{\sim}{b} = -4\underset{\sim}{i} - 2\underset{\sim}{j} - 3\underset{\sim}{k}$.

(i) Find $3\underset{\sim}{a} - 2\underset{\sim}{b}$. **1**

(ii) Evaluate $\underset{\sim}{a} \cdot \hat{\underset{\sim}{b}}$ showing full working. **2**

(iii) It is known that $\lambda\underset{\sim}{a} + \mu\underset{\sim}{b}$ is a vector parallel to the y/z plane. Find the smallest possible positive integer values of λ and μ. **1**

(b) The points $A = (3, 2, 7)$, $B = (1, -3, -3)$ and $C = (\lambda, 5, 13)$ are co-linear. Find the value of λ. **2**

(c) Consider the following vectors $\underset{\sim}{a} = 2\underset{\sim}{i} - \underset{\sim}{j} + 4\underset{\sim}{k}$ and $\underset{\sim}{b} = -2\underset{\sim}{i} - 2\underset{\sim}{j} + 3\underset{\sim}{k}$.

(i) Find the vector projection of $\underset{\sim}{b}$ onto $\underset{\sim}{a}$. **2**

(ii) Hence or otherwise, find the values of λ and $\underset{\sim}{c}$ where $\underset{\sim}{b} = \lambda\underset{\sim}{a} + \underset{\sim}{c}$ and $\underset{\sim}{a} \cdot \underset{\sim}{c} = 0$ **2**

(d) Consider the position vectors $\overrightarrow{OA} = \begin{bmatrix} -2 \\ -1 \\ 2 \end{bmatrix}$, $\overrightarrow{OB} = \begin{bmatrix} -3 \\ 1 \\ 1 \end{bmatrix}$ and $\overrightarrow{OC} = \begin{bmatrix} 3 \\ 3 \\ -1 \end{bmatrix}$.

(i) Find the parametric equations of the line passing through A and B. **3**

(ii) Does C lie on the line found in part (i)? Justify your answer. **1**

End of Question 6

Question 7 (15 marks) Please begin a new Writing Booklet.

(a) Consider the following two lines.

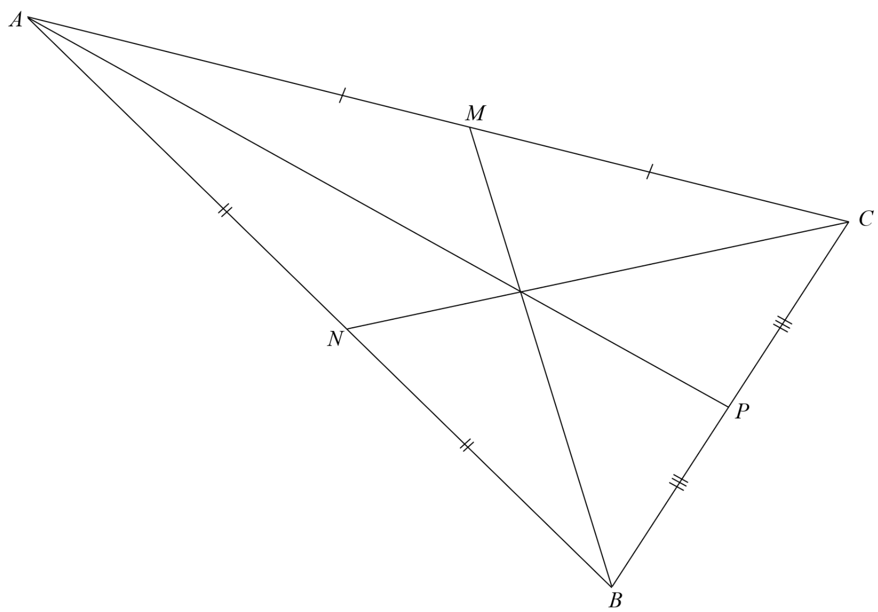
$$L_1 : \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix} + \lambda \begin{bmatrix} 2 \\ 2 \\ -1 \end{bmatrix} \text{ and } L_2 : \begin{bmatrix} -2 \\ 1 \\ -3 \end{bmatrix} + \mu \begin{bmatrix} 3 \\ -4 \\ -2 \end{bmatrix}.$$

(i) If it exists, find the point of intersection between L_1 and L_2. **3**

(ii) Hence or otherwise state the relationship/s between L_1 and L_2 justifying your answer. **1**

(b) Consider $\triangle ABC$, where A, B and C have position vectors $\underset{\sim}{a}$, $\underset{\sim}{b}$ and $\underset{\sim}{c}$ respectively.

The points M, N and P are the midpoints of AC, AB and BC respectively.



(i) Show that $\overrightarrow{BM} = \frac{1}{2}(\underset{\sim}{a} + \underset{\sim}{c}) - \underset{\sim}{b}$. **1**

(ii) Show that $\overrightarrow{BM} + \overrightarrow{CN} + \overrightarrow{AP} = 0$ **2**

(c) The plane P_1 contains the points $A = (1, 4, -2)$, $B = (-3, 0, 6)$ and $C = (-1, 2, -4)$

(i) Find the Cartesian equation of the P_1. **3**

(ii) P_2 is a different plane that contains the points R, S and T. However, those three points are not sufficient to find the equation of P_2. State the relationship between R, S and T. **1**

(iii) P_3 is another plane with Cartesian equation $3x - 2y + z = 5$. Find the acute angle between P_1 and P_3 correct to the nearest degree. **2**

(d) State the centre and radius of the sphere $2x^2 + 2y^2 + 2z^2 - 4x + y - 12z = -2$. **2**

End of Question 7

Question 8 (14 marks) Please begin a new Writing Booklet.

(a) Consider the vector equation $\underset{\sim}{r}(t) = \begin{bmatrix} 2\sin t \\ 2\cos t \\ \sin 2t \end{bmatrix}$.

(i) Find the Cartesian equation of the curve. **1**

(ii) Find the cartesian equation of the tangent to the curve when $t = \dfrac{\pi}{6}$ **3**

(iii) Find the point/s at which the tangent to the curve is parallel to the x-y plane. **1**

(b) Consider the following two lines.

$$\underset{\sim}{r}_1 = \begin{bmatrix} 3 \\ -4 \\ 3 \end{bmatrix} + \lambda \begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix} \text{ and } \underset{\sim}{r}_2 = \begin{bmatrix} 4 \\ 0 \\ -2 \end{bmatrix} + \mu \begin{bmatrix} 2 \\ -2 \\ -1 \end{bmatrix} \text{ where } \lambda, \mu \in \mathbb{R}.$$

Let the point P lay on $\underset{\sim}{r}_1$ with parameter p and let the point Q lie on $\underset{\sim}{r}_2$ with parameter q.

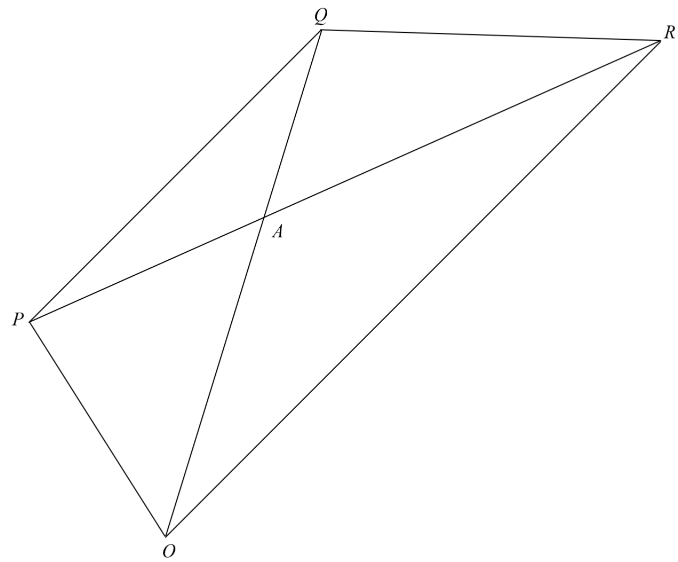
(i) Find $\overrightarrow{PQ}$ in terms of p and q. **1**

(ii) Find the values of p and q when $\overrightarrow{PQ}$ is perpendicular to both $\underset{\sim}{r}_1$ and $\underset{\sim}{r}_2$. **3**

(iii) Hence find the minimum distance between the lines. **1**

(c) In the diagram, $OPQR$ is a trapezium where $\overrightarrow{OP} = \underset{\sim}{p}$, $\overrightarrow{OR} = \underset{\sim}{r}$ and $\overrightarrow{OR}$ is parallel to $\overrightarrow{PQ}$.

Let the diagonals of the trapezium intersect at A.



(i) Show that the vector equation of the line OQ can be written as $\underset{\sim}{l}(t) = t\left(\underset{\sim}{p} + \lambda \underset{\sim}{r}\right)$ where $t, \lambda \in \mathbb{R}$. **1**

(ii) Prove by vector methods that the point of intersection of the diagonals of a trapezium lies on the line passing through the midpoint of the parallel sides. **3**

End of Exam

NSW GOVERNMENT

NSW Education Standards Authority

2020 HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Advanced
Mathematics Extension 1
Mathematics Extension 2

REFERENCE SHEET

Measurement

Length

$$l = \frac{\theta}{360} \times 2\pi r$$

Area

$$A = \frac{\theta}{360} \times \pi r^2$$

$$A = \frac{h}{2}(a + b)$$

Surface area

$$A = 2\pi r^2 + 2\pi rh$$

$$A = 4\pi r^2$$

Volume

$$V = \frac{1}{3}Ah$$

$$V = \frac{4}{3}\pi r^3$$

Functions

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

For $ax^3 + bx^2 + cx + d = 0$:

$$\alpha + \beta + \gamma = -\frac{b}{a}$$

$$\alpha\beta + \alpha\gamma + \beta\gamma = \frac{c}{a}$$

and $\alpha\beta\gamma = -\frac{d}{a}$

Relations

$$(x - h)^2 + (y - k)^2 = r^2$$

Financial Mathematics

$$A = P(1 + r)^n$$

Sequences and series

$$T_n = a + (n - 1)d$$

$$S_n = \frac{n}{2}[2a + (n - 1)d] = \frac{n}{2}(a + l)$$

$$T_n = ar^{n-1}$$

$$S_n = \frac{a(1 - r^n)}{1 - r} = \frac{a(r^n - 1)}{r - 1}, \; r \neq 1$$

$$S = \frac{a}{1 - r}, \; |r| < 1$$

Logarithmic and Exponential Functions

$$\log_a a^x = x = a^{\log_a x}$$

$$\log_a x = \frac{\log_b x}{\log_b a}$$

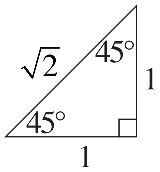
$$a^x = e^{x \ln a}$$

Trigonometric Functions

$$\sin A = \frac{\text{opp}}{\text{hyp}}, \quad \cos A = \frac{\text{adj}}{\text{hyp}}, \quad \tan A = \frac{\text{opp}}{\text{adj}}$$

$$A = \frac{1}{2}ab\sin C$$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

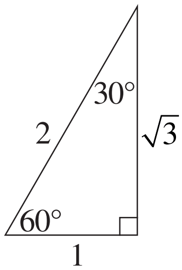


$$c^2 = a^2 + b^2 - 2ab\cos C$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

$$l = r\theta$$

$$A = \frac{1}{2}r^2\theta$$



Trigonometric identities

$$\sec A = \frac{1}{\cos A}, \ \cos A \neq 0$$

$$\operatorname{cosec} A = \frac{1}{\sin A}, \ \sin A \neq 0$$

$$\cot A = \frac{\cos A}{\sin A}, \ \sin A \neq 0$$

$$\cos^2 x + \sin^2 x = 1$$

Compound angles

$$\sin(A + B) = \sin A\cos B + \cos A\sin B$$

$$\cos(A + B) = \cos A\cos B - \sin A\sin B$$

$$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}$$

If $t = \tan\frac{A}{2}$ then $\sin A = \frac{2t}{1+t^2}$

$$\cos A = \frac{1-t^2}{1+t^2}$$

$$\tan A = \frac{2t}{1-t^2}$$

$$\cos A\cos B = \frac{1}{2}\left[\cos(A - B) + \cos(A + B)\right]$$

$$\sin A\sin B = \frac{1}{2}\left[\cos(A - B) - \cos(A + B)\right]$$

$$\sin A\cos B = \frac{1}{2}\left[\sin(A + B) + \sin(A - B)\right]$$

$$\cos A\sin B = \frac{1}{2}\left[\sin(A + B) - \sin(A - B)\right]$$

$$\sin^2 nx = \frac{1}{2}(1 - \cos 2nx)$$

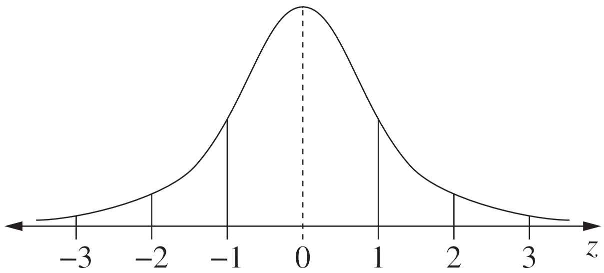
$$\cos^2 nx = \frac{1}{2}(1 + \cos 2nx)$$

Statistical Analysis

$$z = \frac{x - \mu}{\sigma}$$

An outlier is a score
less than $Q_1 - 1.5 \times IQR$
or
more than $Q_3 + 1.5 \times IQR$

Normal distribution



- approximately 68% of scores have z-scores between –1 and 1
- approximately 95% of scores have z-scores between –2 and 2
- approximately 99.7% of scores have z-scores between –3 and 3

$$E(X) = \mu$$

$$\operatorname{Var}(X) = E\left[(X - \mu)^2\right] = E\left(X^2\right) - \mu^2$$

Probability

$$P(A \cap B) = P(A)P(B)$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}, \ P(B) \neq 0$$

Continuous random variables

$$P(X \leq r) = \int_a^r f(x)\,dx$$

$$P(a < X < b) = \int_a^b f(x)\,dx$$

Binomial distribution

$$P(X = r) = {}^nC_r p^r (1 - p)^{n-r}$$

$$X \sim \operatorname{Bin}(n, p)$$

$$\Rightarrow \ P(X = x)$$

$$= \binom{n}{x} p^x (1 - p)^{n-x}, \ x = 0, 1, \ldots, n$$

$$E(X) = np$$

$$\operatorname{Var}(X) = np(1 - p)$$

Differential Calculus

Function	Derivative
$y = f(x)^n$	$\dfrac{dy}{dx} = nf'(x)[f(x)]^{n-1}$
$y = uv$	$\dfrac{dy}{dx} = u\dfrac{dv}{dx} + v\dfrac{du}{dx}$
$y = g(u)$ where $u = f(x)$	$\dfrac{dy}{dx} = \dfrac{dy}{du} \times \dfrac{du}{dx}$
$y = \dfrac{u}{v}$	$\dfrac{dy}{dx} = \dfrac{v\dfrac{du}{dx} - u\dfrac{dv}{dx}}{v^2}$
$y = \sin f(x)$	$\dfrac{dy}{dx} = f'(x)\cos f(x)$
$y = \cos f(x)$	$\dfrac{dy}{dx} = -f'(x)\sin f(x)$
$y = \tan f(x)$	$\dfrac{dy}{dx} = f'(x)\sec^2 f(x)$
$y = e^{f(x)}$	$\dfrac{dy}{dx} = f'(x)e^{f(x)}$
$y = \ln f(x)$	$\dfrac{dy}{dx} = \dfrac{f'(x)}{f(x)}$
$y = a^{f(x)}$	$\dfrac{dy}{dx} = (\ln a)f'(x)a^{f(x)}$
$y = \log_a f(x)$	$\dfrac{dy}{dx} = \dfrac{f'(x)}{(\ln a)f(x)}$
$y = \sin^{-1} f(x)$	$\dfrac{dy}{dx} = \dfrac{f'(x)}{\sqrt{1-[f(x)]^2}}$
$y = \cos^{-1} f(x)$	$\dfrac{dy}{dx} = -\dfrac{f'(x)}{\sqrt{1-[f(x)]^2}}$
$y = \tan^{-1} f(x)$	$\dfrac{dy}{dx} = \dfrac{f'(x)}{1+[f(x)]^2}$

Integral Calculus

$$\int f'(x)[f(x)]^n dx = \frac{1}{n+1}[f(x)]^{n+1} + c$$

where $n \neq -1$

$$\int f'(x)\sin f(x)\,dx = -\cos f(x) + c$$

$$\int f'(x)\cos f(x)\,dx = \sin f(x) + c$$

$$\int f'(x)\sec^2 f(x)\,dx = \tan f(x) + c$$

$$\int f'(x)e^{f(x)} dx = e^{f(x)} + c$$

$$\int \frac{f'(x)}{f(x)} dx = \ln|f(x)| + c$$

$$\int f'(x)a^{f(x)} dx = \frac{a^{f(x)}}{\ln a} + c$$

$$\int \frac{f'(x)}{\sqrt{a^2 - [f(x)]^2}} dx = \sin^{-1}\frac{f(x)}{a} + c$$

$$\int \frac{f'(x)}{a^2 + [f(x)]^2} dx = \frac{1}{a}\tan^{-1}\frac{f(x)}{a} + c$$

$$\int u\frac{dv}{dx} dx = uv - \int v\frac{du}{dx} dx$$

$$\int_a^b f(x)\,dx$$

$$\approx \frac{b-a}{2n}\left\{f(a) + f(b) + 2\left[f(x_1) + \cdots + f(x_{n-1})\right]\right\}$$

where $a = x_0$ and $b = x_n$

Combinatorics

$${}^nP_r = \frac{n!}{(n-r)!}$$

$$\binom{n}{r} = {}^nC_r = \frac{n!}{r!\,(n-r)!}$$

$$(x+a)^n = x^n + \binom{n}{1}x^{n-1}a + \cdots + \binom{n}{r}x^{n-r}a^r + \cdots + a^n$$

Vectors

$$\left|\underset{\sim}{u}\right| = \left|x\underset{\sim}{i} + y\underset{\sim}{j}\right| = \sqrt{x^2 + y^2}$$

$$\underset{\sim}{u} \cdot \underset{\sim}{v} = \left|\underset{\sim}{u}\right|\left|\underset{\sim}{v}\right|\cos\theta = x_1x_2 + y_1y_2,$$

where $\underset{\sim}{u} = x_1\underset{\sim}{i} + y_1\underset{\sim}{j}$

and $\underset{\sim}{v} = x_2\underset{\sim}{i} + y_2\underset{\sim}{j}$

$$\underset{\sim}{r} = \underset{\sim}{a} + \lambda\underset{\sim}{b}$$

Complex Numbers

$$\begin{aligned} z = a + ib &= r(\cos\theta + i\sin\theta) \\ &= re^{i\theta} \end{aligned}$$

$$\begin{aligned} \left[r(\cos\theta + i\sin\theta)\right]^n &= r^n(\cos n\theta + i\sin n\theta) \\ &= r^n e^{in\theta} \end{aligned}$$

Mechanics

$$\frac{d^2x}{dt^2} = \frac{dv}{dt} = v\frac{dv}{dx} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$$

$$x = a\cos(nt + \alpha) + c$$

$$x = a\sin(nt + \alpha) + c$$

$$\ddot{x} = -n^2(x - c)$$

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12 Ext 2 2023 - Task 1 MC Solutions RDS

Monday, 23 October 2023 12:08 PM

Q1 $|\underset{\sim}{a}| = \sqrt{14}$ $\quad |\underset{\sim}{b}| = \sqrt{19}$

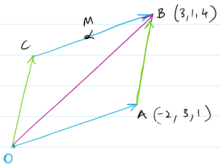
$\underset{\sim}{a} \cdot \underset{\sim}{b} = -9 - 2 - 3$

$= -14$

$\therefore \cos\theta = \dfrac{-14}{\sqrt{14} \times \sqrt{19}}$

$\theta \doteqdot 2.6029\ldots \quad \therefore$ (B)

Q2



$\overrightarrow{OM} = \overrightarrow{OB} + \overrightarrow{BM}$ $\quad \overrightarrow{BM} = -\frac{1}{2}(\overrightarrow{OA})$

$= \begin{bmatrix} 3 \\ 1 \\ 4 \end{bmatrix} - \frac{1}{2}\begin{bmatrix} -2 \\ 3 \\ 1 \end{bmatrix}$

$= \begin{bmatrix} 4 \\ -\frac{1}{2} \\ \frac{7}{2} \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 8 \\ -1 \\ 7 \end{bmatrix} \quad \therefore$ (A)

Q3 (D) $\quad \underset{\sim}{b_1} \cdot \underset{\sim}{b_2} \neq 0 \quad \therefore$ Not perpendicular

No solution $\therefore$ No point of intersection

$\underset{\sim}{a_2} : \underset{\sim}{a_1} = 3:1$ Not direction vector

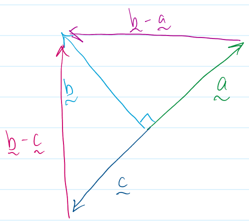
$\therefore$ no relationship.

$\therefore$ Can't tell if Skew or Perpendicular

$\therefore$ Not enough information. (D)

Q4 $\underset{\sim}{a} \cdot \underset{\sim}{b} = 0 \quad \underset{\sim}{b} \cdot \underset{\sim}{c} = 0$ $\quad$ unit vectors. $\quad \underset{\sim}{a} \neq \underset{\sim}{c}$

$\underset{\sim}{a}$ & $\underset{\sim}{c}$ must be parallel but in opposite directions.



$\therefore \underset{\sim}{a} \cdot \underset{\sim}{c} = -1 \quad \therefore$ (A) is not true.

$(\underset{\sim}{b} - \underset{\sim}{c}) \cdot (\underset{\sim}{b} - \underset{\sim}{a}) = 0$ Not (b)

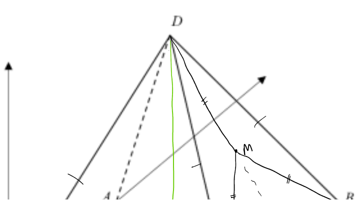
$(\underset{\sim}{a} - \underset{\sim}{c}) = 2\underset{\sim}{a}$

$2\underset{\sim}{a} \cdot \underset{\sim}{b} = 0$ (As they are perpendicular) $\therefore$ Not (c)

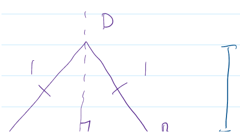
$|\underset{\sim}{a} - \underset{\sim}{c}| = 2$

$|\underset{\sim}{b}| = 1 \quad \therefore \quad |\underset{\sim}{a} - \underset{\sim}{c}| = 2|\underset{\sim}{b}| \quad \therefore$ Not (d)

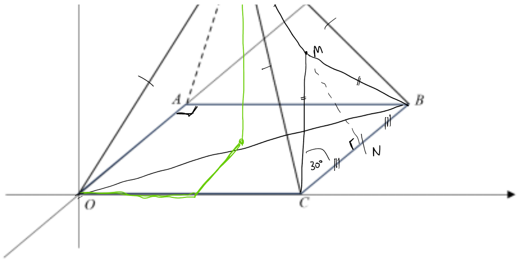
Q5

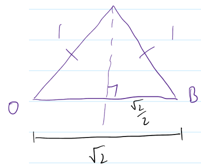


$|OB| = \sqrt{2}$



$= \sqrt{1^2 - \frac{\sqrt{2}}{2}^2}$





$= \sqrt{1^2 - \frac{\sqrt{2}}{2}^2}$

$= \frac{\sqrt{2}}{2}$

$\therefore \overrightarrow{OD} = \frac{1}{2}i + \frac{1}{2}j + \frac{1}{\sqrt{2}}k$

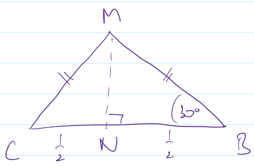
Let N be the midpoint of $\overrightarrow{CB}$

$\overrightarrow{CN} = \frac{1}{2}j$

$\therefore \overrightarrow{ND} = -\frac{1}{2}i + \frac{1}{\sqrt{2}}k$

$\overrightarrow{NM} = \lambda \overrightarrow{ND}$

In $\triangle CMB$



$\frac{|NM|}{\frac{1}{2}} = \tan 30°$

$|NM| = \frac{1}{2} \times \frac{1}{\sqrt{3}}$

$= \frac{1}{2\sqrt{3}}$

$\therefore |NM| = \frac{1}{2\sqrt{3}}$

k

$|ND| = \sqrt{\frac{1}{2}^2 + \left(\frac{1}{\sqrt{2}}\right)^2}$

$= \frac{\sqrt{3}}{2}$

$\therefore \overrightarrow{NM} = \left(-\frac{1}{2}i + \frac{1}{\sqrt{2}}k\right) \div \frac{\sqrt{3}}{\cancel{2}} \times \frac{1}{\cancel{2}\sqrt{3}}$

$= \frac{1}{3}\left(-\frac{1}{2}i + \frac{1}{\sqrt{2}}k\right)$

$\therefore \overrightarrow{OM} = \overrightarrow{OC} + \overrightarrow{CN} + \overrightarrow{NM}$

$= i + \frac{1}{2}j + \left(-\frac{1}{6}i + \frac{1}{3\sqrt{2}}k\right)$

$= \frac{5}{6}i + \frac{1}{2}j + \frac{1}{3\sqrt{2}}k$

$= \frac{1}{6}\left[5i + 3j + \sqrt{2}k\right]$

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(a) $\underset{\sim}{a} = 6\underset{\sim}{i} - 3\underset{\sim}{j} + \underset{\sim}{k}$ $\quad \underset{\sim}{b} = -4\underset{\sim}{i} - 2\underset{\sim}{j} - 3\underset{\sim}{k}$

(i) $$3\underset{\sim}{a} - 2\underset{\sim}{b} = \begin{bmatrix} 3\times 6 - 2\times -4 \\ 3\times -3 - 2\times -2 \\ 3\times 1 - 2\times -3 \end{bmatrix}$$

$$= \begin{bmatrix} 26 \\ -5 \\ 9 \end{bmatrix}$$

(ii) $$\underset{\sim}{a} \cdot \hat{\underset{\sim}{b}} = (6\times -4 + 3\times 2 - 1\times 3) \div \sqrt{16+4+9}$$

1 or

$$= \frac{-21}{\sqrt{29}}$$ 1

(iii) $$\lambda \underset{\sim}{a} + \mu \underset{\sim}{b} = \begin{bmatrix} 6\lambda - 4\mu \\ -3\lambda - 2\mu \\ \lambda - 3\mu \end{bmatrix}$$

Parallel to the y/z plane

$\therefore$ x component is zero.

$$6\lambda - 4\mu = 0$$

$$\lambda = \frac{2}{3}\mu$$

Smallest positive solution $\mu = 3, \ \lambda = 2$ 1

(b) $$\overrightarrow{AB} = \begin{bmatrix} 1-3 \\ -3-2 \\ -3-7 \end{bmatrix} \qquad \overrightarrow{AC} = \begin{bmatrix} \lambda - 3 \\ 3 \\ 6 \end{bmatrix} \qquad \overrightarrow{BC} = \begin{bmatrix} \lambda - 1 \\ 8 \\ 16 \end{bmatrix}$$

$$= \begin{bmatrix} -2 \\ -5 \\ -10 \end{bmatrix}$$ 1

Only one needed.

let $\overrightarrow{AC} = k\overrightarrow{AB}$ $\qquad$ let $\overrightarrow{BC} = m\overrightarrow{AB}$

$$\therefore \begin{bmatrix} \lambda - 3 \\ 3 \\ 6 \end{bmatrix} = \begin{bmatrix} -2k \\ -5k \\ -10k \end{bmatrix} \qquad \therefore \begin{bmatrix} \lambda - 1 \\ 8 \\ 16 \end{bmatrix} = \begin{bmatrix} -2k \\ -5k \\ -10k \end{bmatrix}$$

(from $\underset{\sim}{j}$) $k = -\frac{3}{5}$ $\qquad$ (from $\underset{\sim}{j}$) $k = -\frac{8}{5}$

$\therefore \lambda - 3 = -2 \times \frac{-3}{5}$ $\therefore \lambda = 21/5$ 1 $\therefore \lambda - 1 = +16/5$ $\lambda = 21/5$

(c) $\underset{\sim}{a} = 2\underset{\sim}{i} - \underset{\sim}{j} + 4\underset{\sim}{k}$ $\underset{\sim}{b} = -2\underset{\sim}{i} - 2\underset{\sim}{j} + 3\underset{\sim}{k}$

(i) Vector projection of $\underset{\sim}{b}$ onto $\underset{\sim}{a}$ $= \frac{\underset{\sim}{b} \cdot \underset{\sim}{a}}{|\underset{\sim}{a}|^2} \cdot \underset{\sim}{a}$

$$= \frac{-4+2+12}{21} \cdot \begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix}$$

$$= \frac{10}{21} \begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix}$$

(ii) If $\underset{\sim}{b} = \lambda \underset{\sim}{a} + \underset{\sim}{c}$ then $\lambda \underset{\sim}{a}$ is the vector projection of $\underset{\sim}{b}$ onto $\underset{\sim}{a}$

$\therefore \lambda = \frac{10}{21}$

$$\underset{\sim}{c} = \underset{\sim}{b} - \lambda \underset{\sim}{a}$$

$$= \begin{bmatrix} -2 \\ -2 \\ 3 \end{bmatrix} - \frac{10}{21} \begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix}$$

$$= \begin{bmatrix} -62/21 \\ -32/21 \\ 23/21 \end{bmatrix}$$

(d) (i) $\overrightarrow{AB} = \begin{bmatrix} -3+2 \\ 1+1 \\ 1-2 \end{bmatrix}$

$$= \begin{bmatrix} -1 \\ 2 \\ -1 \end{bmatrix}$$ 1

Vector equation $= \begin{bmatrix} -2 \\ -1 \\ 2 \end{bmatrix} + \lambda \begin{bmatrix} -1 \\ 2 \\ -1 \end{bmatrix}$ 1

$\therefore$ $x = -2 - \lambda$ $y = 2\lambda - 1$ $z = 2 - \lambda$ 1

(ii) $\vec{OC} = \begin{bmatrix} 3 \\ 3 \\ -1 \end{bmatrix}$

When $x = 3$, $3 = -2 - \lambda$

$\lambda = -5$

When $\lambda = -5$, $y = -11$ & $z = 7$

$\therefore$ C does not lie on the line. 1

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(a) $L_1 = \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix} + \lambda \begin{bmatrix} 2 \\ 2 \\ -1 \end{bmatrix}$, $L_2 = \begin{bmatrix} -2 \\ 1 \\ -3 \end{bmatrix} + \mu \begin{bmatrix} 3 \\ -4 \\ -2 \end{bmatrix}$

(i)

$$\begin{array}{lcl} L_1 & & L_2 \\ x = 2 + 2\lambda & \longleftrightarrow & x = 3\mu - 2 \\ y = 2\lambda - 1 & \longleftrightarrow & y = 1 - 4\mu \\ z = 3 - \lambda & & z = -2\mu - 3 \end{array}$$

1

Solve x & y

$$2 + 2\lambda = 3\mu - 2 \qquad 2\lambda - 3\mu = -4 \quad \dots 1$$

$$2\lambda - 1 = 1 - 4\mu \qquad 2\lambda + 4\mu = 2 \quad \dots 2$$

1 − 2

$$-7\mu = -6$$

$$\mu = 6/7 \quad \dots 3$$

3 ⇒ 1

$$2 \times \frac{6}{7} - 3\mu = -4$$

$$40/7 = 3\mu$$

$$\mu = 40/21$$

1

Check in z equations

L_1: $z = 3 - \frac{6}{7}$ $\qquad$ L_2: $z = -2 \times \frac{40}{21} - 3$

$z = 15/7$ $\qquad$ $= -23/3$

Not the same. ∴ No point of intersection.

1

(ii) $\begin{bmatrix} 2 \\ 2 \\ -1 \end{bmatrix} \neq k \begin{bmatrix} 3 \\ -4 \\ -2 \end{bmatrix}$ $k \in \mathbb{R}$ ∴ Not parallel

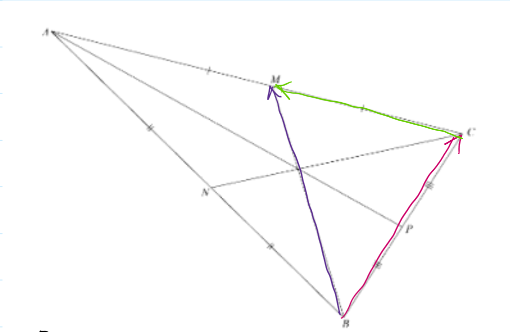
$$\begin{bmatrix} 2 \\ 2 \\ -1 \end{bmatrix} \cdot \begin{bmatrix} 3 \\ -4 \\ -2 \end{bmatrix} = 6 - 8 + 2$$

$$= 0$$

∴ They are perpendicular, but do not intersect ∴ skew & perpendicular.

1

(b)



(i) $\overrightarrow{OA} = \underset{\sim}{a}$ $\quad \overrightarrow{OB} = \underset{\sim}{b}$ $\quad \overrightarrow{OC} = \underset{\sim}{c}$

$\overrightarrow{CA} = \underset{\sim}{a} - \underset{\sim}{c} \quad \therefore \quad \overrightarrow{CM} = \frac{1}{2}(\underset{\sim}{a} - \underset{\sim}{c})$

$\overrightarrow{BC} = \underset{\sim}{c} - \underset{\sim}{b}$

$\therefore \overrightarrow{BM} = \overrightarrow{BC} + \overrightarrow{CM}$

$= \underset{\sim}{c} - \underset{\sim}{b} + \frac{1}{2}\underset{\sim}{a} - \underset{\sim}{c}$

$= \frac{1}{2}\underset{\sim}{a} + \frac{1}{2}\underset{\sim}{c} - \underset{\sim}{b}$

$= \frac{1}{2}(\underset{\sim}{a} + \underset{\sim}{c}) - \underset{\sim}{b}$

1

(ii) Generalising ...

$\overrightarrow{BM} = \frac{1}{2}(\underset{\sim}{a} + \underset{\sim}{c}) - \underset{\sim}{b}$

$\overrightarrow{CN} = \frac{1}{2}(\underset{\sim}{a} + \underset{\sim}{b}) - \underset{\sim}{c}$

$\overrightarrow{AP} = \frac{1}{2}(\underset{\sim}{b} + \underset{\sim}{c}) - \underset{\sim}{a}$

Generalise 1

$\overrightarrow{BM} + \overrightarrow{CN} + \overrightarrow{AP} = \frac{1}{2}(\underset{\sim}{a} + \underset{\sim}{c}) - \underset{\sim}{b} + \frac{1}{2}(\underset{\sim}{a} + \underset{\sim}{b}) - \underset{\sim}{c} + \frac{1}{2}(\underset{\sim}{b} + \underset{\sim}{c}) - \underset{\sim}{a}$

$= \left(\frac{1}{2} + \frac{1}{2} - 1\right)\underset{\sim}{a} + \left(\frac{1}{2} + \frac{1}{2} - 1\right)\underset{\sim}{b} + \left(\frac{1}{2} + \frac{1}{2} - 1\right)\underset{\sim}{c}$

$= 0 + 0 + 0$ As required.

1 Show.

(c) $A(1, 4, -2) \quad B(-3, 0, 6) \quad C(-1, 2, -4)$

(i) let $\underset{\sim}{n}$ be perpendicular to P_1

$\therefore \underset{\sim}{n} \cdot \overrightarrow{AB} = \underset{\sim}{n} \cdot \overrightarrow{AC} = 0$

let $\underset{\sim}{n} = \begin{bmatrix} 1 \\ y \end{bmatrix} \quad \overrightarrow{AB} = \begin{bmatrix} -4 \\ -4 \end{bmatrix} \quad \overrightarrow{AC} = \begin{bmatrix} -2 \\ -2 \end{bmatrix}$

let $\underset{\sim}{n} = \begin{bmatrix} 1 \\ y \\ z \end{bmatrix}$ $\quad \vec{AB} = \begin{bmatrix} -4 \\ -4 \\ 8 \end{bmatrix}$ $\quad \vec{AC} = \begin{bmatrix} -2 \\ -2 \\ -2 \end{bmatrix}$

$\underset{\sim}{n} \cdot \vec{AB} = -4 - 4y + 8z = 0$

$\underset{\sim}{n} \cdot \vec{AC} = -2 - 2y - 2z = 0$

$\therefore \quad y + z = -1 \quad \ldots (1)$

$4y - 8z = -4 \quad \ldots (2)$

$(2) - 4 \times (1)$

$-12z = 0$

$z = 0$

$\therefore \quad y = -1$

$\therefore \quad \underset{\sim}{n} = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$ (1)

(1) or cross product.

Equation of P_1 : $\begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} \cdot \begin{bmatrix} x-1 \\ y-4 \\ z+2 \end{bmatrix} = 0$

$x - 1 - (y-4) + 0 = 0$

$x - 1 - y + 4 = 0$

$x - y = -3$ (1)

(ii) If R, S & T are not sufficient then they must be co linear. (1)

(iii) $\underset{\sim}{n} = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$ $\quad$ let $\underset{\sim}{m}$ be the vector perpendicular to P_3

$\underset{\sim}{m} = \begin{bmatrix} 3 \\ -2 \\ 1 \end{bmatrix}$ (1) (from equation)

Angle between $\underset{\sim}{m}$ & $\underset{\sim}{n}$ is the angle between the

Angle between $\underset{\sim}{m}$ & $\underset{\sim}{n}$ is the angle between the planes.

$$\cos\theta = \frac{\underset{\sim}{m} \cdot \underset{\sim}{n}}{|\underset{\sim}{m}||\underset{\sim}{n}|}$$

$|\underset{\sim}{n}| = \sqrt{2}$

$|\underset{\sim}{m}| = \sqrt{14}$

$\underset{\sim}{m} \cdot \underset{\sim}{n} = 3 + 2 + 0$

$= 5$

$$\therefore \cos\theta = \frac{5}{2\sqrt{7}}$$

$\theta = 19^\circ$

1

(d)

$$2x^2 + 2y^2 + 2z^2 - 4x + y - 12z = -2$$

$$x^2 - 2x + 1 + y^2 + \frac{y}{2} + \frac{1}{16} + z^2 - 6z + 9 = -1 + 1 + \frac{1}{16} + 9$$

$$(x-1)^2 + \left(y + \frac{1}{4}\right)^2 + (z-3)^2 = \frac{145}{16}$$

Centre: $\left(1, -\frac{1}{4}, 3\right)$, radius: $\frac{\sqrt{145}}{4} \doteqdot 3.01$

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(a) (i) $\underset{\sim}{r} = \begin{bmatrix} 2\sin t \\ 2\cos t \\ \sin 2t \end{bmatrix}$

$\frac{x}{2} = \sin t \qquad \frac{y}{2} = \cos t \qquad z = 2\sin t\cos t$

$\therefore \quad z = \frac{xy}{2}$ (1)

(ii) $\underset{\sim}{r}'(t) = \begin{bmatrix} 2\cos t \\ -2\sin t \\ 2\cos 2t \end{bmatrix}$ (1)

@ $t = \frac{\pi}{6}$ $\qquad \underset{\sim}{r}\left(\frac{\pi}{6}\right) = \begin{bmatrix} 1 \\ \sqrt{3} \\ \frac{\sqrt{3}}{2} \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 2 \\ 2\sqrt{3} \\ \sqrt{3} \end{bmatrix}$

$r'\left(\frac{\pi}{6}\right) = \begin{bmatrix} \sqrt{3} \\ -1 \\ 1 \end{bmatrix}$ (1)

$\therefore$ Equation of tangent $= \frac{1}{2}\begin{bmatrix} 2 \\ 2\sqrt{3} \\ \sqrt{3} \end{bmatrix} + \lambda \begin{bmatrix} \sqrt{3} \\ -1 \\ 1 \end{bmatrix}$ Where $\lambda \in \mathbb{R}$

$\therefore \quad x = 1 + \sqrt{3}\lambda \qquad y = \sqrt{3} - \lambda \qquad z = \frac{\sqrt{3}}{2} + \lambda$

$\frac{x-1}{\sqrt{3}} = \sqrt{3} - y = \frac{2z - \sqrt{3}}{2}$ (1)

(iii) Gradient vector parallel to the x/y plane has a 'z' component $= 0$

$\therefore \quad 2\cos 2t = 0$

$2t = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2} \ldots$

$t = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

$\therefore$ 4 points $\quad P_{1/2} = \pm\begin{bmatrix} \sqrt{2} \\ \sqrt{2} \\ 0 \end{bmatrix} \qquad P_{3/4} = \pm\begin{bmatrix} \sqrt{2} \\ -\sqrt{2} \\ 0 \end{bmatrix}$ (1)

$t = \frac{\pi}{4}, \frac{5\pi}{4} \qquad t = \frac{3\pi}{4}, \frac{7\pi}{4}$

$$t = \frac{\pi}{4}, \frac{5\pi}{4} \qquad\qquad t = \frac{3\pi}{4}, \frac{7\pi}{4}$$

(b) $r_1 = \begin{bmatrix} 3 \\ -4 \\ 3 \end{bmatrix} + \lambda \begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix} \qquad r_2 = \begin{bmatrix} 4 \\ 0 \\ -2 \end{bmatrix} + \mu \begin{bmatrix} 2 \\ -2 \\ -1 \end{bmatrix}$

(i) $\therefore P = \begin{bmatrix} 3-p \\ 2p-4 \\ 3p+3 \end{bmatrix} \qquad Q = \begin{bmatrix} 4+2q \\ -2q \\ -2-q \end{bmatrix}$

$$\therefore \overrightarrow{PQ} = \begin{bmatrix} 4+2q-(3-p) \\ -2q-(2p-4) \\ -2-q-(3p+3) \end{bmatrix}$$

$$= \begin{bmatrix} 1+2q+p \\ 4-2q-2p \\ -5-q-3p \end{bmatrix} \quad ①$$

(ii) $\underbrace{\overrightarrow{PQ} \cdot \begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix} = 0}_{①} \quad \& \quad \underbrace{\overrightarrow{PQ} \cdot \begin{bmatrix} 2 \\ -2 \\ -1 \end{bmatrix} = 0}_{②} \quad ①$

① $\therefore -(1+2q+p) + 2(4-2q-2p) + 3(-5-q-3p) = 0$

$$-8 - 9q - 14p = 0$$
$$14p + 9q = -8 \quad \ldots ①$$

② $2(1+2q+p) - 2(4-2q-2p) - (-5-q-3p) = 0$

$$-1 + 9q + 9p = 0$$
$$9p + 9q = 1 \quad \ldots ②$$

(1) ① $-$ ②

$$5p = -9$$
$$p = -9/5 \qquad \therefore q = 86/45 \quad ①$$

(iii) Minimum distance occurs when $\overrightarrow{PQ}$ is perpendicular to both lines.

$$\therefore \overrightarrow{PQ} = \begin{bmatrix} 136/45 \\ 34/9 \\ -68/45 \end{bmatrix}$$

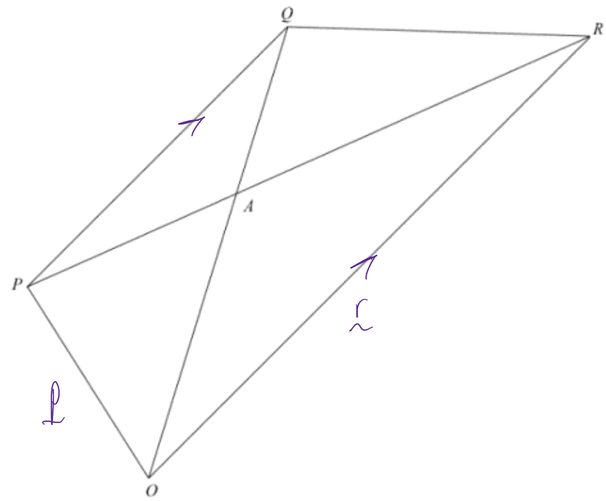
$\therefore$ Minimum distance $= |\overrightarrow{PQ}|$

[.45]

$\therefore$ Minimum distance $= |\overrightarrow{PQ}|$

$\doteq 9.3535\ldots$

(c)



(i) $\overrightarrow{OQ} = \overrightarrow{OP} + \overrightarrow{PQ}$

$\overrightarrow{PQ}$ is parallel to $\overrightarrow{OR}$

$\therefore \overrightarrow{OQ} = \overrightarrow{OP} + \lambda \overrightarrow{OR} \qquad \lambda \in \mathbb{R}$

$= \underset{\sim}{p} + \lambda \underset{\sim}{r}$

The line OQ is parallel to $\overrightarrow{OQ}$ but goes through origin

$\therefore \ell_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} + t(\overrightarrow{OQ})$

$= t(\underset{\sim}{p} + \lambda \underset{\sim}{r})$ As required.

1

(ii) let the midpoint of $\overrightarrow{PQ}$ be M

$\overrightarrow{OR}$ be N

$\therefore$ RTP A lies on $\overrightarrow{MN}$

Equation of the line PR $= \ell_2$

$\ell_2 = \overrightarrow{OP} + k\overrightarrow{PR}$

$= \underset{\sim}{p} + k(\underset{\sim}{r} - \underset{\sim}{p})$

A is the point of intersection between those lines.

$\therefore$ Solve simultaneously.

$$\underset{\sim}{p} + k(\underset{\sim}{r} - \underset{\sim}{p}) = t(\underset{\sim}{p} + \lambda \underset{\sim}{r})$$

$$\underset{\sim}{p}(1-k) + k\underset{\sim}{r} = t\underset{\sim}{p} + \lambda t \underset{\sim}{r}$$

As $\underset{\sim}{p}$ & $\underset{\sim}{r}$ are not parallel

$1 - k = t$ ... ① & $k = \lambda t$ ... ②

② ⇒ ①

$$1 - \lambda t = t$$
$$1 = \lambda t + t$$
$$t = \frac{1}{1+\lambda} \qquad \therefore \quad k = \frac{\lambda}{1+\lambda}$$

$\therefore \quad \overrightarrow{OP} = t(\underset{\sim}{p} + \lambda \underset{\sim}{r})$ As P lies on l_1

$$= \frac{1}{1+\lambda}(\underset{\sim}{p} + \lambda \underset{\sim}{r})$$

$$= \frac{1}{1+\lambda}\underset{\sim}{p} + \frac{\lambda}{1+\lambda}\underset{\sim}{r} \quad ①$$

$\overrightarrow{OM} = \overrightarrow{OP} + \frac{1}{2}\overrightarrow{PQ} \qquad \overrightarrow{ON} = \frac{1}{2}\underset{\sim}{r}$

$$= \underset{\sim}{p} + \frac{1}{2}\lambda \underset{\sim}{r}$$

$$\overrightarrow{NM} = \underset{\sim}{p} + \frac{1}{2}\lambda\underset{\sim}{r} - \frac{1}{2}\underset{\sim}{r}$$

$$= \underset{\sim}{p} + \frac{1}{2}(\lambda - 1)\underset{\sim}{r}$$

suppose $\overrightarrow{NP} = \mu \overrightarrow{NM}$

$\therefore \quad \overrightarrow{OP} = \overrightarrow{ON} + \overrightarrow{NM}$

$$= \frac{1}{2}\underset{\sim}{r} + \mu\left[\underset{\sim}{p} + \frac{1}{2}(\lambda-1)\underset{\sim}{r}\right]$$

$$= \mu\underset{\sim}{p} + \left[\frac{1}{2} + \frac{\mu}{2}(\lambda-1)\right]\underset{\sim}{r} \quad ①$$

If $\mu = \frac{1}{1+\lambda}$

then $\overrightarrow{OP} = \frac{1}{1+\lambda}\underset{\sim}{p} + \left[\frac{1}{2}\cdot\frac{1+\lambda}{1+\lambda} + \frac{1}{2}\left(\frac{1}{1+\lambda}\right)(\lambda-1)\right]\underset{\sim}{r}$

$$= \frac{1}{1+\lambda}\underset{\sim}{p} + \left[\frac{(1+\lambda) + (\lambda - 1)}{2(1+\lambda)}\right]\underset{\sim}{r}$$

$$= \frac{1}{1+\lambda}\underset{\sim}{p} + \left[\frac{(1+\lambda)+(\lambda-1)}{2(1+\lambda)}\right]\underset{\sim}{r}$$

$$= \frac{1}{1+\lambda}\underset{\sim}{p} + \frac{\cancel{2}\lambda}{\cancel{2}(1+\lambda)}\underset{\sim}{r}$$

$$= \frac{1}{1+\lambda}\underset{\sim}{p} + \frac{\lambda}{(1+\lambda)}\underset{\sim}{r} = \overrightarrow{OP}$$ from point of intersection

1