GIRRAWEEN HIGH SCHOOL

YEAR 12 MATHEMATICS EXTENSION 2

TASK 1 2021

(December 2020)

ANSWERS COVER SHEET

Student Number:

____

QUESTION	MARK	MEX12-1	MEX12-2	MEX12-3	MEX12-4	MEX12-5	MEX12-6	MEX12-7	MEX12-8
1-5	**/5**				√				√
6	**/17**				√				√
7	**/10**				√				√
8	**/6**				√				√
9	**/15**				√				√
10	**/14**				√				√
11	**/10**				√				√
TOTALS	**/77**				**/77**				**/77**

Year 12 Mathematics Extension 2 outcomes

A student:

MEX12-1	understands and uses different representations of numbers and functions to model, prove results and find solutions to problems in a variety of contexts
MEX12-2	chooses appropriate strategies to construct arguments and proofs in both practical and abstract settings
MEX12-3	uses vectors to model and solve problems in two and three dimensions
MEX12-4	uses the relationship between algebraic and geometric representations of complex numbers and complex number techniques to prove results, model and solve problems
MEX12-5	applies techniques of integration to structured and unstructured problems
MEX12-6	uses mechanics to model and solve practical problems
MEX12-7	applies various mathematical techniques and concepts to model and solve structured, unstructured and multi-step problems
MEX12-8	communicates and justifies abstract ideas and relationships using appropriate language, notation and logical argument

GIRRAWEEN HIGH SCHOOL

MATHEMATICS EXTENSION 2
2021 HSC Task 1 (December 2020)

Student No:____

This Booklet contains the answer sheet for Part A and Writing Booklet for Part B.

Part A ANSWER SHEET

Select the alternative A, B, C or D that best answers the question.

1.	A	○	B	○	C	○	D	○
2.	A	○	B	○	C	○	D	○
3.	A	○	B	○	C	○	D	○
4.	A	○	B	○	C	○	D	○
5.	A	○	B	○	C	○	D	○

Instructions

- If you need more paper for Part B, please ask your supervisor.
- Write your name on every booklet you use.
- Write on both sides of each sheet of paper.

Total number of booklets used ______________.

GIRRAWEEN HIGH SCHOOL

TASK 1 2021 *(December 2020)*

MATHEMATICS

EXTENSION 2

Complex Numbers

Time allowed – 90 Minutes

DIRECTIONS TO CANDIDATES

- Attempt ALL questions.
- All necessary working should be shown in every question. Marks may be deducted for careless or badly arranged work.
- Board-approved calculators may be used.
- Each question attempted is to be marked clearly Question 6, Question 7 etc. Each question is to be returned on a separate page in your answer booklet.
- You may ask for spare answer booklets if you need them.
- For Multiple choice: Fill in the circle corresponding to the correct answer on the multiple choice answer sheet in your answer booklet.
- For Question 9(a) please draw your diagrams on the page of number planes provided in your answer booklet.
- The notation $cis\ \boldsymbol{\theta}$ is acceptable in this examination.

Multiple Choice begins on the following page

Question 1

$i^{2021} =$

(A) i (B) -1 (C) $-i$ (D) 1

Question 2

$2e^{\frac{5\pi i}{6}} =$

(A) $\sqrt{3}+i$ (B) $\sqrt{3}-i$ (C) $-\sqrt{3}+i$ (D) $-\sqrt{3}-i$

Question 3

$\frac{z}{\bar{z}} =$

(A) $\frac{z^2}{|z|^2}$ (B) $\frac{(\bar{z})^2}{|z|^2}$ (C) $\frac{z\bar{z}}{|z|^2}$ (D) $\frac{1}{z}$

Question 4

In Modulus/argument form to the nearest degree, $-2+7\mathrm{i} =$

(A) $\sqrt{53}\ cis\ 16^o$ (B) $\sqrt{53}\ cis\ 74^o$ (C) $\sqrt{53}\ cis\ 106^o$ (D) $\sqrt{53}\ cis\ 286^o$

Question 5

$\frac{6+8i}{2+i} =$

(A) $3+8i$ (B) $\frac{4+22i}{5}$ (C) $\frac{20+10i}{3}$ (D) $4+2i$

Question 6 ***(17 Marks – show all of your working in your answer booklet)*** **Marks**

(a) **(i)** Find $\frac{-1+i}{1+i\sqrt{3}}$ in Cartesian form. **2**

(ii) Express $-1+i$ and $1+i\sqrt{3}$ in modulus/argument form. **4**

(iii) Hence find an exact value for $sin\frac{5\pi}{12}$. **2**

QUESTION 6 CONTINUES ON THE FOLLOWING PAGE

Question 6 (continued) **Marks**

(b) **(i)** By letting $(x + iy)^2 = -16 + 30i$, find $\sqrt{-16 + 30i}$ in Cartesian form. **4**

(ii) Hence solve the equation $z^2 - (1 + i)z + (4 - 7i) = 0$. Give your answer in Cartesian form. **3**

(c) Find the six sixth roots of $32 - 32i\sqrt{3}$. Leave your answers in modulus/argument form. **2**

Question 7 ***(10 Marks – show all of your working in your answer booklet)***

(a) Sketch these curves on separate Argand diagrams:

(i) $|z - 1 - 2i| = \sqrt{5}$ **4**

(ii) $Arg(z - 2 - i) = \frac{3\pi}{4}$ **2**

(b) Sketch the region on the complex plane where $z\bar{z} \leq 4$ and $-\frac{\pi}{4} < Arg\ z \leq \frac{\pi}{2}$ **4**

Question 8 ***(6 Marks – show all of your working in your answer booklet)***

If α, β and γ are the roots of the equation $2z^3 - 3z^2 + 8z + 5 = 0$,

(a) Without finding the roots, find the value of $\alpha^2 + \beta^2 + \gamma^2$. **2**

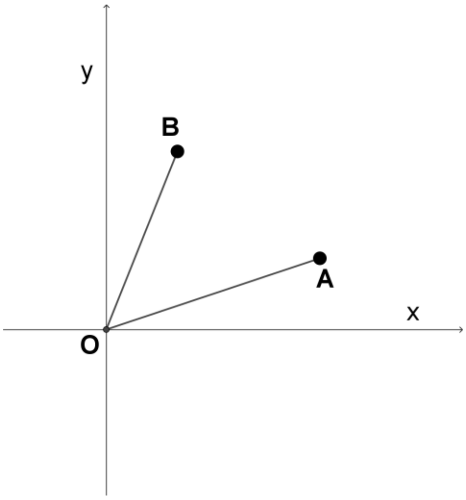
(b) HENCE (using your answer to (a), explain why $2z^3 - 3z^2 + 8z + 5 = 0$ has exactly one real root. **2**

(c) If one of the roots of $2z^3 - 3z^2 + 8z + 5 = 0$ is $z = 1 + 2i$, find, with reasons, the other two roots. **2**

EXAMINATION CONTINUES ON THE FOLLOWING PAGE

Question 9 ***(15 Marks – show all of your working in your answer booklet)*** **MARKS**

(a) In the diagram below, $\overrightarrow{OA} = z$ and $\overrightarrow{OB} = w$.



Draw and label on the separate Argand diagrams provided in your answer booklet (note that all of these are to be drawn as vectors from O). **6**

(i) $\overrightarrow{OC} = z + w$

(ii) $\overrightarrow{OD} = z - w$

(iii) $\overrightarrow{OE} = z + 1$

(iv) $\overrightarrow{OF} = w + i$

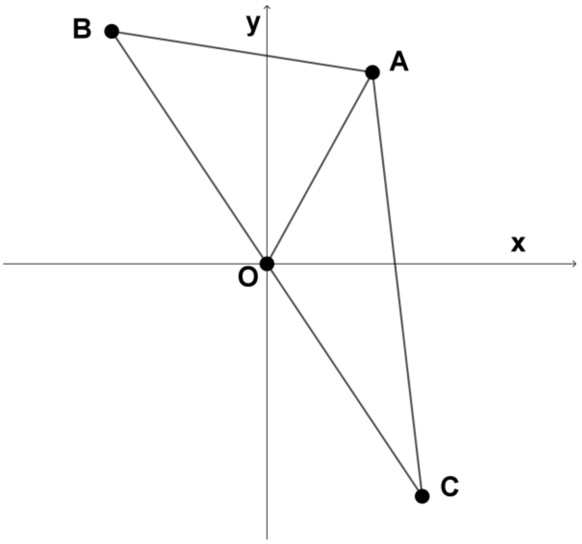
(v) $\overrightarrow{OG} = iw$

(vi) $\overrightarrow{OH} = z\ cis(-\frac{\pi}{3})$

QUESTION 9 CONTINUES ON THE FOLLOWING PAGE

Question 9 (continued) **Marks**

(b) In the Argand diagram below, $\overrightarrow{OA} = z_1$ and $\overrightarrow{OB} = z_2$. O is the midpoint of B and C. *(see diagram)*



Show that $(\overrightarrow{AB})^2 + (\overrightarrow{AC})^2 = 2{z_1}^2 + 2{z_2}^2$. **4**

(c) The locus $Arg\ \{\frac{z-3}{z+1}\} = \frac{\pi}{6}$ represents part of a circle. **5**

Draw this circle part on an Argand diagram and find its centre, radius and equation.

Question 10 ***(14 Marks – show all of your working in your answer booklet)***

(a) Use DeMoivre's Theorem to show that $cos3\boldsymbol{\theta} = \boldsymbol{4cos^3\theta - 3cos\theta}$. **3**

(b) **(i)** Use Euler's Theorem to show that $e^{ni\theta} + e^{-ni\theta} = 2cos\ n\boldsymbol{\theta}$. **2**

(ii) HENCE show that $16cos^5\theta = cos5\boldsymbol{\theta} + \boldsymbol{5cos3\theta + 10cos\theta}$. **3**

(iii) HENCE show that $cos\frac{\pi}{10}$ is a root of the equation **3**

$16x^4 - 20x^2 + 5 = 0$ *(you may use your answer to Question 10(a) here)*

(iv) HENCE find the exact value of $cos\frac{\pi}{10}$ *(you may leave your answer under a square root sign).* **3**

EXAMINATION CONTINUES ON THE FOLLOWING PAGE

Question11 ***(10 Marks – show all of your working in your answer booklet)*** **Marks**

(a) Solve $z^7 - 1 = 0$. *(Leave your answers in cis$\boldsymbol{\theta}$ form).* **1**

(b) If w is the root of $z^7 - 1 = 0$ with the smallest positive argument, show that w^n is also a root if n is a positive integer. **1**

(c) Show that $1 + w + w^2 + w^3 + w^4 + w^5 + w^6 = 0$ **1**

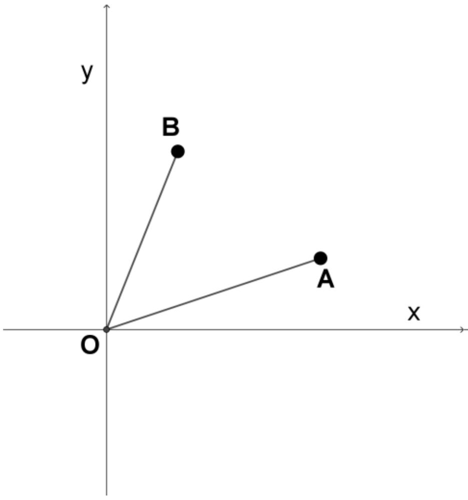
(d) Form the monic quadratic equation with roots $w + w^2 + w^4$ and $w^3 + w^5 + w^6$ **3**

(e) Hence show that $sin\frac{2\pi}{7} + sin\frac{3\pi}{7} - sin\frac{\pi}{7} = \frac{\sqrt{7}}{2}$ **4**

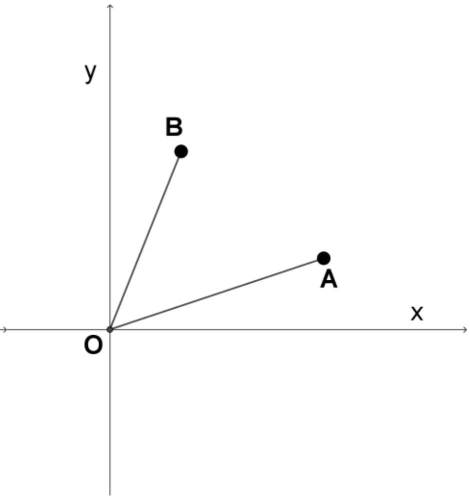
END OF EXAMINATION

Question 9(a) Diagrams

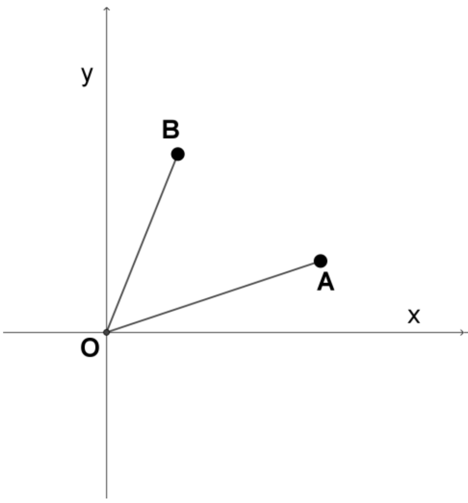
(i)



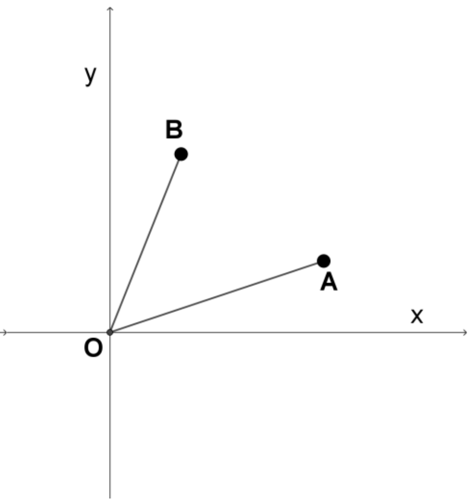
(ii)



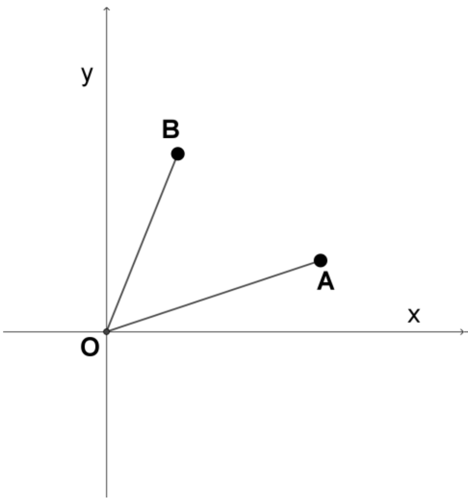
(iii)



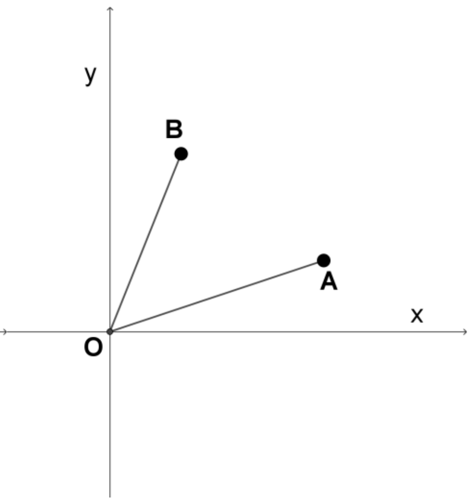
(iv)



(v)



(vi)



Solutions: Ext 2 Task 1 2021 (p.1)

[Dec. 2020]

Complex Numbers

Multiple Choice

(1) A (2) C (3) A (4) C (5) D

(1) i^{2021}

$= (i^{2020}) \times i$

$= 1 \times i$

$= i$ (A)

(2) $2e^{\frac{5\pi i}{6}}$

$= 2\left(\cos\frac{5\pi}{6} + i\sin\frac{5\pi}{6}\right)$

$= 2\left(-\frac{\sqrt{3}}{2} + \frac{1}{2}i\right)$

$= -\sqrt{3} + i$. (C)

(3) $\frac{z}{\bar{z}} \times \frac{z}{z}$

$= \frac{z^2}{z\bar{z}}$ (A)

$= \frac{z^2}{|z|^2}$

(4) $|-2+7i|$

$= \sqrt{2^2+7^2}$

$= \sqrt{53}$.

$\text{Arg}(-2+7i)$

$= 180^\circ - \tan^{-1}\left(\frac{7}{2}\right)$

$= 106^\circ$.

$\therefore -2+7i = \sqrt{53}\,\text{cis}\,106^\circ$

[nearest degree] (C)

(5) $\frac{6+8i}{2+i} \times \frac{(2-i)}{(2-i)}$

$= \frac{20+10i}{2^2+1^2}$ (D)

$= 4+2i$

Q.(6)(a)(i) $\frac{-1+i}{1+i\sqrt{3}} \times \frac{(1-i\sqrt{3})}{(1-i\sqrt{3})}$

$= \frac{-1+i\sqrt{3}+i+\sqrt{3}}{1^2+(\sqrt{3})^2}$ 2

$= \frac{(\sqrt{3}-1)}{4} + i\frac{(\sqrt{3}+1)}{4}$.

(ii) $-1+i = \sqrt{2}\,\text{cis}\,\frac{3\pi}{4}$ 4

$1+i\sqrt{3} = 2\,\text{cis}\,\frac{\pi}{3}$

(iii) $\frac{-1+i}{1+i\sqrt{3}}$

$= \frac{\sqrt{2}\,\text{cis}\,\frac{3\pi}{4}}{2\,\text{cis}\,\frac{\pi}{3}}$ 2

$= \frac{1}{\sqrt{2}}\,\text{cis}\,\frac{5\pi}{12}$

Equating imaginary parts of (i) & (iii)

$\frac{\sqrt{3}+1}{4} = \frac{1}{\sqrt{2}}\sin\frac{5\pi}{12}$.

$\therefore \sin\frac{5\pi}{12} = \frac{\sqrt{6}+\sqrt{2}}{4}$. 2

Q.(6)(b)(i) $(x+iy)^2 = -16+30i$, x, y real.

$(x^2-y^2)+2ixy = -16+30i.$

Equating reals, Equating imaginaries

$x^2-y^2 = -16$ (1) $\quad 2xy = 30$

$y = \frac{15}{x}$ (2)

Sub. (2) in (1)

$x^2 - \left(\frac{15}{x}\right)^2 = -16$

$x^2 - \frac{225}{x^2} = -16$

$x^4+16x^2-225=0$

$(x^2+25)(x^2-9)=0$

$x = \pm 3$ as $x \in \mathbb{R}$.

Sub. $x=3$ in (2) $y = \frac{15}{3} = 5.$

Sub. $x=-3$ in (2) $y = \frac{15}{-3} = -5.$

$\sqrt{-16+30i} = \pm(3+5i).$

4

(ii) $z^2-(1+i)z+(4-7i)=0$

$z = \dfrac{(1+i) \pm \sqrt{(1+i)^2 - 4\times1\times(4-7i)}}{2\times1}$

$= \dfrac{(1+i) \pm \sqrt{-16+30i}}{2}$

$z = \dfrac{1+i+(3+5i)}{2}$ or $z = \dfrac{1+i-3-5i}{2}$ 3

$z = 2+3i$ or $z = -1-2i.$

(c) $32-32i\sqrt{3} = 64\,\text{cis}\left(\frac{-\pi}{3}\right)$

$\therefore \sqrt[6]{32-32i\sqrt{3}} = 2\,\text{cis}\left(\frac{-\pi}{18} + \frac{2k\pi}{6}\right)$, $k=0,1,2,3,4,5.$ By De Moivre. 2

$= 2\text{cis}\left(-\frac{\pi}{18}\right), 2\text{cis}\left(\frac{5\pi}{18}\right), 2\text{cis}\left(\frac{11\pi}{18}\right), 2\text{cis}\left(\frac{17\pi}{18}\right), 2\text{cis}\left(\frac{23\pi}{18}\right), 2\text{cis}\left(\frac{29\pi}{18}\right)$

or if $-\pi < \text{Arg}\, z \le \pi$ $= 2\text{cis}\left(-\frac{\pi}{18}\right), 2\text{cis}\left(\frac{5\pi}{18}\right), 2\text{cis}\frac{11\pi}{18}, 2\text{cis}\frac{17\pi}{18}, 2\text{cis}\frac{-13\pi}{18}, 2\text{cis}\frac{-7\pi}{18}.$

Q.(7)(a)(i) $|z-1-2i| = \sqrt{5}$

$\Rightarrow$ Circle centre $1+2i$, $r = \sqrt{5}$.

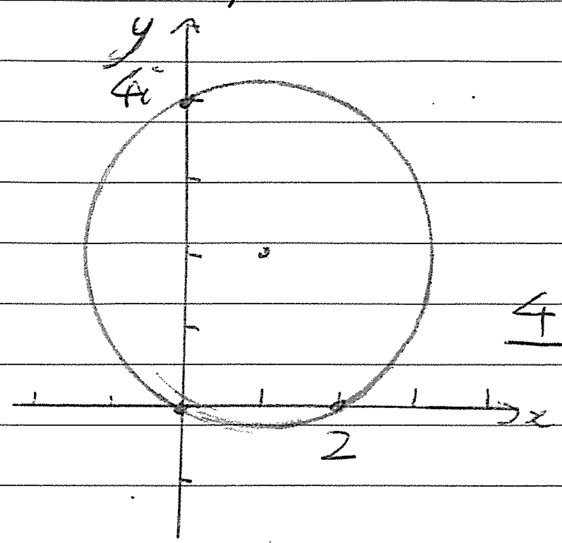
Equation $(x-1)^2+(y-2)^2 = 5$.

x intercepts: $(x-1)^2+(0-2)^2 = 5$

$x = 2$ or $x = 0$

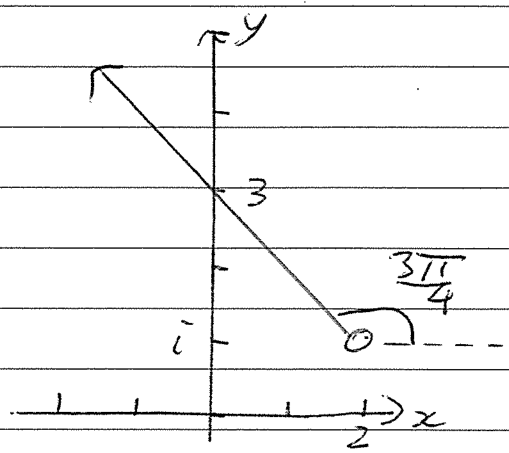
y intercepts: $(0-1)^2+(y-2)^2 = 5$

$y = 0$ or $4i$



4

(ii) $\text{Arg}(z-2-i) = \frac{3\pi}{4}$.



2

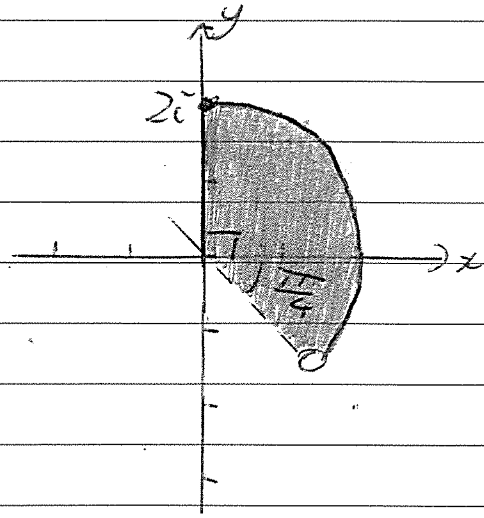
(b) $z\bar{z} \leq 4$

If $z = x+iy$,

$z\bar{z} = (x+iy)(x-iy)$

$= x^2+y^2$.

So $x^2+y^2 \leq 4$.



4

Q.(8)(a) $\alpha^2+\beta^2+\gamma^2$

$= (\alpha+\beta+\gamma)^2 - 2(\alpha\beta+\alpha\gamma+\beta\gamma)$

$= \left(\frac{3}{2}\right)^2 - 2\times 4$

$= \frac{-23}{4}.$

Note: $\alpha+\beta+\gamma = \frac{3}{2}$

$\alpha\beta+\alpha\gamma+\beta\gamma = \frac{8}{2} = 4.$

$\underline{2}$

(b) As $\alpha^2+\beta^2+\gamma^2 < 0$, $2z^3-3z^2+8z+5=0$ must have at least 1 non-real root.

As all co-efficients are real, the CONJUGATE of that non-real root must also be a root. $\underline{2}$

$\therefore$ $2z^3-3z^2+8z+5=0$ has 3 roots (fundamental theorem of algebra).

$2z^3-3z^2+8z+5=0$ has 1 real root (& 2 non real roots).

(c) If $1+2i$ is a root, then its conjugate $(1-2i)$ is also a root

$\therefore$ As $\alpha+\beta+\gamma = \frac{3}{2}$

$(1+2i)+(1-2i)+\gamma = \frac{3}{2}$

$\gamma = \frac{-1}{2}.$

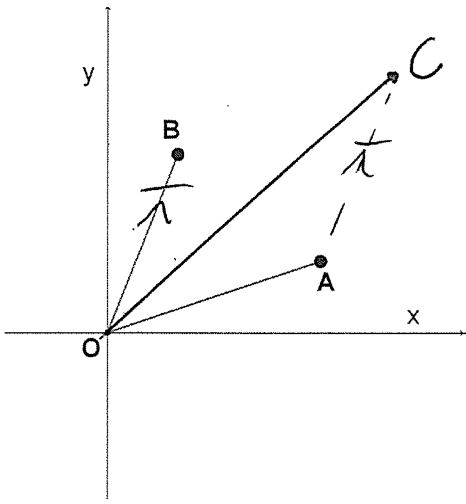
$\underline{2}$

$\therefore$ Roots of $2z^3-3z^2+8z+5=0$ are $1+2i, 1-2i, -\frac{1}{2}.$

For Q.(9)(a) PTO →

Question 9(a) Diagrams

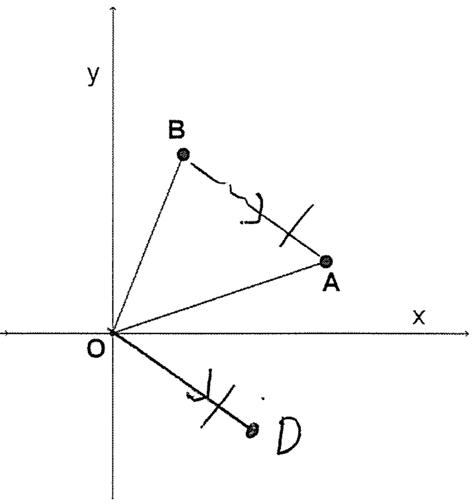
(i)



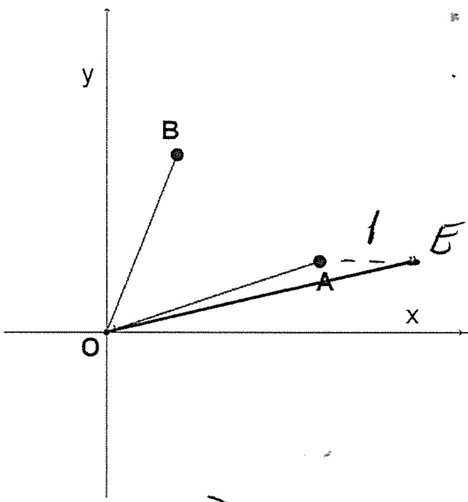
(ii)

$\vec{OD} = \vec{BA}$

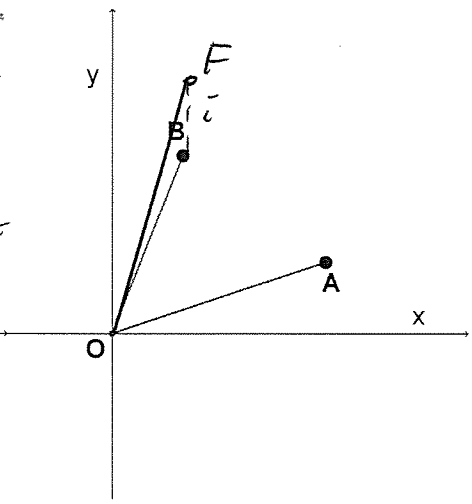
6



(iii)

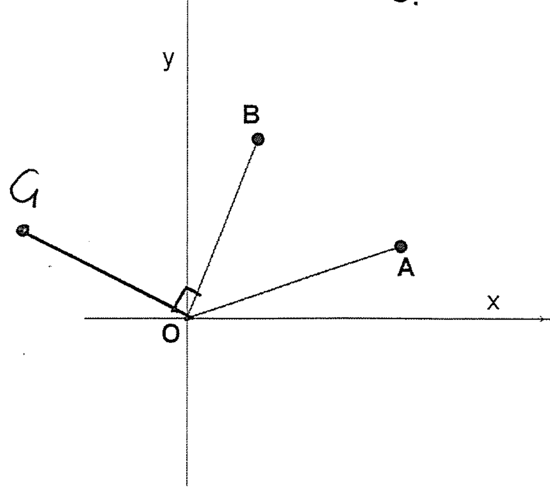


(iv)

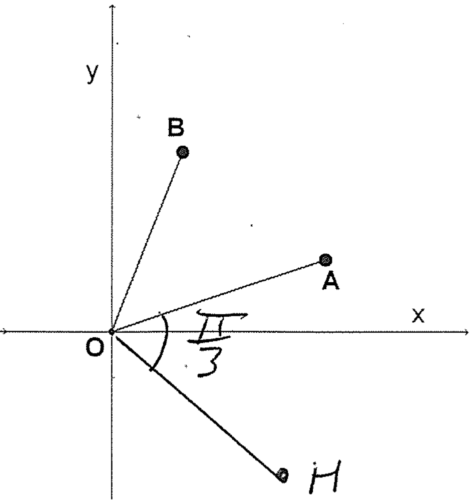


(v)

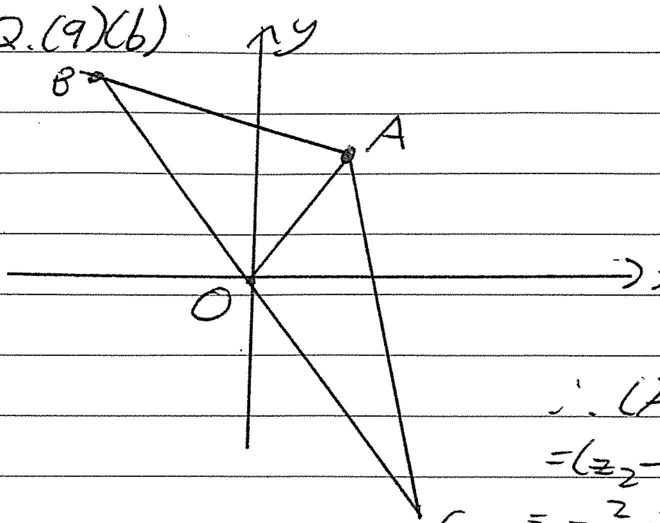
$\vec{OG} = iw$
$= w \operatorname{cis} \frac{\pi}{2}.$



(vi)



Q.(9)(b)



Note: $\overrightarrow{OC} = -\overrightarrow{OB} = -z_2$.

$\overrightarrow{AB} = z_2 - z_1$.

$\overrightarrow{AC} = -z_2 - z_1$

$\therefore (\overrightarrow{AB})^2 + (\overrightarrow{AC})^2$

$= (z_2 - z_1)^2 + (-z_2 - z_1)^2$

$= z_2^2 - 2z_1z_2 + z_1^2 + z_2^2 + 2z_1z_2 + z_1^2$

$= 2z_1^2 + 2z_2^2$

$=$ RHS QED.

4

(c) Let $\overrightarrow{OA} = z$.

$\overrightarrow{OB} = 3$

$\overrightarrow{OC} = -1$.

Let H = circle centre.

$\angle BHC = \frac{\pi}{3}$ [$\angle$ at centre $= 2 \times \angle$ at circumference subtended by arc AB].

As $BH = CH$ [H is circle centre],

$\triangle BHC$ is equilateral.

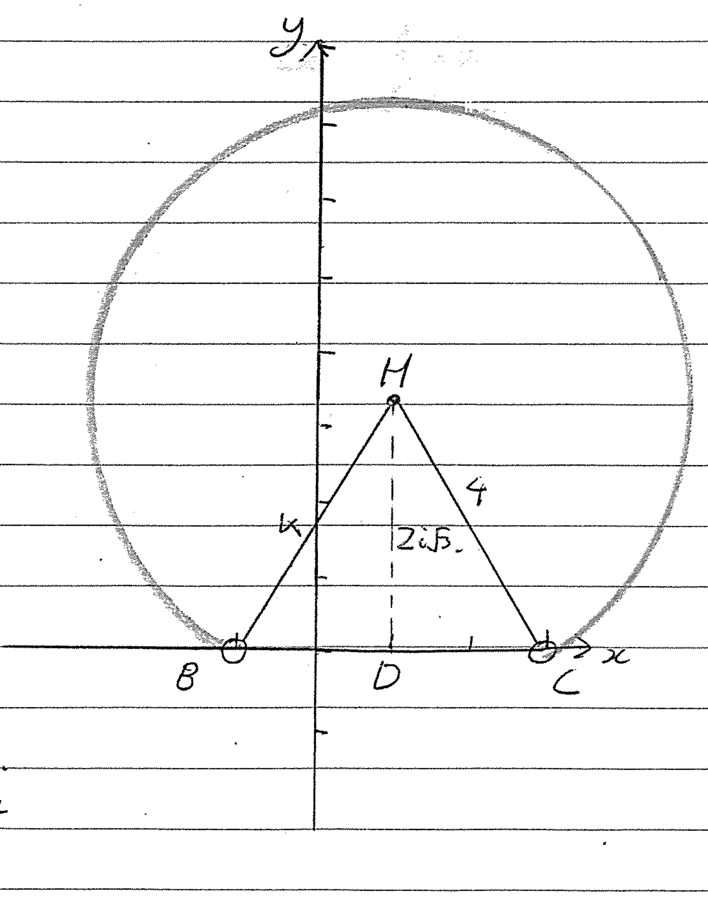
Also, H is on $x = 1$ by symmetry.

Let $D = 1 + 0i$ be the midpoint of B & C.

$BH = CH = 4$.

$DH = 4\sin\frac{\pi}{3} = 2\sqrt{3}$

So $\overrightarrow{OH} = 1 + 2i\sqrt{3}$.



Circle is $(x-1)^2 + (y - 2\sqrt{3})^2 = 16$, $y > 0$.

(10)(a) By DeMoivre's Theorem

$(\cos\theta + i\sin\theta)^3 \qquad = \cos 3\theta + i\sin 3\theta$ 3

$\cos^3\theta + 3i\cos^2\theta\sin\theta - 3\cos\theta\sin^2\theta - i\sin^3\theta \quad = \cos 3\theta + i\sin 3\theta$

Equating real parts,

$\cos 3\theta = \cos^3\theta - 3\cos\theta\sin^2\theta$

$= \cos^3\theta - 3\cos\theta(1-\cos^2\theta)$

$= \cos^3\theta - 3\cos\theta + 3\cos^3\theta$

$\cos 3\theta = 4\cos^3\theta - 3\cos\theta.$

(b)(i) As $e^{ni\theta} = \cos n\theta + i\sin n\theta \quad (1)$ [Euler]

& $e^{-ni\theta} = \cos n\theta - i\sin n\theta \quad (2)$ + 2

$e^{ni\theta} + e^{-ni\theta} = 2\cos n\theta$

(ii) $(e^{i\theta} + e^{-i\theta})^5 = e^{5i\theta} + 5e^{3i\theta} + 10e^{i\theta} + 10e^{-i\theta} + 5e^{-3i\theta} + e^{-5i\theta}$

$= (e^{5i\theta} + e^{-5i\theta}) + 5(e^{3i\theta} + e^{-3i\theta}) + 10(e^{i\theta} + e^{-i\theta})$

Using Euler's theorem, 3

$[2\cos\theta]^5 = 2\cos 5\theta + 5\times 2\cos 3\theta + 10\times 2\cos\theta$

$32\cos^5\theta = 2\cos 5\theta + 10\cos 3\theta + 20\cos\theta$

$\div 2$

$16\cos^5\theta = \cos 5\theta + 5\cos 3\theta + 10\cos\theta.$

(iii) If $\theta = \frac{\pi}{10}$, $5\theta = \frac{\pi}{2}$ & $\cos 5\theta = \cos\frac{\pi}{2} = 0$

& $3\theta = \frac{3\pi}{10} \rightarrow \cos\frac{3\pi}{10} = 4\cos^3\frac{\pi}{10} - 3\cos\frac{\pi}{10}$ [from (a)]. 3

Sub. $\cos 5\theta = 0$ & $\cos\frac{3\pi}{10} = 4\cos^3\frac{\pi}{10} - 3\cos\frac{\pi}{10}$ in (b)(ii)

$16\cos^5\frac{\pi}{10} = 0 + 5\left(4\cos^3\frac{\pi}{10} - 3\cos\frac{\pi}{10}\right) + 10\cos\frac{\pi}{10}$

$16\cos^5\frac{\pi}{10} = 20\cos^3\frac{\pi}{10} - 5\cos\frac{\pi}{10}$ $\quad \div \cos\frac{\pi}{10}$, as $\cos\frac{\pi}{10} \neq 0$

$16\cos^4\frac{\pi}{10} = 20\cos^2\frac{\pi}{10} - 5$

$16\cos^4\frac{\pi}{10} - 20\cos^2\frac{\pi}{10} + 5 = 0$

$\therefore \cos\frac{\pi}{10}$ is a root of $16x^4 - 20x^2 + 5 = 0$

PTO →

Solutions: Complex N^{os} Dec 2020 p.8

Q.(10)(b)(iv) $\cos\frac{\pi}{10}$ is a solution of $16x^4-20x^2+5=0$

By quadratic formula, $x^2 = \dfrac{20 \pm \sqrt{(-20)^2-4\times16\times5}}{2\times16}$

$$x^2 = \frac{5\pm\sqrt{5}}{8}.$$

3

Note: As $\frac{\pi}{10}$ is in Q_1, $\cos\frac{\pi}{10}$ will be positive.

Also, $\frac{5-\sqrt5}{8} < \frac{1}{2}$ so $\sqrt{\frac{5-\sqrt5}{8}} < \frac{1}{\sqrt2}$

which means $\cos^{-1}\left(\sqrt{\frac{5-\sqrt5}{8}}\right) > \frac{\pi}{4}$ & $\neq \frac{\pi}{10}$

So $\cos\frac{\pi}{10} = \sqrt{\dfrac{5+\sqrt5}{8}}.$

Q.(11)(a) $z^7-1=0$

$z = cis\frac{2k\pi}{7}$, $k=0,1,2,3,4,5,6.$ 1

(b) $w = cis\frac{2\pi}{7}$

$w^n = cis\frac{2n\pi}{7}$ [de Moivre]

$(w^n)^7 - 1 = cis\,2n\pi - 1$ 1

$= 1-1$

$=0$

w^n is a solution.

(c) Roots of $z^n-1=0$ $=1, cis\frac{2\pi}{7}, cis\frac{4\pi}{7}, cis\frac{6\pi}{7}, cis\frac{8\pi}{7}, cis\frac{10\pi}{7}, cis\frac{12\pi}{7}$

$=1, w, w^2, w^3, w^4, w^5, w^6$

By sum of roots of $z^n-1=0$, $1+w+w^2+w^3+w^4+w^5+w^6=0$ 1

(d) Sum of roots: $=w+w^2+w^4+w^3+w^5+w^6 = -1$ (using (c)).

Product of roots $=(w+w^2+w^4)(w^3+w^5+w^6)$

$= w^4+w^6+w^7+w^5+w^7+w^8+w^7+w^9+w^{10}$

$= w^4+w^6+1+w^5+1+w+1+w^2+w^3$ 3

$=(1+w+w^2+w^3+w^4+w^5+w^6)+2$

$=2$ [as $1+w+\dots+w^6=0$ from (c)].

$\therefore$ Monic quadratic equation is $\underline{z^2+z+2=0.}$

Q.(11)(e) if $w = cis\frac{2\pi}{7}$,

$$w + w^2 + w^4$$

$$= \left(\cos\frac{2\pi}{7} + i\sin\frac{2\pi}{7}\right) + \left(\cos\frac{4\pi}{7} + i\sin\frac{4\pi}{7}\right) + \left(\cos\frac{8\pi}{7} + i\sin\frac{8\pi}{7}\right)$$

$$= \cos\frac{2\pi}{7} + i\sin\frac{2\pi}{7} - \cos\frac{3\pi}{7} + i\sin\frac{3\pi}{7} - \cos\frac{\pi}{7} - i\sin\frac{\pi}{7}.$$

Imaginary part of $w + w^2 + w^4 = \left[\sin\frac{2\pi}{7} + \sin\frac{3\pi}{7} - \sin\frac{\pi}{7}\right]$.

Roots of $z^2 + z + 2 = 0$

are z $= \dfrac{-1 \pm \sqrt{1^2 - 4\times1\times2}}{2\times1}$.

$$= -\frac{1}{2} \pm \frac{i\sqrt{7}}{2}.$$

4

Equating imaginary part of $w + w^2 + w^4$ with solution,

$$\sin\frac{2\pi}{7} + \sin\frac{3\pi}{7} - \sin\frac{\pi}{7} = \pm\frac{\sqrt{7}}{2}.$$

As $\sin\frac{2\pi}{7}$ & $\sin\frac{3\pi}{7} > \sin\frac{\pi}{7}$ (all in Q_1 & $\frac{2\pi}{7}, \frac{3\pi}{7}$ close to $\frac{\pi}{2}$)

$$\sin\frac{2\pi}{7} + \sin\frac{3\pi}{7} - \sin\frac{\pi}{7} > 0$$

$$\underline{\sin\frac{2\pi}{7} + \sin\frac{3\pi}{7} - \sin\frac{\pi}{7} = \frac{\sqrt{7}}{2}.}$$