GIRRAWEEN HIGH SCHOOL

TASK 1 2022 *(December 2021)*

MATHEMATICS

EXTENSION 2

Complex Numbers

Time allowed – 90 Minutes

DIRECTIONS TO CANDIDATES

- Attempt ALL questions.
- All necessary working should be shown in every question. Marks may be deducted for careless or badly arranged work.
- Board-approved calculators may be used.
- Each question attempted is to be marked clearly Question 6, Question 7 etc. Each question is to be returned on a separate page in your answer booklet.
- You may ask for spare answer booklets if you need them.
- For Multiple choice: Fill in the circle corresponding to the correct answer on the multiple choice answer sheet in your answer booklet.
- For Question 9(a) please draw your diagrams on the page of number planes provided in your answer booklet.
- The notation $cis\ \theta$ is acceptable in this examination.

Multiple Choice begins on the following page

Question 1

$(-i)^{2022} =$

(A) i **(B)** $-i$ **(C)** 1 **(D)** -1

Question 2

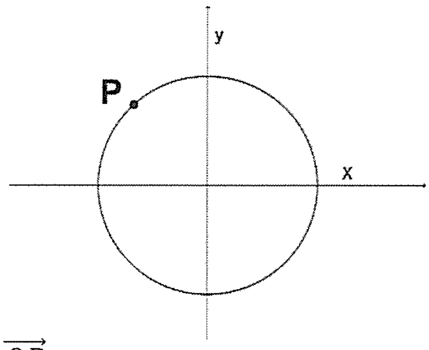
$4e^{-\frac{\pi}{3}} =$

(A) $2\sqrt{3} - 2i$ **(B)** $2 - 2i\sqrt{3}$ **(C)** $-2\sqrt{3} + 2i$ **(D)** $-2 + 2i\sqrt{3}$

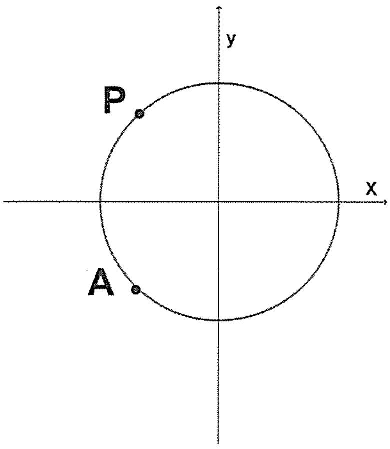
Question 3

If $\overrightarrow{OP} = z, \frac{\pi}{2} < Arg\ z < \pi$ and $|z| > 1 (see\ diagram)$,

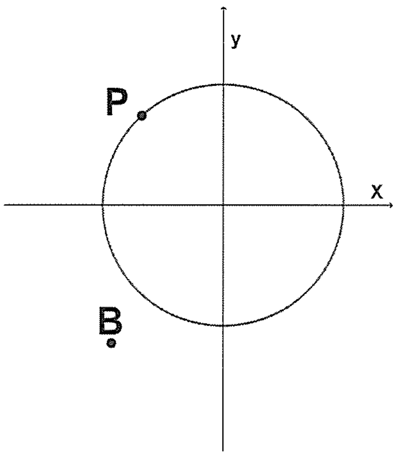
then $\sqrt{z}$ would be



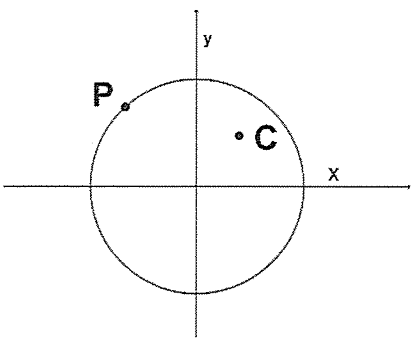
$(A)\overrightarrow{OA}$



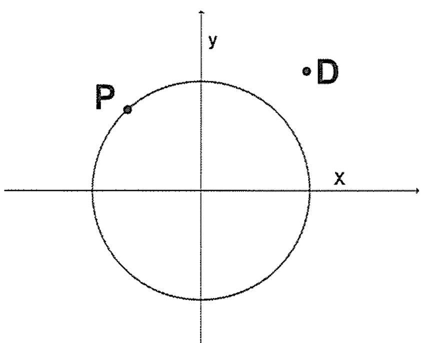
$(B)\overrightarrow{OB}$



$(C)\overrightarrow{OC}$



$(D)\overrightarrow{OD}$



Multiple Choice continues on the following page

Multiple Choice ***(continued)***

Question 4

The solutiions to $z^2 - 2z + 5 = 0$ are

(A)$z = 1 \pm 2i$ **(B)** $z = -1 \pm 2i$ **(C)** $z = 1 \pm 4i$ **(D)** $z = 1 \pm 4i$

Question 5

$z \times \frac{\bar{z}}{|z|^2} =$

(A) $\frac{z}{\bar{z}}$ **(B)** $\frac{1}{z}$ **(C)** z **(D) 1**

Question 6 ***(20 Marks – show all of your working in your answer booklet)*** **Marks**

(a) If $z = -\sqrt{3} - i$ and $w = 1 - i$

(i) Find $\frac{z}{w}$ in Cartesian form. **2**

(ii) Convert z and w to modulus/argument form. **4**

(iii) Find $\frac{z}{w}$ in modulus/argument form. **2**

(iv) Use your result to find the exact value of $cos(-\frac{7\pi}{12})$. **2**

(b) (i) By using $(x + iy)^2 = 15 - 8i$, find the square roots of $15 - 8i$ in Cartesian form. **3**

(ii) Hence solve $z^2 + (2 - i)z + (i - 3) = 0$. **3**

(c) Find the five fifth roots of $-16\sqrt{3} + 16i$. Leave your answers in modulus/argument form. **4**

Question 7 ***(7 Marks – show all of your working in your answer booklet)***

(a) If the roots of the polynomial equation $4z^3 - 13z^2 + 40z + 39 = 0$ are α, β and λ, find $\alpha^2 + \beta^2 + \lambda^2$ WITHOUT finding α, β or λ. **2**

(b) Using your result from (a), state why $4z^3 - 13z^2 + 40z + 39 = 0$ has exactly one real root. **2**

(c) If one of the roots of $4z^3 - 13z^2 + 40z + 39 = 0$ is $z = 2 + 3i$, solve $4z^3 - 13z^2 + 40z + 39 = 0$ over the complex field. **3**

Examination continues on the following page

Question 8 *(12 Marks – show all of your working in your answer booklet)* **Marks**

(a) Sketch the following curves and regions on separate complex planes.

(i) $|z-3+i|=2$ **2**

(ii) $Arg\ (z+1)=-\frac{2\pi}{3}$ **2**

(iii) $|z|=4\ and\ -\frac{\pi}{4}<Arg z\leq\frac{\pi}{4}$ **3**

(b) The locus $Arg\left(\frac{z-3-i}{z+1-i}\right)=\frac{\pi}{4}$ represents part of a circle. Find the centre and radius of this circle and write the Cartesian equation for the locus. **5**

Question 9 *(10 Marks – show all of your working in your answer booklet)*

(a) If $\overrightarrow{OA}=z$ and $\overrightarrow{OB}=w$ with $0\leq Arg\ z\ \leq\frac{\pi}{2}$,

$0\leq Arg\ w\ \leq\frac{\pi}{2}$ and both $|z|$ and $|w|>1$ *(see diagram),*

sketch on the Argand diagrams provided for 9(a) in your answer booklet

(Note that these vectors all start from O)

(i) C so that $\overrightarrow{OC}=z+w$ **1**

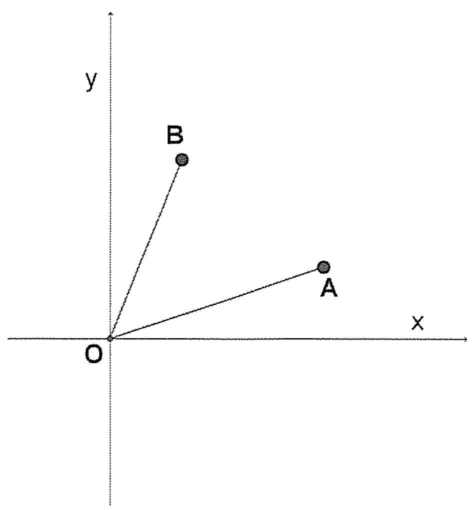
(ii) D so that $\overrightarrow{OD}=w-z$ **1**

(iii) E so that $\overrightarrow{OE}=z+1$ **1**

(iv) F so that $\overrightarrow{OF}=z-2i$ **1**

(v) G so that $\overrightarrow{OG}=-iz$ **1**

(vi) H so that $\overrightarrow{OH}=wcis\frac{\pi}{4}$ **1**

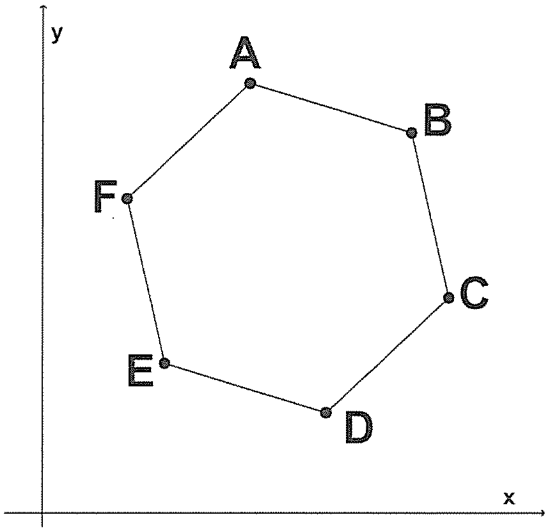


Question 9 continues on the following page

Question 9 ***(continued)*** **Marks**

(b) $ABCDEF$ is a regular hexagon in Quadrant 1 of the complex plane

(see diagram)



If $\overrightarrow{OA} = z_1$, $\overrightarrow{OB} = z_2$, $\overrightarrow{OC} = z_3$, $\overrightarrow{OD} = z_4$, $\overrightarrow{OE} = z_5$ and $\overrightarrow{OF} = z_6$

(i) Explain why $Arg\left(\frac{z_2 - z_3}{z_2 - z_5}\right) = \frac{\pi}{3}$ **2**

(ii) Show that $\left(\frac{z_2 - z_5}{z_3 - z_1}\right)$ is entirely imaginary. **2**

Question 10 ***(13 Marks – show all of your working in your answer booklet)***

(a)(i)Solve $\cos 4\theta = 0, 0 \le \theta \le 2\pi$. **2**

(ii) Using DeMoivre's Theorem and the expansion of $(cos\theta + isin\theta)^4$ **3**

show that $\cos 4\theta = 8cos^4\theta - 8cos^2\theta + 1$.

(iii) Hence find the exact value of $cos\frac{\pi}{8}$. **2**

(you may leave your answer as a fourth root).

(b)(i) Using Euler's Theorem, show that $e^{ni\theta} + e^{-ni\theta} = 2\cos n\theta$. **2**

(ii) Hence show that $cos^4\theta = \frac{1}{8}cos4\theta + \frac{1}{2}cos2\theta + \frac{3}{8}$. **2**

(iii) Hence find the exact value of $cos\frac{\pi}{8}$ **2**

(you may leave your answer as a fourth root).

Examination continues on the following page

Question 11 ***(7 Marks – show all of your working in your answer booklet)*** **Marks**

(a) Solve $z^9 - 1 = 0$. **2**

(b) If w is the root of $z^9 - 1 = 0$ with the smallest positive argument, show that

(i) $w + w^2 + w^3 + w^4 + w^5 + w^6 + w^7 + w^8 = -1$ **1**

(you may assume that w^2, w^3etc. are also roots of $z^9 - 1 = 0$).

(ii) Show that $(w^2 + w^7)(w^4 + w^5)(w + w^8) = -1$ **2**

(c) Hence show that $cos\frac{\pi}{9}\ cos\frac{2\pi}{9}\ cos\frac{4\pi}{9} = \frac{1}{8}$. **2**

(You may assume that if $z = cis\theta, z^n + \frac{1}{z^n} = 2cosn\theta$)

END OF EXAMINATION!!!

GIRRAWEEN HIGH SCHOOL

MATHEMATICS EXTENSION 2
2022 HSC Task 1 (December 2021)

Student No: Solutions. [Final].

This Booklet contains the answer sheet for Part A and Writing Booklet for Part B.

Part A ANSWER SHEET

Select the alternative A, B, C or D that best answers the question.

1.	A	○	B	○	C	○	D	●
2.	A	○	B	●	C	○	D	○
3.	A	○	B	○	C	●	D	○
4.	A	●	B	○	C	○	D	○
5.	A	○	B	○	C	○	D	●

Instructions

- If you need more paper for Part B, please ask your supervisor.
- Write your name on every booklet you use.
- Write on both sides of each sheet of paper.

Total number of booklets used ______________.

Multiple Choice:

(1) $(-i)^{2022}$

$= (-i)^{2020} \times (-i)^2$

$= 1 \times -1$

$= -1.$ D

(2) $4e^{-\frac{\pi}{3}}$

$= 4[\cos(-\frac{\pi}{3}) + i\sin(-\frac{\pi}{3})]$.

$= 4\left[\frac{1}{2} - i\frac{\sqrt{3}}{2}\right]$

$= 2 - 2i\sqrt{3}.$ B

(3) $|\sqrt{z}| < |z|$ if $|z| > 1$

so $\sqrt{z}$ is INSIDE circle.

If $\frac{\pi}{2} < \text{Arg } z < \pi$ then

$0 < \text{Arg}(\sqrt{z}) \leq \frac{\pi}{2}$

as $\text{Arg}(\sqrt{z}) = \frac{1}{2}\text{Arg } z.$ C

(4) $z^2 - 2z + 5 = 0$

$(z-1)^2 + 4 = 0$

$[z - 1 - 2i][z - 1 + 2i] = 0$

$z = 1 \pm 2i.$ A

[Could also be done using the quadratic formula].

(5) $z \times \frac{\bar{z}}{|z|^2}$

$= \frac{z\bar{z}}{|z|^2}$

$= \frac{|z|^2}{|z|^2}$

$= 1$ D

Q. (6)(a)(i) $\frac{z}{w}$

$= \frac{\sqrt{3} - i}{1 - i} \times \frac{(1+i)}{(1+i)}$

$= \frac{\sqrt{3} + i\sqrt{3} - i + 1}{2}$ 2

$= \left(\frac{1+\sqrt{3}}{2}\right) + i\left(\frac{\sqrt{3}-1}{2}\right)$

(ii) $z = 2\,\text{cis}\left(-\frac{5\pi}{6}\right)$ 4

$w = \sqrt{2}\,\text{cis}\left(-\frac{\pi}{4}\right)$

(iii) $\frac{z}{w} = \sqrt{2}\,\text{cis}\left[-\frac{7\pi}{12}\right]$ 2

(iv) Equating real parts of (i) & (iii)

$\frac{1-\sqrt{3}}{2} = \sqrt{2}\cos\left(-\frac{7\pi}{12}\right)$ 2

$\therefore \cos\left(-\frac{7\pi}{12}\right) = \frac{1-\sqrt{3}}{2\sqrt{2}}.$

(b)(i) $(x+iy)^2 = 15 - 8i$, x, y REAL.

$(x^2 - y^2) + 2ixy = 15 - 8i$.

Equating reals, $x^2 - y^2 = 15$ (1)

Equating imaginaries $2xy = -8$

$y = -\frac{4}{x}$ (2)

Sub. (2) in (1):

$x^2 - \left(-\frac{4}{x}\right)^2 = 15$

$x^2 - \frac{16}{x^2} = 15$

$x^4 - 15x^2 - 16 = 0$

$(x^2 - 16)(x^2 + 1) = 0$

As x is real, $x^2 - 16 = 0$

$x = \pm 4.$

As $y = -\frac{4}{x}$ 3

$y = -\frac{4}{4}$ or $-\frac{4}{-4}$.

$= -1$ or 1.

So $\sqrt{15 - 8i}$

$= \pm(4 - i).$

Complex Numbers: p.2

Q.(6)(b)(ii) $z^2+(2-i)z+(i-3)=0$

$$z = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$= \frac{-2+i \pm \sqrt{15-8i}}{2\times 1}.$$

Note: b^2-4ac

$= (2-i)^2 - 4\times 1\times(i-3)$

$= 4-4i-1-4i+12$

$= 15-8i.$

$$= \frac{-2+i \pm (4-i)}{2} \text{ [from (b)(i)]}.$$

3

$$z = \frac{-2+i+4-i}{2} \quad \text{or} \quad z = \frac{-2+i-4+i}{2}$$

$$\underline{z = 1 \quad \text{or} \quad z = -3+i.}$$

(c) $-16\sqrt{3}+16i$

$= 32 \operatorname{cis} \frac{5\pi}{6}.$

$\therefore$ The five 5^{th} roots of $-16\sqrt{3}+16i$

$= 2\operatorname{cis}\left[\frac{\pi}{6}+\frac{2k\pi}{5},\ k=0,1,2,3,4\right].$

4

or $= 2\operatorname{cis}\frac{\pi}{6},\ 2\operatorname{cis}\frac{17\pi}{30},\ 2\operatorname{cis}\frac{29\pi}{30},\ 2\operatorname{cis}\frac{41\pi}{30},\ 2\operatorname{cis}\frac{53\pi}{30}.$

or $= \underline{2\operatorname{cis}\frac{\pi}{6},\ 2\operatorname{cis}\frac{17\pi}{30},\ 2\operatorname{cis}\frac{29\pi}{30},\ 2\operatorname{cis}-\frac{19\pi}{30},\ 2\operatorname{cis}-\frac{7\pi}{30}}.$

Q.(7)(a) $\alpha^2+\beta^2+\lambda^2$

$= (\alpha+\beta+\lambda)^2 - 2(\alpha\beta+\alpha\lambda+\beta\lambda)$

$= \left(\frac{13}{4}\right)^2 - 2\times\frac{40}{4}$ $\underline{2}$

$= -\frac{151}{16}$.

(b) As $\alpha^2+\beta^2+\lambda^2<0$, $4z^3-13z^2+40z+39=0$ must have at least 1 non-real root as a real number squared ≥ 0. $\underline{2}$

As $4z^3-13z^2+40z+39=0$ has all real co-efficients, the CONJUGATE of any non-real root must ALSO be a solution.

Hence $4z^3-13z^2+40z+39=0$ has 1 real & 2 non-real roots

(c) One root of $4z^3-13z^2+40z+39=0$ is $z=2+3i$.

$\therefore$ Another root is $z=2-3i$

So by $\alpha+\beta+\lambda = -\frac{b}{a}$.

$(2+3i)+(2-3i)+\lambda = \frac{13}{4}$. $\underline{3}$

$\lambda = -\frac{3}{4}$.

Solutions to $4z^3-13z^2+40z+39=0$ are

$\underline{z = 2\pm 3i, -\frac{3}{4}}$.

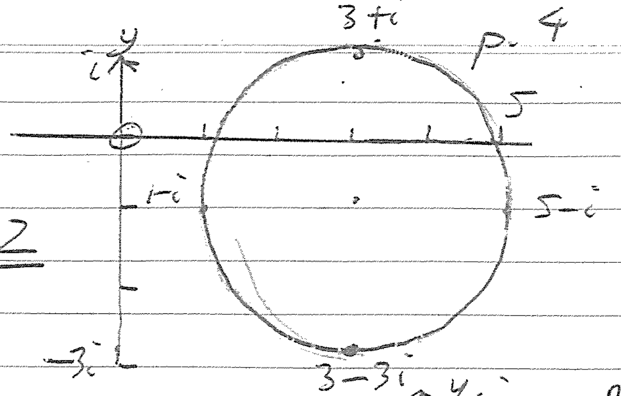
Q.(8) (a)

(i) $|z-3+i| = 2.$

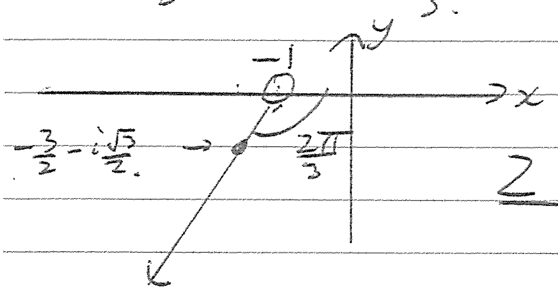
Circle centre $3-i$, $r=2$.

Equation:

$(x-3)^2+(y+1)^2=4.$ $\underline{2}$

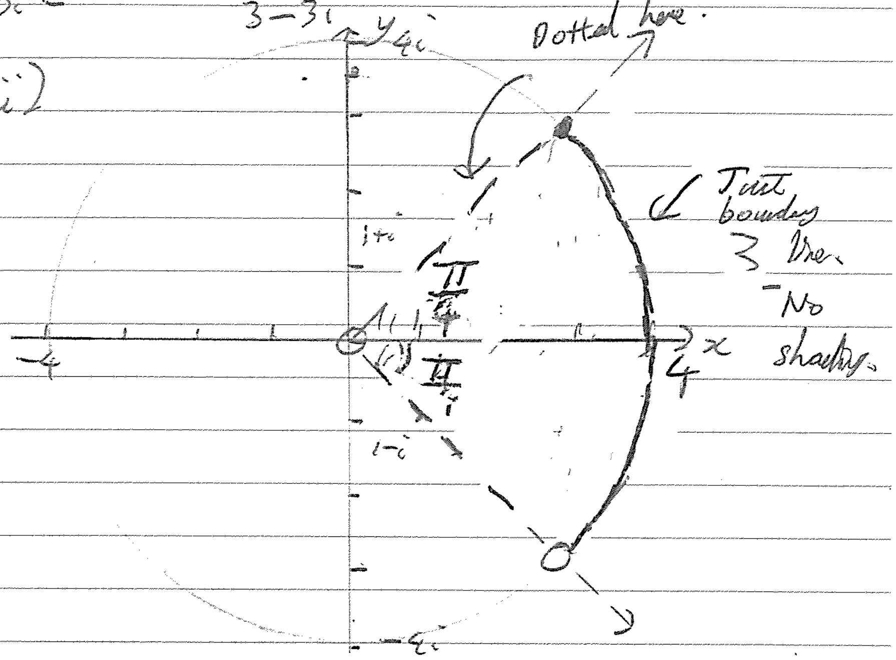


(ii) $\mathrm{Arg}(z+i) = -\frac{2\pi}{3}$.

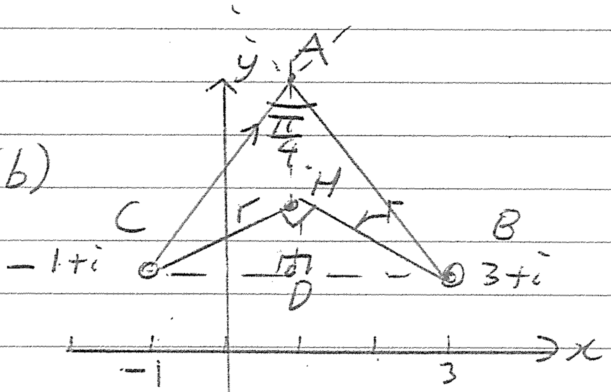


$\underline{2}$

(iii)



Q.(8)(b)



$\vec{OA} = z$

$\vec{OB} = 3+i$

$\vec{OC} = -1+i.$

$\therefore \vec{BA} = z-3-i.$ $\frac{\pi}{2} \le \mathrm{Arg}\,\vec{BA} < \pi.$

$\vec{CA} = z+1-i$ $0 \le \mathrm{Arg}\,\vec{CA} < \frac{\pi}{2}.$

$\therefore \mathrm{Arg}\left(\frac{z-3-i}{z+1-i}\right) = \angle CAB = \frac{\pi}{4}.$

Let circle centre $= H$.

x co-ordinate of $H = 1$.

$\angle CHB = \frac{\pi}{2}$ ($\angle$ at centre $= 2\times\angle$ at circumference subtended by arc CB).

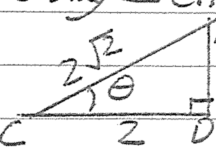
$CH = BH = r$ [radius of circle]

By Pythagoras on $\triangle CHB$, $r^2+r^2 = (CB)^2 = 4^2.$

$r = 2\sqrt{2}.$

Let D be midpoint of $CB = 1+i$

Using $\triangle CHD$:



$(DH)^2 = (2\sqrt{2})^2 - 2^2$

$= 4.$

$DH = 2.$ $\underline{5}$

$\therefore H = \underline{1+3i}$

Locus is

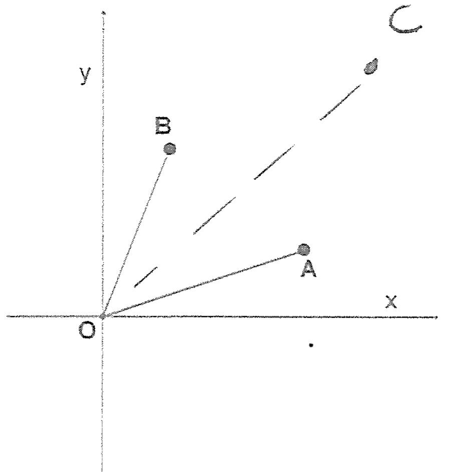
$(x-1)^2+(y-3)^2 = 8,$

$\underline{y > 1.}$

p-5

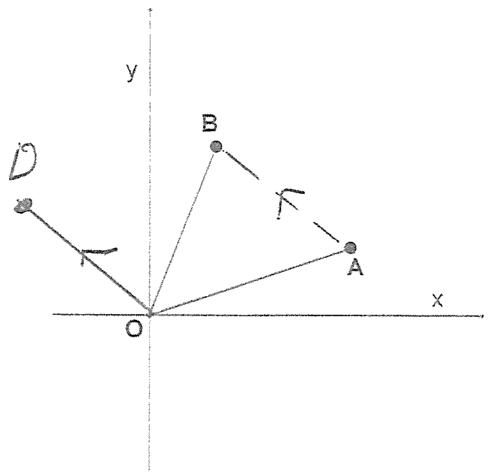
Answer sheet for Question 9(a)

(i)



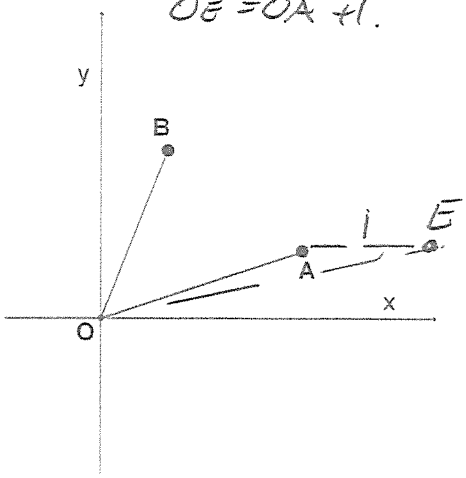
(ii)

$\vec{OD} = \vec{AB} = w - z.$

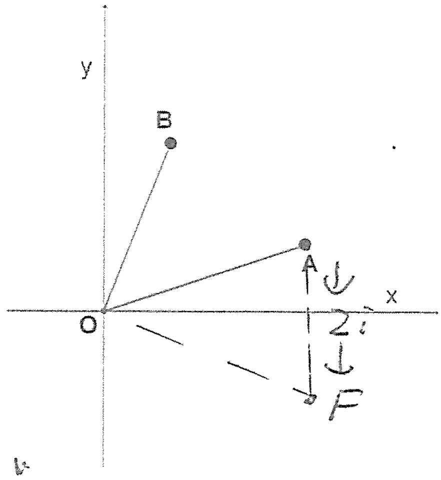


(iii)

$\vec{OE} = \vec{OA} + 1.$

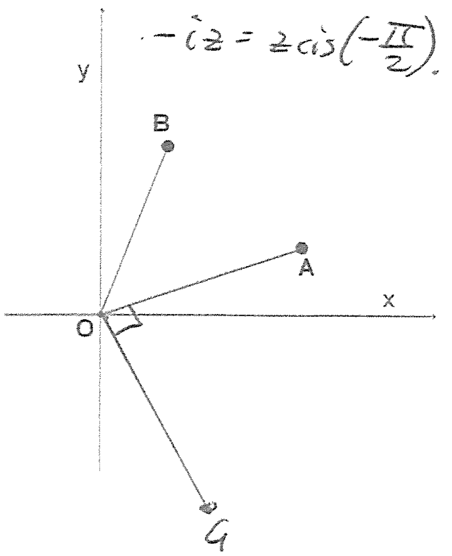


(iv)

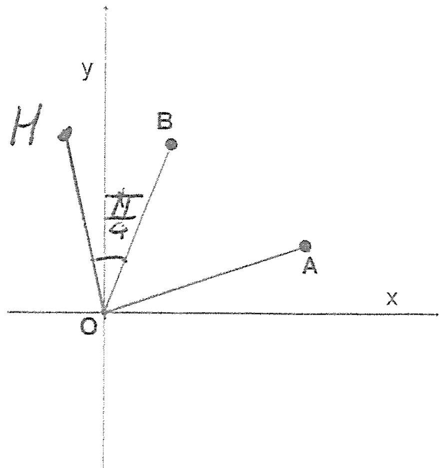


(v)

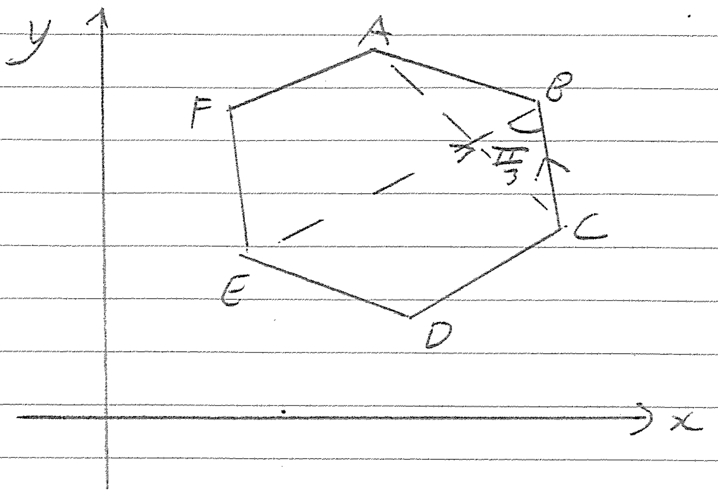
$-iz = z\,\text{cis}\left(-\frac{\pi}{2}\right).$



(vi)



Q.(9)(b)



(i) $\text{Arg}\left(\dfrac{z_2 - z_3}{z_2 - z_5}\right) = \angle EBC$ as $z_2 - z_3 = \overrightarrow{CB}$
& $z_2 - z_5 = \overrightarrow{EB}$.

$\angle ABC = \frac{2\pi}{3}$ (interior $\angle$ of regular hexagon). $\underline{2}$
As EB bisects $\angle ABC$, $\angle EBC = \frac{\pi}{3}$

(ii) $z_3 - z_1 = \overrightarrow{AC}$
Letting G be the point of intersection of AC & EB:

Using $\triangle BCO$.
$\angle BCA = \frac{\pi}{6}$ [$\angle$'s opposite = sides in isosceles $\triangle ABC$].
Using $\triangle BCG$
$\angle BGC = \frac{\pi}{2}$ [$\angle$ sum of $\triangle BGC$]. $\underline{2}$

$\therefore$ As $\text{Arg}\left(\dfrac{z_2 - z_5}{z_3 - z_1}\right) = \angle BCG$.

$\text{Arg}\left(\dfrac{z_2 - z_5}{z_3 - z_1}\right) = \frac{\pi}{2}$.

$\dfrac{z_2 - z_5}{z_3 - z_1}$ is entirely imaginary.

Q.(10)(a)(i) $\cos 4\theta = 0$

$4\theta = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}, \frac{9\pi}{2}, \frac{11\pi}{2}, \frac{13\pi}{2}, \frac{15\pi}{2}$

$\theta = \frac{\pi}{8}, \frac{3\pi}{8}, \frac{5\pi}{8}, \frac{7\pi}{8}, \frac{9\pi}{8}, \frac{11\pi}{8}, \frac{13\pi}{8}, \frac{15\pi}{8}$ 2

(ii) By de Moivre

$(\cos\theta + i\sin\theta)^4 = \cos 4\theta + i\sin 4\theta$

$\cos^4\theta + 4i\cos^3\theta\sin\theta - 6\cos^2\theta\sin^2\theta - 4i\cos\theta\sin^3\theta + \sin^4\theta = \cos 4\theta + i\sin 4\theta$. 4

Equating real parts,

$\cos 4\theta = \cos^4\theta - 6\cos^2\theta\sin^2\theta + \sin^4\theta$

$= \cos^4\theta - 6\cos^2\theta(1-\cos^2\theta) + (1-\cos^2\theta)^2$

$= 8\cos^4\theta - 8\cos^2\theta + 1$, as required. 3

(iii) Hence $\frac{\pi}{8}, \frac{3\pi}{8}, \frac{5\pi}{8}, \frac{7\pi}{8}$ etc. are solutions to $\cos 4\theta = 0$

i.e. $8\cos^4\theta - 8\cos^2\theta + 1 = 0$

$$\cos^2\theta = \frac{8 \pm \sqrt{(-8)^2 - 4\times 8\times 1}}{2\times 8}$$

$$\cos^2\theta = \frac{2 \pm \sqrt{2}}{4}.$$ 2

$$\cos\theta = \pm\sqrt{\frac{2\pm\sqrt{2}}{4}}.$$

As $\frac{\pi}{8}$ is in Q_1, $\cos\frac{\pi}{8}$ will be positive & will be $\sqrt{\frac{2+\sqrt{2}}{4}}$ or $\sqrt{\frac{2-\sqrt{2}}{4}}$.

As $\cos\frac{\pi}{8} > \cos\frac{3\pi}{8}$, $\cos\frac{\pi}{8} = \sqrt{\frac{2+\sqrt{2}}{4}}$.

Q.(10)(b)(i) By Euler's theorem,

$e^{ni\theta} + e^{-ni\theta}$

$= (\cos n\theta + i\sin n\theta) + (\cos(-n\theta) + i\sin(-n\theta))$ $\underline{2}$

$= \cos n\theta + i\sin n\theta + \cos n\theta - i\sin n\theta$ [as cos even, sin odd]

$= 2\cos n\theta$ as required.

(ii) Hence $[2\cos\theta]^4 = (e^{i\theta} + e^{-i\theta})^4$

$\Rightarrow$

$16\cos^4\theta = e^{4i\theta} + 4e^{2i\theta} + 6 + 4e^{-2i\theta} + e^{-4i\theta}$ $\underline{2}$

$= (e^{4i\theta} + e^{-4i\theta}) + 4(e^{2i\theta} + e^{-2i\theta}) + 6$

$16\cos^4\theta = 2\cos 4\theta + 8\cos 2\theta + 6$

$\cos^4\theta = \frac{1}{8}\cos 4\theta + \frac{1}{2}\cos 2\theta + \frac{3}{8}$ as required.

(iii) Hence $\cos^4\frac{\pi}{8} = \frac{1}{8}\cos\frac{\pi}{2} + \frac{1}{2}\cos\frac{\pi}{4} + \frac{3}{8}$

$\cos^4\frac{\pi}{8} = \frac{1}{2\sqrt{2}} + \frac{3}{8}$ $\underline{2}$

$\cos^4\frac{\pi}{8} = \frac{2\sqrt{2} + 3}{8}$.

$\cos\frac{\pi}{8} = \sqrt[4]{\frac{2\sqrt{2} + 3}{8}}$.

as cos positive in Q_1, just positive $\sqrt[4]{\ }$.

[Note: Can also do $\cos^4\frac{\pi}{8} = \frac{4\sqrt{2} + 6}{16}$

$\cos\frac{\pi}{8} = \frac{\sqrt[4]{4\sqrt{2} + 6}}{2}$.

Q.(11)(a) $z^9 - 1 = 0$

$z = \text{cis}\,\frac{2k\pi}{9}$, $k = 0, 1, 2, 3, 4, 5, 6, 7, 8$.

(b) (i) In $z^9 - 1 = 0$

By $\alpha + \beta + \dots + \lambda = \frac{-b}{a}$

$1 + \omega + \omega^2 + \omega^3 + \omega^4 + \omega^5 + \omega^6 + \omega^7 + \omega^8 = 0$

$\omega + \omega^2 + \omega^3 + \omega^4 + \omega^5 + \omega^6 + \omega^7 + \omega^8 = -1.$

Note: Can also use

$z^9 - 1 = 0$

$(z-1)(1 + z + z^2 + z^3 + z^4 + z^5 + z^6 + z^7 + z^8) = 0$

sub. in ω

$(\omega - 1)(1 + \omega + \omega^2 + \dots + \omega^8) = 0$

As $\omega \neq 1$, $1 + \omega + \omega^2 + \omega^3 + \omega^4 + \omega^5 + \omega^6 + \omega^7 + \omega^8 = 0.$

(ii) $(\omega^2 + \omega^7)(\omega^4 + \omega^5)(\omega + \omega^8)$

$= (\omega^6 + \omega^7 + \omega^{11} + \omega^{12})(\omega + \omega^8)$

$= \omega^7 + \omega^{14} + \omega^8 + \omega^{15} + \omega^{12} + \omega^{19} + \omega^{13} + \omega^{20}$

$= \omega^7 + \omega^5 + \omega^8 + \omega^6 + \omega^3 + \omega + \omega^4 + \omega^2$

$= -1$ (using (i)).

(iii) As $\omega + \omega^8 = \text{cis}\,\frac{2\pi}{9} + \text{cis}\,\frac{16\pi}{9}$

$= 2\cos\frac{2\pi}{9}$

Similarly $\omega^2 + \omega^7 = 2\cos\frac{4\pi}{9}$ & $\omega^4 + \omega^5 = 2\cos\frac{8\pi}{9}$

$= -2\cos\frac{\pi}{9}$

By $(\omega^2 + \omega^7)(\omega^4 + \omega^5)(\omega + \omega^8) = -1.$

$2\cos\frac{4\pi}{9} \times -2\cos\frac{\pi}{9} \times 2\cos\frac{2\pi}{9} = -1$

$-8\cos\frac{\pi}{9}\cos\frac{2\pi}{9}\cos\frac{4\pi}{9} = -1. \quad \div -8$

$\cos\frac{\pi}{9}\cos\frac{2\pi}{9}\cos\frac{4\pi}{9} = \frac{1}{8}.$

END OF SOLUTION!!!