HURLSTONE AGRICULTURAL HIGH SCHOOL

PRO PATRIA

MATHEMATICS – EXTENSION TWO

2013 HSC

ASSESSMENT TASK 3

Examiners ~ G Huxley, S Gee

GENERAL INSTRUCTIONS

- Reading Time – 3 minutes.
- Working Time – 40 minutes.
- Attempt **all** questions.
- **All** necessary working should be shown in every question.
- This paper contains four (4) multiple choice questions, and 2 extended response questions, worth 15 marks each. Total = 34 marks.
- Multiple choice is to be answered on the sheet provided. The extended response questions are to be answered in the booklets provided.
- Marks may not be awarded for careless or badly arranged work.
- Board approved calculators may be used.
- **Each question is to be started in a new booklet.**
- This examination paper must **NOT** be removed from the examination room.

STUDENT NAME:____________________________

TEACHER: ____________________________

Part A Multiple Choice (Complete on the answer sheet provided)

QUESTION 1

The minimum degree of the polynomial $y = A(x)$, given that $y = A(x)$, is an even polynomial with a double root at $x = a$ ($a \neq 0$), is

A. 1 B. 2 C. 4 D. 8

QUESTION 2

The polynomial equation $x^3 - 3x^2 - x + 2 = 0$ has roots $\alpha,\ \beta$ and γ. Which of the following polynomial equations have roots $\frac{1}{\alpha},\ \frac{1}{\beta}$ and $\frac{1}{\gamma}$?

A $x^3 - x^2 - 3x + 1 = 0$

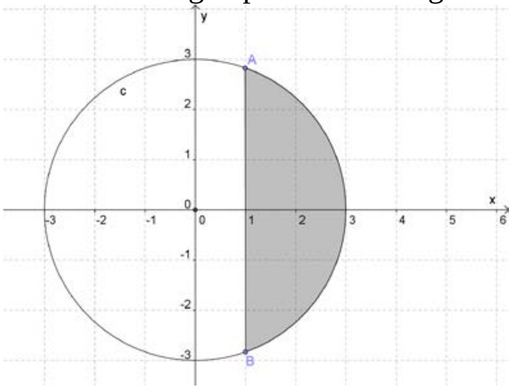
B $x^3 - 2x^2 - 3x + 1 = 0$

C $2x^3 - x^2 - 3x + 1 = 0$

D $2x^3 - 2x^2 - 3x + 1 = 0$

QUESTION 3

Which of the following expressions will give the correct value for the shaded area shown below?



A $\int_1^3 \sqrt{3 - x^2}\, dx$

B $2\int_1^3 \sqrt{3 - x^2}\, dx$

C $\int_1^3 \sqrt{9 - x^2}\, dx$

D $2\int_1^3 \sqrt{9 - x^2}\, dx$

QUESTION 4

$\int_{-\pi}^{\pi} \sin x \cos x \ dx =$

A π B −π C 0 D 1

Part B:

QUESTION 5 **15 marks** *Start a SEPARATE booklet.* **Marks**

(a) Find the polynomial $T(x)$ of degree three that has zeros at $x=-1, x=1$ and $x=2$ and the graph of $y = T(x)$ has a y intercept of 4. **2**

(b) Find the values of k for which $P(x)=x^3-3x^2-9x+k$ has exactly 3 unique solutions **3**

(c) Find the values of α and β if $z^3+3z+2i=(z-\alpha)^2(z-\beta)$ **3**

(d) Suppose that b and d are real numbers and $d \neq 0$.
Consider the polynomial $h(x)=x^4+bx^2+d$. It is given that $h(\alpha) = 0$ and is a double root.

(i) Prove $h'(x)$ is an odd function. **2**

(ii) Prove that $h(-\alpha)=0$ is also a double root of $h(x)$. **2**

(iii) Prove $d=\dfrac{b^2}{4}$ **1**

(iv) For what values of b does $h(x)$ have a double root equal to $i\sqrt{3}$? **1**

(v) For what values of b does $h(x)$ have real roots? **1**

QUESTION 6 **15 marks** *Start a SEPARATE booklet.* **Marks**

(a) Find $\int \cos^3 x\, dx$ **2**

(b) Using the substitution $u = \ln x$, or otherwise,

evaluate: $\int_1^e \frac{(\ln x)^3}{x}\, dx$. **2**

(c) When finding $\int \sec^2 x \tan x\, dx$;

substituting $u = \tan x$ gives the solution: $\frac{1}{2}\tan^2 x + c_1$

but substituting $u = \sec x$ gives the solution: $\frac{1}{2}\sec^2 x + c_2$.

(There is no need to prove the results above)

Explain why both these solutions are valid,
and determine a relationship between c_1 and c_2. **2**

(d) (i) Show that $\int_0^2 \frac{x+4}{x^2+2x+4}\, dx = \frac{1}{2}\ln 3 + \frac{\pi\sqrt{3}}{6}$ **3**

(ii) Find constants A, B and C such that:

$$\frac{3x}{(x+1)(x^2+2x+4)} \equiv \frac{Ax+B}{x^2+2x+4} + \frac{C}{x+1}$$ **3**

(iii) Hence or otherwise, evaluate:

$$\int_0^2 \frac{3x}{(x+1)(x^2+2x+4)}\, dx$$ **3**

STUDENT NUMBER:........................ ....

HURLSTONE AGRICULTURAL HIGH SCHOOL

2013 Assessment Task 3

YEAR 12
EXTENSION 2
MATHEMATICS

PART A Answers

1	a	b	c	d
2	a	b	c	d
3	a	b	c	d
4	a	b	c	d

Year 12 Mathematics Extension 2 Task 3 Examination 2013		
Question No.5 Solutions and Marking Guidelines		
Outcomes Addressed in this Question		
E4 uses efficient techniques for the algebraic manipulation required in dealing with questions such as those involving conic sections and polynomials		
Outcome	**Solutions**	**Marking Guidelines**
	$T(x)=a(x-1)(x+1)(x-2)$ $T(0)=4$ $T(0)=a(0-1)(0+1)(0-2)$ $4=2a$ $a=2$ $T(x)=2(x-1)(x+1)(x-2)$	2 marks correct method leading to correct solution 1 mark substantially correct method
	$P(x)=x^3-3x^2-9x+k$ $P'(x)=3x^2-6x-9$ $P'(x)=3(x^2-2x-3)$ Stationary points occur when $P'(x)=0$ $(x^2-2x-3)=0$ $(x-3)(x+1)=0$ $x=-1,3$ $P''(x)=6x-6$ $P''(-1)=6(-1)-6<0 \therefore$ relative max at $x=-1$ $P''(3)=6(3)-6>0 \therefore$ relative min at $x=3$ When $P(-1)=(-1)^3-3(-1)^2-9(-1)+k=0$ then a double root on the x axis. When $P(3)=(3)^3-3(3)^2-9(3)+k=0$ then a double root on the x axis. The product of the y values must be negative to be on opposite sides of the x axis to enable 3 distinct solutions to $P(x)=0$ $\therefore (k+5)(k-27)<0$ $-5<k<27$	3 marks correct method 2 mark substantially correct method 1 mark some progress towards correct solution
	(c) $P(z)=z^3+3z+2i$ $P(z)=(z-\alpha)^2(z-\beta)$ $P'(z)=3z^2+3$ $P'(\alpha)=3\alpha^2+3=0$ (double root) $\alpha^2=-1$ $\alpha=\pm i$ $P(i)=i^3+3i+2i=5i$ $P(-i)=(-i)^3+3(-i)+2i=0$ $\therefore \alpha=-i$ $\alpha^2\beta=-2i$ $\therefore \beta=2i$	3 marks correct method 2 marks substantially correct method 1 mark some progress towards correct solution

	(d)(i) $h(x) = x^4 + bx^2 + d$ $h'(x) = 4x^3 + 2bx$ $h'(-x) = 4(-x)^3 + 2b(-x)$ $h'(-x) = -(4x^3 + 2bx)$ $h'(-x) = -h'(x) \therefore$ odd function	2 marks correct method leading to correct solution 1 mark substantially correct method
	*(ii)*Since $h'(x)$ is an odd function then $h'(-\alpha)$ is also a root. Since $h(x) = x^4 + bx^2 + d$ $h(-x) = (-x)^4 + b(-x)^2 + d$ $h(-x) = h(x) \therefore$ an even fuction $h(\alpha)$ is a double root then $h(-\alpha)$ is a double root	2 marks correct method leading to correct solution 1 mark substantially correct method
	*(iii)*The roots of $h'(x)$ are $\alpha, -\alpha$ and 0. $\frac{2b}{4}$ is the coefficient of x. $\therefore \alpha \times (-\alpha) + \alpha \times (0) + 0 \times (-\alpha) = \frac{b}{2}$ $\alpha^2 = -\frac{b}{2}$ The product of the roots of $h(x)$ is d. $\therefore \alpha \times \alpha \times -\alpha \times -\alpha = d$ $\alpha^4 = d$ $(\alpha^2)^2 = \left(-\frac{b}{2}\right)^2 = \frac{b^2}{4}$	1 mark correct method leading to correct solution
	$(iv) b = -2\alpha^2$ $b = -2(i\sqrt{3})$ if $\alpha = i\sqrt{3}$ $b = -2 \times -3 = 6$	1 mark correct method leading to correct solution
	(v) Using $\alpha^2 = -\frac{b}{2}$ *if* $b > 0$, then the roots are imaginary $\left(\pm i\sqrt{\frac{b}{2}}\right)$ *if* $b < 0$, then the roots are real $\left(\pm\sqrt{\frac{b}{2}}\right)$ $b \neq 0$ as $d \neq 0$. $\therefore$ for real roots $b < 0$.	1 mark correct method leading to correct solution