STUDENT'S NAME: ______________________________

TEACHER'S NAME: ______________________________

2024

HURLSTONE
AGRICULTURAL
HIGH SCHOOL

Mathematics Extension 2

HSC ASSESSMENT TASK 3

Examiners

- Mr G Rawson
- Mr G Huxley

General Instructions

- Reading time – 3 minutes
- Working time – 45 minutes
- Write using black or blue pen
- NESA-approved calculators may be used
- A Reference Sheet is provided for your use at the back of this question booklet
- In Questions 7 – 8, show relevant mathematical reasoning and/or calculations

Total marks: 30

Section I – 6 marks (pages 2 - 3)

- Attempt Questions 1– 6
- Allow about 9 minutes for this section

Section II – 24 marks (pages 4 – 5)

- Attempt Questions 7 – 8
- Allow about 36 minutes for this section

Section I : 6 marks

Attempt Questions 1 – 6.

Allow about 9 minutes for this section.

Use the multiple choice answer sheet provided for Questions 1 – 6.

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1. Which expression is equal to $\int x^3(x^4-1)^2\,dx$?

A $\frac{1}{12}(x^4-1)^2+C$ **B** $\frac{1}{4}(x^4-1)^3+C$

C $x(x^4-1)+C$ **D** $\frac{1}{12}(x^4-1)^3+C$

2. Which expression is equal to $\int \frac{1}{\sqrt{-x^2+6x-5}}\,dx$?

A $\sin^{-1}\left(\frac{x-3}{2}\right)+C$ **B** $\cos^{-1}\left(\frac{x-3}{2}\right)+C$

C $\ln\left|x-3+\sqrt{(x-3)^2+4}\right|+C$ **D** $\ln\left|x-3+\sqrt{(x-3)^2-4}\right|+C$

3. If $\int_a^b \frac{1}{\sin x+\cos x}\,dx = 1$, where $a, b \in \mathbb{R}$,

What is the value of $\int_{2a}^{2b} \frac{1}{\sin\frac{x}{2}+\cos\frac{x}{2}}\,dx$?

A $\frac{1}{4}$ **B** $\frac{1}{2}$

C 1 **D** 2

4. Which one of the following points lies on the line $\underset{\sim}{r} = \begin{bmatrix} 5 \\ -3 \\ -2 \end{bmatrix} + \lambda \begin{bmatrix} -6 \\ 1 \\ 6 \end{bmatrix}$?

A $(11, -4, 4)$ **B** $(-13, 0, 16)$

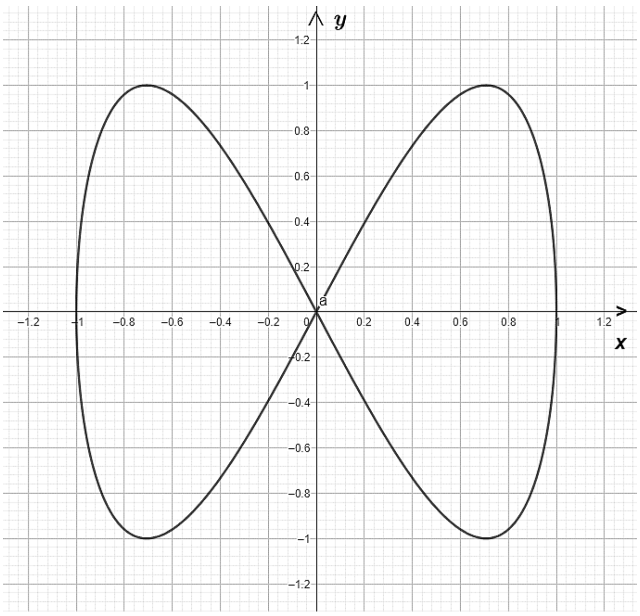
C $(-13, -6, 16)$ **D** $(7, -5, -8)$

5. What is the approximate size of the angle between the vectors $\begin{bmatrix} 2 \\ -3 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$?

A 57° **B** 93°

C 123° **D** 158°

6. Consider the curve drawn below:



Which of the following is the vector equation for the curve above?

A $\underset{\sim}{r}(t) = (\sin t)\underset{\sim}{i} + (2\sin t)\underset{\sim}{j}$ **B** $\underset{\sim}{r}(t) = (\sin t)\underset{\sim}{i} + (\sin 2t)\underset{\sim}{j}$

C $\underset{\sim}{r}(t) = (2\sin t)\underset{\sim}{i} + (2\sin t)\underset{\sim}{j}$ **D** $\underset{\sim}{r}(t) = (2\sin t)\underset{\sim}{i} + (\sin t)\underset{\sim}{j}$

Section II

24 marks

Attempt Questions 7 – 8

Allow about 36 minutes for this section

Answer each question in a separate writing booklet.

All necessary working should be shown in every question.

Question 7 (12 marks) Start a new booklet **Marks**

(a) Find $\int \frac{x^2}{1+4x^2}\,dx$. **3**

(b) Evaluate $\int_0^{\frac{\pi}{2}} \frac{dx}{1+\sin x}$. **4**

(c) (i) Let $I_n = \int_0^x \sec^n t\,dt$, where $0 \le x < \frac{\pi}{2}$. **3**

Show that $I_n = \frac{\sec^{n-2} x \tan x}{n-1} + \frac{n-2}{n-1} I_{n-2}$.

(ii) Hence, find the exact value of **2**

$$\int_0^{\frac{\pi}{3}} \sec^4 t\,dt$$

Question 8 (12 marks) Start a new booklet **Marks**

(a) Find the distance AB, if A and B have the respective position vectors, **2**

$$4\underset{\sim}{\boldsymbol{i}} - 2\underset{\sim}{\boldsymbol{j}} - 7\underset{\sim}{\boldsymbol{k}} \qquad \text{and} \qquad -3\underset{\sim}{\boldsymbol{i}} + 6\underset{\sim}{\boldsymbol{j}} - \underset{\sim}{\boldsymbol{k}}$$

(b) Use vector methods to show that the points: **2**

$A(5, 3, -2)$ $\qquad$ $B(2, -1, 3)$ and $C(-7, -13, 18)$

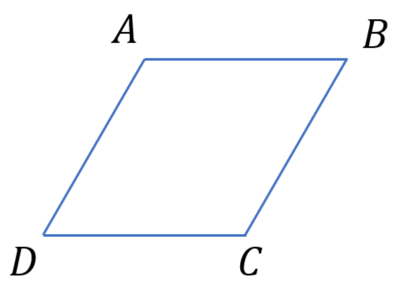
are collinear.

(c) (i) Find the centre and the radius of the sphere which satisfies the following: **2**

$$x^2 + y^2 + z^2 - 2x - 8y + 6z + 1 = 0$$

(ii) Find the equation of the intersection between the sphere in part (i) and the xy plane. **1**

(d) $ABCD$ is a rhombus.



Let P, Q, R and S be the midpoints of AB, BC, CD, and AD respectively.
Use vector methods for the following:

(i) Prove that the points $PQRS$ form a parallelogram. **2**

(ii) Let $\angle ABC = \theta$.
Show that $\overrightarrow{AB} \cdot \overrightarrow{AD} = -\overrightarrow{BA} \cdot \overrightarrow{BC}$ **1**

(iii) Hence or otherwise, prove that the points $PQRS$ form a rectangle. **2**

End of Test

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Nothing to see here….

NSW Education Standards Authority

2020 HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Advanced
Mathematics Extension 1
Mathematics Extension 2

REFERENCE SHEET

Measurement

Length

$$l = \frac{\theta}{360} \times 2\pi r$$

Area

$$A = \frac{\theta}{360} \times \pi r^2$$

$$A = \frac{h}{2}(a+b)$$

Surface area

$$A = 2\pi r^2 + 2\pi rh$$

$$A = 4\pi r^2$$

Volume

$$V = \frac{1}{3}Ah$$

$$V = \frac{4}{3}\pi r^3$$

Financial Mathematics

$$A = P(1+r)^n$$

Sequences and series

$$T_n = a + (n-1)d$$

$$S_n = \frac{n}{2}[2a + (n-1)d] = \frac{n}{2}(a+l)$$

$$T_n = ar^{n-1}$$

$$S_n = \frac{a(1-r^n)}{1-r} = \frac{a(r^n-1)}{r-1},\ r \ne 1$$

$$S = \frac{a}{1-r},\ |r| < 1$$

Functions

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

For $ax^3 + bx^2 + cx + d = 0$:

$$\alpha + \beta + \gamma = -\frac{b}{a}$$

$$\alpha\beta + \alpha\gamma + \beta\gamma = \frac{c}{a}$$

and $\alpha\beta\gamma = -\frac{d}{a}$

Relations

$$(x-h)^2 + (y-k)^2 = r^2$$

Logarithmic and Exponential Functions

$$\log_a a^x = x = a^{\log_a x}$$

$$\log_a x = \frac{\log_b x}{\log_b a}$$

$$a^x = e^{x \ln a}$$

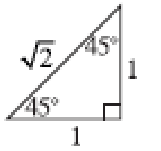
– 1 –

Trigonometric Functions

$$\sin A = \frac{\text{opp}}{\text{hyp}}, \quad \cos A = \frac{\text{adj}}{\text{hyp}}, \quad \tan A = \frac{\text{opp}}{\text{adj}}$$

$$A = \frac{1}{2}ab\sin C$$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

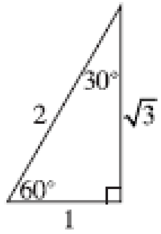


$$c^2 = a^2 + b^2 - 2ab\cos C$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

$$l = r\theta$$

$$A = \frac{1}{2}r^2\theta$$



Trigonometric identities

$$\sec A = \frac{1}{\cos A}, \ \cos A \neq 0$$

$$\operatorname{cosec} A = \frac{1}{\sin A}, \ \sin A \neq 0$$

$$\cot A = \frac{\cos A}{\sin A}, \ \sin A \neq 0$$

$$\cos^2 x + \sin^2 x = 1$$

Compound angles

$$\sin(A + B) = \sin A\cos B + \cos A\sin B$$

$$\cos(A + B) = \cos A\cos B - \sin A\sin B$$

$$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}$$

If $t = \tan\frac{A}{2}$ then $\sin A = \frac{2t}{1+t^2}$

$$\cos A = \frac{1-t^2}{1+t^2}$$

$$\tan A = \frac{2t}{1-t^2}$$

$$\cos A\cos B = \frac{1}{2}\left[\cos(A - B) + \cos(A + B)\right]$$

$$\sin A\sin B = \frac{1}{2}\left[\cos(A - B) - \cos(A + B)\right]$$

$$\sin A\cos B = \frac{1}{2}\left[\sin(A + B) + \sin(A - B)\right]$$

$$\cos A\sin B = \frac{1}{2}\left[\sin(A + B) - \sin(A - B)\right]$$

$$\sin^2 nx = \frac{1}{2}(1 - \cos 2nx)$$

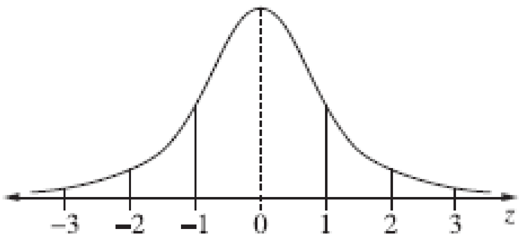
$$\cos^2 nx = \frac{1}{2}(1 + \cos 2nx)$$

Statistical Analysis

$$z = \frac{x - \mu}{\sigma}$$

An outlier is a score
less than $Q_1 - 1.5 \times IQR$
or
more than $Q_3 + 1.5 \times IQR$

Normal distribution



- approximately 68% of scores have z-scores between –1 and 1
- approximately 95% of scores have z-scores between –2 and 2
- approximately 99.7% of scores have z-scores between –3 and 3

$$E(X) = \mu$$

$$\operatorname{Var}(X) = E\left[(X - \mu)^2\right] = E\left(X^2\right) - \mu^2$$

Probability

$$P(A \cap B) = P(A)P(B)$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}, \ P(B) \neq 0$$

Continuous random variables

$$P(X \leq r) = \int_a^r f(x)\,dx$$

$$P(a < X < b) = \int_a^b f(x)\,dx$$

Binomial distribution

$$P(X = r) = {}^nC_r p^r (1-p)^{n-r}$$

$$X \sim \operatorname{Bin}(n, p)$$

$$\Rightarrow \ P(X = x)$$

$$= \binom{n}{x} p^x (1-p)^{n-x}, \ x = 0, 1, \ldots, n$$

$$E(X) = np$$

$$\operatorname{Var}(X) = np(1-p)$$

Differential Calculus

Function	Derivative
$y = f(x)^n$	$\dfrac{dy}{dx} = nf'(x)\left[f(x)\right]^{n-1}$
$y = uv$	$\dfrac{dy}{dx} = u\dfrac{dv}{dx} + v\dfrac{du}{dx}$
$y = g(u)$ where $u = f(x)$	$\dfrac{dy}{dx} = \dfrac{dy}{du} \times \dfrac{du}{dx}$
$y = \dfrac{u}{v}$	$\dfrac{dy}{dx} = \dfrac{v\dfrac{du}{dx} - u\dfrac{dv}{dx}}{v^2}$
$y = \sin f(x)$	$\dfrac{dy}{dx} = f'(x)\cos f(x)$
$y = \cos f(x)$	$\dfrac{dy}{dx} = -f'(x)\sin f(x)$
$y = \tan f(x)$	$\dfrac{dy}{dx} = f'(x)\sec^2 f(x)$
$y = e^{f(x)}$	$\dfrac{dy}{dx} = f'(x)e^{f(x)}$
$y = \ln f(x)$	$\dfrac{dy}{dx} = \dfrac{f'(x)}{f(x)}$
$y = a^{f(x)}$	$\dfrac{dy}{dx} = (\ln a)f'(x)a^{f(x)}$
$y = \log_a f(x)$	$\dfrac{dy}{dx} = \dfrac{f'(x)}{(\ln a)f(x)}$
$y = \sin^{-1} f(x)$	$\dfrac{dy}{dx} = \dfrac{f'(x)}{\sqrt{1-\left[f(x)\right]^2}}$
$y = \cos^{-1} f(x)$	$\dfrac{dy}{dx} = -\dfrac{f'(x)}{\sqrt{1-\left[f(x)\right]^2}}$
$y = \tan^{-1} f(x)$	$\dfrac{dy}{dx} = \dfrac{f'(x)}{1+\left[f(x)\right]^2}$

Integral Calculus

$$\int f'(x)\left[f(x)\right]^n dx = \frac{1}{n+1}\left[f(x)\right]^{n+1} + c$$

where $n \neq -1$

$$\int f'(x)\sin f(x)\,dx = -\cos f(x) + c$$

$$\int f'(x)\cos f(x)\,dx = \sin f(x) + c$$

$$\int f'(x)\sec^2 f(x)\,dx = \tan f(x) + c$$

$$\int f'(x)e^{f(x)}\,dx = e^{f(x)} + c$$

$$\int \frac{f'(x)}{f(x)}\,dx = \ln\left|f(x)\right| + c$$

$$\int f'(x)a^{f(x)}\,dx = \frac{a^{f(x)}}{\ln a} + c$$

$$\int \frac{f'(x)}{\sqrt{a^2-\left[f(x)\right]^2}}\,dx = \sin^{-1}\frac{f(x)}{a} + c$$

$$\int \frac{f'(x)}{a^2+\left[f(x)\right]^2}\,dx = \frac{1}{a}\tan^{-1}\frac{f(x)}{a} + c$$

$$\int u\frac{dv}{dx}\,dx = uv - \int v\frac{du}{dx}\,dx$$

$$\int_a^b f(x)\,dx$$

$$\approx \frac{b-a}{2n}\left\{f(a) + f(b) + 2\left[f(x_1) + \cdots + f(x_{n-1})\right]\right\}$$

where $a = x_0$ and $b = x_n$

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Combinatorics

$${}^{n}P_{r} = \frac{n!}{(n-r)!}$$

$$\binom{n}{r} = {}^{n}C_{r} = \frac{n!}{r!\,(n-r)!}$$

$$(x+a)^{n} = x^{n} + \binom{n}{1}x^{n-1}a + \cdots + \binom{n}{r}x^{n-r}a^{r} + \cdots + a^{n}$$

Vectors

$$\left|\underset{\sim}{u}\right| = \left|x\underset{\sim}{i} + y\underset{\sim}{j}\right| = \sqrt{x^{2}+y^{2}}$$

$$\underset{\sim}{u}\cdot\underset{\sim}{v} = \left|\underset{\sim}{u}\right|\left|\underset{\sim}{v}\right|\cos\theta = x_{1}x_{2} + y_{1}y_{2},$$

where $\underset{\sim}{u} = x_{1}\underset{\sim}{i} + y_{1}\underset{\sim}{j}$

and $\underset{\sim}{v} = x_{2}\underset{\sim}{i} + y_{2}\underset{\sim}{j}$

$$\underset{\sim}{r} = \underset{\sim}{a} + \lambda\underset{\sim}{b}$$

Complex Numbers

$$z = a + ib = r(\cos\theta + i\sin\theta)$$
$$= re^{i\theta}$$

$$\left[r(\cos\theta + i\sin\theta)\right]^{n} = r^{n}(\cos n\theta + i\sin n\theta)$$
$$= r^{n}e^{in\theta}$$

Mechanics

$$\frac{d^{2}x}{dt^{2}} = \frac{dv}{dt} = v\frac{dv}{dx} = \frac{d}{dx}\left(\frac{1}{2}v^{2}\right)$$

$$x = a\cos(nt+\alpha) + c$$

$$x = a\sin(nt+\alpha) + c$$

$$\ddot{x} = -n^{2}(x-c)$$

Year 12 HSC Assessment Task 3

Mathematics Extension 2 – 2024 HSC

Section I – Multiple Choice Answer Sheet

Student Name ______________________________

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample: $2 + 4 =$ (A) 2 (B) 6 (C) 8 (D) 9

A ○ B ● C ○ D ○

- If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ● B ~~●~~ C ○ D ○

- If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word correct and drawing an arrow as follows.

correct

A ● B ~~●~~ C ○ D ○

---

1. A ○ B ○ C ○ D ○

2. A ○ B ○ C ○ D ○

3. A ○ B ○ C ○ D ○

4. A ○ B ○ C ○ D ○

5. A ○ B ○ C ○ D ○

6. A ○ B ○ C ○ D ○

Year 12	Mathematics Extension 2	Ass Task 3 2024 HSC
Multiple choice	Solutions and Marking Guidelines	

Outcomes Addressed in this Question

MEX 12-3 Uses vectors to model and solve problems in two and three dimensions.

MEX 12-5 Applies techniques of integration to structured and unstructured problems.

Question/ Outcome	Solutions	Marking Guidelines
1 **12-5**	$\int x^3(x^4-1)^2\,dx = \frac{1}{4}\int 4x^3(x^4-1)^2 dx$ $= \frac{1}{4}\frac{(x^4-1)^3}{3} + C = \frac{1}{12}(x^4-1)^3 + C$	1 mark: D
2 **12-5**	$\int \frac{1}{\sqrt{-x^2+6x-5}}dx = \int \frac{1}{\sqrt{2^2-(x-3)^2}}dx$ $= \sin^{-1}\left(\frac{x-3}{2}\right) + C$ $-x^2+6x-5$ $= -(x^2-6x+5)$ $= -[(x-3)^2-4]$ $= 4-(x-3)^2$	1 mark: A
3 **12-5**	$u = \frac{x}{2}$ $du = \frac{1}{2}dx$ $2du = dx$ $x = 2a \Rightarrow u = a$ $x = 2b \Rightarrow u = b$ $\int_{2a}^{2b} \frac{1}{\sin\frac{x}{2} + \cos\frac{x}{2}}dx = \int_a^b \frac{1}{\sin u + \cos u}.2du$ $= 2\int_a^b \frac{1}{\sin u + \cos u}du$ $= 2 \times 1 = 2$	1 mark: D
4 **12-3**	The 3 co-ordinates of the point need to satisfy the same value of λ. (-13, 0, 16) works for $\lambda = 3$	1 mark: B
5 **12-3**	Use the dot product formulas from the reference sheet to give a value of $\cos\theta$	1 mark: C
6 **12-3**	All options give parametrisations in sin *t*. Hence we seek the response with the correct period and amplitude for the *x* and *y* co-ordinates. Only B has both domain and range from -1 to +1. Also the *y* values complete 2 periods along the entire curve.	1 mark: B

<table>
<tr><td>Year 12</td><td>Mathematics Extension 2</td><td>Ass Task 3 2024 HSC</td></tr>
<tr><td>Question No. 7</td><td>Solutions and Marking Guidelines</td><td></td></tr>
<tr><td colspan="3">Outcomes Addressed in this Question</td></tr>
<tr><td colspan="3">MEX 12-5 Applies techniques of integration to structured and unstructured problems.</td></tr>
</table>

Part / Outcome	Solutions	Marking Guidelines
(a) **12-3**	$\int \frac{x^2}{1+4x^2}\,dx = \frac{1}{4}\int \frac{4x^2}{1+4x^2}\,dx$ $= \frac{1}{4}\int \frac{1+4x^2-1}{1+4x^2}\,dx$ $= \frac{1}{4}\int \left(1-\frac{1}{1+4x^2}\right)dx$ $= \frac{1}{4}\left(x-\frac{1}{2}\tan^{-1}(2x)\right)+c$ $= \frac{1}{4}x-\frac{1}{8}\tan^{-1}(2x)+c$	3 marks: correct solution 2 marks: substantially correct solution 1 mark: partially correct solution
(b) **12-3**	$\int_0^{\frac{\pi}{2}} \frac{dx}{1+\sin x} = \int_{\tan 0}^{\tan\frac{\pi}{4}} \frac{1}{1+\frac{2t}{1+t^2}} \times \frac{2}{1+t^2}\,dt$ $= \int_0^1 \frac{2}{1+t^2+2t}\,dt$ $= \int_0^1 \frac{2}{(1+t)^2}\,dt$ $= \left[-\frac{2}{1+t}\right]_0^1$ $= -1+2 = 1$ $\boxed{\begin{aligned} t &= \tan\frac{x}{2} \\ \frac{dt}{dx} &= \frac{1}{2}\sec^2\frac{x}{2} \\ &= \frac{1}{2}\left(1+\tan^2\frac{x}{2}\right) \\ &= \frac{1}{2}(1+t^2) \\ dx &= \frac{2dt}{1+t^2} \end{aligned}}$	4 marks: correct solution 3 marks: substantially correct solution 2 marks: partially correct solution 1 mark: attempts t-substitution, or equivalent merit

(c)(i) **12-5**	*Question 7 continued…* $I_n = \int_0^x \sec^n t\,dt = \int_0^x \sec^{n-2} t \sec^2 t\,dt$ $let\ \ u = \sec^{n-2} t \qquad dv = \sec^2 t\,dt$ $du = (n-2)\sec^{n-2} t \tan t\,dt \qquad v = \tan t$ $I_n = uv - \int_0^x v du$ $I_n = [\sec^{n-2} t \tan t]_0^x - (n-2)\int_0^x \sec^{n-2} t \tan^2 t\,dt$ $= \sec^{n-2} x \tan x - (n-2)\int_0^x \sec^{n-2} t\,(\sec^2 t - 1)dt$ $= \sec^{n-2} x \tan x - (n-2)\int_0^x (\sec^n t - \sec^{n-2} t)dt$ $I_n = \sec^{n-2} x \tan x - (n-2)I_n + (n-2)I_{n-2}$ $\therefore (n-1)I_n = \sec^{n-2} x \tan x + (n-2)I_{n-2}$ $\therefore \qquad I_n = \dfrac{\sec^{n-2} x \tan x}{n-1} + \dfrac{n-2}{n-1} I_{n-2}$	<u>3 marks</u>: correct solution <u>2 marks</u>: substantially correct solution <u>1 mark</u>: partially correct solution
(c)(ii)	$[I_4]_0^{\frac{\pi}{3}} = \dfrac{\sec^2 \frac{\pi}{3} \tan \frac{\pi}{3}}{3} + \dfrac{2}{3} I_2$ $= \dfrac{4\sqrt{3}}{3} + \dfrac{2}{3} I_2$ $I_2 = \int_0^{\frac{\pi}{3}} \sec^2 t\,dt$ $= [\tan t]_0^{\frac{\pi}{3}}$ $= \tan \dfrac{\pi}{3} - 0 = \sqrt{3}$ $\therefore I_4 = \dfrac{4\sqrt{3}}{3} + \dfrac{2\sqrt{3}}{3}$	<u>2 marks</u>: correct solution <u>1 mark</u>: substantially correct solution

Year 12	Mathematics Extension 2	Ass Task 3 2023 HSC
Question No. 8	Solutions and Marking Guidelines	

Outcomes Addressed in this Question

MEX 12-3 Uses vectors to model and solve problems in two and three dimensions.

Part / Outcome	Solutions	Marking Guidelines
(a)	$(a)\ \lvert\overrightarrow{AB}\rvert = \sqrt{(-3-4)^2 + (6-2)^2 + (-1-7)^2}$ $= \sqrt{49+64+36} = \sqrt{149}$	**(a)2 marks:** correct solution. **1 mark:** Substantially correct.
(b)	(b) $\overrightarrow{AB} = \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix} - \begin{pmatrix} 5 \\ 3 \\ -2 \end{pmatrix} = \begin{pmatrix} -3 \\ -4 \\ 5 \end{pmatrix}$, which is the gradient vector between the two points. $\overrightarrow{AC} = \begin{pmatrix} -7 \\ -13 \\ 18 \end{pmatrix} - \begin{pmatrix} 5 \\ 3 \\ -2 \end{pmatrix} = \begin{pmatrix} -12 \\ -16 \\ 20 \end{pmatrix} = 4\begin{pmatrix} -3 \\ -4 \\ 5 \end{pmatrix}$ Since $\overrightarrow{AC}$ is a scalar multiple of $\overrightarrow{AB}$ the 2 vectors are parallel. Since they both share the point A, the points A, B, C are collinear.	**(b)2 marks:** Correct solution with complete reason included. **1 mark:** Substantial progress.
(c)	(c) (i) Completing the squares and adding the equivalent amount to the RHS gives: $(x-1)^2 + (y-4)^2 + (z+3)^2 = 25$ Hence sphere's centre is $(1, 4, -3)$ and its radius is 5 units. (ii) The xy plane has the equation $z=0$. Substitute into (i) for intersection. $(x-1)^2 + (y-4)^2 + 3^2 = 25$ So, equation is: $(x-1)^2 + (y-4)^2 = 16$	**(c)(i) 2 marks:** Correct centre and radius. **1 mark:** One correct component. **(ii) 1 mark:** Correct equation for circle.
(d)	$(i)\ \overrightarrow{PQ} = \frac{1}{2}\overrightarrow{AB} + \frac{1}{2}\overrightarrow{BC}$ $\overrightarrow{SR} = \frac{1}{2}\overrightarrow{AD} + \frac{1}{2}\overrightarrow{DC}$ But $ABCD$ is a rhombus, so $\overrightarrow{AB} = \overrightarrow{AD}$ and $\overrightarrow{BC} = \overrightarrow{DC}$ Therefore $\overrightarrow{SR} = \overrightarrow{PQ}$ Therefore $PQRS$ forms a parallelogram, because it has a pair of parallel and equal sidelengths. Note: Finding only one pair of opposite equal vectors is sufficient to provide the above reason for a parallelogram. (ii) $\overrightarrow{AB}\,.\,\overrightarrow{AD} = \lvert\overrightarrow{AB}\rvert\,\lvert\overrightarrow{AD}\rvert\,\cos(180° - \theta)$ $= -\lvert\overrightarrow{AB}\rvert\,\lvert\overrightarrow{AD}\rvert\,\cos\theta$ $= -\lvert\overrightarrow{BA}\rvert\,\lvert\overrightarrow{BC}\rvert\,\cos\theta$ Because all of the sidelengths of the rhombus are equal. $= -\overrightarrow{BA}\,.\,\overrightarrow{BC}$	**(d) 2 marks:** Finding 2 equal vectors and providing the property of a parallelogram. Reasoning and vector method is required. **1 mark:** One component not provided. **(ii) 1 mark:** Correct solution.

	Question 8 continued… (iii) $\overrightarrow{PQ}\,.\overrightarrow{PS} = \frac{1}{2}\left(\overrightarrow{AB} + \overrightarrow{BC}\right).\frac{1}{2}\left(\overrightarrow{BA} + \overrightarrow{AD}\right)$ $= \frac{1}{4}\left(\overrightarrow{AB}\,.\overrightarrow{BA} + \overrightarrow{AB}\,.\overrightarrow{AD} + \overrightarrow{BC}\,.\overrightarrow{BA} + \overrightarrow{BC}\,.\,\overrightarrow{AD}\right)$ Note: $\overrightarrow{AD} = \overrightarrow{BC}$ and the dot product of a vector with itself is the square of its length. So we have: $\frac{1}{4}\left(-\lvert\overrightarrow{AB}\rvert^2 + \overrightarrow{AB}\,.\,\overrightarrow{AD} + \overrightarrow{BC}\,.\overrightarrow{BA} + \lvert\lvert BC\rvert\rvert^2\right)$ $= \frac{1}{4}\left(\overrightarrow{AB}\,.\,\overrightarrow{AD} + \overrightarrow{BC}\,.\overrightarrow{BA}\right)$ since the sidelengths of the rhombus are equal. Which gives $\frac{1}{4}(0)$ from part (ii). Therefore $\overrightarrow{PQ}\,.\overrightarrow{PS} = 0\ \ so\ \ PQ\ \perp PS$ So *PQRS* is a rectangle, because it is a parallelogram with an angle of 90^0. Note that showing one ninety degree angle is required to prove that the parallelogram is a rectangle.	**(iii) 2 marks:** Vector method used properly leading to property of rectangle. **1 mark:** One component not provided.